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Free convection flow of water near 4 °C past a moving plate

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Abstract In the present study the unsteady free convection flow of water near 4 °C in the laminar boundary layer over a vertical moving porous plate is investigated. The effect of the suction/injection parameter at the plate on the velocity is considered. The momentum equation is solved numerically by a fourth-order Runge-Kutta scheme. The numerical results which are obtained for the flow are shown graphically.

Freie Konvektionsströmung von Wasser mit einer Temperatur von ca. 4 °C über eine bewegte Platte

Zusammenfassung In der vorliegenden Arbeit wird die instationäre freie Konvektionsströmung von Wasser mit einer Temperatur nahe 4 °C und einer laminaren Grenzschichtströmung über eine vertikale, bewegte, poröse Platte untersucht. Der Einfluß einer lokalen Ansaugung bzw. Einspritzung auf das Geschwindigkeitsfeld wird berücksichtigt. Die Impulserhaltungsgleichung wird numerisch durch die Anwendung eines Runge-Kutta-Verfahrens vierter Ordnung gelöst. Die erhaltenen Ergebnisse werden graphisch dargestellt.

1 Introduction

It is known, that for a fluid like air or water at ordinary temperature and atmospheric pressure the variation of the density with the temperature is given by

$$\Delta\rho = -\rho\beta(\Delta T') \ , \tag{1}$$

where $\beta = 2.07 \times 10^{-4}\,(^\circ\mathrm{C})^{-1}$ at 20 °C. Goren [1] has shown, that the usual Navier-Stokes equations are not suitable for studying the flow of water near 4 °C and hence the modified form of the above equation applicable to water near 4 °C is very closely given by

$$\Delta\rho = -\rho\gamma(\Delta T')^2 \ , \tag{2}$$

where $\gamma = 8.0 \times 10^{-6}\,(^\circ\mathrm{C})^{-2}$.

Several authors have studies the free convection flow of water near 4 °C past a stationary vertical plate [1–8]. The measurement of the terminal velocity of small particles, working near 4 °C, may be benefit in measuring molecular diffusivities in water and other mass and heat transfer experiments, where one wishes to suppress free convection. On the other hand, rates of heat transfer by free convection in water near 4 °C may be much reduced from that at other temperatures and this might be an important consideration in certain freezing processes.

The purpose of this paper is the study of the unsteady free convection flow of water near 4 °C in the laminar boundary layer over a vertical moving porous plate.

2 Analysis

Let us consider a two dimensional laminar free convection flow of water near 4 °C past a moving vertical porous plate. The x' and y' axes are along and normal to the plate respectively (Fig. 1). The equations governing the problem are

$$\frac{\partial v'}{\partial y'} = 0 \tag{3}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\gamma\left(T' - T'_\infty\right)^2 \tag{4}$$

$$\frac{\partial T'}{\partial t'} + v'\frac{\partial T'}{\partial y'} = \frac{k}{\rho c_p}\frac{\partial^2 T'}{\partial y'^2} \ , \tag{5}$$

with the following boundary conditions

$$t' > 0: \begin{cases} u' = c't',\ v' = v'_w(t'),\ T' = T'_w, & \text{at } y' = 0 \\ u' \to 0,\ T' \to T'_\infty & \text{as } y' \to \infty \ , \end{cases} \tag{6}$$

where u' is the component of the velocity along the plate, v' the component of the velocity normal to the plate, v'_w the velocity of injection at the plate, t' the time, c' a constant, ν the kinematic viscosity, γ the coefficient of thermal expansion of water near 4 °C, g the acceleration due to gravity, T' the temperature of the fluid, T'_w the temperature of the fluid at the plate, T'_∞ the temperature of the fluid far away the plate, k the thermal conductivity, ρ the density of the fluid and c_p the specific heat at constant pressure.

Integrating Eq. (3) we take

$$v' = v'_w(t') = -a\left(\frac{\nu}{t'}\right)^{1/2} \ , \tag{7}$$

where the constant a characterizes the phenomenon of suction at the plate when $a > 0$ and injection when $a < 0$.

Introducing the following non-dimensional quantities

$$t = t'\left(\frac{c'^2}{\nu}\right)^{1/3} \tag{8}$$

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$$y = y'\left(\frac{c'}{\nu^2}\right)^{1/3} \quad (9)$$

$$u = \frac{u'}{(\nu c')^{1/3}} \quad (10)$$

$$T = \frac{T' - T'_\infty}{T'_w - T'_\infty} \quad (11)$$

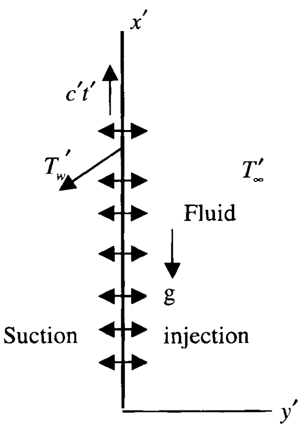
$$\mathrm{Gr} = \frac{\left(T'_w - T'_\infty\right)^2 g\gamma}{c'} \quad (12)$$

$$\mathrm{Pr} = \frac{\rho \nu c_p}{k} \quad (13)$$

into Eqs. (4) and (5), we get

$$\frac{\partial u}{\partial t} - \frac{a}{t^{1/2}}\frac{\partial u}{\partial y} = \mathrm{Gr}T^2 + \frac{\partial^2 u}{\partial y^2} \quad (14)$$

$$\frac{\partial T}{\partial t} - \frac{a}{t^{1/2}}\frac{\partial T}{\partial y} = \frac{1}{\mathrm{Pr}}\frac{\partial^2 T'}{\partial y^2} \ . \quad (15)$$



A sketch of the physical model

Fig. 1. Explanation of the physical model

The boundary conditions of the problem are

$$t > 0: \begin{cases} u = t, \ T = 1 & \text{at } y = 0 \\ u \to 0, \ T \to 0 & \text{as } y \to \infty \end{cases} \quad (16)$$

If we introduce the new variables η and $f(\eta)$

$$\eta = \frac{y}{2t^{1/2}}, \quad u = tf(\eta) \quad (17)$$

into Eqs. (14) and (15), we obtain

$$\frac{\mathrm{d}^2 f}{\mathrm{d}\eta^2} + 2(\eta + a)\frac{\mathrm{d}f}{\mathrm{d}\eta} - 4f = -4\mathrm{Gr}T^2 \quad (18)$$

$$\frac{\mathrm{d}^2 T}{\mathrm{d}\eta^2} + 2\,\mathrm{Pr}(\eta + a)\frac{\mathrm{d}T}{\mathrm{d}\eta} = 0 \ . \quad (19)$$

The corresponding boundary conditions are

$$\begin{aligned} &f = 1, \ T = 1 \quad &\text{at } \eta = 0 \\ &f \to 0, \ T \to 0 \quad &\text{as } \eta \to \infty \end{aligned} \quad (20)$$

The solution of Eq. (19) is given by

$$T(\eta) = \frac{\mathrm{erfc}\left\{\sqrt{\mathrm{Pr}}(\eta + a)\right\}}{\mathrm{erfc}\left(\sqrt{\mathrm{Pr}}a\right)} \ , \quad (21)$$

where erfc( ) is the complementary error function.

3
Conclusions

In order to get a physical insight into the problem we numerically solve equation (18, by a fourth-order Runge-Kutta scheme, taking into account Eq. (21) for different values of the suction/injection parameter a when Pr = 11.4 and Gr = 1. This value of Pr corresponds to water at 4 °C [1]. The velocity variations are shown in Fig. 2. From this figure we observe, that when the parameter a increases the velocity decreases.

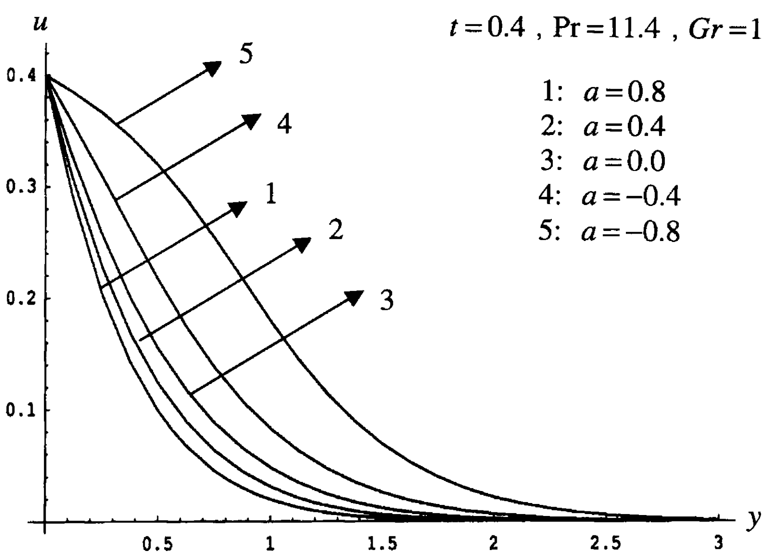


Fig. 2. Velocity profiles for various parameter

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