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Discretization of the contact conditions considering the grain engagement for generating gear grinding

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Abstract

Generating gear grinding is a hard finishing process mainly used to meet the high requirements for gears in terms of geometry and surface quality. However, due to the high friction generated in the contact zone, high temperatures are achieved, which might cause thermal damages. To avoid the generation of these damages, an analysis of the resulting energy conversion is recommended. The energy conversion is significantly influenced by the interaction between each grain engaging with the material in the contact zone. However, current approaches for energy description do not take into consideration the grinding tool topography due to the random and complex distribution of the grains in terms of position and shape in the contact zone. For the specific case of generating gear grinding the challenge is increased due to the complex contact conditions induced by the process kinematics.

The objective of this work is the investigation of the influence of the grains on the material removal behavior and the implementation of this influence on a generating gear grinding energy model. The research is focused on an empirical investigation of a single-grain trial and on the transferability of the findings first onto an analogy trial with surface grinding process and last onto an analogy of the continuous generating gear grinding process.

The energy was analyzed in terms of normal force during generating gear grinding. An alternative approach for the modelling of the normal force taking into consideration the tool topography for the process of generating gear grinding was developed. According to the results, the alternative approach for the calculation of the normal force presented in this work showed a promising method, even though further optimization is still required. Ultimately, the alternative approach for normal force calculation can be further developed for the thermo-mechanical energy determination for generating gear grinding process.

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Diskretisierung der Kontaktbedingungen unter Berücksichtigung des Korneingriffs zur Verzahnung Wälzschleifen

Zusammenfassung

Das Wälzschleifen ist ein Hartbearbeitungsverfahren, das hauptsächlich zur Erfüllung der hohen Anforderungen an Zahnräder in Bezug auf Geometrie und Oberflächenqualität eingesetzt wird. Durch die hohe Reibung in der Kontaktzone werden jedoch hohe Temperaturen erreicht, die zu thermischen Schädigungen führen können. Um die Entstehung dieser Schädigungen zu vermeiden, wird eine Analyse der resultierenden Energieumwandlung empfohlen. Die Energieumwandlung wird maßgeblich durch die Wechselwirkungen zwischen den einzelnen Körnern beeinflusst, die mit dem Material in der Kontaktzone in Eingriff kommen. Aktuelle Ansätze zur Energiebeschreibung berücksichtigen jedoch aufgrund der zufälligen und komplexen Verteilung der Körner in Bezug auf Position und Form in der Kontaktzone, nicht die Schleifwerkzeugtopographie. Für den speziellen Fall des Verzahnungswälzschleifens erhöht sich die Herausforderung durch die komplexen Kontaktbedingungen, die durch die Prozesskinematik verursacht werden.

Ziel dieser Arbeit ist die Untersuchung des Einflusses der Körner auf das Zerspanungsverhalten und die Umsetzung dieses Einflusses auf ein Energiemodell zum Wälzschleifen. Die Forschung konzentriert sich auf eine empirische Untersuchung eines Einkornprozesses und auf die Übertragbarkeit der Ergebnisse zunächst auf einen Analogieprozess eines Flachschleifprozesses und zuletzt auf eine Analogie des kontinuierlichen Verzahnungswälzschleifprozesses.

Die Energie wurde hinsichtlich der Normalkraft beim Verzahnungswälzschleifen analysiert. Ein alternativer Ansatz zur Modellierung der Normalkraft unter Berücksichtigung der Werkzeugtopographie für den Prozess des Wälzschleifens wurde entwickelt. Nach den Ergebnissen zeigte der in dieser Arbeit vorgestellte alternative Ansatz zur Berechnung der Normalkraft eine vielversprechende Methode, auch wenn noch weitere Optimierungen erforderlich sind. Schließlich kann der alternative Ansatz zur Normalkraftberechnung für die thermo-mechanische Energiebestimmung zur Erzeugung des Verzahnungsschleifprozesses weiterentwickelt werden.

1 Introduction

During grinding, a major percentage of the generated energy is converted into heat. The generated heat is distributed mainly into four regions: Environment, tool (grinding worm), chip and workpiece. The portion of heat transmitted to the workpiece leads to the generation of high temperatures in the contact zone, which can cause thermal damages in the workpiece such as metallurgical phase transformations and undesirable residual stress profiles [1]. To avoid the generation of thermal damages during the process, a detailed analysis of the resulting energy conversion is recommended.

The energy conversion is significantly influenced by the interaction between grains of the grinding tool and workpiece material. At each engagement of a grain, mechanical energy from the chip formation is converted to heat [2]. The engagement between grain and workpiece is a result of the combination of several parameters such as grain shape and distribution, process parameters and workpiece material. Therefore, the grinding tool topography, especially in terms of grain shape and distribution, plays an important role in the energy conversion and it needs to be taken into consideration in the energy calculation [3].

Conversion of energy for grinding processes in general was already investigated by several studies over the years [4–7]. The outcome of these studies is normally transferred into different available models. Essentially, the models used in these studies for the transference of their outcome are based on pure kinematic-empirical relationships or on existing models such as from Malkin and Guo [1]. In their work, Malkin and Guo [1] summarized numerous heat flux models for different grinding processes. In his model for surface grinding, Malkin [1] defined that for a specific grinding energy, or power P, the energy conversion to the workpiece is defined as

$$\mathbf{E} = \frac{\boldsymbol{\varepsilon} \cdot \mathbf{P}}{l_c \cdot b} \quad (1)$$

E [W/mm^2]	Energy input
ε [–]	Energy partition
P [W]	Power
L_c [mm]	Length of contact zone
b [mm]	Width of contact zone

In Eq. 1 the numerator represents the fraction of the grinding energy converted into heat entering the workpiece. The denominator represents the area of the grinding zone. The model of Malkin and Guo was already widely applied in several works. For example, in his work Reimann [8] aimed to model the thermo-mechanical energy conversion in generating gear grinding in order to predict the presence of grinding burn. For this purpose, the model of Malkin and Guo was used under some alterations. To calculate the energy input for generating gear grinding, specific parameters must be determined, as shown in Eq. 2.

$$E''_{\mathbf{WSy}} = \frac{F_{cy}.v_{cy}.t_{ky}}{A_{ky}} \tag{2}$$

E''_{WSy} [J/mm²]	Specific energy input
F_{cy} [N]	Cutting force
V_{cy} [m/s]	Cutting speed
T_{ky} [s]	Contact time
A_{ky} [mm²]	Area of contact zone

The four parameters—cutting force, cutting speed, contact time and area of contact zone—form the basis for the approach to thermo-mechanical modelling of the process of generating gear grinding, according to Reimann [8]. Each of these parameters were determined by analytical and empirical methods.

In the approach of Reimann as well as in other current approaches, the description of energy conversion is mainly based on empirical relationships. In addition, due to the random and irregular engagement between grains and material in grinding, the distribution and shape of the grains are neglected. For the process of generating gear grinding, the modelling of the grains engagement is even more challenging due to the additional complex contact conditions induced process kinematics.

2 Objective and approach

In this study, an approach for the modelling of the energy conversion is proposed, considering the tool topography as well as the process parameters for the process for the generating gear grinding. Therefore, the main objective of this work is the investigation of the influence of the grains on the material removal behavior and the implementation of this influence on a generating gear grinding energy model.

In this work, the analysis of the thermo-mechanical energy is performed using the model of Reimann, presented in Eq. 2. The influence of the tool topography is introduced by means of the development of an alternative method for calculating the grinding forces. With this purpose, the normal force is investigated according to the model of Werner [9], Eq. 3.

$$F'_n = \int_0^l k \cdot A_{cu}(l)^n \cdot N_{\mathbf{kin}}(l)\, dl \tag{3}$$

F'_n [N/mm]	Specific normal force
A_{cu} [mm²]	Chip cross-section area
l [mm]	Contact length
k [N/mm²]	Specific energy
N_{kin} [1/mm²]	Number of kinematic cutting edges
n [–]	Material constant

The factor k is called specific energy and is normally determined by cutting force measurements. It represents a local quantity of the instantaneous specific energy, depending on the contact length [2, 9]. The specific energy k is also responsible for representing the material removal mechanism during the process in the force model. In the work of Teixeira et al. [10], an alternative method for determination of the specific energy k taking into consideration the tool topography and process parameters was developed. In their work, the tool topography was analyzed in terms of cutting angles such as opening angle α and apex angle β. The alternative method for the specific energy calculation also takes into consideration the position of the grain along the contact length. In Eq. 4, an alternative approach based on Werner model is proposed for the calculation of the specific energy. The research is focused on the transferability of the findings from the work of Teixeira [10] first onto surface grinding process and last onto an analogy of the continuous generating gear grinding process. Therefore, trials with surface grinding and generating gear grinding must be performed.

$$k_i = \mathbf{f}(A_{cu}, v_c, l_c) \tag{4}$$

K_i [N/mm²]	Specific energy
A_{cu} [mm²]	Grain cross-section area
V_c [m/s]	Cutting speed
L_c [mm]	Contact length

3 Experimental scope

The trials for surface grinding were performed on a surface grinding machine Blohm 6000, Fig. 1. For the tests, segments of a surface grinding wheel were used. The segments had a length of $l_s = 20$ mm, a width of $b_s = 8$ mm, a height of $h_s = 8$ mm and the grains had the same properties as the grains for the single-grain trials. The segment was fixed in a screw and the screw was fixed in the aluminum grinding wheel with diameter of $d_s = 400$ mm, left of Fig. 1. Before each trial, the segment was dressed, in order to eliminate irregularities introduced by the segment fixation procedure. For both trials, the workpiece was a block of 20MnCr5 with length of $l = 100$ mm, width of $b = 70$ and a height of $h = 39$ mm. The workpieces were pre-ground in order to obtain a smoother surface ($R_a = 0.15$ µm and $R_z = 1.18$ µm). The workpiece was fixed on the top of a measurement plat-

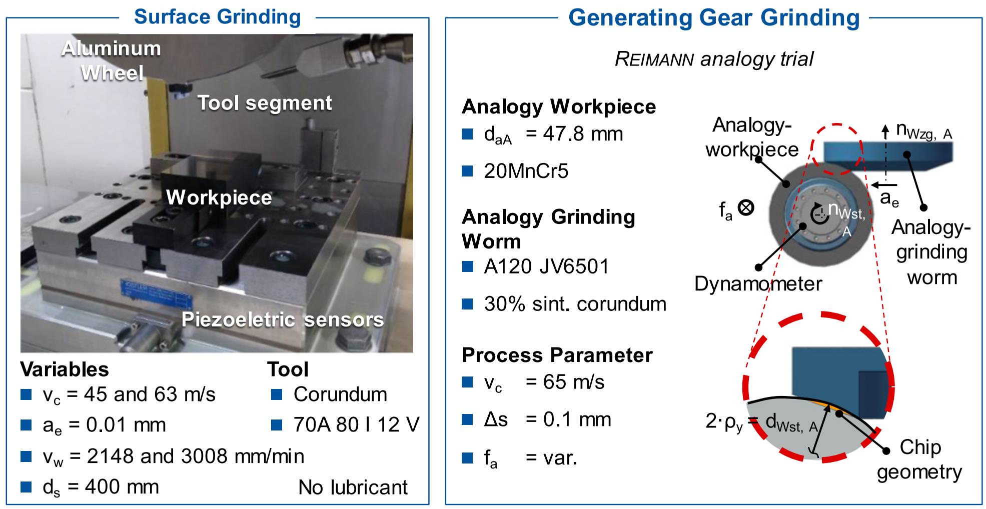


Fig. 1 Description of experimental procedure for surface grinding and generating gear grinding trials

form of piezoelectric sensors from Minidyn 9256C1 type from the company Kistler Instruments, with a sample rate of f = 1 MHz, for the assessment of normal force during the trials.

In the surface grinding trials two factors were varied. Two levels of cutting speed were investigated, v_{c1} = 45 m/s and v_{c2} = 63 m/s, as well as two levels of feed rate, v_{f1} = 3008 mm/min and v_{f2} = 1504 mm/min.

The findings in surface grinding were transferred to the process of generating gear grinding. Due to the complex contact conditions induced by the kinematics of the process, the analogy trial for generating gear grinding developed by Reimann [8] was used. The principle of the analogy trial is shown in right side of Fig. 1. According to Reimann, in a specific point in the gear flank, the involute can be approximated by a circle with a specific local radius of curvature [8]. In his analogy trial, Reimann approximated the contact conditions at the pitch circle. The diameter of the workpiece of the analogy trials is determined as the double radius of curvature at the pitch circle. The grinding tool in the analogy trial is a face wheel with a conic working surface. The parameters for the trials are shown in the right side of Fig. 1. With this analogy trial, the forces can be measured in one contact point in the process of generating gear grinding. The analogy trial represents a generating gear grinding with the following properties: grinding worm with external diameter of d_{a0} = 196 mm, module of m_{n0} = 4.5 mm and pressure angle of α_0 = 20°; workpiece with number of teeth z = 31, module of m_n = 4.5 mm, pressure angle of α = 20°, and tip diameter of d_a = 146.7 mm.

The use of the analogy trial leads to an easier transferability of knowledge from surface to generating gear grinding. Due to this, a comparison between the measured force and the calculated force with the alternative procedure developed by this work is possible. In the end of this work it is expected to validate the alternative approach for normal force calculation.

4 Results and discussion

In this chapter, the findings regarding the specific energy obtained in the works of Teixeira et al. [10] are transferred to a macro-scale level. First, the findings are transferred to the process of surface grinding. Next, the findings will be transferred to the generating gear grinding, a process with a more complex kinematics.

The transferability of findings is first performed from single-grain to surface grinding. The model of Werner for the calculation of normal force for the process of grinding is used, applying an alternative method for the determination of the specific energy. In this alternative method, the specific energy is calculated based on the number of grains engaging in the workpiece in the contact zone, the grains shape as well as their distribution along the contact length. Fig. 2 shows the first steps for the specific energy calculation. First, a step of surface topography analysis is performed. In this step, the tool topography is measured and analyzed by the software TopoTool. The tool was developed in the Laboratory for Machine Tools and Production Engineering (WZL) Aachen [3]. It enables the description of the contact conditions between grinding wheel topography and workpiece based on the resulting kinematic contact conditions [3]. Based on inputs such as tool topography and process parameters, the TopoTool provides information regarding the number of kinematic cutting edges N_{kin} and grain cross-section area A_{cu} in the point of maximum chip thickness. Therefore, an analysis of the shape of each grain engaging in the material is possible. The next step is the determination of a statistical distribution of the kinematic cutting edges N_{kin}, upper right of Fig. 2. The method used

Fig. 2 Analysis of contact conditions for surface grinding and procedure for determination of instantaneous kinematic cutting edges

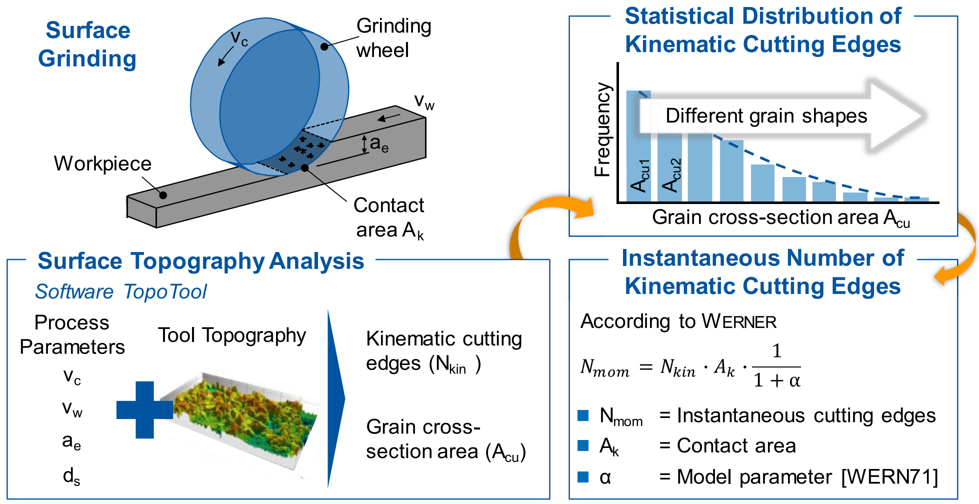
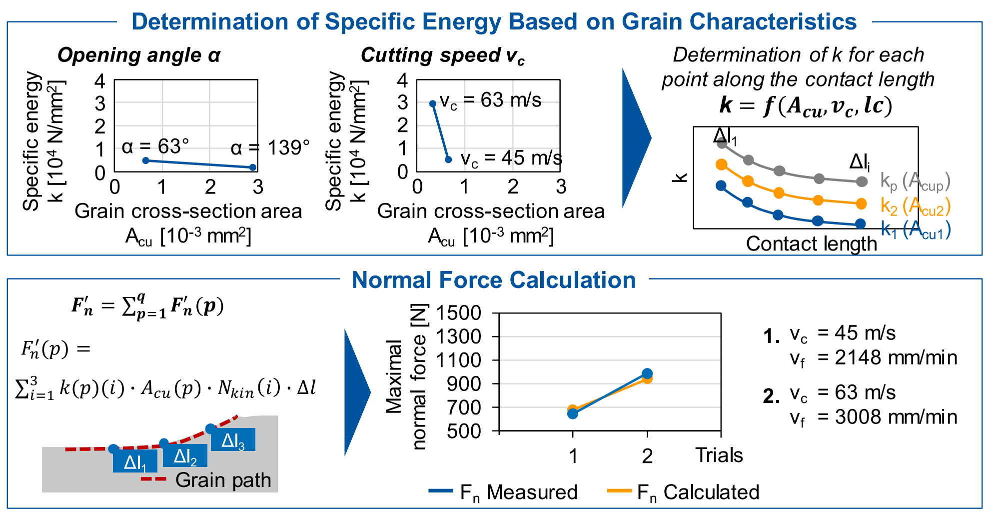


Fig. 3 Procedure for the calculation of normal force in surface grinding



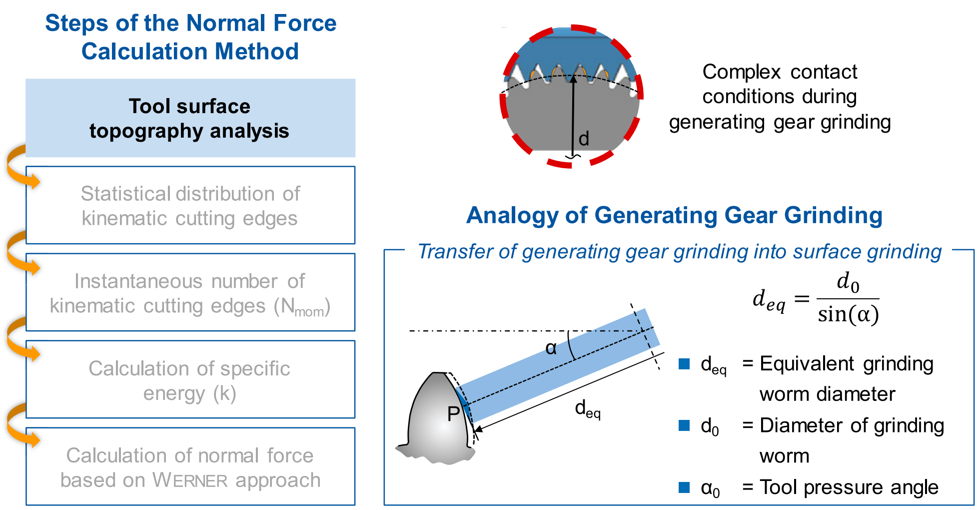
for the determination of the distributional fit is the analysis of the histogram of the actual data of the grains cross-section area A_{cu}. With this information it is possible to analyze the presence of different grain shapes in engagement.

It is important to highlight that the number of N_{kin} given by TopoTool represents the number of all grains engaging in the material during one tool revolution. However, for the application of this parameter in the calculation of normal force according to Werner, the quantity of the grains engaging in the contact area A_k in one specific time frame is has to be known. This parameter is called instantaneous number of kinematic cutting edges N_{mom}, and it is calculated according to the equation shown in the bottom right of Fig. 2 [9]. The parameter α is a model parameter that takes into consideration the form of the grains. For this work, a standard value according to Werner, are used for the calculation of N_{mom}. The combination between the results of the statistical distribution and calculation of instantaneous kinematic cutting edges provides information regarding the shape distribution of the grains that are actually in contact locally with the material.

The next step is to take into consideration the information regarding the quantity and the shape of the grains in contact for the determination of the specific energy k for surface grinding. In the upper left of Fig. 3, two diagrams of the specific energy according to the grain shape, in terms of cross-section area, and cutting speed are shown. The value of the specific energy shown in these both diagrams represents the value of k for a specific grain positioned in the point of the contact length with maximum chip thickness. Once this value is defined, the other values of specific energy along the contact length are defined, as shown in the diagram in the upper right of Fig. 3.

With the information regarding how many grains engage locally in the contact zone, as well as their maximum cross-section area A_{cu}, it is possible to determine the specific energy k for each of these grains. However, the TopoTool calculates the information of the grains in the moment where

Fig. 4 Procedure for calculation of normal force and analogy for contact condition analysis in generating gear grinding according to Schriefer [11]



they have the highest chip thickness, it is not possible to know precisely the position of each of the grains N_{mom} engaging along the contact length. Therefore, in this work it is assumed a distribution of these grains in three different positions along the contact length is assumed: Δl_1, Δl_2 and Δl_3. For each position, a specific energy k is calculated according to the curve shown in the upper right of Fig. 3.

Finally, the findings of the specific energy are applied in an adapted Werner model for the calculation of normal force for the surface grinding process, bottom left of Fig. 3. In the adapted model, a final normal force F'_n is defined. F'_n is the sum of the normal force components $F'_n(p)$ induced by each type of grain detected in the tool topography analysis. The normal force components $F'_n(p)$ are calculated for each individual class $A_{cu}(p)$ of grain shapes determined in the statistical distribution of the kinematic cutting edges. Based on the $A_{cu}(p)$, the specific energy is determined, according to the procedure explained before. $F'_n(p)$ is the sum of the product between specific energy k, grain cross-section area and kinematic cutting edges over the contact length. As already stated, the contact length is discretized in three parts, and for each of them the specific energy is calculated accordingly.

The procedure for the normal force calculation is validated for two different grinding conditions: (1) $v_c = 45$ m/s and $v_f = 2148$ mm/min and (2) $v_c = 63$ m/s and $v_f = 3008$ mm/min. For these parameter set-ups, the feed per rotation remains constant. In the diagram in the bottom right of Fig. 3 the maximum normal force calculated and measured are compared. According to the graphic, the calculated normal force is in accordance with the measured forces for both the grinding conditions investigated. The increase in the process parameters increases the normal forces. This result is expected because higher cutting speed requires higher specific energy [10]. Due to this, the fraction of specific energy input for each grain engaging in the contact zone is higher for higher cutting speeds, and an increase of the total normal force is verified.

Next, the procedure for the calculation of the normal forces applied for the process of surface grinding is adapted for the process of generating gear grinding. According to the procedure showed in the topics before for the calculation of normal force in surface grinding the first step is the analysis of the tool topography by means of the software TopoTool, Fig. 4.

However, due to the complex contact conditions of the generating gear grinding, a direct analysis with the software is not possible. To overcome this challenge and to perform an analysis with the TopoTool, an analogy of the generating gear grinding process is required. This analogy will be called in this work: *contact analogy*. It is important to highlight that the analogy trial of Reimann is different from the contact analogy. In order to avoid confusion between the two analogies, the analogy of Reimann will be called *Reimann analogy*. According to Schriefer [11], the generating gear grinding process can be replicated by finite sequence of surface grinding processes along the discretized movement of the grinding worm. For the contact analogy, an equivalent grinding wheel diameter d_{eq} is used, positioned in one specific point P along the gear flank, as shown in the bottom right of Fig. 4. Therefore, the process of generating gear grinding can be represented by the surface grinding process for one specific point P in the gear flank.

The use of the contact analogy enables the analysis of the tool topography with TopoTool. The process parameters used for the analysis are specified in Sect. 3. The cutting speed is kept constant at $v_c = 65$ m/s, the axial cutting depth is kept constant at $a_e = 0.3$ mm and the axial feed is varied between $f_{a1} = 0.1$ mm, $f_{a2} = 0.2$ mm and $f_{a3} = 0.3$ mm. These three set-ups of parameters were also used by Reimann in his Reimann analogy. Therefore, the force measurement ob-

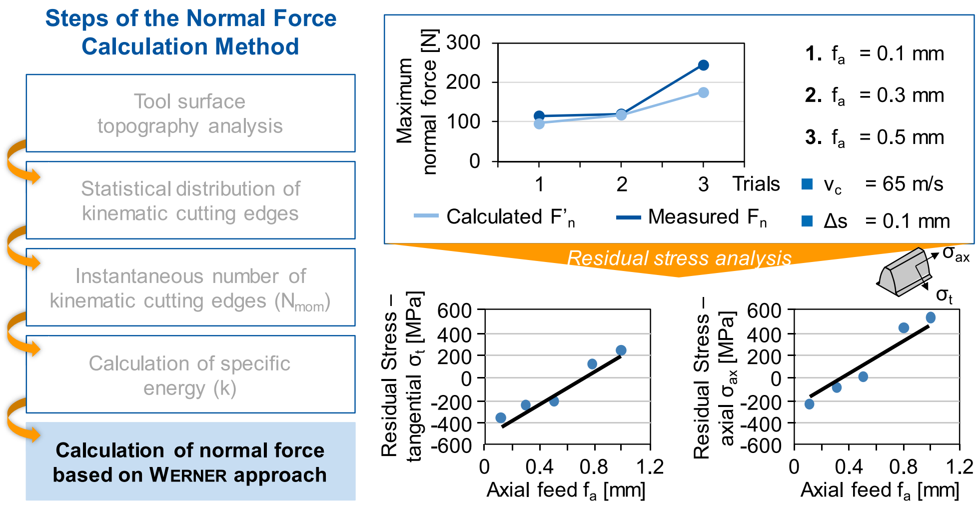


Fig. 5 Comparison between normal force calculated measured for the analogy trials from Reimann [8]

tained by Reimann in his trials can be used to validate the procedure for the force calculation developed in this work. It is important to highlight that the resulting force presented by Reimann in his work had to be adapted. In his work, the resulting force results from a vectorial addition from components which are not assumed to be in peripheral direction. Therefore, in order to obtain the correct value of the normal component from the resulting forces from Reimann, a factor was implemented based on the contact conditions induced by the trials. All the three set-ups are submitted to the analysis of the TopoTool and the number of the kinematic cutting edges as well as the shape of each of them are obtained. Next, cutting edges analysis is performed, following the same procedure showed for the case of surface grinding. The statistical distribution of the kinematic cutting edges is performed, followed by the calculation of the instantaneous number of kinematic cutting edges N_{mom}. In the next step, the specific energy k for each class of the grain shape determined in the statistical distribution is calculated. In these trials, since no variation in the cutting speed is performed, only the influence of the grain shape is taken into consideration for the calculation of the specific energy k. Following the procedure developed for surface grinding, the grains are also distributed in three different positions along the contact length.

Finally, the normal force F'_n is calculated for the process of generating gear grinding. The diagram in the bottom right of Fig. 5 shows the calculated forces as well as the normal forces measured in the Reimann analogy trials.

In both calculated and measured forces, the increase of the axial infeed increases the normal force as well. However, the magnitude of both forces is not the same. It is important to highlight that the tool topography used by Reimann in his trials could not be analyzed in the current work. Therefore, the topography used for the calculation of the forces in the generation gear grinding trials was the topography of the surface grinding trials. Although both tools are from the same material, the grains from both tool have different sizes. In addition, the tools were submitted to different dressing parameters. The combination of these two factors, grain size and tool dressing parameters, can lead to significant differences in the tool topography which can contribute for the difference in the final normal force calculated. In addition, the influence of the wear was also not taken into consideration in this work. The wear of the grains can change the material removal mechanism and, consequently, can change the forces during grinding. The presence of wear can explain in particular the difference in the force magnitude for the trial 3 shown in the graphic in the bottom right of Fig. 5, with the highest axial feed.

The bottom right of Fig. 5 showed an analysis of the influence of the axial feed on the residual stress state at the workpiece surface. It is possible to see that around $f_a = 0.5$ mm, the residual stress state changes from compressive to tensile in the axial direction. An analysis of the normal force shows an increase in the force when axial feed $f_a = 0.5$ mm. From the comparison of results between force and residual stress state it is possible to assume a limit for the axial feed in order to avoid undesirable residual stress state.

Ultimately, the force for the process of grinding has a significant influence on the energy conversion, as it is shown in the models of Malkin and Reimann, Eqs. 1 and 2 respectively. Therefore, the consideration of the tool topography for the calculation of the normal force developed in this work is an initial step towards the understanding of the influence of each grain on the energy conversion.

5 Summary and outlook

In this work, an alternative approach for the modelling of the normal force taking into consideration the tool topography for the process of generating gear grinding was developed. This alternative approach considers that each grain engaging in the material generates a different specific energy k that depends on the grain shape, process parameters and position of the engaging grain along the contact length.

A methodology was developed, in order to take into consideration all the instantaneous cutting edges in the contact zone, as well as the specific energy k for each of these cutting edges for surface grinding. In the end, the calculation of the normal force by the application of the alternative approach showed a good agreement with the force measurements. Finally, the alternative approach for the normal force calculation was transferred to generating gear grinding. Due to the complex contact conditions of the process, an analogy was used for the analysis of the tool topography and kinematic cutting edges distribution. The force during the process was analyzed based on the force measurements from the analogy trials developed in the work of Reimann [8]. The comparison between the calculated and measured force showed a good accordance in terms of the curve tendency for the different axial feed. The magnitude of both forces, on the other hand, presented different results. This difference can be attributed to the tool topography used for the calculation of the normal force, which was not the same topography used in the work of Reimann [8], as well as by the grain wear, which was not considered in this work. Analysis of the residual stress state showed that for the setup where the force increases, an alteration in the residual stress is also detected.

The alternative approach for the calculation of the force presented in this work showed promising results. Further optimization is still required, based on the limitations currently found. Such optimizations can be listed as: consideration the specific shape of the contact area of generating gear grinding for the characterization of the kinematic cutting edges; investigation of the single grain contact path during generating gear grinding; investigation of the influence of different process parameters as well as different workpiece materials on the magnitude of the specific energy.

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Conflict of interest P. de Oliveira Teixeira, C. Brecher and C. Löpenhaus declare that they have no competing interests.

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