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Combining sensitivity and uncertainty analysis to efficiently quantify parametric uncertainties in NVH system simulation models

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Abstract
The acoustic and structural dynamic properties of vehicles—often referred to as Noise, Vibration, Harshness (NVH)—form a crucial criterion during product development. To reduce iterations with physical prototypes, NVH simulation models are well established. In early development phases, many parameters of NVH models, such as material and contact properties, are either assumed based on empirical values or have to be measured. In both cases, the value of these parameters is uncertain. Thus, the output of NVH system simulation models such as structure borne or air borne sound is also uncertain and must be quantified. However, applying state-of-the-art uncertainty analysis methods to NVH simulation models considering all uncertain input parameters is inefficient due to their high computation time. Therefore, this paper presents a method of coupled sensitivity (SA) and uncertainty analysis (UA), which enables the efficient uncertainty calculation for NVH simulations. In this method, firstly the most influential parameters are determined using a SA to reduce the number of input parameters. Depending on the number of parameters and the computation time of the NVH simulation model, either the Morris SA or an EFAST SA is chosen. Finally, a fuzzy UA is performed, which quantifies the uncertainty of the output of the NVH simulation and provides its possible ranges. The procedure is applied to the NVH model for predicting air borne sound of an electric drive with 53 uncertain input parameters.

Recheneffiziente Quantifizierung parametrischer Unsicherheiten in NVH-Systemsimulationsmodellen mittels Sensitivitäts- und Unsicherheitsanalyse

Zusammenfassung
Die unter dem Begriff Noise, Vibration and Harshness (NVH) zusammengefassten akustischen und strukturdynamischen Eigenschaften von Fahrzeugen sind ein entscheidendes Kriterium bei der Produktentwicklung. Um zeit- und kostenintensive Iterationen mit physischen Prototypen zu reduzieren, haben NVH-Simulationsmodelle in den Entwicklungsalltag Einzug erhalten. In frühen Entwicklungsphasen werden viele Parameter von NVH-Modellen, wie z. B. Material- und Kontakteigenschaften, entweder auf der Grundlage empirischer Werte angenommen oder müssen gemessen werden. In beiden Fällen ist der Wert dieser Parameter mit Unsicherheiten behaftet. Somit ist auch die Ausgangsgröße von NVH-System-Simulationsmodellen, wie bspw. Körper- oder Luftschall, Unsicherheiten unterworfen, die quantifiziert werden müssen. Die Anwendung moderner Methoden der Unsicherheitsanalyse auf NVH-Simulationsmodelle unter Berücksichtigung aller unsicheren Eingangsparameter ist jedoch aufgrund der hohen Rechenzeit ineffizient. Daher wird in diesem Beitrag eine Methode der gekoppelten Sensitivitäts- (SA) und Unsicherheitsanalyse (UA) vorgestellt, die eine effiziente Unsicherheitsberechnung für NVH-Simulationen ermöglicht. Bei dieser Methode werden zunächst die einflussreichsten Parameter mit Hilfe einer SA bestimmt, um die Anzahl der Eingabeparameter zu reduzieren. Abhängig von der Anzahl der Parameter und der Berechnungszeit des NVH-Simulationsmodells wird entweder die Morris SA oder eine EFAST SA gewählt. Abschließend wird eine Fuzzy-UA durchgeführt, die die Unsicherheit der Ausgabe der NVH-Simulation quantifiziert und ihre möglichen Bereiche angibt. Das Verfahren wird auf das NVH-Modell zur Vorhersage des Luftschalls eines elektrifizierten Antriebsstrangs mit 53 unsicheren Eingangsparametern angewandt.

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1 Introduction

The structural dynamic and acoustic behavior of drivetrains—often referred to as Noise, Vibration and Harshness (NVH)—is of ever-increasing importance when evaluating a product's requirements. It is a decisive quality feature of the product which often is crucial in a customer's decision making process of which product to buy [1]. The continuous trend towards shorter product development times has imposed the need for shorter and more cost-effective iterations on the product development process. In order to reduce the effort involved in designing products, methods of virtual product development (VPD) such as virtual prototypes are nowadays well-established in the product development process also for evaluating a virtual prototype's NVH behavior as they reduce the amount of time- and cost-intensive physical prototypes [2]. Within early phases of VPD assuring the fulfillment of requirements is mainly based on the evaluation of simulation models. Therefore, these models should not only be sufficiently detailed to facilitate design choices but they should furthermore be suitable to give a quantitative indication on the product's performance.

As a result, developing reliable, sophisticated NVH models which are usable already at an early phase of the development process as well as developing appropriate workflows to use the NVH models to evaluate the fulfillment of acoustic requirements have been a key focus of recent research [3–6]. Special focus needs to be given onto the key points of structural-dynamic sound propagation: Excitation (such as gear stages, electric engines, inverters, power-split transmissions), elements in the transfer path (flexible structures of housing and shafts, bearings, elastomeric mounts, joints etc.) and sound radiation. Numerous models of different fidelity levels have therefore been developed to model the structural-dynamic effects of the aforementioned machine elements [7, 8]. However, the parameters of these models are often uncertain, such as material and contact properties. Especially during early phases of the development process these parameters are either assumed based on empirical values or have to be elaborately measured. Thus, also the output of NVH system simulation models such as structure borne or air borne sound is prone to uncertainties. These uncertainties must be quantified considering the influence and interactions of all uncertain input parameters when taking design decisions based on NVH models' results.

Uncertainty analysis (UA) such as Monte Carlo simulations, Bayesian networks and fuzzy logic are well-established methods in order to quantify the uncertainty of a model's output and are applied to a variety of engineering domains, including stress calculation [9, 10] and simplified structural dynamic models [11, 12] but also civil engineering [13], biological models [14] and medicine [15]. UA can be applied when a deterministic model is linking input parameter values to outputs. However, NVH system model's inherent non-linear and cross-coupled transient effects together with a large number of parameters and therefore a highly multidimensional parameter space often renders the application of classical uncertainty analysis unfeasibly computationally expensive. Thus, it is currently not feasible to evaluate the uncertainties of outputs of NVH system simulation models during the VDP. As a result, it is uncertain if a calculated NVH quantity such as a sound-pressure level is reliably meeting the requirements imposed on it. To overcome these limitations, this paper presents a method of coupled sensitivity (SA) and uncertainty analysis (UA), which enables the efficient quantification of uncertainties of NVH simulations based on uncertain input parameters. The advantages of the proposed method are demonstrated on an NVH simulation model of a battery-electric vehicle (BEV) which calculates the air borne sound.

The paper is structured as follows: In Sect. 2, an overview over the state of the art of UA and SA as well as NVH system simulation models is given. Section 3 describes the problem of efficiently calculating uncertainties of NVH simulation models. The solution approach is presented in Sect. 4 and applied to an electro-mechanic drivetrain in Sect. 5. Conclusion and outlook are given in Sect. 6.

2 State of the art

2.1 Uncertainty analysis (UA) and sensitivity analysis (SA) methods

Both UA and SA methods are used to evaluate parameter uncertainties in numerical models. UA methods aim at quantifying the influence of one or several uncertain input parameters onto the model's output's uncertainty. If multi-domain models are used, UA aims at quantifying the uncertainty throughout the entire model toolchain. The result of a UA therefore is a range of possible output values which are weighted e.g. with probabilities of occurrence of individual values within the output range. An UA will however not allow for deducing the influence of individual input parameters on the outputs range [12, 16].

To allow for the identification of the most relevant input parameters (i.e. the parameters, which should be given the most care during parameter identification), conducting a SA is necessary. The result of a SA is the identification of parameters which are most relevant for a change in the output as well as the cross-influence between multiple uncertain input parameters. The result of a SA is quantified either as the main effect of an uncertain input parameter, which describes the change in the output when varying the specific uncertain input parameter and leaving every other input parameter constant, or as a total effect, which additionally considers cross influences between uncertain parameters and which is therefore the sum of the main effect and the cross influencing effects [17].

In order to understand and evaluate existing SA and UA methods, a second differentiation is necessary: Uncertainties can generally be classified into aleatory and epistemic uncertainties. Aleatory uncertainties are arbitrary and can be described by a probability distribution. They most commonly originate within the variance of material properties or environmental influences around a known mean value. Epistemic uncertainties on the other hand are a result of a lack of information on a parameter which is subsequently estimated (e.g. based on experience) during the modelling process [12, 18].

Uncertainty analysis is most commonly done by using one of the following methods:

- Monte-Carlo simulation [19]: Based on an initially assigned probability distribution, which is often assumed either normal, logarithmic or uniformly distributed, a fraction of the full parameter space is evaluated either based on (Quasi-)random or Latin hypercube Sampling [16]. While it is an easy-to-use method, it requires a large amount of calculations to be carried out and the output's quality heavily depends on the chosen fraction of the parameter space.
- Bayesian networks [20] are a combination of Bayesian statistics with neural networks which calculate a likelihood function based on a training dataset. The likelihood function is subsequently multiplied with an initial assumption about the parameter's probability distribution, the so-called priori function. Bayesian networks are mainly used to quantify aleatory uncertainties because the required priori functions are difficult to obtain for parameters prone to epistemic uncertainties. Training the neural network adds an additional step to the UA which imposes additional effort as well as additional uncertainties in the quality of the trained network.

Fuzzy-Logic and fuzzy transform method [21, 22] model epistemic and aleatory uncertainties by mapping a membership function between 0 and 1 to the parameter range of each individual uncertain input parameter. Parameter value with a membership function's value of 0 are considered as outside of the uncertainty band of the input parameter and thus not considered in the uncertainty calculation. A membership function's value of 1 indicates the most probable value of the input parameter. In order to incorporate fuzzy logic into deterministic simulation models, fuzzy transformation methods [23] are used, which split the continuous interval of uncertain parameters into discrete α-Cuts which are used for simulation. By retransforming the discrete output membership function into an uncertainty band, an evaluation of the output's uncertainty is carried out. Two different variations of the fuzzy-Transformation method exist: A reduced fuzzy-transformation method will only evaluate the minimum and maximum value of a parameter range for a given value of the membership function (i.e. for each α-Cut), while a generalized fuzzy-transformation method will also consider parameter values in between. The variations are visualized in Fig. 1 for one uncertain input parameter. The result of a fuzzy UA again is a membership function of the model's output. The output parameter's value which is assigned to a membership function of 1 is considered as the most probable output, while membership function's values of 0 are considered as impossible to occur given the chosen range of the input parameters.

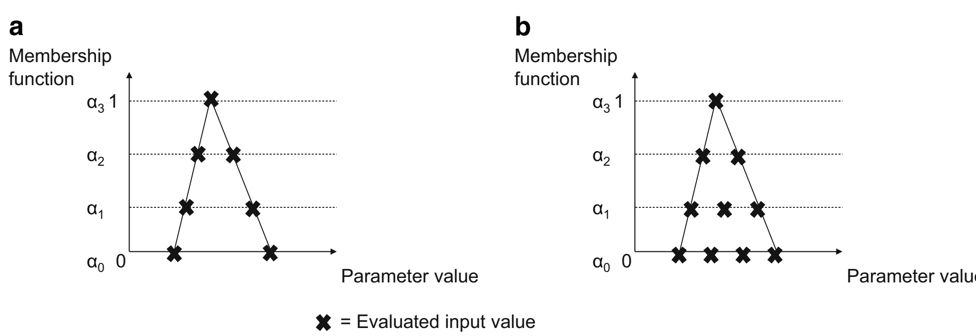


Fig. 1 Reduced (**a**) and generalized (**b**) fuzzy Transformation method

Table 1 Example calculations for SA according to Morris

Simulation	Value of Parameter X_1	Value of Parameter X_2	Value of Parameter X_3
1	x_1	x_2	x_3
2	$x_1+\Delta$	x_2	x_3
3	$x_1+\Delta$	$x_2+\Delta$	x_3
4	$x_1+\Delta$	$x_2+\Delta$	$x_3+\Delta$

The following methods are widely used for sensitivity analysis:

- Linear regression is an approach suitable for the identification of linear models by calculating regression coefficients between input and output. Similarly, correlation analysis can be applied to identify the relationship between input- and output variables where a correlation coefficient of 1 indicates the highest sensitivity [16].
- Extended Fourier Amplitude Sensitivity (EFAST) [24], which is based on the Fourier Amplitude Sensitivity method [25], compares the variance of the output parameter to the variance of all input parameters by assigning each individual input parameter a frequency and calculating the spectrum of the variance of the output. EFAST allows for both the calculation of the total and the main effect. While the total effect allows for identifying negligible input parameters, the main effect is used to rank the relevance of the input parameters with respect to each other.
- Morris [26] proposes a method based on calculating elementary effects which is the change of the output's parameter when one individual parameter is changed. Each individual uncertain input parameter is varied by a value Δ. For the example of a system with three input parameters X_1, X_2 and X_3, the calculations listed in Table 1 are carried out.
  The elementary effect is then calculated by dividing the output of two consecutive simulations by Δ. The process is repeated r times (with r being a tuning parameter of the method) with different values of Δ. Evaluating the elementary effect's standard deviation yields the indication for the sensitivity of the input parameter. The proposed method is computationally very efficient, compared to the Fourier Amplitude Sensitivity methods. However, as it is a screening-based method, it only allows for the identification of negligible input parameters, but not a ranking of individual parameters.

UA and SA are numerously applied on simple structural dynamic models in literature, so that only an excerpt of relevant engineering applications can be given. Monte-Carlo and fuzzy-logic methods are used in [9] to evaluate the uncertainty of a crash test model based on the uncertainty of sheet metal thickness [12]. demonstrates the application and limitations of applying Monte-Carlo methods and fuzzy-transformation method onto an NVH model of the influence of joints onto the dynamics of a motorcycle's engine. Other applications of uncertainty or sensitivity analysis on structural dynamic models can be found in [11, 27, 28].

2.2 NVH system simulation models

NVH system simulation models aim at predicting the air borne or structure borne sound of systems such as drivetrains especially already at an early stage of the development process. This allows the design engineer to evaluate requirements of the acoustic behavior before manufacturing the first physical prototype. NVH models therefore comprise of models for the relevant effects of excitation, sound transfer and sound radiation along the chain of acoustic transmission. The NVH model of a battery electric vehicle (BEV), as presented in [4, 29], on which the developed UA method is demonstrated in Sect. 5, is shown as an example

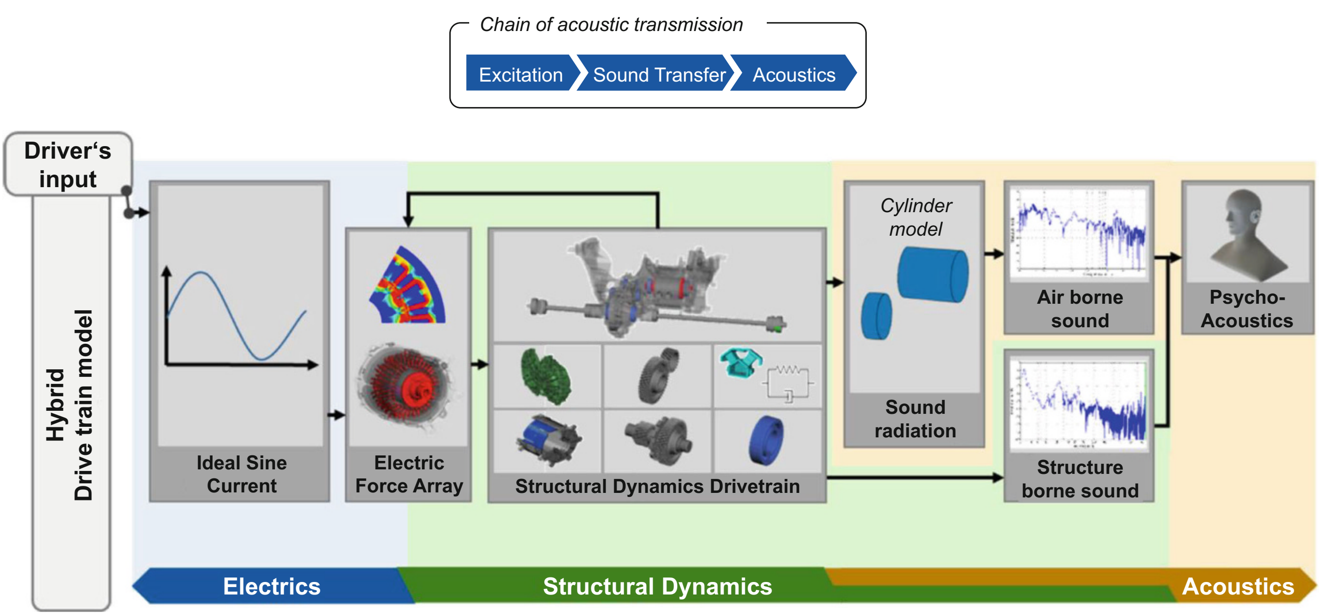


Fig. 2 Model toolchain for deriving the NVH behavior of a BEV [30]

for NVH system models in Fig. 2. The individual parts of the model will be explained in what follows.

Especially in mechatronic systems such as BEVs, NVH simulation models often combine models of different domains such as electromagnetics, mechanics and acoustics. The models of excitation mechanisms (such as gear stages, electric or combustion engines) are usually nonlinear and thus defined by cross-influences between multiple input parameters. The NVH model studied in this work contains electrics models which mimics the control strategy of the system and a quasi-static force look up array, which includes the dependency of the radial and tangential forces in the electric machine in dependency of the currents and eccentricities. As the transfer-path from the excitation to a receiver point (typically the driver's position) is also influenced by non-linear elements such as rolling bearings, elastomeric mounts and joints [7, 31, 32], the entire system model needs to be solved in time-domain which is computationally expensive compared to linear, frequency-domain models. Solving in time-domain also accounts for the nonlinearities of the electric engine's excitation. While this allows to consider transient effects during rpm runup and nonlinearities in the excitation and transfer behavior of the system, solving NVH models in time domain is computationally expensive, as it requires implicitly solving the system's state for each time-domain. Additionally, a high sampling rate is necessary to capture all effects in the audible frequency range up to 20 kHz which further increases the computational effort, thereby posing strict requirements concerning efficiency on the UA and SA method.

The structural dynamic model of the demonstrator example is an EMBS model including linearized rolling-bearing and elastomeric mount models in form of stiffness and damping matrices and modally reduced flexible bodies of the shafts, gears and housing. Mostly, a linear calculation of the sound radiation based on the surface velocities concludes the model chain, which can either be modeled analytically or using boundary element method (BEM) [33]. A rpm-runup under constant torque is simulated. Other applications for such complex NVH system simulation models can be found in hydro-mechanical powersplit drives [3] or gearbox models of drivetrains [34].

Because of the multi-physical effects and the large scope of NVH models, these models usually contain many nonlinear, cross-influencing parameters such as material properties, stiffnesses and damping of rolling bearings or elastomeric mounts and contact stiffnesses and damping in spline couplings which leads to a large design space while simultaneously taking relatively long calculating times (up to several days) therefore posing strong requirements on the application of UA and SA methods. The uncertainties of NVH models' parameters are mostly epistemic, as these parameters are often times estimated based on analytical calculations or similar designs during the development process before the manufacturing of physical prototypes.

3 Problem formulation

For complex NVH simulation models such as the ones presented in Chap. 2.2 there are today no methods for applying uncertainty analysis which are sufficiently fast enough to integrate them into the virtual product development process because of the nonlinear nature of NVH models with a large number of parameters with a strong cross-influence. Uncertainty analysis methods for NVH models are however needed, as generating reliable measures to quantify the acoustic behavior of a virtual prototype is crucial during the development process. Many input parameters to NVH models are however unknown or at least uncertain during the early phases of product development, so that the influence of their uncertainty on the model's output quality has to be evaluated.

The following requirements are posed onto a UA method to account for the uncertainties in NVH models:

- Consideration of both aleatory and epistemic uncertainties because in the early phase of a development processes parameter uncertainties are not only caused by the stochastic variation of parameters but also because of a lack of knowledge of e.g. elastomeric mount stiffnesses or similar material properties
- Integration of the UA on existing numerical models because NVH simulation models are generated for discrete parameter sets during the development process to aid in design choices and existing models should be used and integrated into a UA method.

4 Solution

In order to evaluate the uncertainty of NVH system simulation models, a solution approach is presented which combines sensitivity analysis (SA) and uncertainty analysis (UA) as shown in Fig. 3. It allows for the first time to quantify an NVH simulation model's uncertainty with respect to many influencing input parameters by reducing the parameter space to the most important (i.e. sensitive) parameters using computationally efficient SA methods before executing the UA. This approach reduces the calculation time sufficiently to make the model usable during the product development process without the need to neglect the influence of certain input parameters a priori. It considers the time required to execute the simulation model for each given set of input parameters as well as the model's number of uncertain input parameters to be considered in

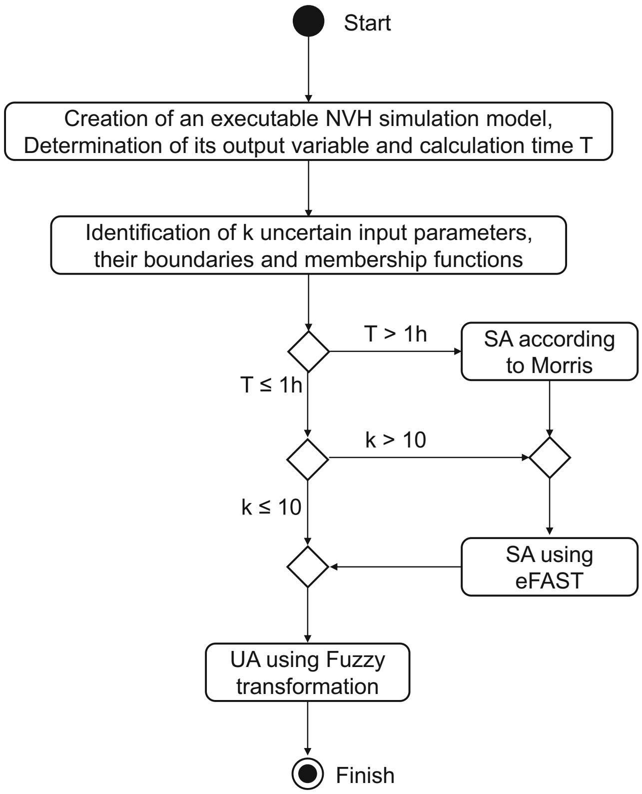


Fig. 3 Proposed method of combining SA and UA to quantify uncertainties of NVH system simulation models

the UA. The choice of the individual SA and UA methods will be explained in what follows.

In short, the process is as follows: Firstly, an executable NVH simulation model is created. It combines all excitation mechanisms, transfer path properties and sound radiation calculation which are necessary to evaluate the input parameter's uncertainty and therefore usually is comprised of a model toolchain integrating the structural dynamics domain (using e.g. eMBS) with the acoustic domain (using e.g. BEM). The output variable, of which the uncertainty is to be quantified, is chosen and the model calculation time T is determined. Then, the amount k of uncertain input parameters is identified. For each of these input parameters, the boundaries (i.e. maximum and minimum value) are determined and the membership function in between the boundaries for the UA is defined. The complexity of the NVH model is described by the parameters k and T and an appropriate method for parameter reduction is chosen. Three cases can be distinguished:

1. If the model is sufficiently fast (a simulation time of less than 1 h has empirically been determined as a useful boundary value) and has a sufficiently small amount of uncertain input parameters (i.e. not more than 10), an uncertainty analysis can directly be carried out. The limit values for T and k can be increased by the design engineer, if more time is available during the product development process, thereby increasing the algorithms accuracy, or vice versa, reduced, if faster results are required, accepting a decrease in accuracy.
2. If either the limit for T or k is exceeded, a sensitivity analysis is carried out before conducting the UA in order to reduce the parameter space by ranking the parameters by their sensitivity and only considering the most relevant once. If the model runs fast enough (T less than 1 h) but the number of parameters exceeds 10, a SA using EFAST is executed. The EFAST method is chosen because it allows for a thorough ranking of the uncertain input parameters by using both total and main effect. The choice of using EFAST is elaborated in greater detail in what follows by comparing the method to other SA methods.
3. If, however, the model running time exceeds 1 h, executing an EFAST consumes too much time, so that a SA according to Morris is conducted first, as it is more computationally efficient but only capable of computing the total effect of an uncertain input parameter. This abstraction is required on long running models, as more thorough SA analysis cannot be executed within a reasonable time because of the large number of model evaluations required for calculating both total and main effects of uncertain input parameters.

Therefore, the method combines up to three algorithms: UA using fuzzy transformation, SA using EFAST and SA according to Morris. The choice of these algorithms is explained in what follows.

A comparison of the requirements on UA methods as described in Chap. 3 and the properties of the UA methods introduced in Chap. 2 is presented in Table 2.

The fuzzy transformation method is chosen because it is the only one being capable of both considering aleatory and epistemic uncertainties and being able to operate on discrete parameter values. Monte Carlo Simulation and Bayesian networks only include aleatory uncertainties and the fuzzy

Table 2 Requirements and properties of UA methods

Requirement	Monte Carlo Simulation	Bayesian Networks	Fuzzy Logic	Fuzzy Transformation method
Consideration of aleatory and epistemic uncertainties	X	X	✔	✔
Integrability into existing NVH models	✔	✔	X	✔

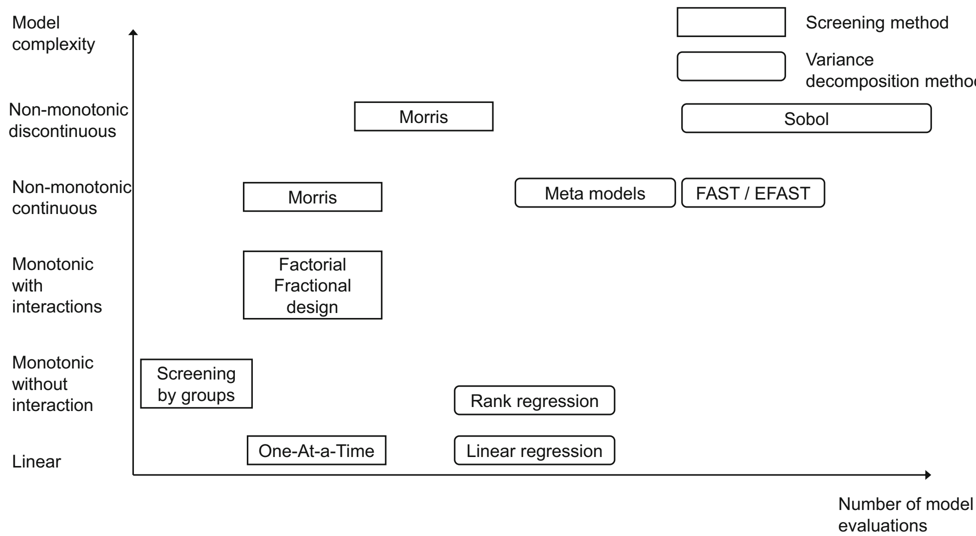


Fig. 4 Classification of Sensitivity Analysis methods according to [35]

logic requires continuous membership functions of the uncertain input parameters which prohibits it from being applied onto existing, discrete NVH simulation models. The advantage of choosing the fuzzy transformation method as the UA method is furthermore, that it allows to define a trade-off between accuracy and computational effort by choosing the number of α-Cuts as well as choosing either a reduced or generalized method.

The disadvantage of the fuzzy transformation method however is, that with a large number of input parameters, the calculation effort scales with a factor of 2^k. Therefore, for 10 uncertain input parameters, 2049 ($2 * 2^{10} + 1$) evaluations of the model have to be carried out. As a result, for models with more than 10 uncertain input parameters, SA has to be applied in order to reduce the parameter space. As NVH models include non-linear interdependencies between parameters, linear regression methods cannot capture the model's sensitivities accurately. Additionally, the parameters' influences do not have to be monotonic, as shifting system resonances with respect to excitation frequencies by changing stiffnesses or mass might cause a rise in sound pressure levels first, as a resonance is moved towards the excitation frequencies, and subsequently the sound pressure level can decrease again, as the resonance passes the excitation frequency. Therefore, methods for sensitivity analysis, which are able to include non-monotonic behavior, needs to be applied. Figure 4 shows the classification of existing SA methods according to [35] with respect to the requirements to include non-monotonic dependencies.

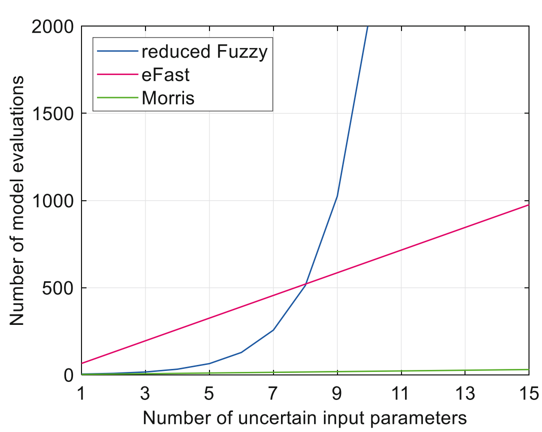


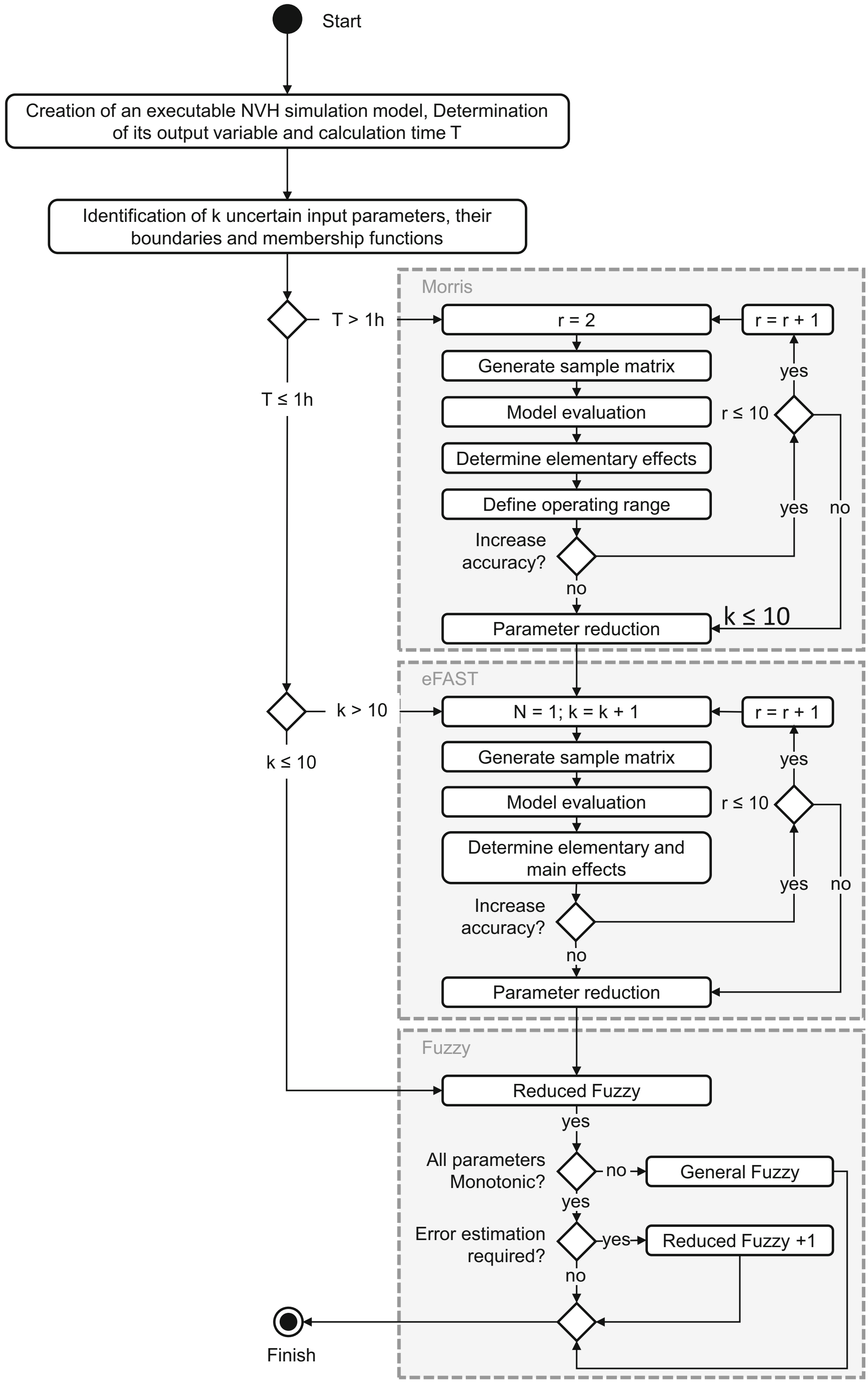
Fig. 5 Influence of number of uncertain input parameters on the number of model evaluations for Fuzzy UA, Morris SA and EFAST SA

Variance decomposition methods allow for a ranking of the sensitivity of input parameters. This is helpful in order to reduce the parameter space. As the parameter's influence usually is continuous in NVH models and no metamodels should be developed, the EFAST method is chosen as a method for sensitivity analysis. The number of model evaluations using EFAST only shows a linear dependency on the number of input parameters. However, as it scales with a Factor of 65k, conducting a SA using EFAST still is not practicably feasible, see Fig. 5. Therefore, a screening SA is needed to further reduce the number of model evaluations.

Screening SA methods allow for a computationally efficient identification of uncertain input parameters with negligible effect on the output parameter. Therefore, a SA according to Morris is carried out before a SA of EFAST if the number of uncertain input parameters becomes too large. The method according to Morris is chosen because it allows for non-monotonic parameter interdependencies with a high computational efficiency.

In detail, the entire method is presented in Fig. 6. After having created the executable NVH model, identified the uncertain input parameters and their membership function (which is assumed uniformly distributed for the SA and normally distributed for UA if no further information is available) as well as the uncertain output parameter, the model execution time is determined. If it is larger than 1 h,

Fig. 6 Proposed method of combining UA and SA in detail



a SA according to Morris is carried out. This consist of generating a sample matrix for a number of repetitions r, which is initially set to 2. The model evaluations according to the sample matrix are carried out and the elementary effects are determined. The execution of an NVH model usually aims at deriving the behavior of an output variable (such as air borne sound) as a function of an rpm runup. Therefore, the rpm runup is conducted and the sensitivity analysis is carried out for each individual sampling point of the rpm runup. In order to identify negligible input parameters based on the elementary effects, the entire operating range (i.e. the range from minimum to maximum rpm) is split into operating ranges in which similar parameters can be neglected. The identification of the operating ranges is carried out as follows: A boundary between two operating ranges is identified at rpm sampling points at which 33% of the uncertain input parameters are causing less than 85% of the outputs' variation. At these points, a new operating range is started. The standard deviation of the elementary effects, which is considered the indicator for the sensitivity of an input parameter, is integrated along the rpm axis of each operating range to yield scalar, comparable parameters.

Next, a SA according to EFAST is executed. The evaluation of the total effect allows for a ranking of the uncertain input parameters and thus for the further reduction of the parameter space by only considering the most relevant uncertain input parameters in the following UA. Again, as already done for the SA according to Morris, the total effects, which are calculated as a function of the system's rpm, are integrated over the rpm-values of the operating range in order to obtain scalar, comparable variables, which allow for the identification of negligible parameters. As a last step, the fuzzy UA is carried out. Firstly, a reduced fuzzy UA is done, which allows to check if the output parameter is monotonic with respect to all input parameters. If it is monotonic, the result of the reduced fuzzy UA is the model's output uncertainty. If it is not monotonic, a generalized fuzzy method has to be conducted to obtain the model's output's uncertainty.

The main advantage of the method is an improvement in computational efficiency in calculating a model's output's uncertainty. This allows for the adoption during product development processes. For large models, neglecting some uncertain input parameters however reduces the accuracy of the uncertainty calculation. By choosing appropriate SA methods, this error is minimized, but a small difference between a full UA and the proposed method has to be accepted to increase the computational efficiency. Furthermore, the method will be of limited use in a model which has a large number of parameters with very comparable sensitivities, as the SA will not be able to meaningfully reduce the parameter set. The assumption for applying this model is therefore, that only a few uncertain input parameters are responsible for the most of the output's uncertainty.

In Sect. 5, the application of the presented method, its advantages and the aforementioned error will be discussed on an NVH system simulation model of a BEV's drivetrain.

5 Application to an E-Motive NVH simulation model

The developed method is demonstrated on the NVH model of a battery electric vehicle's (BEV) drivetrain as presented in [4, 29] and introduced in Sect. 2.2. Modeling three different domains (electrics, structural dynamics and acoustics) is necessary to calculate the air borne sound at the left driver's ear induced by the drivetrain which is defined as its output value.

While each domain model induces its own parametric uncertainties, the method is demonstrated on the uncertain parameters of the structural dynamic models. These parameters are:

- Mass and Modal damping values of the two coolant pipes (4 parameters)
- Stiffness of the three elastomeric mounts (3 parameters)
- Damping of the three elastomeric mounts (3 parameters)
- Stiffness and damping of torque support arm (2 parameters)
- Stiffness and damping of spline coupling (2 parameters)
- Mass and Damping of the electric connector (2 parameters)
- Stiffness and damping of the six bearings (12 parameters)
- Damping of two tapered bearings (2 parameters)
- Stiffness and damping of four parking brake components (8 parameters)
- Stiffness and damping of two side shafts (4 parameters)
- Average modal damping of eleven modally reduced assemblies (11 parameters)

These uncertain input parameters are identified because they fulfilled one of the following criteria:

- Either, the parameter is known to exhibit strong variance in series production such as elastomer components
- Or, the parameters could not be physically measured directly, such as the stiffness and damping of the park brake components and the electric connectors and no established literature values could be obtained
- Or, the parameter is part of a linearization of a non-linear behavior, such as the stiffness of the spline coupling and the bearings

To motivate the necessity for the proposed method, the effort for directly conducting a UA using fuzzy transformation method is determined: One simulation takes 16 h.

Table 3 Operating ranges and most sensitive input parameters

Operating range [rotor rpm]	Most sensitive input parameter	2nd most sensitive input parameter	3rd most sensitive input parameter
1000 ... 2670	Stiffness of left elastomeric mount	Stiffness of right elastomeric mount	Modal damping of housing
2671 ... 4890	Stiffness of left side shaft	Modal damping of housing	Stiffness of right elastomeric mount
4891 ... 6000	Stiffness of rotor bearing	Stiffness of spline coupling	Modal damping of housing

For the 53 uncertain input parameters, conducting an uncertainty analysis using fuzzy transformation would require $3.6 * 10^{16}$ model evaluations which leads to a calculation time of $2.4 * 10^{16}$ days. Even considering the possibility of parallel execution of the model, the UA is taking orders of magnitude too long to include it into VPD processes.

On the other hand, conducting a SA according to Morris only requires 108 model evaluations. While 2592 simulation hours may seem a long time at first, using parallel execution of 25 models reduces the time required for a SA according to Morris to 2.88 days. The sensitivity analysis is carried out for different operation ranges sampling points individually, so that the sensitivity of the parameters is a function of the system's rpm. Three distinctive operating ranges can be identified. Within each of these operating ranges, the most sensitive input parameters are the same. Table 3 shows the identified operating ranges from the SA according to Morris and the corresponding three most relevant input parameters.

The SA according to Morris is evaluated in order to identify the 10 most significant input parameters for further evaluation using EFAST and later on UA, see Fig. 7. The number of input parameters to retain in the further steps of the method is a value that allows to adapt the method in

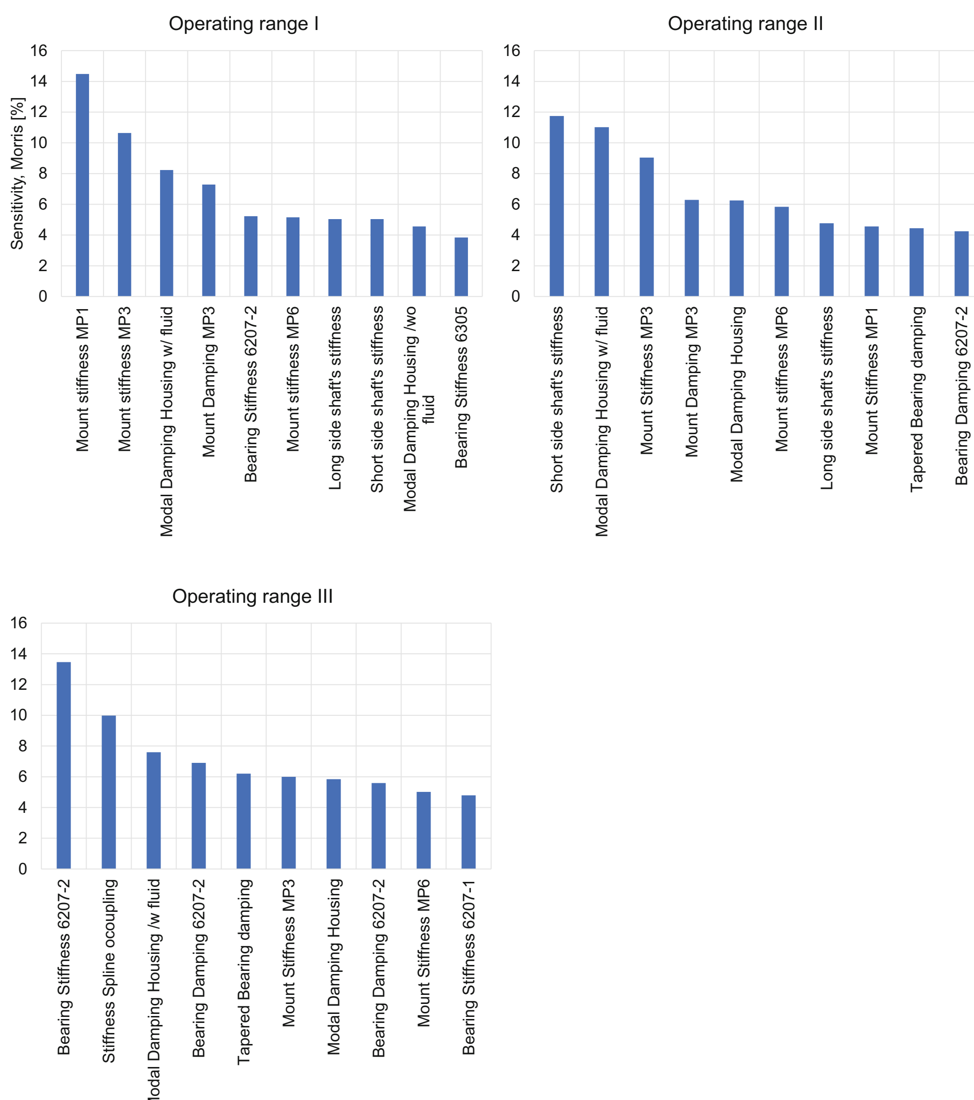


Fig. 7 10 most influencing input parameters for the three operating ranges according to SA of Morris

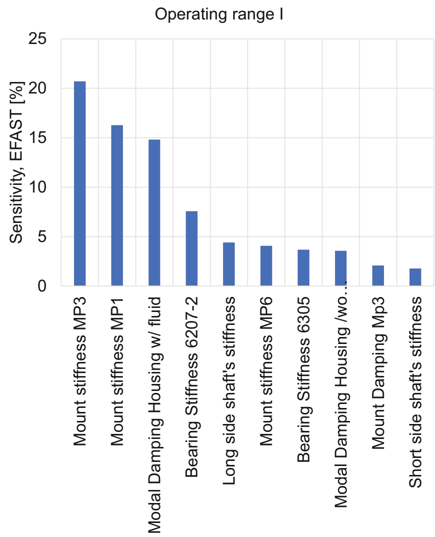


Fig. 8 Uncertain input parameters for the first operating range according to EFAST

a tradeoff between accuracy and computational efficiency depending on the simulation time of the model and the time available for conducting the SA. 10 significant input parameters have been empirically determined to be suitable for the demonstration of the following steps. The 10 most significant input parameters retained for the three operating ranges are shown in Fig. 7.

For each of the operating ranges, the 10 uncertain input parameters are used as the input for an EFAST sensitivity analysis. Using the EFAST sensitivity analysis, the total effect of the uncertain input parameters is evaluated for each rpm sampling point and subsequently integrated along the rpm-axis to yield the sensitivity which is used to create the ranking of the parameters. Figure 8 shows the relative total effects of the 10 input parameters used in EFAST for the first operating range. By defining a threshold of 5% of relative sensitivity, the four most relevant parameters are subsequently used in a fuzzy algorithm in order to evaluate the uncertainty.

Finally, for the 4 identified uncertain parameters, the UA can be carried out. In order to conduct the fuzzy transformation method, the uncertain input parameters have to be assigned to membership functions which represent the range and distribution of the parameter's values. The assigned membership functions are exemplarily shown in Fig. 9.

They are assumed to be normally distributed. The mean value is assigned a membership function of 1, the boundaries are set to be 0 at three times of the standard deviation. The chosen boundaries are $-50\%/+100\%$ for the stiffness of the elastomeric mounts, $\pm 25\%$ for the stiffness of the bearing and 0.5% as well as 5.5% for the modal damping.

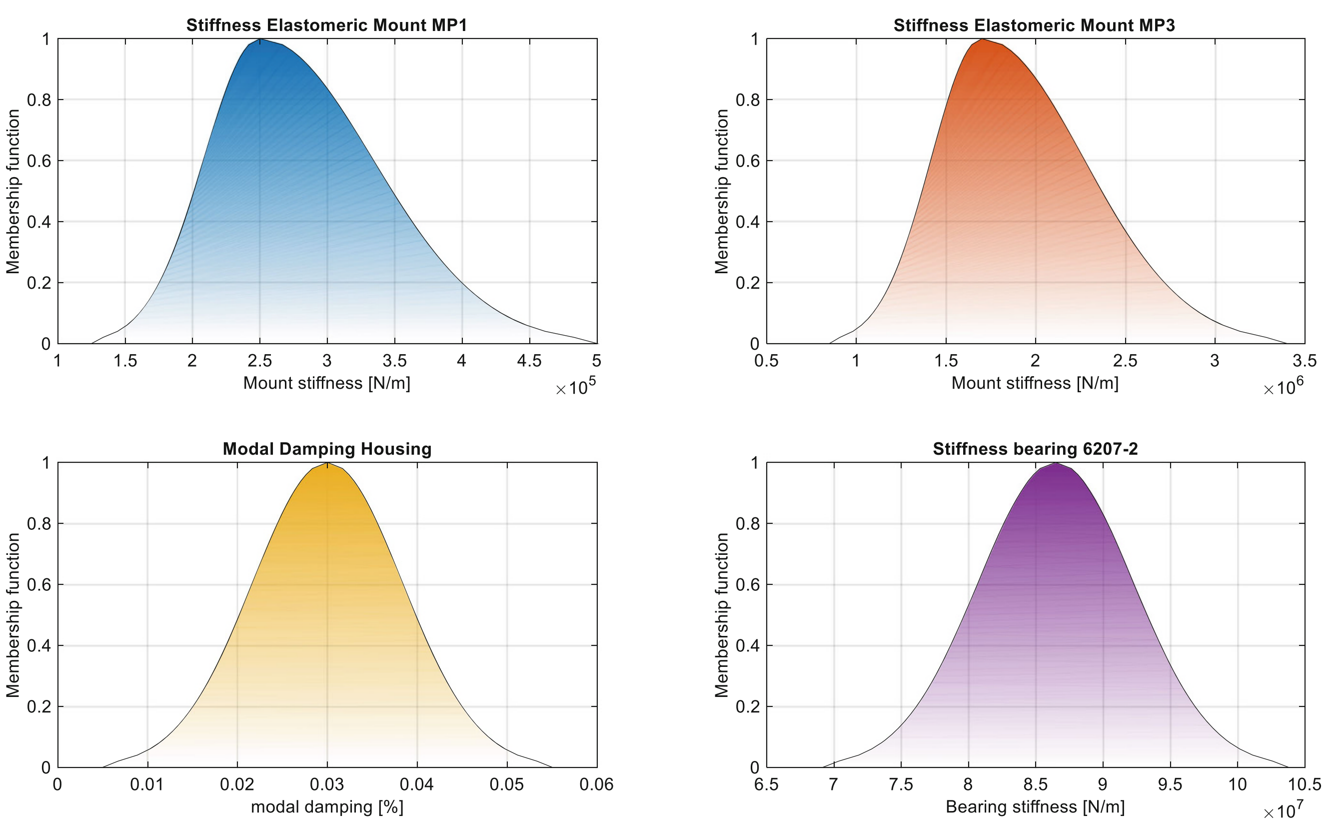


Fig. 9 Membership functions of the four main influencing parameters

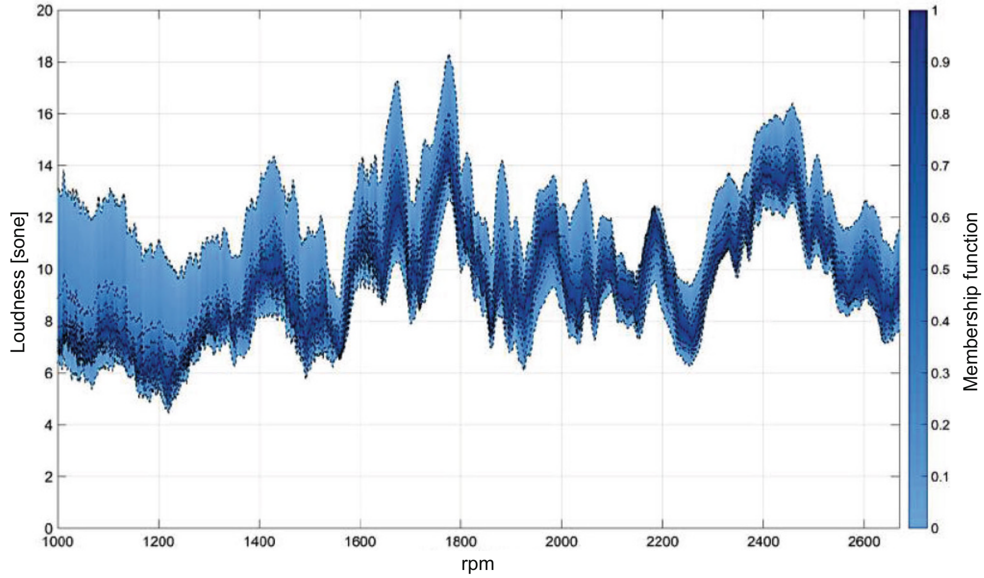


Fig. 10 Membership function of the air borne sound at the driver's ear for the first operating range

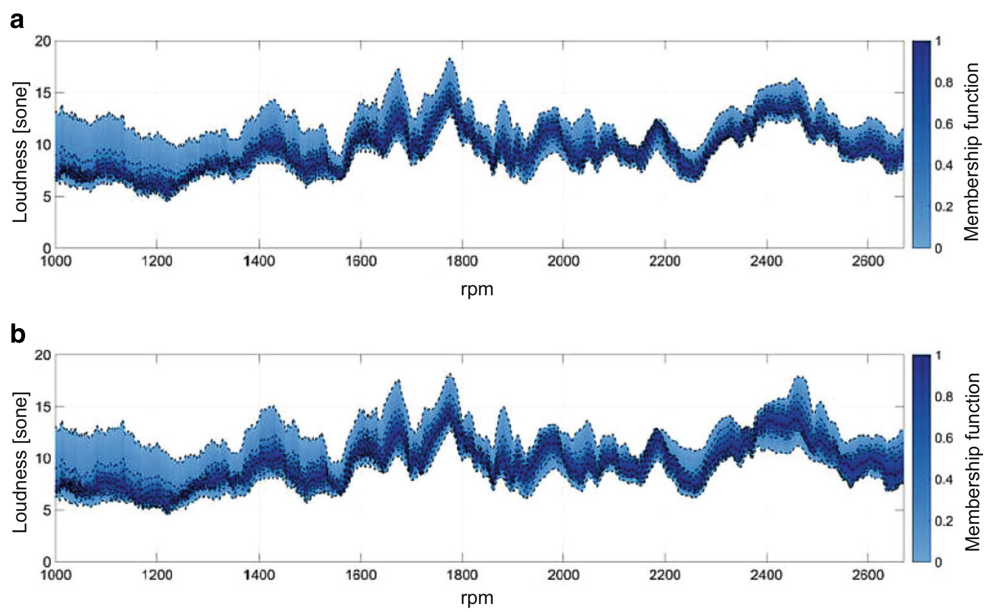


Fig. 11 Membership function of the air borne sound for four (**a**) and five (**b**) uncertain input parameters

This results in an asymmetric membership function for the elastomeric mounts. The dynamic stiffening of elastomers towards higher frequencies is known to be in the order of magnitude of the quasi-static stiffness itself, therefore +100% (factor 2) has been chosen as the upper boundary. The lower boundary has been set at half of the nominal value (factor ½), respectively. Therefore, the absolute difference between upper boundary and nominal value is larger than between the nominal value and the lower boundary, which results in an asymmetric stiffness. Five α-Cuts are used to transform the continuous membership function to discrete input values for the uncertainty analysis. The fuzzy transformation UA is carried out using the FAMOUS toolbox [36].

The result of the UA for the defined uncertain input parameters is shown in Fig. 10 in Form of the membership function of the loudness in sone depending on the rpm. A membership function value of 1 indicates the most probable output, whereas a membership function of 0 indicates that the output values cannot occur based on the given intervals of the uncertain input parameters. In between, higher membership function values indicate more probable output parameter values. It can be seen that for the chosen demonstrator case, a spread of 4 to 7 sone around the nominal value can be observed. It can also be seen that the uncertainty is largest at low rpm ranges and decreases towards higher rpms. Furthermore, the uncertainty of the air borne sound is asymmetrically distributed towards higher loudness values with a maximum of 5 sone above the nominal value at the rpm range below 1200 rpm as well as at 1700 rpm.

The demonstrated method is evaluated using an additional fifth uncertain input parameter for the UA, which is the stiffness of the side shafts. The uncertainty bands are compared in Fig. 11. Up to an rpm range of 2200 rpm, no significant differences can be observed. However, in the rpm range from 2400 to 2550 rpm, the worst-case values (membership function with a value of 0) show a larger spread of 8 sone compared to 4 sone with only four input parameters. It can therefore be concluded, that the developed method is capable of evaluating the overall system's uncertainty, while for individual operating points of interest, a study of the next influencing parameter should be included to gain insights about the error in the quantitative evaluations.

6 Summary and outlook

The structural dynamic and acoustic behavior of drivetrains—often referred to as Noise, Vibration and Harshness (NVH)—is of ever-increasing importance when evaluating a product's requirements. It is a decisive quality feature of the product which often is crucial during a customer's decision of which product to buy. As a result, developing reliable, sophisticated NVH models has been a key focus of recent research. Special focus needs to be given onto the key points of structural-dynamic sound propagation: Excitation (such as gear stages, combustion and electric engines, inverters, power-split transmissions), elements in the transfer path (bearings, elastomeric mounts, joints etc.) and sound radiation. Numerous models of different fidelity levels have therefore been developed to model the structural-dynamic effects of the aforementioned machine elements. The parameters of these models are often uncertain, such as material and contact properties. Especially during early phases of the development process these parameters are either assumed based on empirical values or have to be elaborately measured. Thus, also the output of NVH system simulation models such as structure borne or air borne sound is prone to uncertainties, which must be quantified considering the influence and interactions of all uncertain input parameters. These models are characterized by nonlinear cross-influences between a large number of uncertain parameters and long simulation times which renders the application of existing UA methods unusable.

Therefore, in this work a method is presented, to conduct uncertainty analysis (UA) for NVH simulation models. The proposed method therefore combines the advantages of sensitivity analysis (SA) and UA. Using SA, the uncertain input parameters are ranked according to the relevance to the output. Only the most relevant input parameters are then included in the uncertainty analysis. By applying the proposed method, it is for the first time possible, to quantify the uncertainty of an NVH model within a time period of a few days.

The method is demonstrated on the NVH model of a battery electric vehicle's (BEV) drivetrain with 53 uncertain input parameters. The simulation time needed to conduct a UA can be reduced from $2.4 * 10^{16}$ days to 43 days. By conducting 25 model evaluations in parallel, the simulation was carried out in 1.7 days. This enables the NVH engineer to include uncertainty analysis during the development process and obtain information on the uncertainty of the model's output. A spread of the air borne sound for the given input parameters' distribution of 4 to 7 sone depending on the operating point can be observed. A second advantage of the proposed method is that the application of SA allows for the identification of parameters which influence the output the most. The stiffness of the elastomeric mounts was found to contribute most to the uncertainty of the output signal. Therefore, the proposed method can also be used to derive which parts of the model should be carefully considered during the modelling process and enhance NVH model's quality in a target-oriented way.

In future works, special focus should be given to the methodological identification of operating ranges for the input parameter's ranking using SA methods. Furthermore, an approach for identifying the required number of input parameters considered during the UA depending on the desired reliability of the uncertainty results should be developed. With increasing model fidelity of NVH system simulation models, additional parameters should be analyzed in future works: The consideration of eccentricity effects on the NVH behavior allows for the consideration of manufacturing tolerances and modeling joint behavior introduces parameters concerning the contact and the assembly process. Theses parameters should further be studied using the proposed method to analyze their influence on the uncertainty of the calculated sound. Additionally, the applicability of the developed model to other models which are not used to evaluate the NVH behavior of systems but share similar characteristics such as considering multiple domains using numerous, nonlinear parameters, should be explored. This will allow a broader understanding of the advantages of the proposed method for analyzing the uncertainties in fulfilling the requirements during early phases of the development process.

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