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ORIGINAL ARTICLE

Melnikov integral formula for beam sea roll motion utilizing a non-Hamiltonian exact heteroclinic orbit

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Abstract The chaos that appears in the ship roll equation in beam seas known as the escape equation has been intensively investigated because it is closely related to capsizing incidents. In particular, many applications of the Melnikov integral formula have been reported in the literature; however, in all the analytical works concerning the escape equation, the Melnikov integral is formulated utilizing a separatrix for the Hamiltonian part or a numerically obtained heteroclinic orbit for the non-Hamiltonian part of the original escape equation. To overcome such limitations, this article attempts to utilise an analytical expression for the non-Hamiltonian part. As a result, an analytical procedure is provided that makes use of a heteroclinic orbit of the non-Hamiltonian part within the framework of the Melnikov integral formula.

Keywords Escape equation · Chaos phenomenon · Melnikov integral formula · Analytical formulae · Non-Hamiltonian heteroclinic orbit

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1 Introduction

In the research field of nonlinear dynamical system theory, it is well known that the Feigenbaum cascade of period-doubling bifurcation can lead to chaos [1], and considerable research with regard to this phenomenon has been reported. The chaotic behaviour of ship roll motion in beam seas has been studied by Virgin [2], Thompson [3, 4], and Kan and Taguchi [5, 6], among others. Thompson used the escape equation with a second-order polynomial fitting the restoring term and discussed the occurrence of capsize (or escape) and chaos. Kan and Taguchi observed capsizing phenomena caused by period-doubling bifurcation in their model experiment [5]. Further, they investigated the escape equation with a nonlinear cubic restoring term using numerical time simulation and confirmed a close relationship between capsize and chaos [6]. On the other hand, Murashige and Aihara [7] calculated the Lyapunov exponents from the measured time history of a flooded ship model, and they confirmed that the ship rolling motion could tend to a chaotic attractor. Moreover, they did detailed numerical studies with a theoretical model [8].

The Melnikov integral formula enables us to test for the existence of a transverse homoclinic connection of an invariant manifold of a saddle [9, 10]. The existence of such a connection implies the beginning of the fractal metamorphosis and is one of the prerequisites of chaotic behaviour. As an example of the direct application of this method to the ship roll problem using the escape equation with a cubic restoring term, Kan and Taguchi [6] analytically estimated the conditions of chaotic behaviour. Although it is necessary to analytically or numerically obtain the heteroclinic orbit in the time domain to calculate the Melnikov integral of a highly dissipative system, an analytical expression of the non-Hamiltonian part is not easily obtained in general. Therefore, Wu and McCue [11] calculated the Melnikov integral using a numerically obtained heteroclinic orbit. Although numerical integration requires verification of its accuracy for infinite integrals, it seems to be an extremely powerful technique even for high-dimensional systems.

A general solution for nonlinear differential equations is not always available. In the case of the escape equation with a cubic restoring term, a general solution is not available. Therefore, Kan and Taguchi [6] calculated the Melnikov integral based on the separatrix of the Hamiltonian part of the escape equation as an alternative to solving its non-Hamiltonian part. However, although we cannot find a general solution of the escape equation, heteroclinic orbits themselves can be obtained using a solution technique that is used for analysing nonlinear waves.

A heteroclinic orbit is an orbit connecting two saddles, and such an orbit can be realized for certain parameters, i.e., the set of heteroclinic bifurcation points [12]. For instance, it is well known that the surf-riding threshold in following and quartering seas can be represented as a heteroclinic bifurcation point [13]. Considerable effort has been expended on research of the surf-riding threshold, and one of the current authors has proposed an analytical technique for estimating this threshold. In this technique, the sinusoidal periodic surge force induced by waves was approximated using a third-order polynomial, and then an analytical formula to estimate the surf-riding threshold was obtained [14]. This approximated surge equation is identical to the non-Hamiltonian part of the escape equation. Therefore, in the present article, we attempt to apply the same procedure to the escape equation in an attempt to provide an analytical formula for the threshold of chaos.

2 Escape equation

Uncoupled roll motion with wave excitation is modelled as [6]:

$$I\frac{\mathrm{d}^2\Phi}{\mathrm{d}t^2}+N\frac{\mathrm{d}\Phi}{\mathrm{d}t}+W\cdot GM\cdot\Phi(1-\Phi/\Phi_\mathrm{V})(1+\Phi/\Phi_\mathrm{V}) = M_\mathrm{rs}+M_\mathrm{r}\sin(\omega t+\delta) \tag{1}$$

where Φ is roll the angle, I is the moment of inertia in roll, N is the damping coefficient in roll, W is the ship mass, and GM is the metacentric height. In the forcing term, i.e., the right side of Eq. 1, M_rs denotes the second-order steady wave-induced roll moment, whereas M_r denotes the amplitude of the first-order wave-induced roll moment. It is assumed that M_rs and M_r have relatively small values compared to the terms on the left side of Eq. 1. Obviously, $\Phi=\pm\Phi_\mathrm{V}$ are saddles. Here we divide M_rs into two parts:

$$M_\mathrm{rs}=M_1+M_2 \tag{2}$$

This separation is done to create the heteroclinic orbit. In practice, value M_1 is determined by the condition of the heteroclinic bifurcation described in the next section. Factorisation yields:

$$W\cdot GM\cdot\Phi(1-\Phi/\Phi_\mathrm{V})(1+\Phi/\Phi_\mathrm{V})-M_1 = W\cdot GM(\Phi-\Phi_1)(\Phi_2-\Phi)(\Phi-\Phi_3)/\Phi_\mathrm{V}^2 \tag{3}$$

where the relation $\Phi_1<\Phi_2<\Phi_3$ applies. Considering the following transformation:

$$\varphi=\frac{\Phi-\Phi_1}{\Phi_3-\Phi_1} \tag{4}$$

the restoring terms becomes:

$$\frac{W\cdot GM(\Phi-\Phi_1)(\Phi_2-\Phi)(\Phi-\Phi_3)}{\Phi_\mathrm{V}^2} = \frac{W\cdot GM(\Phi_3-\Phi_1)^3}{\Phi_\mathrm{V}^2}\varphi\left(\varphi-\frac{\Phi_2-\Phi_1}{\Phi_3-\Phi_1}\right)(1-\varphi) \tag{5}$$

Therefore we have:

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}+\frac{N\mathrm{d}\varphi}{I\,\mathrm{d}t}+\frac{W\cdot GM(\Phi_3-\Phi_1)^2}{I\Phi_\mathrm{V}^2}\varphi\left(\varphi-\frac{\Phi_2-\Phi_1}{\Phi_3-\Phi_1}\right)(1-\varphi) = \frac{M_2}{I(\Phi_3-\Phi_1)}+\frac{M_\mathrm{r}}{I(\Phi_3-\Phi_1)}\sin(\omega t+\delta) \tag{6}$$

Defining new variables as:

$$\begin{cases}\tilde\beta\equiv N/I, \quad \tilde\mu\equiv W\cdot GM(\Phi_3-\Phi_1)^2/I\Phi_V^2,\\ \tilde a\equiv(\Phi_2-\Phi_1)/(\Phi_3-\Phi_1)\\ b_0\equiv M_2/I(\Phi_3-\Phi_1), \quad b\equiv M_\mathrm{r}/I(\Phi_3-\Phi_1)\end{cases} \tag{7}$$

yields the following equation:

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}+\tilde\beta\frac{\mathrm{d}\varphi}{\mathrm{d}t}+\tilde\mu\cdot\varphi(1-\varphi)(\varphi-\tilde a)=b_0+b\sin(\omega t+\delta) \tag{8}$$

Note that if $\tilde a=0.5$, then the left side of this equation becomes symmetrical. This equation is utilised for all the considerations in this article.

3 Solution of the non-Hamiltonian heteroclinic orbit

Considering the case $b_0=0$ and $b=0$ in Eq. 8:

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2}+\tilde\beta\frac{\mathrm{d}\varphi}{\mathrm{d}t}+\tilde\mu\cdot\varphi(1-\varphi)(\varphi-\tilde a)=0 \tag{9}$$

This equation is identical to the FitzHugh–Nagumo (FHN) equation, [15, 16] except for some coefficients (see Appendix 1), so that the solution method for nonlinear waves [17] to find a travelling wave is applicable. Here, assuming that Eq. 9 has a heteroclinic orbit, then let us postulate a non-Hamiltonian heteroclinic orbit:

$$\dot{\varphi} = \tilde{c}\varphi(1-\varphi) \tag{10}$$

Differentiation of Eq. 10 with regard to time yields:

$$\begin{aligned}\ddot{\varphi} &= \tilde{c}\frac{\mathrm{d}(\varphi-\varphi^2)}{\mathrm{d}t} = \tilde{c}(\dot{\varphi}-2\varphi\dot{\varphi}) = \tilde{c}\dot{\varphi}(1-2\varphi)\\ &= \tilde{c}^2\varphi(1-\varphi)(1-2\varphi)\end{aligned} \tag{11}$$

If we substitute above equation into Eq. 9, then we obtain:

$$\begin{aligned}&\tilde{c}^2\varphi(1-\varphi)(1-2\varphi) + \tilde{\beta}\tilde{c}\varphi(1-\varphi) + \tilde{\mu}\cdot\varphi(1-\varphi)(\varphi-\tilde{a})\\ &= 0\end{aligned} \tag{12}$$

Here, taking a monomial order of $^{\forall}\varphi : \varphi \in (0,1)$, then:

$$\varphi\left(\tilde{\mu}-2\tilde{c}^2\right) + \left(\tilde{c}^2+\tilde{\beta}\tilde{c}-\tilde{\mu}\tilde{a}\right) = 0 \tag{13}$$

In order to satisfy the above equation for $^{\forall}\varphi : \varphi \in (0,1)$, the following relations are required:

$$\begin{cases}\tilde{\mu}-2\tilde{c}^2 = 0\\ \tilde{c}^2+\tilde{\beta}\tilde{c}-\tilde{\mu}\tilde{a} = 0\end{cases} \tag{14a, b}$$

From Eq. 14a we have:

$$\tilde{c} = \pm\sqrt{\tilde{\mu}/2} \tag{15}$$

The positive sign corresponds to a heteroclinic orbit on the upper part of phase plane, whereas the negative sign corresponds to an orbit on the lower part of the phase plane. Substituting this condition into Eq. 14b, we obtain:

$$\frac{\tilde{\mu}}{2} \pm \tilde{\beta}\sqrt{\frac{\tilde{\mu}}{2}} - \tilde{\mu}\tilde{a} = 0 \Rightarrow \tilde{\mu}\left(\frac{1}{2}-\tilde{a}\right) \pm \tilde{\beta}\sqrt{\frac{\tilde{\mu}}{2}} = 0 \tag{16}$$

Here, the condition $\tilde{a} > 0.5$ corresponds to the heteroclinic orbit on the upper plane, whereas $\tilde{a} < 0.5$ corresponds to that on the lower plane because positive roll damping ($\tilde{\beta} > 0$) and a positive metacentric height ($\tilde{\mu} > 0$) should apply to a normal intact ship in general. Equation 16 can be solved using a simple iteration procedure with respect to a single variable, such as M_1, when the bifurcation point is required as a function of M_1. Table 1 provides a comparison of the critical values of σ_C providing the heteroclinic orbit. Here σ_C denotes the non-dimensionalized value M_1 as $\sigma = M_1/W \cdot GM \cdot \Phi_V$. Further calculation conditions were set to be the same as those of Table 3 in Wu and McCue, [11] and the values σ_C obtained by Wu and McCue are noted. Since the results obtained by the present procedure agree well with the numerical results of Wu and McCue [11], it is concluded that the analytical method proposed here is verified and the numerical results of Wu and McCue [11] have sufficiently high accuracy. If we solve Eq. 10, we can easily obtain as a solution in the time domain:

Table 1 Comparison between the numerical results of Wu and McCue [11] and the present analytical results

$\tilde{\beta}$	σ_C (numerical results)	σ_C (analytical results)
0.05	0.023577	0.023557
0.1	0.047036	0.047036

$\tilde{\beta}$, (damping coefficient in roll)/(moment of inertia in roll), σ_C critical value of non-dimensionalized value M_1

$$\varphi^0(t) = \frac{1}{1+\exp\left(-\tilde{c}t+\tilde{d}\right)} = \frac{1}{2}+\frac{1}{2}\tanh\left(\frac{\tilde{c}t-\tilde{d}}{2}\right) \tag{17}$$

Here, $\tilde{d} \in (-\infty, \infty)$ denotes the arbitrary integral constant determined by an initial condition. Taking $\varphi = 0.5$ at $t = 0$ yields $\tilde{d} = 0$, then Eq. 17 becomes:

$$\varphi^0(t) = \frac{1}{2}+\frac{1}{2}\tanh\frac{\tilde{c}t}{2} \tag{18}$$

This equation is utilized for calculating the Melnikov integral in the next section. Moreover, for the rolling equation with a fourth-order polynomial, we can similarly obtain an analytical solution of the heteroclinic orbit in the limiting condition. A separate publication describing this approach is planned for the future.

If we consider the case $\tilde{a} = 0.5$, the solution of Eq. 16 is $\tilde{\beta} = 0$ or $\tilde{\mu} = 0$. $\tilde{\mu} = 0$ implies the non-existence of a restoring term, so that this solution is not relevant to the current problem. Therefore $\tilde{\beta} = 0$ should be regarded as a solution. If $\tilde{\beta} = 0$, i.e., the Hamiltonian system applies, the separatrix connecting $\varphi = 0$ and $\varphi = 1$ is realised only when $\tilde{a} = 0.5$. This can be easily proved as follows. If we consider the case of $\tilde{\beta} = 0$ in Eq. 9, simple manipulation yields:

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = \tilde{\mu}\left[\varphi^3-(\tilde{a}+1)\varphi^2+\tilde{a}\varphi\right] \tag{19}$$

Multiplying each side of Eq. 19 by $\mathrm{d}\varphi/\mathrm{d}t$ and integrating with regard to time t, we obtain:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \pm\varphi\sqrt{2\tilde{\mu}\left(\frac{1}{4}\varphi^2-\frac{\tilde{a}+1}{3}\varphi+\frac{\tilde{a}}{2}\right)} \tag{20}$$

Here, $\varphi = 0$ at $\mathrm{d}\varphi/\mathrm{d}t = 0$ is assumed. Then, at $\varphi = 1$, $\mathrm{d}\varphi/\mathrm{d}t$ takes the value of:

$$\frac{\mathrm{d}\varphi}{\mathrm{d}t} = \pm\sqrt{\tilde{\mu}\left(\frac{2\tilde{a}-1}{6}\right)} \tag{21}$$

If we require $\mathrm{d}\varphi/\mathrm{d}t = 0$, the condition of $\tilde{a} = 0.5$ is necessary. Therefore the symmetrical equation has a separatrix connecting two saddles only for the case $\tilde{\beta} = 0$, otherwise the separatrix becomes a homoclinic orbit.

Finally we briefly consider whether Eq. 10 with $\tilde{a} \neq 0.5$ can represent all the heteroclinic orbits. Equation 16 denotes the set consisting of heteroclinic bifurcation points in a parameter plane spanned by $\tilde{\beta}$ and $\tilde{\mu}$ as two solution one-dimensional manifolds. The system has only one heteroclinic orbit for certain parameter combinations of $\tilde{\beta}_0$ and $\tilde{\mu}_0$ by the uniqueness of the solution. It cannot be denied that there could exist other heteroclinic orbits for other parameter combinations. The heteroclinic orbit introduced here, however, becomes identical to Eq. A5 in Kan and Taguchi [6] when $\tilde{\beta}$ is zero (see Appendix 2), so that it is supposed that the heteroclinic orbit in which we are interested is realised for the parameter combination of $\tilde{\beta}$ and $\tilde{\mu}$ obtained from Eq. 16. Therefore, it is concluded that Eq. 16 is the required solution for our analysis within the framework of the present research.

4 Calculation of the Melnikov integral

Using the heteroclinic orbit obtained above and following the methodology introduced by Salam [10], the Melnikov integral can be calculated. State Eq. 8 can be rewritten in vectorial representation as:

$$\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix}\varphi\\ \dot{\varphi}\end{pmatrix} = \begin{pmatrix}\dot{\varphi}\\ -\tilde{\beta}\dot{\varphi} - \tilde{\mu}\varphi(1-\varphi)(\varphi-\tilde{a})\end{pmatrix} + \begin{pmatrix}0\\ b_0 + b\sin\omega t\end{pmatrix} \equiv \mathbf{F}(\mathbf{x}) + \mathbf{G}(t) \tag{22}$$

As shown above, the solution for $b_0 = 0$ and $b = 0$ can be obtained as Eq. 21. Here, we apply the Melnikov integral method:

$$D\mathbf{F}(\mathbf{x}) = \begin{pmatrix}0 & 1\\ \tilde{\mu}[3\varphi^2 - 2(\tilde{a}+1)\varphi + \tilde{a}] & -\tilde{\beta}\end{pmatrix} \tag{23}$$

yields

$$\mathrm{tr}D\mathbf{F}(\mathbf{x}) = -\tilde{\beta} \tag{24}$$

Note that the wedge product is defined as $\mathbf{a} \wedge \mathbf{b} = a_1 b_2 - a_2 b_1$. Then $\mathbf{F}(q^0(t)) \wedge \mathbf{G}(t+t_0)$ can be calculated as:

$$\mathbf{F}(q^0(t)) \wedge \mathbf{G}(t+t_0) = \dot{\varphi}(b_0 + b\sin\omega(t+t_0)) \tag{25}$$

Therefore, Melnikov function $M(t_0)$ is determined as:

$$\begin{aligned} M(t_0) &= \int_{-\infty}^{\infty} \mathbf{F}(\varphi^0(t)) \wedge \mathbf{G}(t+t_0) \cdot \exp\left(-\int_0^t \mathrm{tr}D\mathbf{F}(\mathbf{x})\mathrm{d}s\right)\mathrm{d}t \\ &= \int_{-\infty}^{\infty} b\sin\omega(t+t_0)\tilde{c}\varphi^0(t)(1-\varphi^0(t)) \cdot \exp\left(\int_0^t \tilde{\beta}\mathrm{d}s\right)\mathrm{d}t \\ &\quad + \int_{-\infty}^{\infty} b_0\tilde{c}\varphi^0(t)(1-\varphi^0(t)) \cdot \exp\left(\int_0^t \tilde{\beta}\mathrm{d}s\right)\mathrm{d}t \\ &= b\tilde{c}(I_\mathrm{i}\cos\omega t_0 + I_\mathrm{r}\sin\omega t_0) + b_0\tilde{c}I(0) \\ &= b\tilde{c}\sqrt{I_\mathrm{r}^2 + I_\mathrm{i}^2}\sin(\omega t_0 + \tan^{-1}(I_\mathrm{i}/I_\mathrm{r})) + b_0\tilde{c}I(0) \end{aligned} \tag{26}$$

Here, $I(\omega)$ is defined as the following Fourier transformation:

$$I(\omega) \equiv \int_{-\infty}^{\infty} \varphi^0(t)(1-\varphi^0(t))\mathrm{e}^{\tilde{\beta}t}\mathrm{e}^{i\omega t}\mathrm{d}t \tag{27}$$

where we put $I_\mathrm{r} = \mathrm{Re}[I]$ and $I_\mathrm{i} = \mathrm{Im}[I]$. The condition having the simple zero of Eq. 26 can be represented as:

$$\frac{I(0)}{\sqrt{I_\mathrm{r}^2 + I_\mathrm{i}^2}} = b/b_0 \tag{28}$$

Each component can be calculated by using Cauchy's integral theorem (see Appendix 3). Equations 18 and 34 provide the condition of the onset of chaos in the escape equation discussed here.

5 Concluding remarks

A fully analytical solution of the heteroclinic orbit was used for calculating the Melnikov integral to estimate the onset of chaotic behaviour of the escape equation. This approach is an alternative to the technique using a separatrix of the Hamiltonian part of the escape equation or a numerically obtained heteroclinic orbit of its non-Hamiltonian part. Verification of the proposed technique was achieved by comparison with the results from existing numerical work. The uniqueness of the heteroclinic orbit having the form of Eq. 10 should be mathematically examined in the future.

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Appendix 1

Hodgkin and Huxley [18] have shown that the shape and speed of pulses in the nerve of a squid are well-approximated by numerical solution of the Hodgkin–Huxley equation. Other closely related models were discussed by Fitzhugh and Nagumo [16]. Nagumo et al. [15] simplified the Hodgkin–Huxley equation as follows:

$$\frac{\partial e}{\partial t} = \frac{\partial^2 e}{\partial x^2} + e(1-e)(e-a) - b\int e\mathrm{d}t \tag{29}$$

where e is a function of x and t, and $0 < a < 1$. Assuming that $b = 0$ and $e(x, t) = e(x + ct)$ yields:

$$\frac{\mathrm{d}^2 e}{\mathrm{d}t^2} - c\frac{\mathrm{d}e}{\mathrm{d}t} + e(1-e)(e-a) = 0 \tag{30}$$

This equation is identical to Eq. 9, except for some of the coefficients.

Appendix 2

We explain that for $\tilde{\beta} = 0$, Eq. 21 leads to Eq. A5 in Kan and Taguchi [6]. As mentioned above, $\tilde{d}$ is an arbitrary constant. Putting $\tilde{d} = 0$ and taking $\tilde{c} = \sqrt{\tilde{\mu}/2}$ in Eq. 15, then the following equation can be obtained:

$$\varphi = \frac{1}{2} + \frac{1}{2}\tanh\left(\frac{t\sqrt{\tilde{\mu}}}{2\sqrt{2}}\right) \tag{31}$$

This orbit is defined within the open set $\varphi \in (0, 1)$, so that utilizing the change of variable $\varphi = (\psi + 1)/2$ yields:

$$\psi = \tanh\left(\sqrt{\frac{\mu}{2}}t\right) \tag{32}$$

This result is identical with Eq. A5 in Kan and Taguchi [6].

Appendix 3

Here we briefly consider an integral having the form of Eq. 27. This equation can be rewritten as follows:

$$I(\omega) = \frac{1}{4}\int_{-\infty}^{\infty} \frac{\exp\left(\tilde{\beta}t + i\omega t\right)}{\cosh^2(\tilde{c}t/2)}\mathrm{d}t \tag{33}$$

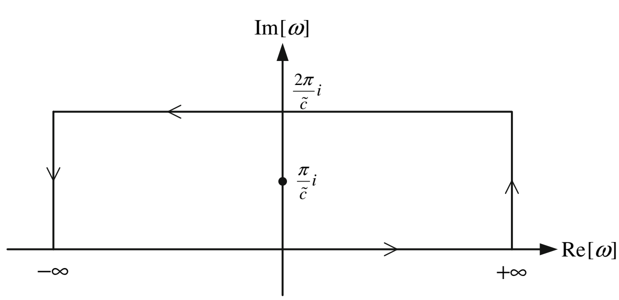

Fig. 1 An integral route for positive $\tilde{c}$

Taking the integral route as shown in Fig. 1, Cauchy's integral theorem easily leads to the following result:

$$I(\omega) = \frac{\pi\left(\tilde{\beta} + i\omega\right)\csc\left[\pi\left(\tilde{\beta} + i\omega\right)/\tilde{c}\right]}{\tilde{c}^2 \mathrm{sgn}\tilde{c}} \tag{34}$$

Note that a singular point of Eq. 33, i.e., $t = \pi i(2n+1)/\tilde{c}$, is a pole of order 2. Here n denotes an arbitrary integer.

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