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ORIGINAL ARTICLE

Mathematical programming basis for ship resistance reduction through the optimization of design waterline

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Abstract It has been more than a decade since Calisal et al. (J Ship Res 46(3):208–213, 2002) presented their concept of wave resistance reduction by allowing an increase in the beam of a ship accompanied by smoothing the shoulders. Since then a series of computational and experimental studies have been performed to provide evidence for the design idea that an increase in the beam with waterline parabolization may give reduced wave resistance for moderate Froude numbers in most cases in contrast to the common understanding among naval architects. The procedure in the design concept mentioned has been based on a systematic search supported by computational work and validated by experimental studies. The present study attempts to rationalize the introduced design concept by establishing a mathematical programming basis which employs the thin-ship approximation for wave resistance and a quadratic programming in finding the optimal shape of the design waterline (DWL), for minimum wave resistance, for the given design constraints. In this case, the shape of the DWL and in turn the hull shape in accordance with DWL is determined by a well-defined optimization procedure, which is dictated by the mathematical programming routine. Computational tools are employed to check the hydrodynamic characteristics of the hull form determined by the optimization routine, before tank testing. An experimental work is carried out to confirm the computational results and to validate the proposed methodology. Both the computational and the experimental work point out that the present design approach is effective particularly for Froude numbers greater than 0.21 and less than 0.4 where the ship-generated wavelength along the hull is less than the body length.

Keywords Resistance reduction · DWL Optimization · Computational and experimental analyses

Abbreviations

B	Beam of the ship/model (m)
C_B	Block coefficient
C_F	Frictional resistance coefficient
C_P	Prismatic coefficient
C_T	Total resistance coefficient
C_W	Wave resistance coefficient
C_{WP}	Waterplane area coefficient
DWL	Design waterline
EHP	Effective horse power (HP)
Fr	Froude number; $V/\sqrt{gL}$
L_{PP}	Length between perpendiculars (m)
L_{WL}	Length on waterline (m)
P_E	Effective power (kW)
T	Draft of the ship/model (m)
V	Ship/model speed (kn)/(m/s)
WSA	Wetted surface area
$(1+k)$	Form factor

1 Introduction

There has been a wide-spread understanding, on a technical design principle, among naval architects that one has to refrain from increasing the beam so as to prevent an

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increase in resistance. This design principle found its basis in the earlier works of Kent [1], Weinblum [2], Gertler [3], and Wehausen et al. [4], which concluded that the residual resistance varies with the beam to the power of 2 within an experimental error. One may bear in mind that those studies belong to an era of relatively low-speed ships and the design studies of that era relied rather on experimental and empirical data. Nevertheless, that conventional design principle appears to be valid until recently.

In contrast to the common understanding mentioned above, Calisal et al. [5] discussed, for the first time, the resistance reduction potential of increasing the beam while smoothing the shoulders of ships for moderate and relatively higher Froude numbers. To show the validity of this concept, a mathematical justification was presented in the same study using the Michell's integral. The earlier design principle mentioned above, which recommends that addition of parallel middle body accompanied by a decrease in beam, when the Froude number is less than 0.21, was already confirmed mathematically by Calisal et al. [5]. Besides, the same study, which bases its mathematical justification on Michell's integral, shows that the new concept of parabolization of waterlines by increasing the beam (which leads to a decrease in the length of the parallel middle body) decreases the wave resistance for the Froude number region, approximately; $0.2 < Fr < 0.4$. It is worth noting here that it was first N.E. Zhukovsky who found out that the ship's optimum waterline form for supercritical speeds is a parabola of the second degree, according to Kostyukov [6]. Gotman's [7] work, which shows that ships with a mid-ship bulb have the least wave resistance, pinpoints the potential of the present concept as well. Afterwards, a series of computational and experimental studies, summarized in Calisal et al. [8], have been performed to provide evidence for the design concept that an increase in the beam with waterline parabolization may give reduced wave resistance for moderate Froude numbers in most cases. The procedure in the design concept mentioned has been rather based on a systematic search supported by computational work and validated by experimental studies. Calisal et al. [9] made an attempt to figure out the optimal position of the maximum beam increment using patches retrofitted on the side of the hull to give reduced wave resistance.

Meantime, there are of course hull form optimization studies—performed globally or locally—which employ high-fidelity computational tools. From a vast number of those studies; Peri et al. [10], Chen and Huang [11], Chrismianto and Kim [12], Huang and Yang [13] may be mentioned as representative works. But the concept presented here becomes dissimilar in that it focuses on the DWL optimization, which aims to keep the wave formation to a minimum.

In the present study in order to have an optimal determination of waterline shape, which allows an increase in the beam, we require a full optimization process, which employs a mathematical programming approach. Thus, the present study attempts to rationalize the above introduced design concept by establishing a mathematical programming basis. This procedure employs the thin-ship formulation for wave resistance calculation and quadratic programming in finding the optimal shape of the design waterline (DWL) for minimum wave resistance under the given design constraints. In this case, the shape of the DWL and in turn the hull shape in accordance with DWL is determined by a well-defined optimization procedure as explained in Sect. 2 and, as a consequence, the degree of the increment in the beam—as well as possible decrements in some parts of DWL—is determined by the mathematical programming routine. Computational tools are used in checking the hydrodynamics of the hull form determined by the optimization routine before tank testing. An experimental validation work is carried out to confirm the findings of the proposed methodology. Both the computational and the experimental work point out that the design concept rationalized using a mathematical programming is very promising and effective in hull form hydrodynamic designs.

2 The design concept and the mathematical programming approach

2.1 The design concept

The design concept under consideration may be presented by an example from an earlier work of Calisal et al. [5]. The hull form of a coaster tanker (with $L_{pp} = 82$ m) was selected with the aim of increasing its speed corresponding to a Froude number of at least 0.26. In this project, the original form was widened systematically, up to 20% increase in the beam, while the length and draft remained constant. In this approach, whereas the length of the parallel middle body was nearly 20% of the total length, the parallel middle body was decreased gradually to see the change in resistance by computational means. The computational results showed that the parallel middle body was almost removed and waterlines were parabolized in the widened ship, Fig. 1. This resulted in 7% increase in the displacement in the widened ship as well.

Experimental work done with the two models (original and widened) indicated a total of 10% reduction in EHP at a speed of 15 knots, corresponding to $Fr = 0.27$. The role of the beam increment accompanied by the parabolization of waterlines can be seen in Fig. 2 where the widened hull

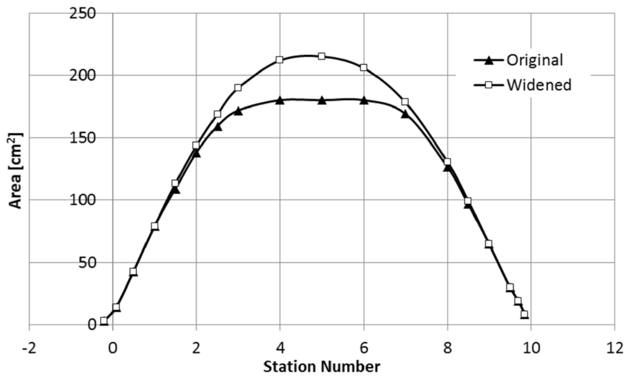


Fig. 1 Sectional area curves of models (scale 1/40) [5]

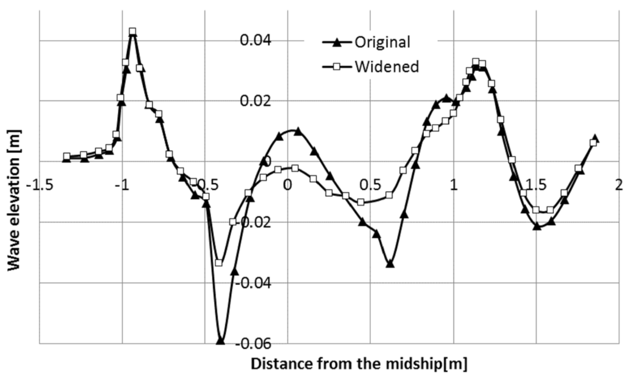


Fig. 2 Numerically evaluated wave deformations of the original and wide models along the body contour; $Fr = 0.27$ [5]

appears to be very effective in reducing the troughs and crests of the wave system along the hull.

There were many attempts by the authors to show that the present concept is not a hull form-dependent approach, but can be applied, at least from the hydrodynamic point of view, to many types of ships operating at moderate to relatively high Froude numbers, as shown in Calisal et al. [8]. There was a need in all these attempts, which necessitates a mathematical programming basis, to determine the optimal shape of the waterline under the given design constraints. Thus, the present study aims at developing a mathematical programming basis to obtain an optimum hull form for minimum calm water resistance.

2.2 Mathematical programming in DWL optimization

To keep the versatility and the easiness of the method, hull geometry is represented by tent functions, which may be regarded as 1st order spline functions, and the wave resistance is calculated by the thin-ship approximation—an approach presented by Hsiung [14] and then adopted by Goren and Calisal [15] who used Wolfe's [16] algorithm in solving the quadratic programming problem. The usage of tent functions in Michell's integral, which yields a quadratic objective function, can be found in full detail in Hsiung [14] and the solution of the quadratic programming by Wolfe's algorithm can be found in Goren and Calisal [15] and Goren et al. [17].

The same mathematical programming approach is considered here and extended for the present study. The procedure developed is summarized as follows:

Grid generation on the centerplane is given in Fig. 3 and according to the coordinate system chosen the hull surface is defined as follows:

$$y = f(x, z) \tag{1}$$

As the hull surface is approximated by tent function, $H(x, z)$, the Eq. 1 turns out to be

$$f(x, z) \cong H(x, z) = \sum_{i=1}^{I} \sum_{j=1}^{J} y_{ij} h^{(i,j)}(x, z)\ , \tag{2}$$

where $h^{(i,j)}$ is the unit tent function and y_{ij} is the hull offset at point (x_i, z_j). I is the number of stations along the hull and J is the number of waterlines as WLJ is the DWL. (See Appendix for the unit tent function). Note that mesh structure need not be necessarily equally spaced.

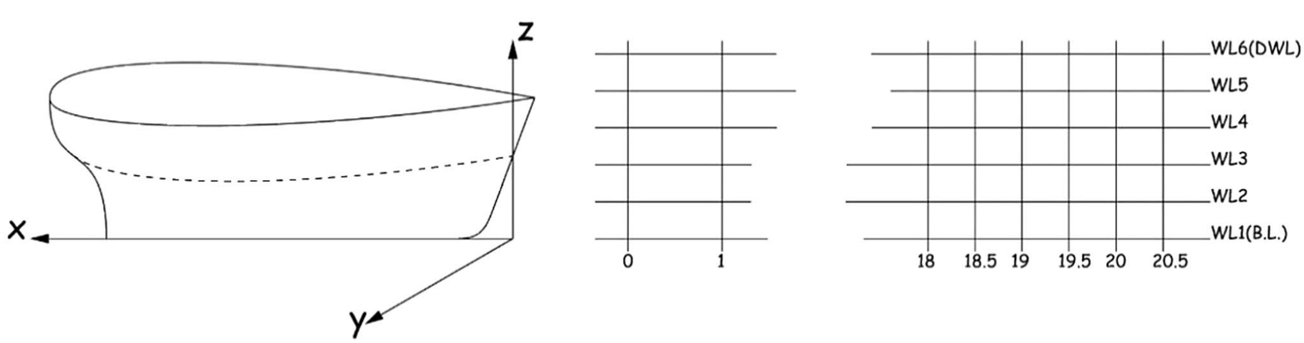


Fig. 3 Coordinate system and the discretization of the centerplane

Thin-ship approximation of the ship's wave resistance is considered here by a well-known version of Michell's integral:

$$R_W = 8\frac{\rho g^2}{\pi V^2}\int_0^\infty \frac{(u^2+1)^2}{(u^2+2)^{1/2}}\left[P^2(u)+Q^2(u)\right]du \tag{3a}$$

$$\begin{pmatrix} P \\ Q \end{pmatrix} = \int_0^T\int_0^L f_x(x,z)\begin{pmatrix} cos \\ sin \end{pmatrix}\left(\frac{g}{V^2}(u^2+1)x\right) \exp\left[\frac{g}{V^2}\left(u^2+1\right)^2(z-T)\right]dxdz, \tag{3b}$$

where V is the ship's speed and L and T are the length and draft, respectively. f_x is the partial derivative of $f(x,z)$ with respect to x. Equations 3, 4 are non-dimensionalized by $8\rho gB^2T^2/(\pi L)$ to give the wave resistance coefficient:

$$C_W = \gamma_0\int_0^\infty \frac{(u^2+1)^2}{(u^2+2)^{1/2}}\left[P^2(u)+Q^2(u)\right]du \tag{4a}$$

$$\begin{pmatrix} P \\ Q \end{pmatrix} = \frac{2}{BT}\int_0^T\int_0^L f_x(x,z)\begin{pmatrix} cos \\ sin \end{pmatrix}\left(4\gamma_0\left(\frac{x}{L}\right)(u^2+1)\right) \exp\left[4\gamma_0\left(u^2+1\right)^2(\frac{T}{L})\left(\frac{z}{T}-1\right)\right]dxdz\,(4.b), \tag{4b}$$

where $\gamma_0 = 1/(2Fr)^2$, with Froude number, Fr. Equation 4a, 4b can be discretized by substituting the tent function approximation of the hull surface in Eq. 2 into Eq. 4a, 4b:

$$C_W \cong \sum_{m=1}^{I\times J}\sum_{n=1}^{I\times J} d_{mn}y_my_n = \boldsymbol{y}^{\boldsymbol{T}}\cdot\boldsymbol{D}\cdot\boldsymbol{y}, \tag{5}$$

where m and n should be interpreted as: $m,n = j+(i-1)J;(j=1,2,\ldots,J;i=1,2,\ldots,I)$, $\boldsymbol{D}$ is a $(I\times J)$ by $(I\times J)$matrix and $\boldsymbol{y}$ is $(I\times J)$ dimensional vector of half-breadths. Frictional resistance can also be included in the optimization process by virtue of ITTC-1957 formula in which wetted surface area, S, may be approximated by a linearized expression:

$$S \cong 2\int_0^T\int_0^L\left[1+\frac{1}{2}f_x^2(x,z)+\frac{1}{2}f_z^2(x,z)\right]^{\frac{1}{2}}dxdz \tag{6}$$

Representation of the hull surface, $f(x,z)$, by tent functions, $h^{(i,j)}$, using ITTC-1957 formula for frictional coefficient, C_f, and finally non-dimensionalization of $R_F = 0.5\rho SV^2C_f$ by $8\rho gB^2T^2/(\pi L)$ gives

$$C_F = \boldsymbol{y}^{\boldsymbol{T}}\cdot\boldsymbol{c_b}+\boldsymbol{y}^{\boldsymbol{T}}\cdot\boldsymbol{F}\cdot\boldsymbol{y}, \tag{7}$$

where $\boldsymbol{c_b}$ is related with the flat bottom area of the ship. The matrix coefficients in Eqs. 5 and 7 can be found in detail in Hsiung [14] and/or Goren and Calisal [15]. Thus, the total resistance coefficient is set by the sum of frictional and wave resistance coefficients:

$$C_T \cong C_F + C_W = \boldsymbol{y}^{\boldsymbol{T}}\cdot\boldsymbol{c_b}+\boldsymbol{y}^{\boldsymbol{T}}\cdot\boldsymbol{C}\cdot\boldsymbol{y}, \tag{8}$$

where $\boldsymbol{C}$ is symmetric positive definite matrix, which guarantees that any local minimum calculated throughout the quadratic programming will be a global minimum. Additionally, in order to complete the optimization problem, design constraints should be organized in inequalities (an equality constraint can be converted into two inequality constraints). We use a set of four different design constraints in optimizing the shape of the DWL:

i. All the unknown offsets (half-breadths) of the DWL are required to be smaller than a designated increment in the beam of ship;

$$y_{ij} < a\left(\frac{B}{2}\right), \tag{9}$$

where a denotes the percentage increase allowed in the beam. (For example, a takes the value of 1.05, when 5% increase in the beam is aimed). Optionally, one may impose that the half-breadths on DWL for the first and the last stations are not greater than the original values; $y_{1J} \le y_{1J}^{(o)}$ and $y_{IJ} \le y_{IJ}^{(o)}$.

ii. The waterline slope is less than a specified value;

$$y_{i+1,j} - y_{ij} < (x_{i+1}-x_i)\tan\theta_{max} \tag{10}$$

- $\theta_{\max}$; may be specified by the designer to have an applicable/acceptable waterline geometry.

iii. Waterplane area coefficient is kept constant or increased/decreased by the designer;

$$C_{WP} = \frac{1}{BL}\sum_{i=1}^{I-1}\left(x_{i+1}-x_i\right)\left(y_{i+1,J}+y_{i,J}\right) \tag{11}$$

iv. Original half-breadths, $y_{ij}^{(o)}$, of the hull on DWL are taken as the lower bounds for the unknown offsets (the lower bound may be adjusted to a percentage of the original offsets);

$$y_{ij} > y_{ij}^{(o)} \tag{12}$$

It is the authors' recommendation to impose the constraints given in Eqs. 9–12. Additional constraints may be imposed, but this confines the feasible region to a more limited space in which the local extremum attained may be less satisfactory as compared to that of the recommended set of constraints 9–12.

The final form of the mathematical programming problem is determined first by defining the unknown vector $\boldsymbol{y}'$;

$$\boldsymbol{y}' = [y_{1,J}, y_{2,J}, \ldots, y_{I,J}]^T, \quad (13)$$

where $y_{i,J}$'s denote half-breadths on DWL and $i = 1, 2, ..., I$ are the stations. Secondly, $\boldsymbol{C}$, in Eq. 8, is a $(I \times J) \times (I \times J)$ square matrix according to the mesh structure given in Fig. 3, but needs to be re-arranged in accordance with $\boldsymbol{y}'$ in Eq. 13 to give $\boldsymbol{C}'$:

$$\boldsymbol{C}' = \begin{bmatrix} c_{J,J} & c_{J,2J} \cdots c_{j,I\times J} \\ c_{2J,J} & c_{2J,2J} \cdots c_{2J,I\times J} \\ & \cdots \\ c_{I\times J,J} & c_{I\times J,2J} \cdots c_{I\times J,I\times J} \end{bmatrix}, \quad (14)$$

where $c_{i,j}$ are the elements of matrix $\boldsymbol{C}$ in Eq. 8. Thus, the quadratic programming, in the present case, is described as in the following:

$$\text{minimize } \boldsymbol{y}'^T \cdot \boldsymbol{p} + \boldsymbol{y}'^T \cdot \boldsymbol{C}' \cdot \boldsymbol{y}', \quad (15)$$

where $\boldsymbol{p}$ is a vector of dimension I, which is equal to the product of $\boldsymbol{C}$, without $\boldsymbol{C}'$ elements in it, and the vector $\boldsymbol{y}$ excluding $\boldsymbol{y}'$ elements in it. The objective function expressed in Eq. 13 is *subject to* the set of constraints—explained in Eqs. 9–12;

$$\boldsymbol{A} \cdot \boldsymbol{y}' < B \text{ with } \boldsymbol{y}' \geqslant 0, \quad (16)$$

where $\boldsymbol{B}$ is an M dimensional vector and M is the total number of constraints. Note that some constraints, as in Eq. 12, are inserted into $\boldsymbol{B}$ as negative elements which can be treated as well by the algorithm of Wolfe [16], which is summarized in the Appendix A as the solution of the quadratic programming problem given in Eq. 15 and 16.

3 Numerical work and experimental validation

To test the algorithm developed for the optimization of the design waterline, allowing an increase in the beam, SL-7 containership form is selected. SL-7 hull has a slender form, which bears difficulties for waterline optimization. The cross sections are obtained from the open literature (see Fig. 4). SL-7 was designed and built for high-speed transportation between the two sides of the Pacific Ocean, then converted to Fast Sealift Ships, Boylstone et al. [18].

For discretization purposes, number of stations is chosen as $I = 24$ and number of waterlines is $J = 6$ which implies a (144×144) coefficient matrix $\boldsymbol{C}$ as explained in Sect. 2. It is intended first to test the sensitivity of the present thin-ship routine to the variations in DWL. During the course of this work, computational support is received from an in-house source-panel code (ITU-Dawson), which we found to be a relatively higher-fidelity computational tool—as compared to Michell's integral—for determining wave-making characteristics of the hull form. ITU-Dawson has been tested in many studies including cooperative research projects such as Diez et al. [19]. A grid sensitivity/convergence analysis with ITU-Dawson is presented in Fig. 5 together with Table 1 for the SL-7 form. The wave elevation around the SL-7 hull form as obtained from ITU-Dawson is given in Fig. 6. It is understandable from Fig. 6 that many results of our systematic search, to increase the beam along the mid-body, naturally failed to find a better shape for DWL according to thin-ship calculations, but the DWL depicted in Fig. 7 was successful. This result was expected due to the fact that the form, given in Fig. 7, has a potential to reduce the wave crest at the stern. It should be noted here that the whole hull geometry is modified according to the percentage changes in the proposed/optimized DWL, as compared to that of the original hull by means of a simple self-adaptive surface generator routine. This is a practical way of not to increase the number of unknowns in the programming phase. The hull form represented by the

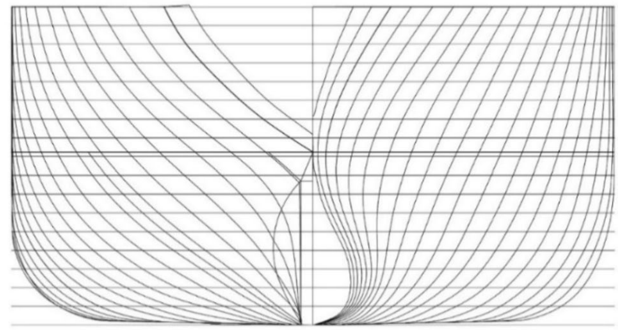


Fig. 4 SL-7 hull form as derived from open literature

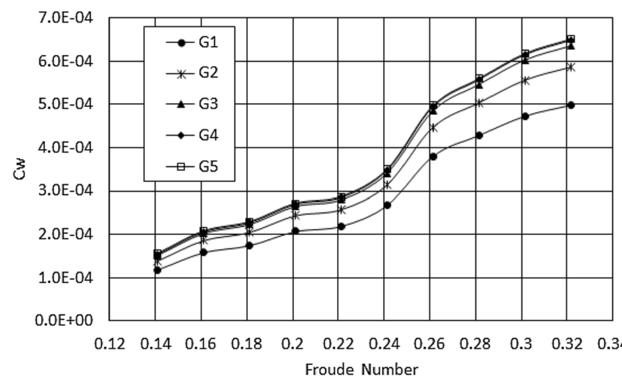


Fig. 5 Change in wave resistance coefficient according to the grid density

Table 1 Grid refinement study for SL-7

Grid	Refinement ratio	Hull grid	Free surface	Total
G1	$\sqrt{2}$	40×10	60×12	1120
G2		48×12	72×14	1584
G3		61×13	85×17	2238
G4		76×15	101×20	3160
G5		90×18	120×24	4500

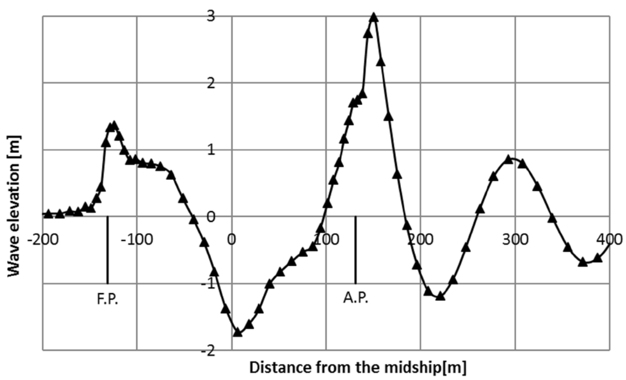


Fig. 6 Wave deformations around the hull, SL-7 ($V = 32$ kn)

proposed DWL appears to reduce the wave resistance by about 12% according to thin-ship calculations. Convinced by the results of this preliminary sensitivity study, which is associated with the relationship between the location of wave troughs and crests of the transverse wave system and the location of side bulbs on the DWL, the optimization problem of DWL is then tackled.

Optimization study for DWL of SL-7 hull was carried out with $I = 24$ number of stations and $J = 6$ number of waterlines which means that quadratic programming had 24 unknowns (as half-breadths on DWL). Constraints set is prescribed in a way that:

i. $y_{iJ} < 1.07\left(\frac{B}{2}\right)$, $(i = 1, 2, ..., 24)$; that is, there is an allowance of beam increment of 7% compared to the original beam. y_{1J} and $y_{24,J}$ are fixed as they are in the original hull.

ii. $\theta_{\max} < 20.5°$, chosen by taking into account the original DWL and to limit the slopes.

iii. $C_{WP} = 1.02C_{WL}^{(o)}$; waterplane area coefficient is assumed to be 2% greater than the original value, to provide an additional waterplane area to increase the beam. (2% increase in C_{WL} is found adequate and no further search beyond this point is carried out on C_{WL}, as it is understood from the results of the optimization).

iv. $y_{iJ} > 0.95y_{iJ}^{(o)}$; optimized half-breadths are allowed to be less than the original offsets by 5%.

The optimization process achieved the optimal solution at the 154th iteration with 24 unknowns taken on the DWL to give a form with its fore part slightly protruding sidewise and the aft part yields a bulbous-like geometry, Fig. 8. This is in agreement with the findings of the systematic search given in Fig. 7. We may remind here that despite a 2% increase given to C_{WP} and an allowance of 7% increase in the beam, the optimization process does not end up with an increase in the beam but with a bulging aft part which is a consequence of the wave system around the body. The geometrical transformation of the whole hull form is achieved by taking into account the changes in DWL and by reflecting those changes to the original hull by means of a simple self-adaptive surface generator routine. This is a reasonable procedure as the number of unknowns is kept limited and the optimal point is not degenerated, as the adaptation of the hull geometry to the optimum DWL always ends up with a better solution. Moreover, the transformed/adapted geometry emphasizes the character and the effect of the optimum DWL.

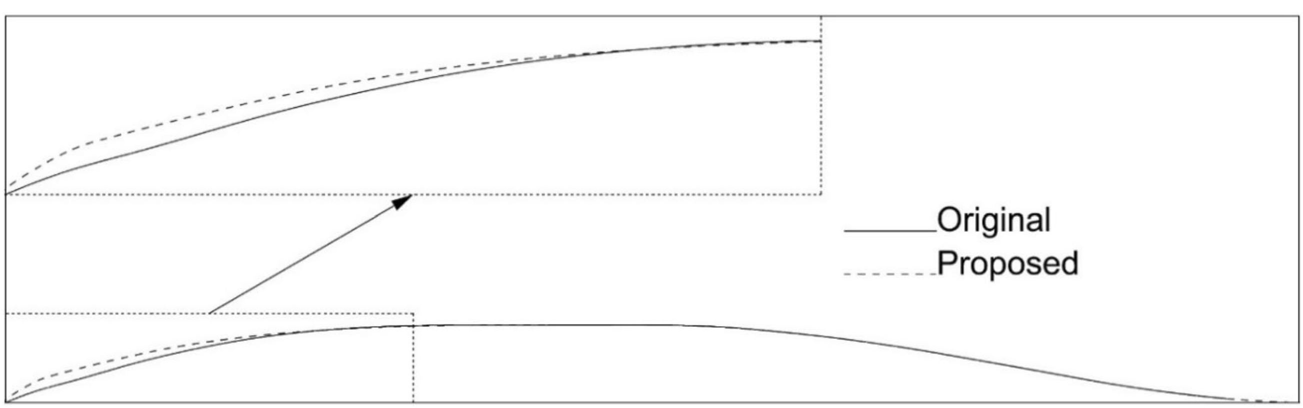


Fig. 7 DWL developed, in the systematic search process, taking into account the wave troughs and crests around the hull

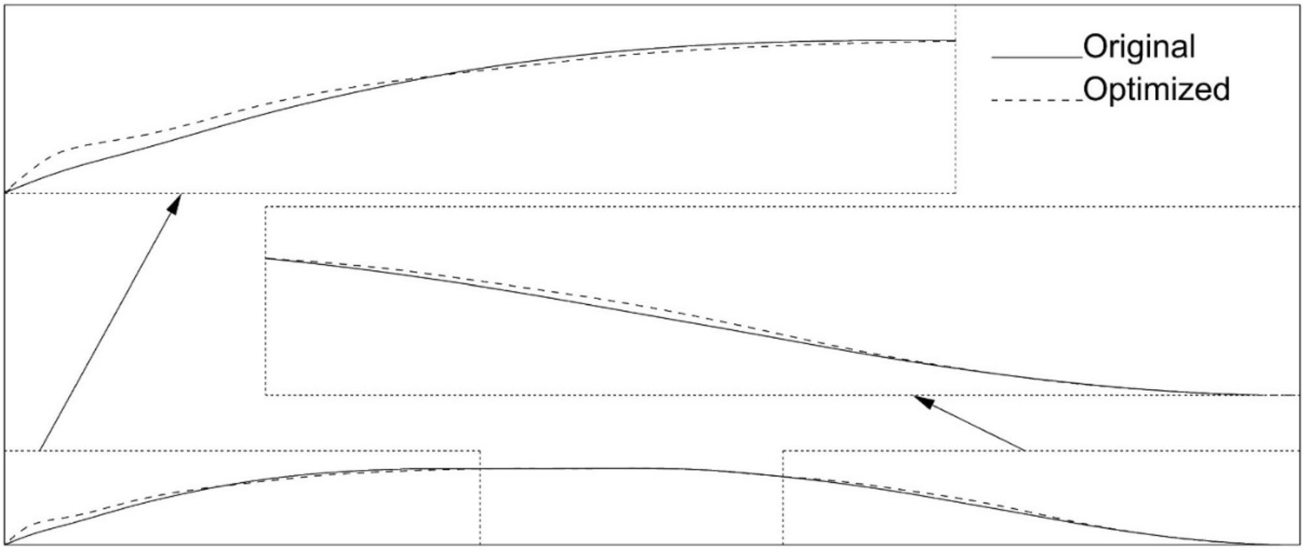


Fig. 8 Optimized and original SL-7 DWLs

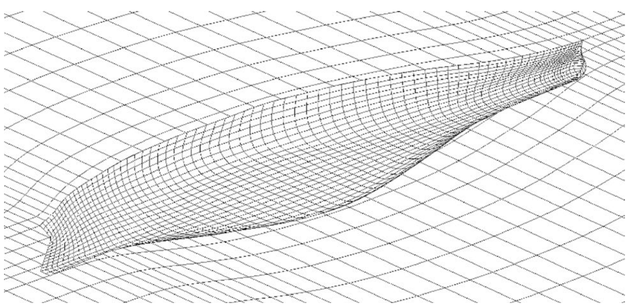


Fig. 9 Quadrilateral panel distribution over the DWL-optimized hull and its free surface vicinity

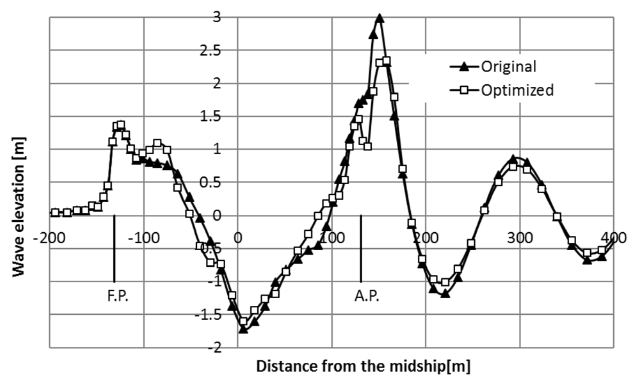


Fig. 10 Wave deformations along the periphery of the original and optimal SL-7 hulls at 32 kn

The computational analyses performed by the in-house potential (free surface) flow solver ITU-Dawson, (an example of hull form discretization can be seen in Fig. 9), show that optimized DWL of SL-7 form has favorable wave pattern particularly around the aft form as compared to the original hull (see Fig. 10). The interesting part of this study is that SL-7 has a very slender hull form and the generated wavelength along the body (at 32 kn) is close to the body length (see Fig. 6). Note that the relationship between the location of wave troughs and crests of the transverse wave system and the location of side bulbs on the DWL is an important fact which makes the implementation of the beam increment strategy from hydrodynamics point of view a difficult one in the case of SL-7. The mathematical programming procedure responds to this physical issue accordingly requiring an apparent sidewise-protruding geometry just at the aft/running part of the hull. As given in Table 2, computational analyses point out that around 10% reduction in the wave resistance is attained by DWL-optimized form of SL-7.

Table 2 Computed wave resistances of SL-7 forms

	R_W [kN]	
V (knot)	Original	DWL-Optimized
32	798.4	702.6

Convinced by the fact that the present optimization approach works properly, BC Ferries hull form is chosen for further demonstration and validation. BC Ferries hull form, designed to operate at 12.5 kn ($Fr = 0.33$), is a plausible form to demonstrate the capabilities of the present procedure. The generated wavelength along its body at 12 kn is far lesser than the body length on the one hand, its wave

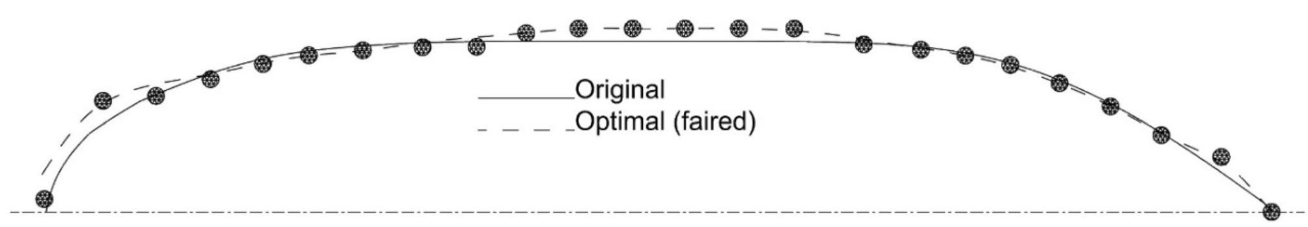


Fig. 11 Original and optimal (faired) DWLs of BC Ferry hull. (*Bold dots* show the optimum points of DWL as obtained from the mathematical programming code)

resistance percentage in the total resistance is considerable on the other.

The optimization study for the BC Ferry hull is carried out using 24 stations along the body and 6 waterlines within the draft. The set of constraints is imposed in a similar manner, that is:

i. $y_{iJ} < 1.075\left(\frac{B}{2}\right)$, $(i = 1, 2, ..., 24)$; that is we allow 7.5% increase in the beam. (In fact, we started the optimization procedure first without any increment and with 5% and ended up with 7.5%.)
ii. $\theta_{max} < 35°$, chosen by taking into account of the slopes of the original DWL and to limit the slopes to a certain degree.
iii. $C_{WL} = 1.02C_{WL}^{(o)}$; the best choice as understood from the optimization procedure, as we started with original waterplane area coefficient, $C_{WL}^{(o)}$, then by making successive increments of 0.01. 2% increase in C_{WL} appears to be the most effective one.
iv. $y_{iJ} > 0.97y_{iJ}^{(o)}$, $(i = 1, 2, ..., 24)$; which imposes that the optimized half-breadths are allowed to be less than the original offsets by only 3%.

The optimization code converges to the optimum solution vector of 24 unknowns via Wolfe's algorithm at the 163rd iteration. The optimal points as obtained from the code are depicted in Fig. 11. Note that there is no fairness criterion in the above-mentioned constraints, to see what the code really calculates. In order to have an applicable form, a spline curve can be employed with tolerances set by the designer. The final DWL—optimized and faired—can be seen in Fig. 11, together with the original one. It should be noted here that the whole hull form is then updated by taking the percentage changes in DWL into account. As a consequence, body plan of the amended hull according to the optimum DWL is compared to the original body plan in Fig. 12.

The potential flow solver (ITU-Dawson) points out that the calculated wave resistance—by pressure integration over the body—is decreased in considerable amounts (more than 20%) for the optimal hull form (see Fig. 17).

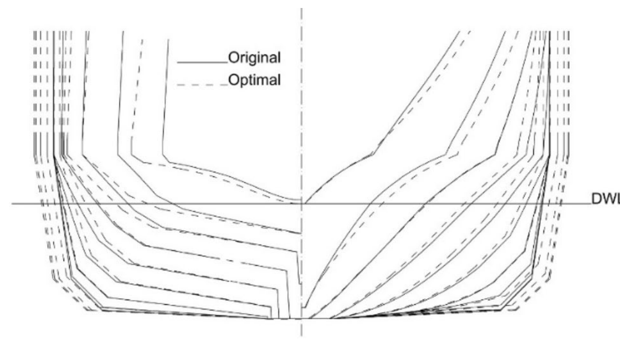


Fig. 12 The body plan of the optimal BC Ferry geometry (*dashed lines*)—obtained by taking the percent changes of the optimal DWL with respect to the original DWL—as compared with the original ones (*solid lines*)

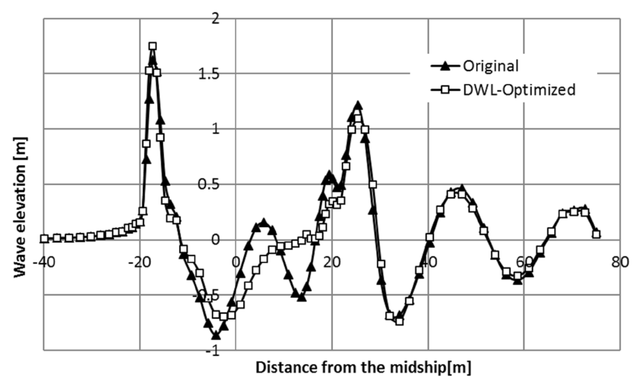


Fig. 13 Wave deformations along the bodies of the original and the optimized hulls at 12.5 kn

The resistance reduction capability of the DWL-optimized hull can be seen by comparing the wave deformations along the bodies, Fig. 13. Computed wave patterns around the original and the DWL-optimized hulls, depicted by contour plots in Fig. 14, can also give an idea about the capability of the DWL-optimized hull in reducing the wave-making resistance.

The DWL-optimized hull with an increase in the beam by 7.5% is then tank tested for validation. The original (parent) model of BC Ferry hull is built with a scale of 14.177

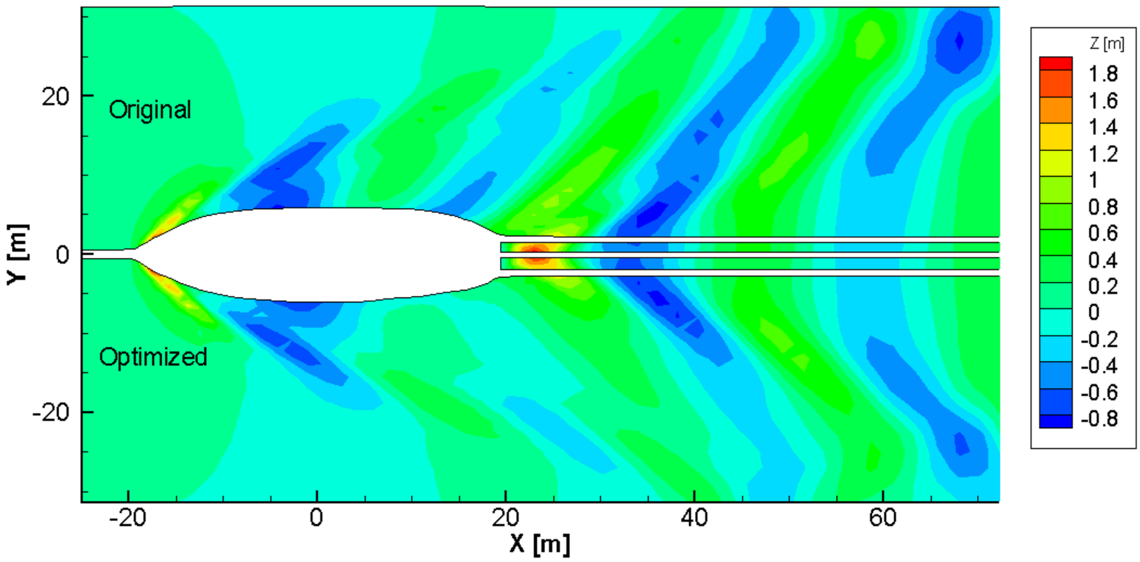


Fig. 14 Contour plot representation of the wave patterns around the hulls; $V = 12.5$ kn

Table 3 Main particulars of the BC Ferry hull forms

	Original form (M318-0)	DWL-Optimized (M318-1)
L_{WL} (m)	38.732	38.732
B (m)	10.888	11.727
T (m)	2.6	2.6
WSA (m^2)	435.74	458.653
Displacement (ton)	655.361	696.122
C_B	0.604	0.573
C_P	0.658	0.626

Fig. 15 Side view of the original Model M318-0 (*above*), DWL-optimized model M318-1 (*below*)

at ITU's A. Nutku Ship Model Testing Laboratory. The particulars of the original BC Ferry model (M318-0) and its DWL-optimized version (M318-1) are given in Table 3. The models built can be seen in Fig. 15. Performance evaluation of the forms are carried out for constant draft which means an increased displacement (6%) for DWL-optimized form.

The model tests were performed at A. Nutku Ship Model Testing Laboratory of ITU. Measurements of total resistance data were acquired by means of electronic dynamometers and models were tested free to sink and trim. (No uncertainty analysis was carried out in this work, but a recent uncertainty analysis according to ITTC standards performed in A. Nutku Ship Model Testing Laboratory gave a global uncertainty of 1.3% of C_{Tm} around $Fr = 0.26$). Measurements in the low-speed region were repeated twice in order to reduce the effect of dispersion in the low-speed experimental data. Thus, only the low-speed experimental data are faired by means of least-squares smoothing, which give form factors of $(1 + k)$, 1.225 and 1.247 for the original hull and the DWL-optimized hull, respectively. Model scale resistance coefficients are given in Fig. 16. The superior resistance performance of the DWL-optimized hull is very obvious and drastic around design Froude number of 0.33 and wave resistance reduction (in terms of coefficients) appears to be nearly 30% as deduced from the experimental results presented in Fig. 16. One can note that the resistance coefficients are non-dimensionalized by $0.5\rho SV^2$, where S denotes WSA. The agreement between the wave resistance results of the potential code—obtained by pressure integration over the

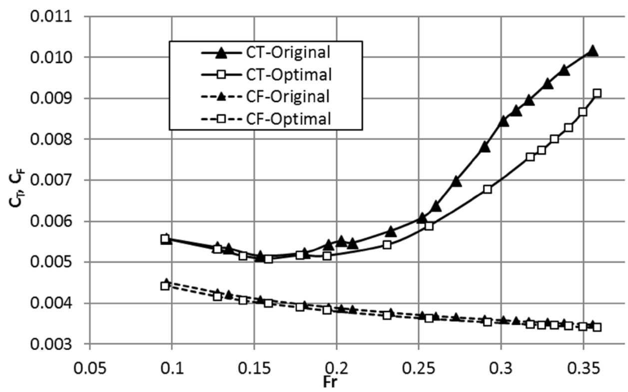


Fig. 16 Model scale resistance coefficients

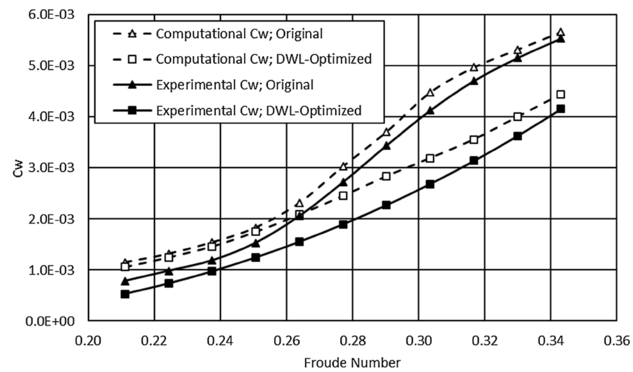


Fig. 17 Comparison of the wave resistance coefficients of the potential flow code (ITU-Dawson) with those of the experimental study

body—with those of the experimental study is demonstrated in Fig. 17. The capability of the DWL-optimized form in reducing the wave resistance can also be seen by comparing the wave elevations around the periphery of the models as shown in Fig. 18a, b. When comparing the wave elevations in Fig. 18, it is very clear that the DWL-optimized model appears to reduce the first wave trough and the second wave crest drastically. This reduction was already noticed from the computational results as can be seen in Figs. 13 and 14. Extrapolation of the experimental data to full scale is performed by ITTC-1978 procedure [20] (air resistance is not included) and the output is given in terms of effective power in Fig. 19. Comparison of the full scale performances of the original and the DWL-optimized forms points out around 12.5% reduction in effective power at the design speed of 12.5 knots.

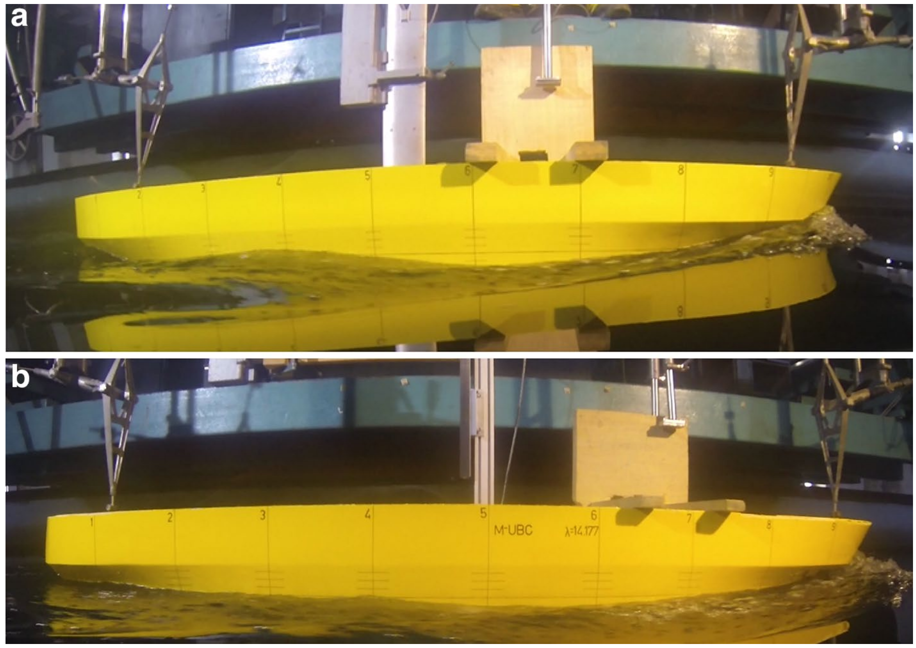
4 Concluding remarks

The present study is carried out to rationalize the design concept of wave resistance reduction by allowing an increase in the beam—accepted in general as an undesirable design practice among the traditional principles of naval architecture. A mathematical programming procedure is developed for this purpose, which includes a quadratic programming with Wolfe's algorithm and the thin-ship formulation for the calculation of wave resistance. These two basic components of the present procedure are chosen to give the designers a versatile and an easy-to-use tool. The present procedure considers the shape optimization of the DWL for minimum resistance and in turn the geometric modification of the entire hull is obtained according to the optimized DWL—which is indeed an easy implementation.

Two applications of the proposed methodology are presented here and an experimental validation study is also reported. Computational and experimental studies show that the proposed methodology is successful in that it can guide the designers in hydrodynamic global hull shape optimization as well as the local shape optimization. The outcomes of the present study support the design concept and the assertion introduced in the previous related works, such as Calisal et al. [5], as well. The limits of the application of the present methodology from wave resistance point of view are established: (i) The hull at the design speed should have a considerable share of wave resistance in total resistance, (ii) generated wavelength along the hull should preferably be less than the body length, which means that the Froude number should be less than 0.4.

Indeed, there is no need to use high-fidelity flow solvers for the present approach. But as the thickness of the boundary layer and the risk of flow separation increases towards the aft of the ship, potential flow codes are no longer adequate to determine the order of merit. Thus, optimized form as a product of the present method may require testing at least computationally for viscous effects, particularly when the aft form is modified substantially. Increases in the beam as well as in the waterplane area could improve the stability and the seakeeping performances as well as payload capacity of the hull. Note that seakeeping aspects of the proposed design concept were already investigated in Calisal et al. [21]. We expect maneuvering effects would be of secondary importance as compared to resistance, stability, and seakeeping concerns. The effects of increasing the beam on the construction cost were already investigated in [22]. This study, carried out for a platform supply vessel, found that there may be some cost increase due to the use of side bulbs and the investment comes into a break-even point just before (between) 9th month and 16th month of the operation time. Environmental benefits are worth mentioning as well.

Fig. 18 Wave elevation around the periphery of the original form (**a**) and of the DWL-optimized form (**b**) at $V = 1.7$ m/s ($Fr = 0.33$), BC Ferry Hull model



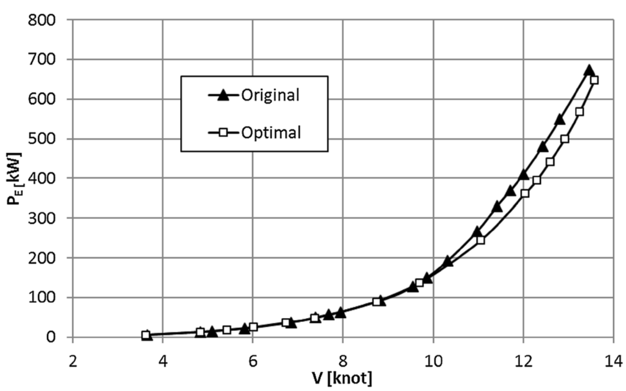


Fig. 19 Effective power comparison, BC Ferry hull (design speed; 12.5 kn)

5 Appendix A

5.1 A.1 Unit Tent Function

The unit tent function at the grid point (x_i, z_j) is defined as:

$$h^{(i,j)}(x,z) = \begin{cases} \left[1 - \frac{x_i - x}{x_i - x_{i-1}}\right]\left[1 - \frac{z_j - z}{z_j - z_{j-1}}\right]; x_{i-1} \le x \le x_i; z_{j-1} \le z \le z_j \\ \left[1 - \frac{x_i - x}{x_i - x_{i-1}}\right]\left[1 - \frac{z_j - z}{z_j - z_{j+1}}\right]; x_{i-1} \le x \le x_i; z_j \le z \le z_{j+1} \\ \left[1 - \frac{x_i - x}{x_i - x_{i+1}}\right]\left[1 - \frac{z_j - z}{z_j - z_{j-1}}\right]; x_i \le x \le x_{i+1}; z_{j-1} \le z \le z_j \\ \left[1 - \frac{x_i - x}{x_i - x_{i+1}}\right]\left[1 - \frac{z_j - z}{z_j - z_{j+1}}\right]; x_i \le x \le x_{i+1}; z_j \le z \le z_{j+1} \\ 0 ; \quad \text{out of the rectangular element bounded by } (i+1)th, \\ (i-1)th, \quad (j+1)th \text{ and } (j-1)th \text{ sections} \end{cases}$$

5.2 A.2 Summary of Wolfe's Algorithm

We adapt the "short form" of the Wolfe's algorithm for the present definition of the quadratic programming problem given in Eqs. 16 and 17:

$$\begin{aligned} &\boldsymbol{A} \cdot \boldsymbol{y'} + \boldsymbol{e} \cdot \boldsymbol{z'} = \boldsymbol{B} \\ &2\boldsymbol{C} \cdot \boldsymbol{y'} + \boldsymbol{A}^{T} \cdot \boldsymbol{u} - \boldsymbol{v} + \boldsymbol{E} \cdot \boldsymbol{z} = -\boldsymbol{p} \\ &\boldsymbol{y'}, \boldsymbol{u}, \boldsymbol{v}, \boldsymbol{z}, \boldsymbol{z'} \geqslant 0 \end{aligned} \tag{A1}$$

In the system (A2); $\boldsymbol{u}$ and $\boldsymbol{v}$ are M dimensional vectors of Lagrange multipliers and $\boldsymbol{z}$ is the N dimensional vector of artificial variables where N is the number of unknowns. Here, artificial variables $\boldsymbol{z'}$ are added to the rows where B_i elements are negative. $\boldsymbol{e}$ and $\boldsymbol{E}$ are obtained as a result of the process to discard slack variables from the system. Wolfe [12] proposes a solution algorithm for the system (A1) which is analogous to the Simplex Method. According to the algorithm, initial basis for the system (A1) can be formed from the variables $\boldsymbol{z}$ and $\boldsymbol{z'}$ and then recursive steps are taken in the Simplex procedure to minimize the linear form:

$$\sum_{k=1}^{n+n^*} z_k \tag{A2}$$

where n and n^* denote the number of positive and negative elements in $\boldsymbol{B}$, respectively. If the form (A2) is positive, recursive step is repeated and terminated when $\boldsymbol{z} = 0$.

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