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A new approach for online multiobjective optimization of mechatronic systems

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Abstract We present a new concept for online multiobjective optimization and its application to the optimization of the operating point assignment for a doubly-fed linear motor. This problem leads to a time-dependent multiobjective optimization problem. In contrast to classical optimization where the aim is to find the (global) minimum of a single function, we want to simultaneously minimize k objective functions. The solution to this problem is given by the set of optimal compromises, the so-called Pareto set. In the case of the linear motor, there are two conflicting aims which both have to be maximized: the degree of efficiency and the inverter utilization factor. The objective functions depend on velocity, force and power, which can be modeled as time-dependent parameters. For a fixed point of time, the entire corresponding Pareto set can be computed by means of a recently developed set-oriented numerical method. An online computation of the time-dependent Pareto sets is not possible, because the computation itself is too complex. Therefore, we combine the computation of the Pareto set with numerical path following techniques. Under certain smoothness assumptions the set of Pareto points can be characterized as the set of zeros of a certain function. Here, path following allows to track the evolution of a given solution point through time.

Keywords Multiobjective online optimization · Time-dependent Pareto sets · Decision making · Numerical path following methods · Linear motor · Operating point assignment · Self-optimization

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1 Introduction

For the optimization of mechatronical systems one has to bear in mind many things.

- It is possible that several objectives have to be optimized at the same time.
- The objectives may depend on time or other parameters.
- The solutions have to be adjusted online—that is, one has to achieve low computational effort for the optimization.

The purpose of this work is to develop an optimization strategy that allows to account for all these aspects in an adequate way and to apply this strategy to the determination of optimized operating points of a linear motor.

The outline of this contribution is as follows: Sect. 2 gives an introduction into the field of multiobjective optimization. The main idea in multiobjective optimization is to develop an optimization theory that allows to optimize several conflicting objectives at the same time. For objective functions that additionally depend on time, our idea was to use so-called numerical path following methods which allow to track solutions of parameter-dependent (nonlinear) systems of equations (see Sect. 3). The following section explains how we combine time-dependent multiobjective optimization and numerical path following methods. The resulting optimization algorithm is applied to develop an optimized strategy for the assignment of an operating point to a linear motor in railway vehicles which is described in detail in Sect. 5. This example shows that the mathematical techniques developed in this work are not only of theoretical interest but are also useful in mechatronical applications.

2 Multiobjective optimization

In classical optimization the goal is to find the (global) minimizer of one single objective function $f : \mathbb{R}^n \to \mathbb{R},\ n \in \mathbb{N}$. But many problems e.g. in engineering show that it is not clear what "the objective function" is. Often there arise several objectives which are (partly) conflicting but which have to be optimized at the same time. The application considered in this work—that is the operating point assignment of a doubly fed linear motor—provides us with two conflicting objective functions. Hence, multiobjective optimization techniques are well qualified to solve the problems. Mathematically, an unconstrained *multiobjective optimization problem* is given by

$$\min\{F(x) : x \in \mathbb{R}^n\}, \tag{MOP}$$

where F is defined as the vector of the objective functions $f_1, \ldots, f_k : \mathbb{R}^n \to \mathbb{R}$,

$$F : \mathbb{R}^n \to \mathbb{R}^k, \quad F(x) = (f_1(x), \ldots, f_k(x)).$$

We obviously have to explain, what 'min' means in this context, because we want to minimize a vector-valued function. Therefore we define the following partial order $\leq_p$ on $\mathbb{R}^k$:

Definition 1 Let $u, v \in \mathbb{R}^k$. Then the vector u is *less than* v, if

$$u_i \leq v_i \quad \text{for all } i \in \{1, \ldots, k\}.$$

In this case we write $u \leq_p v$.

Using this relation, we can define a solution of (MOP). This solution is not a single optimum, but a set of optimal compromises.

Definition 2 A point $x^\star \in \mathbb{R}^n$ is called *globally Pareto optimal* for (MOP) (or a *global Pareto point* of (MOP)), if there is no $x \in \mathbb{R}^n$ with

$$F(x) \leq_p F(x^\star) \quad \text{and} \quad f_j(x) < f_j(x^\star)$$

for at least one $j \in \{1, \ldots, k\}$. If this property is only valid inside a neighborhood $U(x^\star) \subset \mathbb{R}^n$, we call $x^\star$ *locally Pareto optimal* (or a *local Pareto point*). The set of all (global) Pareto points is called *Pareto set*.

From this definition it is not clear how to compute the entire Pareto set efficiently. The following famous theorem of Kuhn and Tucker [9] provides us with a necessary condition for Pareto optimality.

Theorem 1 (Kuhn, Tucker) *Let $x^\star$ be a local Pareto point and let all objectives f_i, $i = 1, \ldots, k$ be continuously differentiable. Then there exist nonnegative multipliers $\alpha_1, \ldots, \alpha_k \in [0, 1]$ such that*

$$\sum_{i=1}^{k} \alpha_i \nabla f_i(x^\star) = 0 \quad \text{and} \quad \sum_{i=1}^{k} \alpha_i = 1. \tag{1}$$

This condition is obviously not sufficient in general, but it is necessary and sufficient, if the objective functions are convex. Numerical methods for the computation of the Pareto set often use this theorem e.g. for the construction of a descent direction [15]. Following [10] we define:

Definition 3 If the vector $x \in \mathbb{R}^n$ satisfies the Kuhn-Tucker condition (1) then it is called a *substationary point*.

Recently, set-oriented numerical methods for the computation of the entire Pareto set in continuous multiobjective optimization problems have been developed. They can be divided into two main classes: the *subdivision techniques* (see [2,16,20]) and the *recovering techniques* (see [17–19]). The subdivision techniques are of global nature and suitable for derivative-free optimization, but restricted to moderate dimensions. The recovering techniques are of local nature, but applicable in higher dimensions both in parameter space ($n > 1000$) and in image space (typically $2 \leq k \leq 5$). Both types of algorithms (and also combinations of them) are very useful for our applications, because they come up with a fine covering of the (global) Pareto set in a comparatively short computational time.

Having computed the entire Pareto set one has to decide which solution within this set "matches best" for the underlying system. This process is called *"decision making"*. The person who has to determine this solution, e.g. the engineer, is called *"decision maker"* (cf. [10]). Due to practical reasons we substitute the decision making process by decision heuristics, which have to be specially designed for the applications considered with the help of the decision maker. Such a decision heuristics can provide us with good operating points online (see for example [14]).

When optimizing mechatronical systems it frequently happens that the objective functions additionally depend on time. In the application we consider (cf. Sect. 5), time does play an important role, as the objective functions depend explicitly on the time. Therefore, we have to optimize

$$f_1(x, t), \ldots, f_k(x, t)$$

for each $t \in [t_{\text{start}}, t_{\text{end}}] \subset \mathbb{R}$, and thus we also have a distinct Pareto set for each t. This means that it is not sufficient for the decision maker to choose one Pareto point for the whole time-interval. Instead, a curve $x(t)$ of Pareto points has to be chosen, parameterized by the time t such that $x(t)$ is a Pareto point for the corresponding MOP.

Such a curve could in principle be computed numerically by solving all the multiobjective optimization problems that correspond to a discrete covering of the time-interval one wants to consider. But this is computationally costly and for the case of mechatronical systems, where solutions have to be adapted online and the time-interval can be arbitrarily long, not of practical interest.

Thus it is more reasonable to investigate in how to compute a "good solution curve" without the global information about every Pareto set. On the other hand we do not want to lose information about the structure of global Pareto sets completely. In this context, numerical path following methods are a useful tool. These methods allow to track the evolution of a given solution point, described as a zero of a certain function, through time. Numerical path following methods will be described in the next section. In Sect. 4 we explain how these methods can be used in terms of online multiobjective optimization.

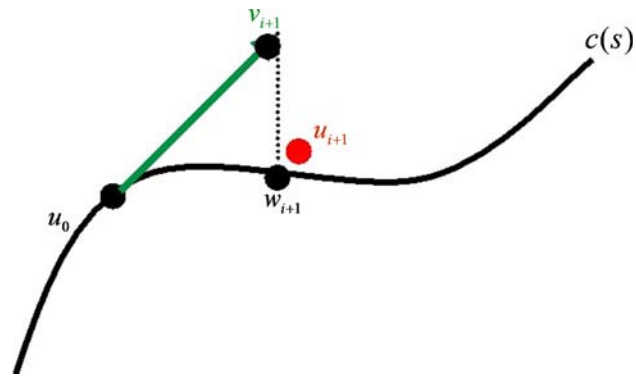


Fig. 1 Illustration of the predictor and corrector step in path following methods

3 Numerical path following methods

The aim of this work is to develop numerical techniques for the treatment of time-dependent multiobjective optimization problems. This task will be reformulated in Sect. 4 into the problem of finding the set of zeros of a smooth parameter-dependent function $H : \mathbb{R}^N \times \mathbb{R} \to \mathbb{R}^N$. Techniques for following solutions under the variation of a parameter, which in our special case is the time t, have already been developed (see e.g. [1,3]). In the following, we shortly outline how these *numerical path following methods*, also known as *continuation methods* [8], work in principle.

We consider a time-dependent nonlinear equation system

$$H(y, t) = 0, \qquad (2)$$

where $y \in \mathbb{R}^N$ and $t \in \mathbb{R}$. In the context of the algorithms for online multiobjective optimization proposed in this work we will have $y = (x, \alpha)$, where α denotes the weight of the objectives defined by the Kuhn–Tucker system (1).

Let $u_0 = (y_0, t_0)$ be a solution of (2), i.e. $H(u_0) = 0$. Suppose that the Jacobian $H'(u_0)$ has full rank. In this case the solution set $H^{-1}(0)$ can locally be parametrized by one parameter s in the neighborhood of u_0. Thus, one obtains a solution curve $c(s)$ with

$$c(0) = u_0 \quad \text{and} \quad H(c(s)) = 0.$$

Differentiating the last equation we obtain

$$H'(c(s))c'(s) = 0.$$

Thus $c'(s)$ spans the one–dimensional kernel of $H'(c(s))$. By choosing s to be the arclength of the curve it follows that

$$\|c'(s)\| = 1.$$

Moreover, one can show that $c(s)$ is the solution of the initial value problem

$$\begin{aligned} \dot{u} &= T(H'(u)) \\ u(0) &= u_0. \end{aligned} \qquad (3)$$

Here, for a matrix $A \in \mathbb{R}^{N\times(N+1)}$ with $rank(A) = N$, $T(A)$ denotes the unique vector T satisfying the following conditions:

$$AT = 0, \quad \|T\| = 1 \quad \text{and} \quad \det \begin{pmatrix} A \\ T^T \end{pmatrix} > 0.$$

The last condition allows to fix the orientation of the tangent vector.

A numerical path following method generates a series of points $u_i,\ i = 0, 1, 2, \ldots$ in the following way. Let u_0 be an initial point satisfying $H(u_0) = 0$. Then u_{i+1} is generated inductively in two steps (cp. Fig. 1).

Predictor step: Compute an approximation to the solution of (3), e.g. by using the explicit Euler method:

$$v_{i+1} = u_i + h \cdot T(H'(u_i)),$$

where $h > 0$ is a certain steplength.

Corrector step: Here one uses the fact that $c(s)$ solves $H(c(s)) = 0$. Define w_{i+1} to be that point on the curve c, which is closest to v_{i+1}, i.e. one solves

$$\min\{\|v_{i+1} - w\| : H(w) = 0\}.$$

The solution of this problem—this is typically done by Newton's method—defines the new point u_{i+1}.

4 Online multiobjective optimization

Path following methods have already been used in the context of multiobjective optimization (see e.g. [7]), but—as far as we know—only for the generation of Pareto sets itself and not for varying parameters (in our case time) in time-dependent multiobjective optimization problems. But especially in the time-dependent case these techniques are very useful, because computing a path between sets of substationary

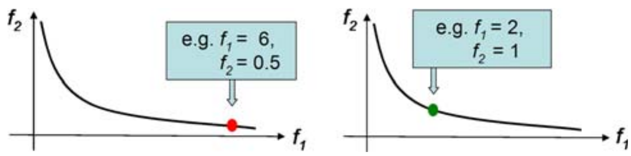


Fig. 2 Schematic visualization of a possible second step in a decision heuristic

points can be performed much faster than computing the entire Pareto sets for "every" point of time.

In order to combine numerical path following methods and time-dependent multiobjective optimization problems we use the necessary condition given by the Kuhn–Tucker-Theorem which states that

$$\sum_{i=1}^{k} \alpha_i(t) \nabla_x \ f_i(x,t) = 0$$
$$\sum_{i=1}^{k} \alpha_i(t) - 1 = 0$$

for every fixed point of time t. Here ∇_x denotes the column vector of partial derivatives with respect to x.

This equation system consists of $n+1$ equations and $n+k+1$ variables ($x \in \mathbb{R}^n$, $\alpha \in \mathbb{R}^k$, $t \in \mathbb{R}$). For a fixed value of the vector α, i.e. $\alpha(t)$ is constant for all t, there are n equations and $n+1$ variables left. For this kind of system, path following methods can be applied directly.

Having computed the entire Pareto set for an initial point of time, e.g. $t = t_{\text{start}} = 0$, it is sensible to choose one point in this set as the most preferred adjustment of the system. The choice of the most preferred adjustment can be realized by a certain decision heuristic, which has to be tailored to the special applications under consideration (cp. [14]). The decision heuristic for the application considered in this work is described in Sect. 5. In a second step this decision heuristic can contain a check how flat (assuming an adequate scalarization) the Pareto set in the image space is. From a mathematical point of view it is not recommendable to choose such a Pareto point as the most preferred adjustment, where the Pareto set in the image space is "flat", because only little losses in one objective can cause great benefits to the other. This fact is visualized schematically in Fig. 2. Of course the "flatness" is relative with respect to scaling and has to be adjusted for the special applications.

In order to update the optimal solutions online, we suggest the following algorithm:

1. Compute the entire Pareto set for the given problem at an initial point in time. Based on a certain decision heuristic the most preferred adjustment for the system at this initial point in time is calculated.
2. The corresponding ratio of the objectives (weight vector α) is set fixed. Using path-following techniques, we compute the solution curve corresponding to this α over time.
3. After a certain time, the entire actual Pareto set is recomputed, the decision heuristic is applied again, and the adjustment of the system is updated. Then we proceed with step 2.

There are different ways how to decide that the new entire Pareto set has to be computed in step 3. The first trivial idea is to compute the Pareto set at fixed points of time, e.g. every minute (assuming that the dimension of the problem is low enough). This computation can be performed in parallel to the computation of the path of solutions, so that the assignment of the optimal solutions never has to be interrupted. The computation of the entire Pareto set can also be started anew in reaction to external influences. As an example in case of the linear motor some precomputed velocity profiles are contained in the optimization. If these profiles cannot be realized any more, e.g. because the vehicle has to brake unexpectedly, a recomputation of the entire Pareto set and the corresponding path (including the new profiles) is necessary.

Including such criteria, the system performs the self-optimizing scheme described in [5].

5 Application and results

5.1 The Railcab system

The development of the online optimization algorithm described in the last sections has been inspired by a novel linear-motor driven railway system, developed by the project Railcab ("Neue Bahntechnik Paderborn") [12]. This system also is one of the demonstrators of the Collaborative Research Center (SFB 614) [11] in which this work has been developed. Figure 3 displays the test vehicle. This vehicle belongs to a test facility with a track length of about 530 m. The track contains an artificial hill with an altitude of about 2.5 m and gradients up to 5.3%, requiring an aligned thrust along the track. The track includes a novel passive switch, which allows the processing of closely following vehicles. The vehicle consists of a superstructure that carries the load and two undercarriages. Figure 4 shows the concept of the undercarriage module, which is one of the basic modules of the vehicle. The undercarriage houses three sub-modules: A driving, an active suspension and a guidance module, the latter based on one single wheel set. The guidance module [4] enables a driving with low attrition and allows to use the novel concept for a passive switch. The active suspension module gives the vehicle the possibility to improve comfort for passengers [6]. The driving module serves two main

Fig. 3 Vehicle of the demonstrator

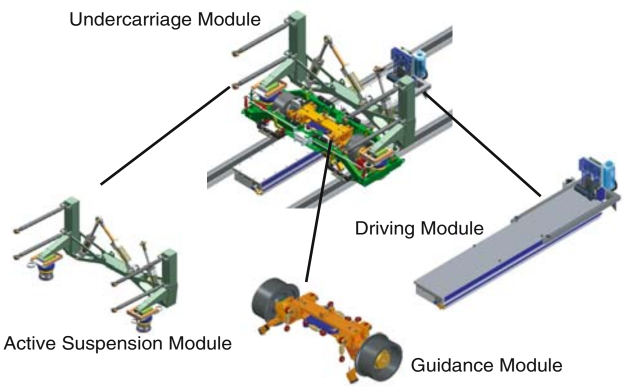


Fig. 4 Concept of the undercarriage

functions: the energy transfer from the track to the vehicle and the generation of thrust described in [11,21]. The doubly fed linear motor partly included in this driving module (cf. Fig. 5) consists of a primary motor part between the rails of the track and a secondary part on the vehicle. Both parts have an own power supply with power inverters for the control of the currents.

5.2 Optimization of the operating points of the doubly-fed linear motor

5.2.1 Goals of the operating point assignment

In the construction of the driving module, several goals should be achieved. Proper operating points should maximize both the efficiency of the drive and the inverter utilization factor. The efficiency considers the mechanical power P_M (4) resulting from the thrust F_M and the electrical power transferred to the vehicle denoted by P_B (5). For thrust generation ($F_M > 0$) the efficiency η_{LM} and the inverter utilization factor η_{SN} are given by:

$$\eta_{LM} = \frac{P_M + P_B}{P_S} \quad \text{and} \quad \eta_{SN} = \frac{P_M + P_B}{S_S + S_L}.$$

In the case of braking ($F_M \leq 0$), the equations change to:

$$\eta_{LM} = \frac{P_B}{P_S + P_M} \quad \text{and} \quad \eta_{SN} = \frac{P_B}{S_S + S_L}.$$

P_S denotes the active power, S_S the apparent power of the primary motor part and S_L the apparent power of the secondary motor part.

$$P_M = v_M F_M \tag{4}$$

$$P_B = 3\left(\pi f_L \frac{L_h N_2}{N_1} I_{2q} I_{1d} - R_2 I_{2q}^2\right) \tag{5}$$

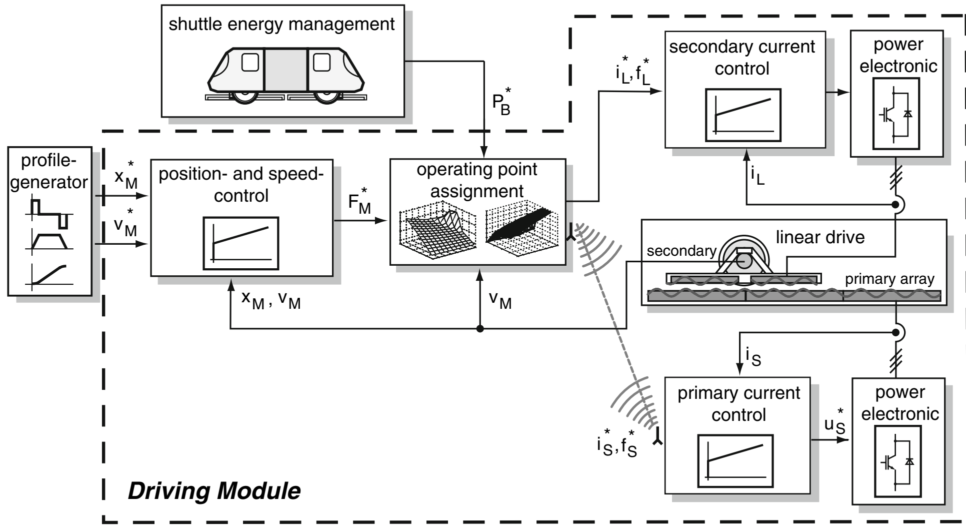
In (4) v_M names the speed of the vehicle that is defined by the velocity profiles of the maneuver. The currents in the primary part I_{1d} and in the secondary part I_{2q} depend on the motor constant K_M and the thrust, which is also specified by the maneuver, in the following way:

$$F_M = K_M I_{1d} I_{2q} \tag{6}$$

R_2, L_h, N_1 and N_2 are constant motor parameter and f_2 denotes the frequency of the secondary current. The current in the primary motor part has to be determined for both driving modules of the vehicle. In case of a second vehicle on the same track section, the assignment has to be done by a master vehicle. The output value of the speed controller of the vehicle is the required thrust for the instantaneous speed. For a given thrust, the primary current I_{1d} is a suitable optimization variable. Figure 5 illustrates the principle of the driving module and the control structure of the doubly fed linear drive. The key function of this structure is the operating point assignment of the vehicle, which is in the focus of this article. We assume that the other functional blocks in the vehicle like current-, speed- and position-control, profile generation and energy management do not influence the operating point assignment. In this case, the assignment alone determines the controlled thrust generation and the power transfer (5) from the track to the vehicle power supply system. For the assignment of the operating point, operation conditions of the vehicle like speed, actual stored energy in the batteries etc. have to be considered. An operating point of the doubly fed linear drive is characterized by the distribution of thrust forming currents and its frequency at the secondary motor part. This frequency and the mechanical speed of the vehicle define the frequency of the current at the primary motor part.

By definition, an optimization of the efficiency leads to minimal required primary real power. The inverter utilization factor represents the ratio of real output power to apparent power of the linear motor. The latter is a gage for the electrical copper losses in the secondary motor part. Figure 6 shows the two objectives at two different mechanical operation conditions for the thrust F_M. The maximum values of the objectives do not belong to the same values of the primary current. Thus the objectives are partly conflicting and an optimal compromise is desirable. As within one maneuver the requested

Fig. 5 Control Structure of the Driving Module



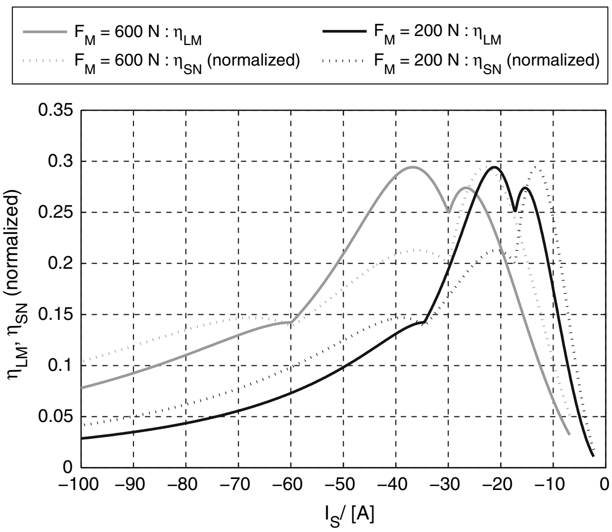


Fig. 6 Objectives and optimization variable for different thrusts

thrust changes and with it also the aligned frequency, the optimization problem is time-dependent (Fig. 7).

The thrust generation has the highest priority, but the power transfer is also very important for the operation of the vehicle. Both goals are constraints for the optimization.

5.2.2 Applying the new optimization algorithm

The problem outlined above with two objectives, one optimization variable and several time-dependent parameters can be solved by the 3-step-algorithm described in Sect. 4. For the demonstration of the algorithm we use profiles for position, speed, thrust and transferred power, cf. Figs. 8 and 9. These profiles stem from a round trip on the test track of the Railcab-Project.

In the first step the algorithm selects the mechanical operating point from the profiles at a special point of time and

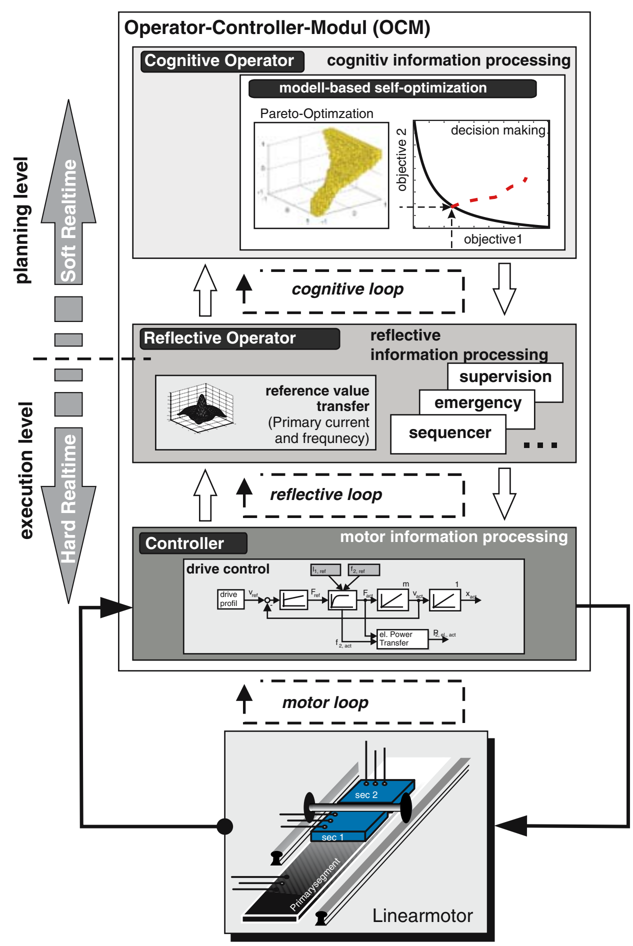


Fig. 7 OCM for the operating point assigment

computes the corresponding Pareto set. The decision heuristic needed for the choice of one of these Pareto points is realized as follows: as the temperature in the secondary motor part and the charging state of the battery play an important role for the system. We use these values as decision criteria.

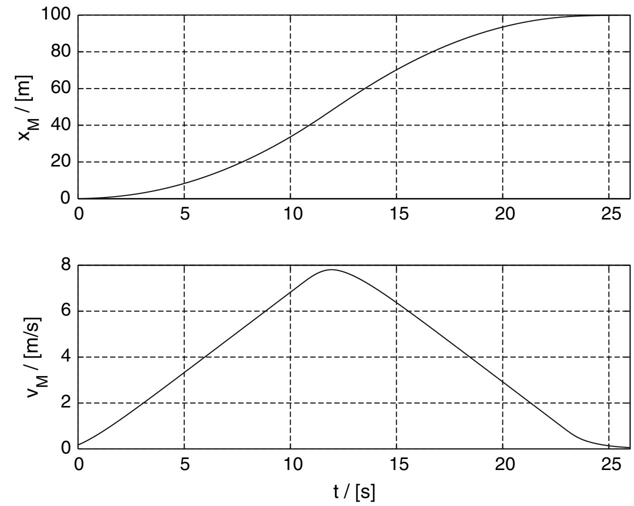


Fig. 8 Profiles for position (above) and velocity (below)

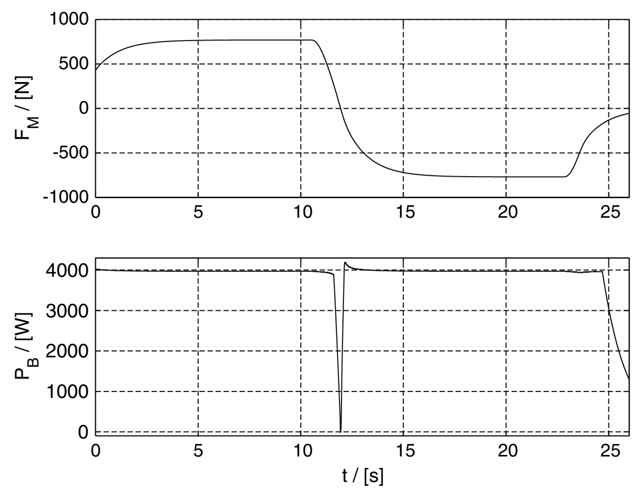


Fig. 9 Thrust (*above*) and resulting transferred power (*below*)

For our application it is desirable to follow solutions with a high efficiency in case of a low charging state of the battery, which guarantees the power supply of the vehicle. Additionally, for high temperatures in the secondary motor part, higher values of the inverter utilization factor η_{SN} should be preferred. Assuming a certain value range for the temperature and the charging state we use this background information for controlling the selection of Pareto points.

Based on the chosen Pareto point, the time-depending curve of solutions—the current values—is computed in the second step using the path following techniques as described on page 226. This solution curve is stored in a buffer for the reference values and fed to the controller at the corresponding positions of the vehicle.

A varying condition, e.g. the required thrust or the energy transfer requires a new operating point (step III).

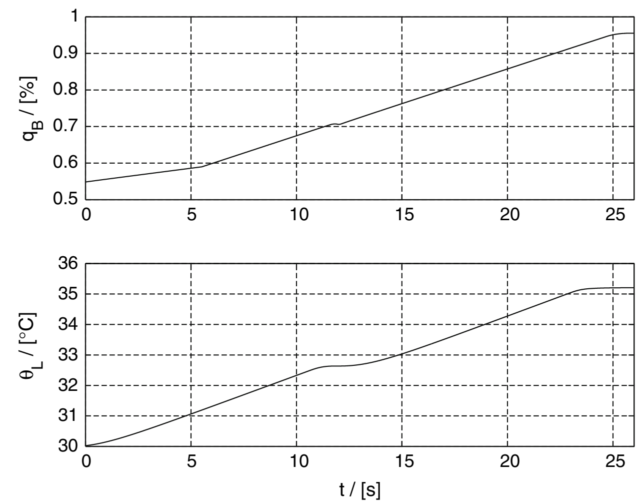


Fig. 10 Energy rate (*above*) and temperature of the secondary motor part (*below*)

5.2.3 Implementation

The implementation of the online multiobjective optimization algorithm is supported by the concept of the operator-controller-model (OCM) (cf. Fig. 7). At the lowest level the controllers perform the current- and speed-control under hard real-time conditions. The reflective operator links the hard real-time of the controller with the soft real-time of the optimization algorithm in the cognitive operator. It buffers the calculated path from the cognitive operator and feeds reference values to the controller. It also supports possible emergency response in cases like unexpected change of these requirements or changing parameters in the linear drive. Until a new optimization is performed and a new path is available, the current optimization value is replaced by a sub-optimal value from a look-up table. In this case, a new strategy for following selected operating points influenced by changing conditions or parameters is required. This has been done by the presented online algorithm (cf. Sect. 4). The main advantage of this very fast algorithm is that it prevents the time-consuming calculation of a new Pareto set and allows to use a multiobjective optimization approach for this complex application.

5.2.4 Results

In order to illustrate the quality of the presented method we compare it to another multiobjective optimization approach described in [14]. This approach—which can only be realized offline—consists of the permanent computation of entire Pareto sets for discretized points in time. The desired adjustments within the Pareto set have been selected with the same decision heuristic that we use in this work. To make the results comparable we consider the same maneuver of the vehicle as the results in [14] were based on, i.e. a round trip on the test

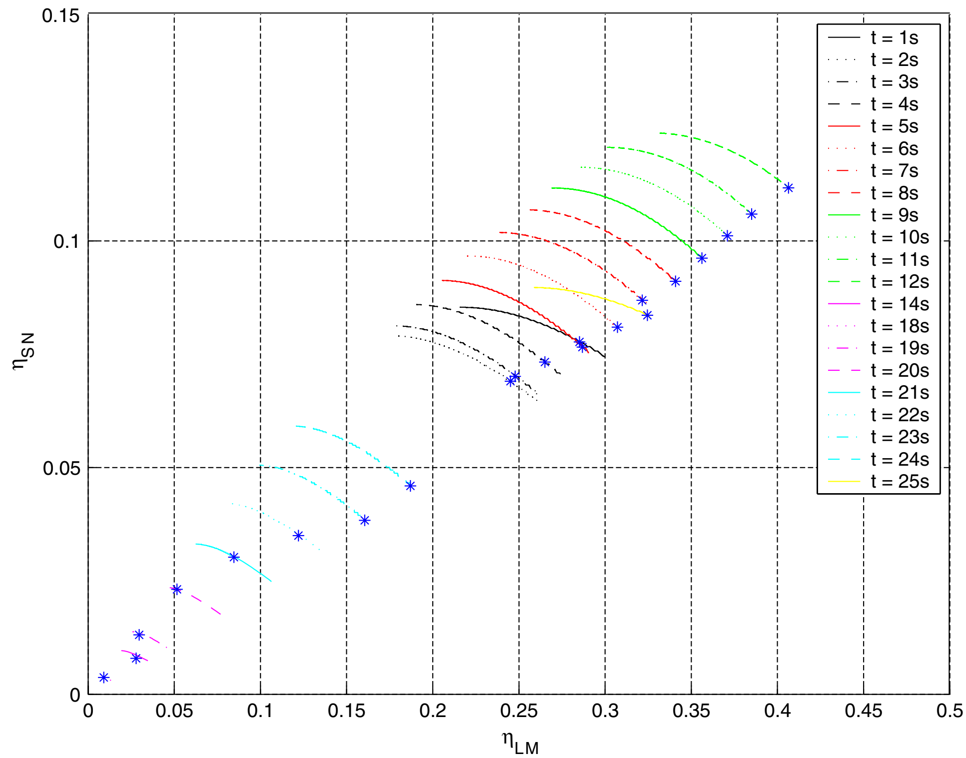


Fig. 11 Pareto optimal values in objective space

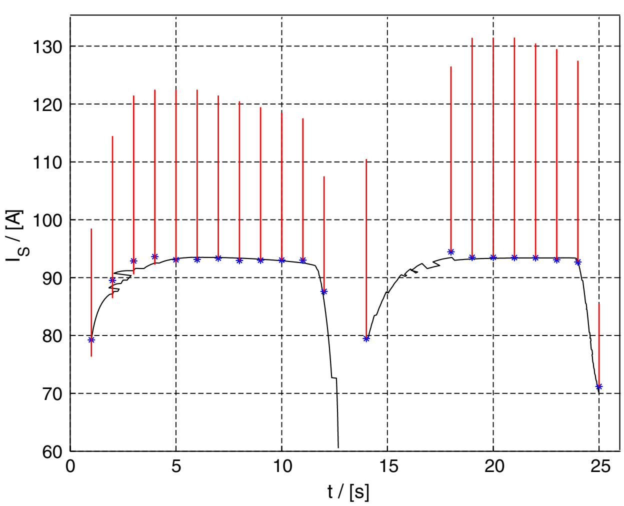


Fig. 12 Comparison of Pareto points selected by the decision heuristic and the computed path in preimage space (t vs. I_S)

track. The maneuver consists of an acceleration and a braking term. While the vehicle accelerates, the thrust shown in Fig. 9 is positive. In the braking term, the thrust becomes negative. If the absolute value of the thrust is small, the required transferred power does not fulfill the constraints (cp. the peak in Fig. 9). In fact, the temperature of the secondary motor part and the stored energy of the batteries depend on the chosen operating point. For this simulative-based application it is assumed that both values can be measured. To simplify the used model and to make the results comparable, the model considers pre-calculated values (see Fig. 10) for the measurements. Figure 12 shows some Pareto sets in preimage space—that is intervals of Pareto–optimal values for the current I_{1d}—(vertical lines), the Pareto points which would have been selected by the decision heuristic (stars) and the computed path (line). In Figure 11 the same Pareto sets and selected solutions (stars) are shown in image space. The quality of the combination of the multiobjective optimization and path following techniques is proven in Figure 12 by the low distance between the followed path and the chosen Pareto points. Because of the low thrust requirements, the transferred power is conflicting to the constrained power transfer. This leads to a disruption of the followed path at $t = 12.5$ s; the path following process has to be restarted with a new Pareto point.

6 Conclusions

In general, a Pareto set (for a fixed point of time) of a time-dependent problem is based on old information at the time when it is completely computed. The problems resulting from this insufficiency increase with time. Therefore, a global Pareto optimization approach is impracticable for online applications. A possible way out of this dilemma is the combination of multiobjective optimization and numerical path following as presented in this work. Having computed one Pareto set (e.g. for $t = 0$) and having applied a decision heuristic (which takes information about the objective values into account) to this set, the path can be computed until the new Pareto set is available. The selected Pareto point and the path of optimized values for the optimization variable I_{1d} can be adjusted online. In case of the optimization studied in this work it took about 3 s to approximate the entire Pareto set sufficiently precise and about 0.5 s to compute a path which provides us with solutions for a time interval of 11 s. The remaining time (in this case 10.5 s) is long enough for the computation of a new Pareto set, e.g. the entire one for $t = 11$ s, assuming known profiles for the maneuver. Although we can only guarantee that the computed points in the path are substationary points, the results for the operating point assignment as described above lie within the Pareto-optimal set. Even if one leaves the Pareto set to some substationary points which are not Pareto points, restarting the path on new entire Pareto sets will correct this problem.

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