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Optimization of contact stress for the high contact ratio spur gears achieved through novel hob cutter

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Abstract

High contact ratio gears are used to minimize the stresses generated on the tooth surface. This research article represents an idea to enrich the contact strength of gear drive using novel high contact ratio (NHCR) spur gear. The increase in contact stress leads to contact fatigue failure, reducing the power transmission capacity of the gear drive. To reduce contact fatigue failure, contact stress needs to be reduced. A high contact ratio spur gear is developed using the novel hob cutter with variable tooth thickness to minimize the contact stress. For a novel hob cutter, the tooth thickness coefficient is greater than 0.5, while the thickness factor of a conventional hob cutter is 0.5. The maximum contact stress is determined through finite element analysis. In addition, a parametric study is executed for the gear parameter such as gear ratio, gear teeth, pressure angle, addendum factor and addendum correction factor to determine optimum contact stress.

Optimierung der Kontaktspannung für Stirnräder mit hohem Kontaktverhältnis durch neuartigen Wälzfräser

Zusammenfassung

Zahnräder mit hoher Überdeckung werden verwendet, um die an der aktiven Zahnflanke vorliegende Beanspruchung zu minimieren. Dieser Forschungsartikel stellt eine Idee vor, die Grübchentragfähigkeit einer Verzahnung durch die Verwendung von Stirnrädern mit großer Profilüberdeckung zu steigern. Der Anstieg der Kontaktspannung führt zu Wälzermüdung und verringert die Leistungsfähigkeit des Zahnradantriebs. Um die Wälzbeanspruchung zu reduzieren, muss die Kontaktspannung reduziert werden. Es werden Stirnräder mit hoher Überdeckung betrachtet, wobei diese unter Verwendung eines neuartigen Wälzfräsers mit variabler Zahndicke erzeugt wurden, um die Kontaktbeanspruchung zu minimieren. Bei dem verwendeten, neuartigen Wälzfräser ist der Zahndickenkoeffizient größer als 0,5, während der Zahndickenkoeffizient eines herkömmlichen Wälzfräsers 0,5 beträgt. Die maximale Kontaktspannung wird durch Finite-Element-Analyse bestimmt. Darüber hinaus wird eine Parametervariation für die Verzahnungsgrößen Übersetzungsverhältnis, Zähnezahl, Eingriffswinkel, Profilverschiebung und Kopfhöhenfaktor durchgeführt, um die optimale Kontaktspannung zu bestimmen.

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1 Introduction

In general, the gear strength enhancement has high demand in transmission gear drives systems considering minimum weight and compactness. Typically, the effect of the contact and root strength of the gear tooth provides an impact on the power transmission capacity [1]. Elkholy [2] proposed a load sharing of high contact ratio (HCR) gear drive through an analytical and experimental method to determine the surface and root stress. Wang and Howard [3] conducted a numerical analysis of profile modified HCR gear to determine the optimum load sharing ratio, surface and fillet stress. Ravivarman et al. [4] determined the ideal

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fillet strength and surface tribology through HCR spur gear with a modified profile, and also, the parametric study was carried out. Mohanty [5] proposed an analytical approach to estimate the load distribution in each tooth of the HCR gear pair, and the respective contact stress is determined. Thirumurugan and Muthuveerappan [6, 7] addressed the effect of contact and root stresses for normal contact ratio (NCR) and HCR gear drive based on the load distribution. The profile shifted gears leads to change in contact ratio and the radius of curvature at the critical points which decreases the contact stress on both face and flank of the gear tooth. In addition, a parametric study is performed to identify the significant contact and root stress using influencing spur gear parameters. The profile modification plays a substantial role in the gear drive system; using the numerical simulation, the authors determined an increment in gear drive efficiency and contact and fillet strength through an asymmetric profile and novel HCR gear [8–11]. Maper et al. [12] investigated the effect of tooth tip modification on surface and root stress of spur gear, also estimated the friction effect on gear stresses. Prabhu Sekar and Muthuveerappan [13] proposed a novel NCR gear to enhance the bending capacity, also varying the influential spur gear parameters to obtain optimum fillet stress. Pedersen [14] proposed an optimized asymmetric tooth profile to enhance the bending strength at the tooth root. A unique concept of the Bezier curve was designed on rack cutter tip to enhance the tooth fillet strength of spur gear [15, 16]. Pedersen [17] optimized the shape of the gear envelope and the rack cutter to minimize the root fillet stress, and extensive studies were made on gear envelope optimization. Spitas et al. [18] proposed an innovative method of combining the cutter tip radii and addendum for improving the fillet strength of 20° involute gear teeth. Dong et al. [19] optimized the rack cutter tip to enrich the fillet strength of the gear using a genetic system and numerical simulation. Pedersen [20] proposed excellent root strength by reducing the fillet stress of the gear through reshaping the profile of the gear cutter tip. A few specialists have utilized a genetic calculation to upgrade the tooth tip and root by altering the coefficients of gear pair to reduce the root pressure and gear mass [21, 22]. Pedrero et al. [23-27] proposed the load sharing based dynamic performance characteristic and contact and root stresses on novel spur gear (NCR and HCR) and helical gear pair with the addition of tooth profile modification for the enhancement of gear drive performance. Pleguezuelos [28] investigated the effect of symmetric lengthy profile changes on HCR spur gear to reduce the peak transmission error and dynamic load. The ideal length of alteration is stated as a factor of contact ratio in both conditions. Karpat [29] addressed the effect of asymmetric HCR spur gear on dynamic load performance, and it shows the least load acting compared to symmetric spur gears. Wang et al. [30] optimized the internal HCR gear by increasing the contact ratio and a novel curved contact path to enhance the load withstanding capacity. Yılmaz et al. [31] proposed a lightweight integrated metal spur gear and examined its root stress and dynamic behaviour through finite element analysis (FEA). Belarhzal et al. [32] proposed a genetic algorithm to optimize the performance of spur gear by addendum modification factor.

Various research studies on HCR spur gears were conducted to estimate the root strength. It was found that varying the hob cutter tooth thickness factor improved the root strength of the gear drive. Limited research is done on evaluating the effect of critical gear parameters on gear drive contact strength using an FEA approach. The implications of estimating the contact strength of the HCR gear to avoid wear, plasticity, scoring, and pitting during operation. This research work aims to determine the optimum contact stress for pinion by varying the hob cutter tooth thickness factor (S_{r2}) of novel high contact ratio (NHCR) spur gear through the FEA. Further, the parametric study on NHCR was performed to evaluate the significance of hob cutter tooth thickness factor.

2 Novel high contact ratio gear

HCR gears are adopted to improve the power transmission capacity and transmit high torque, simultaneously reducing vibration and noise [3]. Generally, in NCR gear, the average teeth contact will be less than two (ε <2), and for HCR gear, the average teeth contact will be greater than two (ε >2). Based on the increase in teeth contact, the load sharing will be more. The NHCR spur gear is generated by altering the hob cutter tooth thickness factor (S_{r2}) results in minimizing the (σ_H)_{max} at the critical contact point.

2.1 Novel hob cutters

The standard hob cutter produces the involute tooth profile of gear and pinion with the tooth thickness $S_{r2}=S_{r1}=0.5$ π m. In the case of a novel hob cutter, the tooth thickness coefficient for gear is represented as $S_{r2}=1-S_{r1}$. Based on the hob cutter tooth thickness factor of gear (S_{r2}), the maximum contact stress is determined for pinion. The S_{r1} indicates the tooth thickness of pinion. The basic standard HCR spur gear and NHCR spur gear hob cutter for pinion are shown in Figs. 1 and 2, respectively.

2.2 Tooth contact points for novel HCR spur gear

Fig. 3a represents five pair contact positions and the critical loading points of NHCR spur gear. The essential contact points on the gear teeth meshing are more predominant to estimate the maximum contact stress that occurred on the



tooth profile. The typical contact points of NHCR gear are highest point of tooth contact (HPTC), second highest point of double tooth contact (SHPDTC), second lowest point of double tooth contact (SLPDTC), first highest point of tooth contact (FHPDTC), first lowest point of double tooth contact (FLPDTC), lowest point of tooth contact (LPTC) which are represented as F, E, D, C, B and A respectively. The contact area between AB, CD and EF are indicated as triple pairs and the double pair contact regions are BC and DE. Fig. 3b shows the critical contact points of NHCR pinion tooth from the start of tooth contact point (F) to the end of tooth contact point (A).

3 FE modeling

The ANSYS parametric design language (APDL) code (ANSYS 12.1) [33] is adopted to develop a two dimensional (2D) five teeth spur gear model is shown in Fig. 4. The gear parameters addressed for this study are shown in Table 1. The mechanical properties considered for this study are modulus of elasticity is 210 GPa, and Poisson's ratio is 0.30. A 6-noded triangular element in PLANE 82 is considered due to a higher-order element. Based on the

convergence study, the crucial fillet area is discretized into 0.20 element size, and the contact region is discretized to 0.002. The surface contact is between the gear and pinion with the target element-TARG169 and contact element-CONTA 172. The backlash is not considered and no contact exists between the wrong tooth flank of the gear drive. The tooth flank was magnified to ensure there is a gap between the non-contact region, as shown in Fig. 4b. The normal force of 10N [34] is applied at the pinion rim, and the gear rim is constrained in all directions. The uniform contact load is applied at each critical contact point using quasi-static finite element analysis.

4 Result and discussion

4.1 Load sharing ratio

The load sharing on the HCR spur gear will minimize the stresses on the mating teeth at a time of contact. The load sharing ratio (LSR) states that the ratio of contact load shared by each pair to the total contact load of gear teeth [35]. The LSR is plotted for NHCR spur gear with hob cutter tooth thickness variation for a single mesh cycle is



0.8

Table 1 Gear parameters

Sl.no	Parameters	Values
1.	Module (m)	1
2.	Pressure angle (α)	20°
3.	Gear ratio (i)	1.5
4.	No. of teeth in pinion (z_1)	40
5.	Addendum factor (ha)	1.3
6.	Dedendum factor (h _f)	$h_a + 0.25$
7.	Addendum modification factor (x)	$x_1 = 0$ and $x_2 = 0$
8.	Rim thickness	5 m
9.	Contact ratio (ɛ)	2.2
10.	Normal force (F _n)	10 <i>N</i>
11	Hob Cutter type	Full round hob cutter

0.7 = 0.520.6 0.5 LSR 0.4 0.3 0.2 27 **3T** 2T 0.1 -1.4-1.2-1.0-0.8-0.6-0.4-0.2 0.0 0.2 0.4 0.6 0.8 1.0 1.2 1.4 LPTC -X/pb - НРТС

Fig. 5 LSR for single mesh cycle

shown in Fig. 5. Fig. 5 illustrates the LSR is maximum at FHPDTC and SLPDTC. The LSR indicate the critical loading points for the s_{r2} =0.52 at the SLPDTC (D).

4.2 Maximum contact stress based on LSR

The comparison of maximum contact stress $(\sigma_H)_{max}$ based on load-sharing ratio is determined through FEA, and the analytical method is shown in Fig. 6. The Hertz equation [35] is adopted to determine the maximum contact stress based on LSR (Eq. 1). The Hertz equation is adopted in the finite element analysis based on load sharing ratio to determine the maximum contact stress at the critical loading points. The finite element based contact stress trend shows a close agreement with the analytical method. The abrupt increase in $(\sigma_H)_{max}$ is observed at the contact points of HPTC (A) and LPTC (F) owing to the incisive tooth tip. This can be eradicated through tip relief at the sharp corners of the gear tooth. It is noticed that the $(\sigma_H)_{max}$ is maximum at the point C and D. The von Mises stress determined for the NHCR gear pair at pitch point is shown in Fig. 7.

$$(\sigma_H)_{\max} = \sqrt{\frac{(\text{LSR} \times F_n) \left[\frac{E_1 \times E_2}{E_2 (1-\mu_1^2) + E_1 (1-\mu_2^2)}\right]}{\pi b \left[\frac{R_1 \times R_2}{R_1 + R_2}\right]}}$$
(1)

4.3 Effect of HCR hob cutter tooth thickness factor

The optimum hob cutter tooth thickness factor for the NHCR spur gear indicates minimum contact stress on the pinion tooth. The $(\sigma_H)_{max}$ is determined for novel HCR by varying the S_{r2} through FEM is shown in Fig. 8. The S_{r2} varies from 0.4 to 0.6. The $(\sigma_H)_{max}$ shows a decrement for S_{r2} from 0.4 to 0.52 and increases for S_{r2} from 0.52 to 0.6. The optimum contact stress for pinion is obtained at $S_{r2}=0.52$.



Fig. 6 Comparison of $(\sigma_H)_{max}$ through FEA and analytical results $(m=1 \text{ mm}, z_1=40, i=1.5, \alpha=20^\circ, \epsilon=2.2, x_1=0, x_2=0)$



Fig. 8 max for various Sr₂ (m = 1 mm, $z_1 = 40$, i = 1.5, $\alpha = 20^\circ$, $\epsilon = 2.2$)

5 Parametric study on symmetric novel HCR spur gears

A parametric investigation on novel HCR spur gear is carried out to estimate the contact stress decrement of pinion based on S_{r2} . The gear parameters examined in this study are detailed in Table 2.

5.1 Effect of gear ratio in HCR gear pairs

The gear ratio (i) impact on $(\sigma_H)_{max}$ based on LSR is determined for novel HCR spur gear with variant tooth thickness coefficient of hob cutter. Fig. 9 indicates the $(\sigma_H)_{max}$ for different gear ratio (i=1.0, 1.5, 2.0) is plotted against the S_{r2} with the parameters of (m=1 mm, z₁=40, α =20°, x₁=0, x₂=0). The percentage reduction in $(\sigma_H)_{max}$ based on gear ratio (i=1 to 2) is 13.82%. The reduction in $(\sigma_H)_{max}$ at the



Fig. 7 von Mises stress plot for contact at pitch point for $(m = 1 \text{ mm}, z_1 = 40, i = 1.5, \alpha = 20^\circ, \epsilon = 2.2, x_1 = 0, x_2 = 0)$

Table 2	Spur	gear	parameters
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Sl.no	Parameters	Values	
1.	Module	1 (mm)	
2.	Pressure angle	17.5°, 20° and 22.5°	
3.	Gear ratio	1, 1.5, and 2	
4.	Pinion teeth number	40, 50 and 60	
5.	Addendum factor	1.25, 1.30 and 1.35	
6.	Dedendum factor	$h_a + 0.25$	
7.	Correction factor x	x1	X2
	S ₊ drive	0.1 0.2	0 0
	S ₋ drive	0 0	-0.1 -0.2
	S _o drive	0.1 0.2	-0.1 -0.2
8.	Normal force (F _n)	10 N	
9.	Cutter	Full round hob cutter	

critical loading point with gear ratio increment based on S_{r2} values. The increase in gear ratio enhances the radius of curvature at the point of contact, resulting in a substantial reduction in $(\sigma_H)_{max}$. Fig. 9 shows that the optimum contact stress $(\sigma_H)_{max}$ obtained for S_{r2} is 0.50, 0.52, and 0.52 for the respective gear ratios i= 1.0, 1.5, and 2.0.

5.2 Influence of pressure angle

The NHCR spur gear with different pressure angle is investigated to determine the maximum contact stress (σ_H)_{max} based on LSR with the modification of S_{r2}. Fig. 10 indicates the (σ_H)_{max} for different pressure angles (α =17.5°, 20° and 22.5°) is plotted against the Sr2 with the parameters of (m=1 mm, z₁=40, i=1.5, x₁=x₂=0). It is noted that the optimum S_{r2} of 0.54, 0.52, 0.50 for the respective pressure angle of 17.5°, 20°, 22.5° shows a reduction in (σ_H)_{max}.



Fig. 9 Influence of gear ratio on $(\sigma_H)_{max}$



Fig. 11 Influence of teeth number on $(\sigma_H)_{max}$

The percentage reduction in $(\sigma_H)_{max}$ based on pressure angle $(\alpha = 17.5^{\circ} \text{ to } 22.5^{\circ})$ is 6.43%. The rise in the pressure angle leads to a decrease in $(\sigma_H)_{max}$ along the contact path, owing to an enhancement in the curvature radius at the contact profile.

5.3 Influence of teeth number

The importance of teeth number on maximum contact stress $(\sigma_H)_{max}$ has been determined based on LSR with S_{r2} modification. The $(\sigma_H)_{max}$ for a different number of teeth on pinion (Z_1 =40, 50 and 60) with the reduction ratio i=1.5 is plotted against S_{r2} as shown in Fig. 11. It is observed that the optimum S_{r2} of 0.52 for the respective teeth numbers shows a reduction in $(\sigma_H)_{max}$ at the crucial loading point. The percentage reduction in $(\sigma_H)_{max}$ based on teeth number (z_1 =40 to 60) is 19.55%. The decrease in $(\sigma_H)_{max}$ with increased teeth number is due to a rise in curve radius at the contact



Fig. 10 Influence of pressure angle on $(\sigma_H)_{max}$



Fig. 12 Influence of addendum height on $(\sigma_H)_{max}$

region, which delays the pitting and scoring on the contact surface.

5.4 Effect of addendum height

Fig. 12 illustrates the influence of addendum factor (h_a =1.25, 1.30, 1.35) on determining the (σ_H)_{max} based on LSR with the modification of hob cutter tooth thickness factor (S_{r2} =0.4 to 0.6). An increase in addendum indicates a reduction in (σ_H)_{max} as the radius of curvature at the critical contact region increases. The optimum S_{r2} of 0.52 for the respective addendum heights of 1.25, 1.30, and 1.35 decreases (σ_H)_{max} for novel HCR spur gear at the critical loading points. The percentage decrease in (σ_H)_{max} due to addendum height (h_a =1.25 to 1.35) is 2.64%.



Fig. 13 Effect of positive addendum correction factor on $(\sigma_H)_{max}$

5.5 Effect of an addendum correction factor

The variation in addendum correction factor influencing the gear tooth radius of curvature which results in higher efficiency and smooth operation in gear drive performance. The positive correction factor leads to increase the radius of curvature on the teeth contact area, which reduce the contact stress. Whereas, the negative correction factor reduces the radius of curvature on teeth contact region and it leads to increase in contact stress. The correction factor is considered for the gear drives, namely S₊, S₋, S₀. The sum of a correction factor of a pinion (x_1) and gear (x_2) will be greater than zero $(x_1+x_2>0)$ is considered to be positive addendum modified factor (S_{+}) of the gear drive. For the negative correction factor (S₋), the addition of modification factor of pinion and gear will be less than zero $(x_1 + x_2 < 0)$ and the other correction factor type S_0 gear drive provides the summation of x_1 and x_2 equals to zero $(x_1 + x_2 = 0)$. The effect of different addendum modification factors of S₊, S₋, S_o is considered to determine the maximum contact stress $(\sigma_{\! H})_{max}$ based on LSR with the modification factor of hob cutter tooth thickness.

5.5.1 S₊ drives

Fig. 13 illustrates the addendum positive correction factor (S+) effect on determining the maximum contact stress $(\sigma_H)_{max}$ based on LSR with the modification S_{r2} . The positive correction factor considered for this study are $x_1 = x_2 = 0$ and $x_1 = 0.1$, $x_2 = 0$ and $x_1 = 0.2$, $x_2 = 0$. A minor reduction in $(\sigma_H)_{max}$ is observed for the considered positive corrected factor to variation in S_{r2} . The percentage reduction in $(\sigma_H)_{max}$ based on S₊ drives is 2%. Fig. 13 states that the optimum S_{r2} of 0.52 for $x_1 = x_2 = 0$ and S_{r2} of 0.50 for $x_1 = 0.1$, $x_2 = 0$ and S_{r2} of 0.50 for $x_1 = 0.1$, $x_2 = 0$ and S_{r2} of 0.50 for $x_1 = 0.2$, $x_2 = 0$. The reason is



Fig. 14 Effect of negative addendum correction factor on $(\sigma_H)_{max}$

that a rise in the radius of curvature of gear is not influential at the contacting teeth surfaces.

5.5.2 S₋ drives

Fig. 14 illustrates the effect of addendum negative correction factor (S_) on determining the $(\sigma_H)_{max}$ based on LSR with the modification Sr₂. The negative correction factor considered for this study are $x_1 = x_2 = 0$ and $x_1 = 0$, $x_2 = -0.1$ and $x_1 = 0$, $x_2 = -0.2$. The effect of the negative correction factor to S_{r2} shows an increase in $(\sigma_H)_{max}$ compared to other correction factors. Fig. 14. States that the optimum S_{r2} of 0.50, 0.52, 0.52 for the negative correction factors for $x_1 = x_2 = 0$ and $x_1 = 0$, $x_2 = -0.1$ and $x_1 = 0$, $x_2 = -0.2$ respectively. The maximum contact stress increases marginally is observed in the addendum negative correction factor of the gear due to a reduction in the radius of curvature of gear. The percentage increment in $(\sigma_H)_{max}$ based on S₋ drives is 1.1%.

5.5.3 S_o drives

The load sharing based $(\sigma_H)_{max}$ is determined for S_o drives with different S_{r2} is shown in Fig. 15. The S_o correction factor considered for this study are $x_1 = x_2 = 0$ and $x_1 = +0.1$, $x_2 = -0.1$ and $x_1 = +0.2$, $x_2 = -0.2$. The influence of S_o drive factor with respect to different S_{r2} decreases $(\sigma_H)_{max}$. From Fig. 15 it is observed that the optimum Sr_2 of 0.52, 0.50, 0.50 for the respective addendum modification factors provide minimized contact stress for novel HCR spur gear at the crucial loading points. The radius of curvature of gear increases is the reason for the maximum contact stress decrement for S_o correction factor with optimum S_{r2} . The percentage reduction in $(\sigma_H)_{max}$ based on S_o drives 1.1%.



Fig. 15 Effect of S_o drive factor on $(\sigma_H)_{max}$

6 Conclusions

The present research work is carried out to determine the maximum contact stress for non-standardized HCR spur gear at the critical loading points. A FE based parametric investigation is done to estimate the ideal contact stress with respect to S_{r2} . The following inferences are drawn from the current study. The variation of S_{r2} shows a reduction in $(\sigma_H)_{max}$ at the critical loading points on the novel HCR spur gear. The surface strength of the gear tooth is increased based on the variation of S_{r2} . The advantages of the novel HCR gear drive are applicable to transmitting more torque and balanced contact stress between the pinion and gear teeth, increasing load bearing capacity.

- 1. The contact strength of the gear drive can be enhanced by altering the hob cutter tooth thickness factor ($S_{r2}=0.52$) instead of the conventional hob cutter.
- 2. The S_{r2} increases based on gear ratio increment. The maximum contact stress decreases substantially at the crucial loading point due to gear ratio increment with optimum $S_{r2}=0.52$.
- 3. The rise in pressure angle shows a remarkable reduction in $(\sigma_H)_{max}$ at the critical loading point of the gear tooth and the S_{r2} decreases, respectively.
- 4. The increment teeth number indicates a substantial decrease in $(\sigma_H)_{max}$ at the crucial contact point of gear with the optimum S_{r2} of 0.52.
- 5. The reduction in maximum contact stress is achieved with an increase in addendum height factor through the increases in non-standard S_{r2} .
- 6. The increment in addendum modifications of S₊ and S₀ drives shows a decrease in $(\sigma_H)_{max}$ leads to enhance the contact strength of novel HCR spur gear. But the negative correction factor S₋ drive provides a marginal decrement in $(\sigma_H)_{max}$.

7. NHCR gears are more advantageous than HCR gears in increasing the load carrying capacity, contact fatigue life, and contact strength by varying the hob cutter tooth thickness factor. Typically, the root strength of HCR gear predicts the breaking failure of the gear tooth. Whereas the contact strength of NHCR gear deals with the failure due to wear, pitting and scoring.

Apart from findings, the effect of tooth stiffness and dynamic response will be performed along with fatigue analysis for novel high contact ratio spur gear drive as a future scope.

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Conflict of interest S. Rajesh, P. Marimuthu and P. Dinesh Babu declare that they have no competing interests.

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