#### **ORIGINAL PAPER**



# **Thermal rectifcation in ultra‑narrow hydrogen functionalized graphene: a non‑equilibrium molecular dynamics study**

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#### **Abstract**

In this study, the non-equilibrium molecular dynamics simulation (NEMD) has been used to evaluate the thermal properties, especially the rectifcation of ultra-narrow edge-functionalized graphene with hydrogen atoms. The system's small width equals  $4.91 \text{ Å}$  (equivalent to two hexagonal rings). The dependence of the thermal rectification on the mean temperature, hydrogen concentration, and temperature diference between the two baths was investigated. Results reveal that the thermal rectifcation increases to 100% at 550 K by increasing the mean temperature. Also, it is disclosed that hydrogen concentration plays a vibrant role in thermal rectifcation. As a result of maximum phonon scattering at the interface, a thorough rectifcation is obtained in a half-fully hydrogenated system. As well, the efects of temperature diference of baths *ΔT* on thermal rectifcation has been calculated. As a result, the thermal rectifcation decreases even though the current heat increases with *ΔT*. Finally, the thermal resistance at the interface using a mismatching factor between the two-phonon density of states (DOS) on both sides of the interface has been explained.

**Keywords** Thermal rectifcation · Ultra-narrow graphene · Edge-functionalized · Molecular dynamics

# **Introduction**

Graphene [[1](#page-4-0)], a two-dimensional (2D) monolayer sheet with honeycomb form, has drawn more attention in the past decade due to its outstanding properties. Graphene shows excellent optical and electronic [\[2](#page-4-1), [3\]](#page-4-2), mechanical [[4\]](#page-4-3), and thermal properties [\[5](#page-4-4)]. Besides graphene, novel 2D materials have been discovered or synthesized functionalized graphene for diferent purposes, such as carbon nitride [\[6](#page-4-5)], molybdenum disulfde [[7\]](#page-4-6), borophene [[8\]](#page-4-7), phosphorene [\[9](#page-4-8)], and graphene [[10](#page-4-9)]. Functional atoms or groups can afect physical properties. For example, graphane was theoretically predicted [[10](#page-4-9)] and synthesized [[11,](#page-4-10) [12\]](#page-4-11) by hydrogenating both sides of the graphene in an alternating manner. In this situation, the sp<sup>2</sup> bond type is transformed to  $sp<sup>3</sup>$  and affects all properties. In addition, the number of hydrogen atoms on graphane can be changed to control properties. Moreover, we investigated the thermal conductivity of other systems and explored the parameters that afect the thermal conductivity [[13–](#page-4-12)[15\]](#page-4-13).

The hybrid of 2D monolayers such as graphene and graphane (functionalized graphene with an arbitrary number of hydrogen atoms) is also enjoyable. Thermal rectification in such systems was investigated [[16,](#page-4-14) [17\]](#page-5-0) recently. In thermal rectifer (thermal diode) systems, the heat current across the system depends signifcantly on the thermal gradient direction. As electrical diodes playing a vital role in electronics, thermal diodes have substantial implications for many thermal management applications, from nanoscale calorimeters to microelectronic processors to macroscopic refrigerators and energy-saving buildings [\[18](#page-5-1)]. Therefore, the heat current prefers one direction to another one. Rajabpour et al. [\[16\]](#page-4-14) have studied the graphene and graphane junction to estimate thermal rectifcation. They obtained thermal rectifcation of about 20% through NEMD simulation. Moreover, they also examined thermal properties at the interface, which is responsible for phonon scattering, leading to thermal resistance (known as Kapitza resistance). They also showed that the Kapitza resistance depends on the

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temperature gradient, which causes the heat current to prefer one direction to the opposite.

Shavikloo et al. [[14\]](#page-4-15) studied thermal rectification in graphene and graphane with grain boundary interfaces between two sides. They examined thermal rectification by randomly removing hydrogen atoms from graphane. They found that the thermal rectification is less than 10% at room temperature. Furthermore, they investigated the effect of the hydrogen concentration on thermal rectification in the hybrid system. Removing hydrogen helps us tune the thermal properties so that they have succeeded in increasing thermal rectification by 17% by removing 40% more hydrogen atoms than fully hydrogenated graphane. In the two above studies, the system's width was about 100 Å, and periodic boundary conditions were applied to in-plane directions. The width and boundary condition can be influenced the thermal properties. Functionalization with hydrogen has been used in other systems such as single-walled carbon nanotube [[19\]](#page-5-2), nanoporous graphene [[20\]](#page-5-3), and pristine graphene [[21](#page-5-4)] with particular patterns. Moreover, thermal rectification has also been seen in other systems, in graphene- $C_3N$  junction [[22,](#page-5-5) [23](#page-5-6)], graphene- $C_3B$ [[24\]](#page-5-7), graphene-pillared  $[25]$  $[25]$ , radial rectification  $[26]$  $[26]$ , telescopic silicon nanowire [[27](#page-5-10)], and so on. Thermal rectification in ultra-narrow graphene with functionalized edge via NEMD simulation has been investigated. The dependency of thermal rectification on temperature and the temperature difference between baths and hydrogen concentration was studied. Additionally, the phonon density of states was obtained to explain thermal resistance at the interface.

### **Computational method**

In semiconductors, the contribution of electrons in the thermal transport is less than the phonon's contribution. Therefore, we ignore the electrons and compute phonon thermal current in our ultra-narrow system. This study implemented all NEMD simulations using a Large-scale Atomic/ Molecular Massively Parallel Simulator (LAMMPS) [[28\]](#page-5-11) as a molecular dynamics program. The desired structure is indicated in Fig. [1](#page-1-0) with a length of  $200 \text{ Å}$  and a width of 4.91 Å. Both edges in the left half of the structure are fully functionalized with hydrogen atoms, while the right side has no hydrogen.

The hydrogen concentration can be different in some simulations due to investigating the effect of that on thermal rectification. Periodic boundary conditions along the x-direction, but free boundary conditions were assumed in other directions. Moreover, to model the interactions in this



<span id="page-1-0"></span>**Fig. 1** Schematic representation of the simulated system. Green and gray-colored atoms are hydrogen and carbon, respectively. F, H, and C indicate the fxed regions, hot and cold baths, respectively

system between C–C and C–H, we employed the Adaptive Intermolecular Reactive Empirical Bond Order (AIREBO) [\[29](#page-5-12)] potential with a cutoff 3.0 ( $\sigma$  scale factor). This potential is a three-body type and is appropriate for modeling breaking, creating bonds, and describing hydrocarbon systems. The Velocity Verlet algorithm integrates the equation of motion of atoms with a time step of 0.2 fs. The time step was considered small because this system has hydrogen atoms that can oscillate with high frequency.

First, to relax the system and eliminate all extra stress, the NPT ensemble with Nose–Hoover [\[30\]](#page-5-13) barostat and thermostat was applied to the system along *x*-direction so that the pressure tensor element  $P_{xx}$  becomes zero at room temperature. In the mentioned situation, the system can expand or shrink if desired. After relaxing, as represented in Fig. [1](#page-1-0), two slabs at two ends of the system were considered fxed atoms so that the particles inside of them could not move in during simulation time. Moreover, two other slabs near two fxed layers were selected as thermal baths (hot and cold sources). We used an NVT ensemble with a Nose–Hoover thermostat to keep the temperature fxed in thermal sources. The NVE ensemble was applied elsewhere.

Here, to estimate the thermal rectifcation, we use the equation below [[20\]](#page-5-3),

<span id="page-1-1"></span>
$$
TR(\%) = \frac{Q_{L \to R} - Q_{R \to L}}{Q_{R \to L}} \times 100
$$
 (1)

where  $Q_{L\rightarrow R}$  ( $Q_{R\rightarrow L}$ ) is the heat current from left to right (right to the left). We assume that in Eq. [1,](#page-1-1)  $Q_{L\rightarrow R} > Q_{R\rightarrow L}$ .

Moreover, the phonon density of states (DOS) on both sides of the interface was calculated using the Fourier transform of the velocity autocorrelation function. Therefore, we choose two slabs on both sides of the interface and use the equation below [\[31](#page-5-14), [32\]](#page-5-15),

$$
DOS(\omega) = \sum_{s} \frac{m_s}{k_B T} \int_0^{\infty} e^{-i\omega t} < v(0).v(t) > {}_{s}dt \tag{2}
$$

where *s* is the atom type and symbol  $\lt$   $>$  indicates the velocity autocorrelation function. Parameters of  $m$ ,  $k_B$  and  $T$  are the mass, Boltzmann constant, and system temperature. The  $\omega$  is the angular frequency.

## **Results and discussion**

Obtaining heat current across a system (in the *x*-direction), the accumulative energies added to and removed from the heat baths were calculated (see Fig. [2](#page-2-0)). Indeed, the slope of the linear ft to energy profle *dE*∕*dt* is the heat current. As indicated in Fig. [2](#page-2-0), the input and output energies from the baths and their slope are equal. It shows that the heat current reached to steady state.

The heat current depends signifcantly on the temperature gradient sign because there is a mass asymmetry across the system. It causes the phonons to prefer to move more in one direction than the opposite one, usually from thick to thin.

Moreover, we present the sensibility of the heat current and the thermal rectifcation of the mean temperature of the system. Therefore, we enhance the temperature from 200 to 650 K.

As shown in Fig. [3a,](#page-2-1) the heat current decreases with increasing the temperature. The behavior of the heat current



<span id="page-2-0"></span>**Fig. 2** Accumulative energy is added to the hot or removed from the cold bath as a function of time. The slope of the curve equals to heat current

<span id="page-2-1"></span>**Fig. 3 a** Heat current and **b** thermal rectifcation versus the mean temperature of the system in both temperature gradient biases arises from two phenomena. First, the phonon–phonon scattering will increase by increasing temperature, which is a negative factor for heat current. On the other hand, by increasing temperature, the high frequency (high energy) phonons will excite, leading to increased heat current (positive factor). Competition between these two factors specifes the behavior of the heat current. Here, the simulation results indicate that the phonon–phonon scattering has been dominant. Furthermore, the heat current along the positive direction is greater than the negative one. It is the origin of thermal rectifcation.

In Fig. [3b](#page-2-1), the thermal rectifcation increases with increasing temperature. It should be noted that the decreasing rate of negative heat current than positive leads to increase thermal rectifcation. The magnitude of thermal rectifcation is more signifcant than 30% in this case. Figure [3b](#page-2-1) shows the estimated trend via two fitted lines in Fig. [3a](#page-2-1). Two linear fits calculated the estimated trend in Fig. [3a.](#page-2-1) Consider L1 and L2 in Fig. [3a;](#page-2-1) for a given temperature, we calculated (L1-L2)/(L2)\*100 for the estimated trend. The estimated trend remains valid as long as the heat fux behavior is linear. In analogy with other studies by Rajabpour [[16](#page-4-14)], Shavikloo [[17\]](#page-5-0), Yousefi [\[20](#page-5-3)], Cartoixà [\[27](#page-5-10)], we get a higher thermal rectifcation. Therefore, we introduce a promising thermal rectifer.

Thermal rectifcation under various conditions, for example, diferent *ΔT*. This parameter can be changed under application circumstances. Therefore, we check its efect on heat current and thermal rectifcation.

According to Fourier's law ( $J = -\kappa \nabla T$ ), the heat flux is proportional to *ΔT*. Therefore, we expect that *J* increases with increasing *ΔT*. The NEMD result confrms this proportionality, and the heat current increases with increasing *ΔT,* as shown in Fig. [4a.](#page-3-0) For the positive direction, the slope of the ftted line is greater than the negative due to thermal conductivity depending on the temperature gradient sign. Although the diference between two heat currents gets larger, the heat current for the negative direction gets more significant too. Therefore, we obtain a decreasing behavior of thermal rectifcation as indicated in Fig. [4b.](#page-3-0) Thermal





<span id="page-3-0"></span>

rectifications are in the range of  $(-20\%, -80\%)$  for the range of *ΔT* (20 K, 50 K) we studied.

One of the crucial factors that help us tune the thermal rectifcation is hydrogen concentration in half of the system. Figure [5a](#page-3-1) and [b](#page-3-1) explored the effect of removed hydrogen atoms in the system in 5–95%. For any amount of removed hydrogen, the heat current in the positive direction is more signifcant than the negative. For 95%, due to the dilution of hydrogen atoms on the left side, the heat currents increase in both directions and close to each other because the interface is vanishing. On the other hand, the diference between the heat currents arises from thermal rectifcation. Figure [5b](#page-3-1) shows that the NEMD result indicates that the thermal rectifcation suppresses in the mentioned range by removing hydrogen. With a slight decrease in hydrogen (5%), the thermal rectifcation drops from 40 to 18%. With more removing the hydrogen atoms up to 50%, thermal rectifcation increases slightly. This behavior was reported already elsewhere [[17\]](#page-5-0). This shows that the hydrogens are responsible for phonon scattering and thermal rectifcation. Thermal rectifcation beyond 65% decreases because the phonon scattering decreases at the interface in a negative direction, leading to an increase in heat current.

To understand the underlying mechanism of the thermal rectifcation, we should investigate the origin of the thermal resistance at the interface (known as Kapitza) [[33](#page-5-16), [34](#page-5-17)]. Kapitza originates from phonon scattering at the interface.

Therefore, a formalism called interface conductance modal analysis (ICMA) developed by Gordiz and Henry [[35](#page-5-18), [36\]](#page-5-19) applies to knowing the number of phonons is conductance or thermal resistance at the interface. The phonon density of states (DOS) on both sides of the interface in positive and negative temperature gradient signs specify mismatching (see Fig. [6](#page-4-16)). The mismatching indicates that some phonon modes are present on only one side. This is attributed to the Kapitza resistance at the interface, which depends on thermal gradient bias. Moreover, the mismatching factor between two DOSs on both sides of the interface can be calculated through the equation below [[20\]](#page-5-3),

$$
\Psi = \frac{\int_0^\infty D_1(\omega) D_2(\omega) d\omega}{\int_0^\infty D_1(\omega) d\omega \int_0^\infty D_2(\omega) d\omega}
$$
(3)

where  $D_1$  and  $D_2$  are the DOS on the left and the right side, respectively. For positive and negative temperature gradients, the mismatching factor is 0.01253 and 0.01332, respectively, for  $T = 300K$  and  $\Delta T = 40K$ .

#### **Conclusion**

Using a series of NEMD simulations, we reported the thermal rectifcation in ultra-narrow hydrogen functionalized graphene. Thermal rectification is significantly high

<span id="page-3-1"></span>**Fig. 5 a** Heat current and **b** thermal rectifcation versus removed H



<span id="page-4-16"></span>**Fig. 6** DOS for **a** positive and **b** negative direction. Red- and blue-colored curves are for the left and right sides of the interface, respectively



and even increases up to 100% in temperature 550 K. The dependency of thermal rectifcation on mean temperature, *ΔT*, and hydrogen atoms concentration was investigated. We found that the thermal rectifcation in the narrow structure in this work increases with increasing mean temperature while then decreases with increasing *ΔT*. Furthermore, we can tune the thermal rectifcation between 10–30% by removing some percent of hydrogen atoms. Finally, by calculating the phonon density of states on both sides of the interface, the underlying mechanism of the phonon scattering at the interface was shown.

**Author contribution** All authors have the same contribution.

**Data availability** N/A.

#### **Declarations**

**Ethics approval** On behalf of all authors, the corresponding author approves.

**Consent to participate** The corresponding author consents on behalf of all authors.

**Consent for publication** On behalf of all authors, the corresponding author consents to publish under the subscription model.

**Competing interests** N/A.

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