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Fourier NUBS method to express ship hull form

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Abstract This article presents a method of numerical expression to draw ship hull forms. Recent developments in research into ship hull optimization need a method to express ship hull form as precisely as possible with a small number of design parameters. This method is based on a combination of the Fourier–sine series expansion and nonuniform B-spline (NUBS) interpolation. The merit of the combination is that it removes the wiggles which are often found when generating a rectangular-type curve, such as the midship section of tanker ship, with a simple Fourier series expansion. Here, the procedure is explained, and then a tanker ship hull is generated. Through the discussion, we show the effectiveness of the method.

Key words Fourier NUBS method · Hull form expression · Ship design

Introduction

Recently, as computer models have improved, more and more research into the optimization of ship hulls has been carried out. In ship hull optimization, it is very important to develop a method to express the ship hull precisely with as few design parameters as possible, because any increase in the number of parameters results in a large amount of computation time in the course of the optimization.

Roughly speaking, there are two ways in which a ship hull can be deformed. One is when the hull is changed by use of weight functions that are applied to a known ship hull, and the other is when the ship hull is deformed directly. Suzuki and Iokamori,[1] Hino,[2] and Minami and Hinatsu[3] used the weight function method. In their work, the weight functions are based on the sine function. For the weight function method, successful changes to the ship hull depend on the type of weight function, for instance, a weight function based on the sine function does not seem to be able to modify a ship hull while keeping the essential features of a practical hull form. On the other hand, for a direct deformation method, many researchers use a B-spline net to express the hull form. For instance, Koyama et al.[4] used a surface B-spline net to design a ship hull form. Chen and Huang[5] also used a B-spline surface net to generate the hull form when optimizing the wake of a ship, and Ragab[6] used a similar method to express a submerged body. Peri et al.[7] adopted a Bezier patch to modify a ship hull. In their method, they added the patch to the original ship hull to achieve deformation. In these cases, the design parameters are the locations of the control points. In general, one control point has three degrees of freedom, and this may cause an increase in the number of design parameters. To avoid this, we can use some combination of mathematical functions. For example, a Fourier series expansion is one choice for generating an arbitrary shape. However, it is not easy to express a curve with a large curvature, because unnatural wiggles are likely to occur. The B-spline curve has the property of a convex hull. This property ensures that the interpolated B-spline curve does not go outside the convex hull surrounded by the control points. This means that the B-spline curve can be used to give a smoother function.

In the present method, the basic idea consists of a combination of the favorable features of both methods. A Fourier series expansion can approximately represent the girth line with some scalar coefficients under mathematically assured accuracy, while the wiggles and overshoot often found in a Fourier series expansion can be avoided because of the convex hull property of the B-spline function. In this work the nonuniform B-spline (NUBS) function is used.

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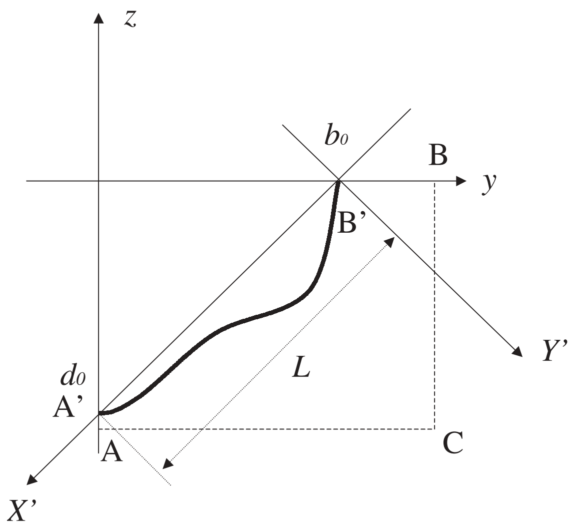

Fig. 1. Coordinate system to express the girth line

In the following section, a method to fit a girth line is presented, and its application to a tanker ship form is shown. Then, the effectiveness of the method is clarified in the discussion.

Body plan fitting with the Fourier NUBS method

Coordinate system

The coordinate system which explains the present method is shown in Fig. 1. The y-axis is the breadth-wise coordinate, and is positive toward starboard. The z-axis is the vertical coordinate positive upward. In the figure, a girth line is shown as a bold line. Here, the depth d_0 is arbitrary, and we may take it as the design load draft, or as a much greater depth if we fit the girth line from above the load water line to the bottom. b_0 is the local breadth at the deck top (including the case when $z = 0$). The letter A indicates the bottom of the ship, and B indicates the breadth of the ship. Then lines AC and BC are the limiting lines to prevent unnatural wiggles when expressing the hull form, as long as we treat common ship hull forms only.

Coordinate transformation

We now consider the new coordinate system shown in Fig. 1. The origin B′ is set on the girth line on the y-axis. Then the X'-axis is chosen such that it goes from the origin toward the point of the girth line on the ship's center plane, A′. The Y'-axis is normal to the X'-axis. Furthermore, the X', Y' coordinate system is normalized by L, the length of A′B′, and the newly derived coordinate system is written in the X, Y system, as shown in Fig. 2.

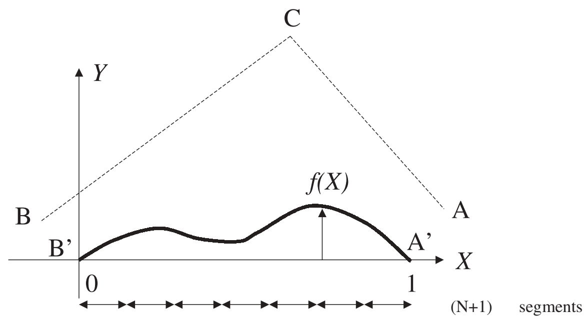

Fig. 2. Transformed coordinate system for the girth line

Fourier expansion

The basic idea of this method is that the control points of the NUBS function for expressing girth lines should be evaluated by the Fourier series. In order to set the control points for determining girth lines, we need suitable initial values of the coefficients of the Fourier series. In order to give suitable Fourier coefficients for the girth lines, we expand the basic ship's girth lines, say $Y = f_{\mathrm{orig}}(X)$, into a Fourier series as follows:

$$f_{\mathrm{approx}}(X) = \sum_{n=1}^{N} A(n)\sin(n\pi X),$$
$$A(n) = 2\int_0^1 f_{\mathrm{orig}}(X)\sin(n\pi X)\,\mathrm{d}X \tag{1}$$

Note that $f_{\mathrm{orig}}(X)$ is the girth line of the basic ship in the transformed coordinates X, Y, $f_{\mathrm{approx}}(X)$ represents the approximate girth line, which may contain some wiggles, and N is the number of terms of the Fourier series expansion.

Set of control points for NUBS interpolation

We then set the control points of the NUBS curve to approximate the girth line. The definition of a NUBS function[8,9] is

$$P(t) = \sum_{i=0}^{n-1} N_{i,m}(t)q_i \tag{2}$$

$$N_{i,m}(t) = \frac{t - t_i}{t_{i+m-1} - t_i} N_{i,m-1}(t) + \frac{t_{i+m} - t}{t_{i+m} - t_{i+1}} N_{i+1,m-1}(t),$$
$$N_{i,1}(t) = \begin{cases} 1 & (t_i \le t < t_{i+1}) \\ 0 & (t < t_i, \quad t_{i+1} \le t) \end{cases} \tag{3}$$

where q_i is the coordinates of the set of control points, m is the order of the NUBS function, which is set to 4 in this work, t_i is the set of knot vectors, and $P(t)$ is the interpolated function. In this method, the X coordinates of the control points are determined as described below.

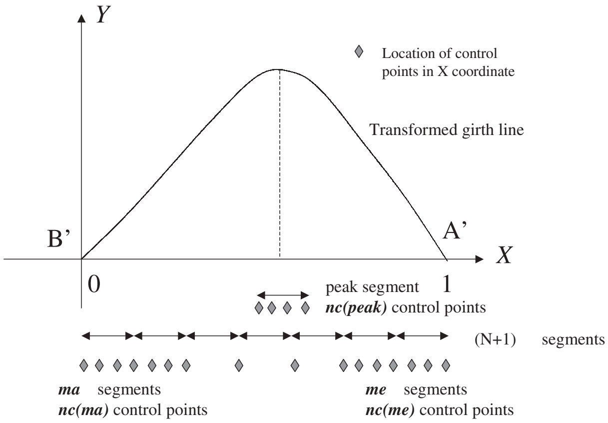

Fig. 3. Schematic figure for the location of control points on the noruri form B-spline (NUBS) curve

1. Divide the $0 \leq X \leq 1$ domain into $N + 1$ equilength segments. Since the highest order of a Fourier series expansion is N, the number of segments should suit set N and not set $N + 1$. However, numerical trials have shown that $N + 1$ divisions gives a much better result. Then the control points are located on both ends of each segment.
2. Near $X = 0$ and $X = 1$, Fourier series expansions may not follow an original curve with a very large or small gradient, and in order to protect against this drawback, more control points are put around these regions. In this study, the *ms* and *me* segments adjacent to $X = 0$ and $X = 1$ are set, and within in these segments, the number of control points are set at *nc*(*ms*) and *nc*(*me*), respectively (Fig. 3).
3. The region around a peak point with a large curvature is also difficult to express without enough control points. In this work, only the region containing the highest peak point is used to set an additional *nc* (peak) control points. Here, the segment whose center coincides with the peak point is found, and then as additional *nc* (peak) control points are set in that segment. It may be necessary to use 4 or 5 points to reflect a sharp corner (Fig. 3). In this study, the parameters *ms* and *me* are set to 2 for $N \leq 10$, and 3 for $N \geq 11$. In both cases, *nc*(*ma*) and *nc*(*me*) are 5 and *nc* (peak) is set to 4.
4. After determining all the X coordinates of the control points, the Y coordinates of the control points can be computed by the first part of Eq. 1. In the computation of Y, if a control point exceeds the limiting lines AC or BC, the control point is reset on the line. This process ensures that the computed girth line does not pass over the limitation line owing to the convex hull properties of B-spline interpolation. After all the procedures are complete, the control points are established and then we can compute the girth lines with the NUBS functions in Eqs. 2 and 3. In this study, a uniformly distributed knot vector was used. Then the NUBS function becomes a simple B-spline function. Alternatively, we can use the equation[9]

$$t_0 = \ldots = t_{m-1} = 0, \qquad t_n = \ldots = t_{n+m-1} = 1,$$
$$t_{j+m-1} = \frac{1}{m-1} \sum_{i=j}^{j+m-2} s_j, \qquad (j = 1, \ldots, n-m) \tag{4}$$

where n is the number of control points, m is the order of NUBS, and s_i is the parameter of the given points on which the NUBS curve passes. s_i is often taken as the distance along the set of the given points.

Inverse transformation of the coordinate system

After setting all the control points, a new girth line is computed by the NUBS function. Then the computed curve in X,Y space is inversely transformed so that the Fourier NUBS-computed girth line can be obtained in (y,z) space. In this method, note that one girth line can be expressed with $N + 2$ design parameters, i.e., breadth at the deck top b_0, and depth d_0, and N coefficients of the Fourier sine series.

Limitation of the present method

Since we express the girth line using the Fourier series in X,Y space, only the curve of a single-value function can be fitted. Hence, we cannot interpolate a line such as a twin-skeg stern form and a top-flat bulbous bow. This is a limitation of the present method, but most parts of an ordinary ship hull with a single propeller are not likely to have such a property.

Set of the surface net of control points for surface NUBS interpolation[9]

Consider the net of points on the surface $P_{i,j}$ and the net of control points $q_{k,l}$. Then the NUBS surface interpolation can be written as

$$P_{i,j} = \sum_{k=0}^{nu-1} N_{k,mu}(u_i) \sum_{l=0}^{nv-1} N_{l,mv}(v_j) q_{k,l} \equiv \sum_{k=0}^{nu-1} N_{k,mu}(u_i) R_{k,j}$$
$$(i = 0, \ldots, npu-1, \qquad j = 0, \ldots, npv-1) \tag{5}$$

Here, *mu* and *mv* are the order of the spline functions in the u and v directions, respectively, *nu* and *nv* are the number of control points in the u and v directions, respectively, and *npu* and *npv* are the number of points of the $P_{i,j}$ net in the u and v directions, respectively. u_i and v_j are the parameters of $P_{i,j}$, which is often given as the distance parameter. In Eq. 5, $R_{k,j} = \sum_{l=0}^{nv-1} N_{l,mv}(v_j) q_{k,l}$ is used. Then, if the surface points $P_{i,j}$ are given, we can compute the net of control points $q_{k,l}$ by inversing Eq. 5 in the following manner. First we solve the inverse equation

$$R_{k,j} = \left(\sum_{k=0}^{nu-1} N_{k,mu}(u_i) \right)^{-1} P_{i,j} \tag{6}$$

This equation shows that the computed $R_{k,j}$ can be regarded as the control points for the isoparameter u line, especially in this study, since it corresponds to each set of control points along the girth line, and this process can be skipped. Next we solve the inverse equation

$$q_{k,l} = \left(\sum_{l=0}^{nv-1} N_{l,mv}(v_j) \right)^{-1} R_{k,j} \tag{7}$$

Then the surface net of control points can be evaluated. However, in order to use the above method, we need to adjust the number of control points for each girth line. In the present method, we newly interpolate $(N + 1) \times nc$ (peak) control points for each girth line using the control points computed with the above-mentioned process (Eqs. 1–4). After attaining the net of control points, the hull surface interpolation can be carried out by computing Eq. 5.

Results and discussion

In this work, a KVLCC2 tanker ship hull[10] was used to show the effectiveness of a new method. First, it is important to find an appropriate value for N. Generally, as N gets larger, the accuracy of the girth line fitting should become better. However, a large value of N is not satisfactory when applying this method to a ship hull optimization. In order to investigate a suitable value for N, we define the area error due to the discrepancy of the girth lines as

$$\delta S/S_0 = \int_{-d_0}^{\text{deck top}} \left| F_{\text{approx}}(x,z) - F_{\text{orig}}(x,z) \right| \mathrm{d}z \Big/ \int_{-d_0}^{\text{deck top}} \left| F_{\text{orig}}(x,z) \right| \mathrm{d}z \tag{8}$$

where $F_{\text{approx}}(x,z)$, $F_{\text{orig}}(x,z)$ are the breadths of the computed and original girth lines at (x,z), respectively. This value is the ratio of the small area surrounded by the computed and original girth lines to the original sectional area, and it may be regarded as an index of how accurately this method can express the original girth line. Although we may choose the standard deviation of the discrepancy $| F_{\text{approx}}(x,z) - F_{\text{orig}}(x,z) |$ as another index, the value defined by Eq. 8 gives the error relative to the original sectional area, and we adopt this as the accuracy index in this study. In this work, we used the average of these indices of the three girth lines at equidivided locations between $x = -0.495$ and 0.405 as the index. Since these control points are evaluated by using the expanded Fourier coefficients of the original ship hull, in theory the computed girth lines do not completely coincide with the original girth line. We now realize the accuracy of the present method in expressing a practical ship hull form through this index.

Figure 4 shows the change in this index as N varies from 5 to 20. From this figure, N = 11 seems large enough, since the reduction in the error becomes very small above N = 11. Therefore, we can draw an accurate girth line with a total of 13(=N + 2) unknown parameters by this method.

Figure 5 shows the mapped girth lines with N = 11 at x = –0.495, –0.045, and 0.405 in X,Y space, and corresponds to Fig. 3. These lines can be regarded as typical forms of the girth lines, and the agreement of the com-

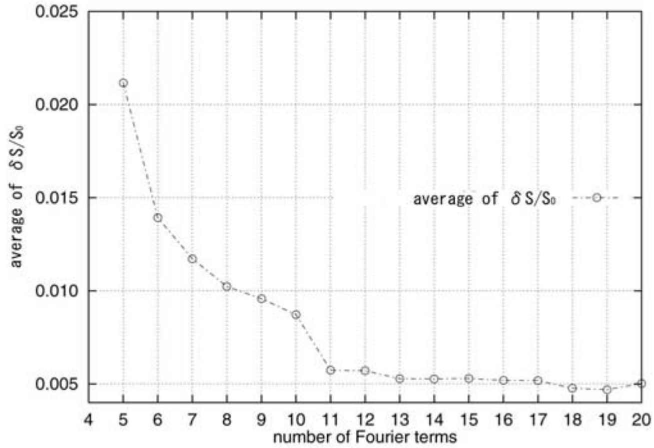

Fig. 4. Accuracy of curve fitting for various numbers of Fourier terms using NUBS functions

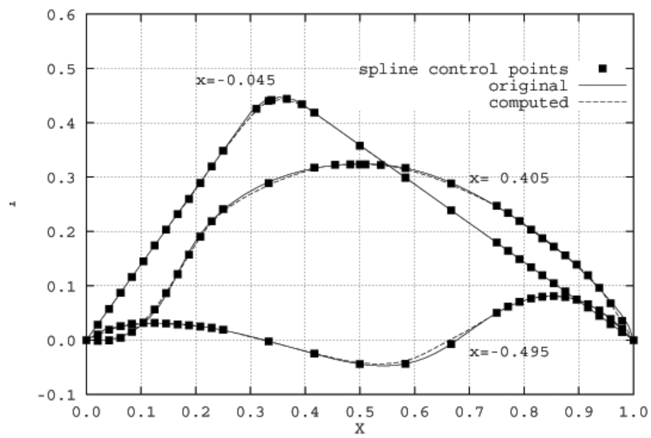

Fig. 5. Original and computed girth lines and control points of NUBS on transformed coordinates for KVLCC2

Table 1. Coefficients of a Fourier sine series to fit girth lines at x = –0.495, –0.045, and 0.405

x	–0.495	–0.045	0.405
Breadth b_0	0.00922	0.09063	0.065
Depth d_0	0.06282	0.06711	0.065
Fourier coefficients for different numbers of terms			
1	0.32868	0.37302	0.00157
2	–0.01915	0.08879	–0.01342
3	–0.00383	–0.00267	0.05320
4	–0.02493	–0.02236	–0.01605
5	–0.01239	–0.01238	0.00752
6	–0.01673	0.00108	0.00225
7	–0.00505	0.00629	0.00133
8	–0.00569	0.00383	0.00013
9	0.00113	–0.00033	0.00203
10	–0.00020	–0.00206	4.14E-05
11	0.00278	–0.00127	0.00107

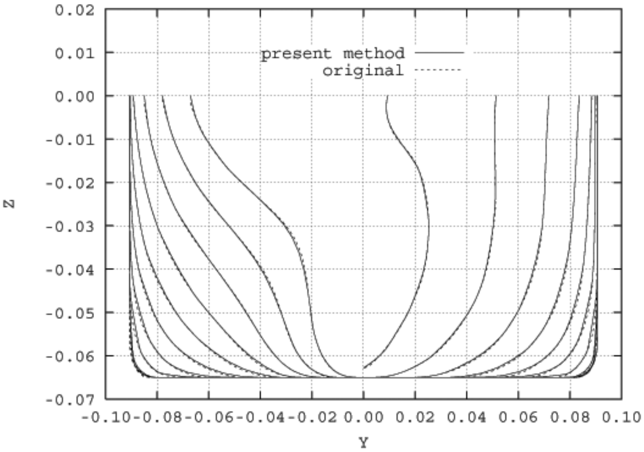

Fig. 6. Comparison of girth lines obtained by the present method with original lines (KVLCC2, $N = 11$, from the bottom to the full-load waterline)

puted hull form generation with the original girth lines can be regarded as satisfactory for the effectiveness of this method, as is well shown in Fig. 5. The locations of the control points are also suitably arranged to deal with sharp corners such as the bilge circle and the portions adjacent to the bottom and deck top near the bow section (x = –0.495). Hence, this method is capable of expressing hull form numerically.

In Table 1 shows 13 parameters corresponding to the three girth lines in Fig. 5. For girth lines around the ship's stern and mid-ship, only the first coefficient is relatively large, while for the line near the bow, all coefficients are of similar magnitude.

A comparison of the original and computed girth lines with the Fourier NUBS method is shown in Fig. 6. In this figure, the case where N = 11 is shown. The computed girth lines agree well with the original lines. Furthermore, no wiggles appear in the computed lines. We can also draw the girth line from the deck top, as shown in Fig. 7. In this case, the breadth at the load water line is determined by a combination of the Fourier coefficients. In other words, in this case the breadth at the load water line is implicitly determined. Since the breadth at the deck top could easily be given in advance by comparing it with that of the load water line, b_0 at the deck top may be regarded as a given parameter, and then the total number of design parameters for ship hull optimization decreases from $N + 2$ to $N + 1$.

From the control points at each girth line, we can set the surface net of control points using the method given above. Figure 8 and 9 show the front and aft views of the surface of an interpolated ship's hull. Although the bulbous bow and the stern end are not drawn here, the other parts can be expressed without any unnatural wiggles. Therefore, we can see that this method could

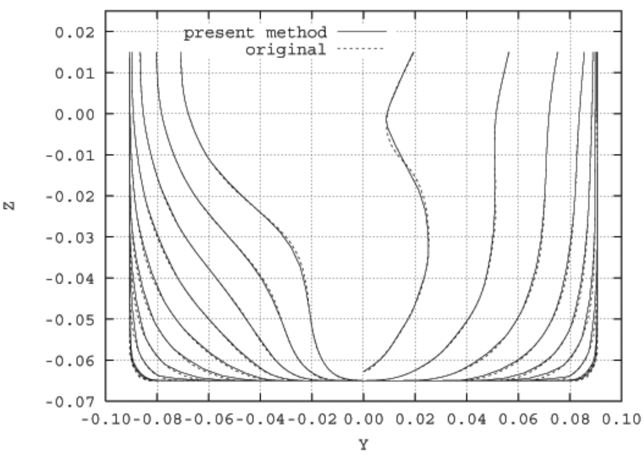

Fig. 7. Comparison of girth lines obtained by the present method with original lines (KVLCC2, $N = 11$, from the bottom to above the full-load waterline)

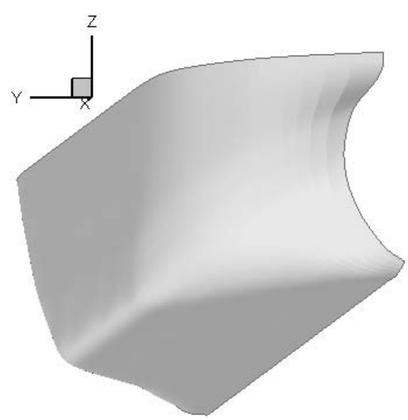

Fig. 8. Hull surface obtained by the present method (view from bow)

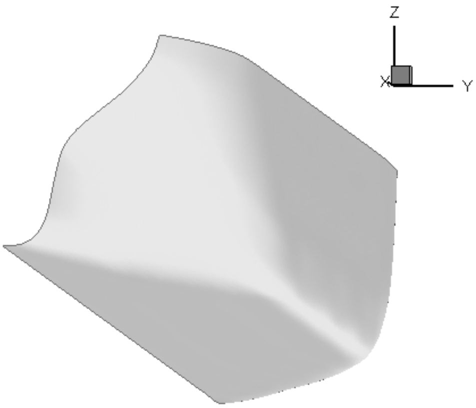

Fig. 9. Hull surface obtained by the present method (view from stern)

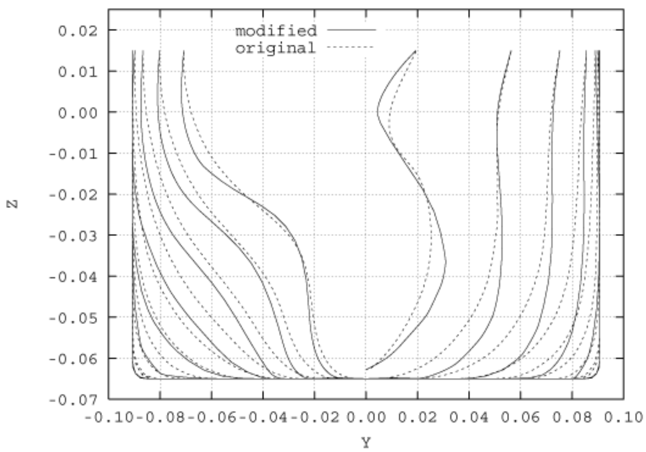

Fig. 10. Deformation of girth lines by changing the first three Fourier coefficients (KVLCC2, $N = 11, f_1 = 1.2, f_2 = 1.5, f_3 = 1.5$)

be useful to express a practical hull form by a set of parameters.

Lastly, we show the sensitivity of the Fourier coefficients to changes in the ship's hull form. This gives important information about how good the present method is for hull shape optimization. Here, we changed the first three Fourier coefficients by multiplying by constants

$$A'(n) = f_n A(n), \qquad (n = 1, 2, 3) \tag{9}$$

where $A'(n)$ is the new Fourier coefficient, f_n is the multiplier, and $A(n)$ is the original Fourier coefficient. Figure 10 shows the case where $f_1 = 1.2$, $f_2 = 1.5$, and $f_3 = 1.5$. The original girth lines around the ship stern are modified into a U- or T-shape. Figure 11 shows the case where $f_1 = 1.2$, $f_2 = 0.5$, and $f_3 = 0.5$. The original girth lines around the ship stern are then modified into a V-shape. Therefore, for the optimization of the ship stern

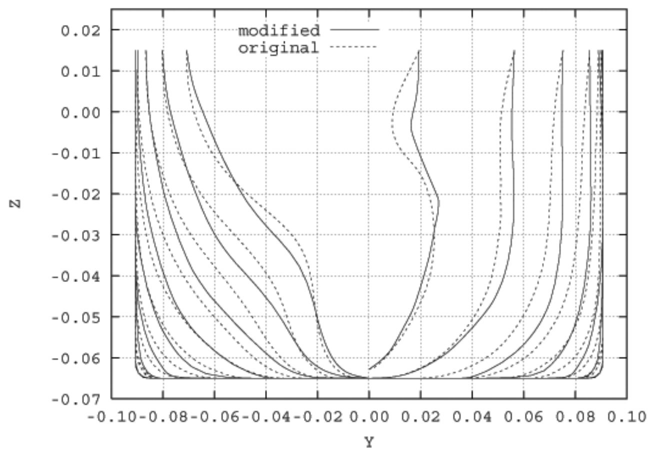

Fig. 11. Deformation of girth lines by changing the first three Fourier coefficients (KVLCC2, $N = 11, f_1 = 1.2, f_2 = 0.5, f_3 = 0.5$)

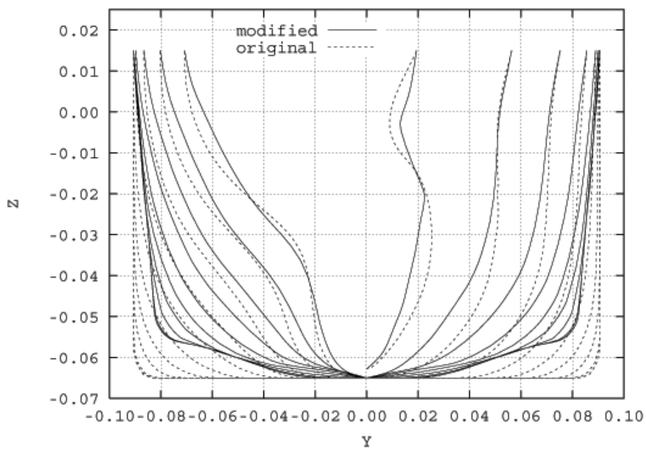

Fig. 12. Deformation of girth lines by changing the first three Fourier coefficients (KVLCC2, $N = 11, f_1 = 0.8, f_2 = 0.5, f_3 = 0.5$)

shape, $A(2)$ and $A(3)$, could be used as the main design parameters. In this case, the number of design parameters for each girth line is only two.

On the other hand, the above combination only makes the bow broad. In order to make the bow slender, it is necessary to make $A(1)$ decrease. Figure 12 shows the case where $f_1 = 0.8$, $f_2 = 0.5$, and $f_3 = 0.5$. In this case, the bow frame line becomes a V-shape. However, the decrease in $A(1)$ makes hull shape extremely fine, and this choice should be limited to either the bow or the stern only. From these figures, typical changes in the girth lines around the bow and stern can be obtained by changing the first three Fourier coefficients only. This means that only three design parameters are required. Therefore, when we use four girth lines to modify the aft or bow part of a ship hull form, for instance, we need a total of 12 design parameters. This number seems to be practical. Therefore, we believe that the present method is promising for hull form modification in ship hull optimizations.

Conclusions

A new method based on a combination of the Fourier series and a NUBS function is introduced to express a ship hull form numerically. By using this combined Fourier NUBS method, we can express a practical hull form with acceptable accuracy. By changing the first three Fourier coefficient terms, we can modify a ship hull form from a V-shape to a U- or T-shape. This means that the method can be expected to be a promising tool for hull form expression in ship hull form optimization. In future, a hull form expression including both the bow and stern end portions should be developed.

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