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Licensing of a new technology by an outside and uninformed licensor

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Abstract
We examine the licensing decision of a non-producer innovator with a new technology that enables the manufacture of a saleable product. The technology is licensed and each user privately knows its innovation-related production cost, whereas the licensor only knows, with a certain probability, that this cost may be low (the user is efficient) or high (the user is inefficient). When a single licence is granted through separating contracts, only the contract intended for the inefficient user involves a per-unit royalty, but when two licences are granted through separating contracts, the contracts intended for the inefficient and efficient users both feature a per-unit royalty. However, screening is less likely as the number of licences increases, to the point that the licensor does not screen users when granting three licences. Additionally, whereas the diffusion of the innovation is socially insufficient under symmetric information, with asymmetric information it may be socially optimal. Finally, when licensing with contracts involving an ad-valorem royalty is also feasible the licensor finds it less attractive than licensing with a per-unit royalty.

Keywords New technology · Exclusive and non-exclusive licensing · Asymmetric information · Screening · Per-unit royalty · Ad-valorem royalty · Welfare

JEL Classification D43 · D82 · L24

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1 Introduction

Licensing is particularly important for innovators that lack productive capacity to implement their innovations themselves. This is the case, for example, of universities, since the US Bayh-Dole Act[1] of 1980 facilitated their ability to control the patents emanating from federally funded research. With patent control, universities and other non-producer innovators can commercialize their innovations by licensing them or using other means, such as by creating start-ups, spin-offs, etc. More generally, licensing is an integral part of technology innovation (Doganoglu and Inceoglu 2014) and has become a relevant topic of debate in the open innovation era, whereby innovation processes are opened up beyond firms' boundaries to enhance innovative potential.

The price of a licence is, however, a matter of dispute. In the absence of established rules, the parties often differ in their valuation of the innovation to be licensed. Because intellectual property assets are often unique, valuation is inherently uncertain, while licensing terms are typically confidential and vary widely (Hickey et al. 2018). It is therefore the natural aim of the licensor to seek how to frame a contract to obtain the highest possible return, yet the contracting scheme is likely to be very dependent on the circumstances of both the licensor and licensees. In addition to other factors, the contractual structure may be affected by the information each party has regarding the value of the innovation, the number of licences granted, and the mode of marketplace competition between licensees.

In this paper, we consider the simultaneous choice regarding both the number of licences and the contractual scheme faced by an outside and uninformed licensor in transferring a new technology to one or several firms capable of producing a saleable good with it. We assume that while each technology user privately knows its own innovation-related marginal cost of production, third parties only know that such a cost has been selected by nature from a set that is high in value (i.e., the technology is inefficiently used in producing a certain good) or low in value (i.e., the technology is efficiently used in production).[2] To ascertain the technology value, the licensor can either offer a single licence, which leads the user to produce as a monopolist, or several licences, which leads the users to play as Cournot firms. In both cases, we allow for two-part tariff (2PT) contracts consisting of a fixed payment combined with a royalty, which in turn can be a fee per unit produced (per-unit royalty) or a percentage of sales (ad-valorem royalty).[3]

In this context, we explore the role played by the number and structure of contracts in extracting private information about the technology's value from its users, and report some relevant results. First, when granting a single licence and both the

[1] The university and small business patent procedures (Bayh-Dole act) of 1980. Public Law 96–517. 1980; 94 Stat. 3015.

[2] Macho-Stadler et al. (2005), using a framework where firms have incomplete information on the quality of inventions, provided a reputation argument for the licensor to reduce the asymmetric information problem.

[3] Both schemes are widely explored in the literature (see, among others, Erutku and Richelle 2007; Sen and Tauman 2007; Hagiu and Wright 2019; or Hsu et al. 2019). Licensing through contracts that may feature an ad-valorem royalty instead of a per-unit royalty is examined in Sect. 7.

probability and cost advantage of being an efficient user are low, the licensor offers a menu of contracts, that leads the user to disclose its type (i.e., it offers a separating screening contract) and involves a per-unit royalty. In this menu, the contract intended for an inefficient user features a royalty rate and therefore will distort market behaviour (distortion at the bottom), but not the contract intended for an efficient user (no distortion at the top). Otherwise, the licence consists of a non-distorting payment only accessible to the user if it is efficient and therefore leads the innovation to only be marketed with a certain probability.

However, when granting two licences by means of a menu of separating screening contracts for each user, both the contract intended for the inefficient and the efficient user include a per-unit royalty, which distorts production for both user types. The rationale is as follows. When the licensor issues a single licence by means of a separating contract, the user is screened by the contract intended for the inefficient type being rendered unattractive to the efficient type. That contract, but not the contract intended for the efficient type, features a per-unit royalty that distorts the market behaviour of the inefficient type (Poddar and Sinha 2008)[4] and that also minimizes the information rents to be granted by the licensor to the efficient user. However, when the licensor trades with two users, the offer of a menu of user-specific contracts that induces each one to self-select can be seen as a two-stage decision. In the first stage, the licensor tries to induce monopoly-like market behaviour (collusion effect), which requires that the contracts intended for both user types involves a per-unit royalty. This allows the licensor to reduce the negative externality caused by users' competition (rent dissipation effect). In the second stage, the situation resembles an exclusive-licensing context: the contractual menu features both a contract intended for efficient users that does not distort the production quantity, and a contract intended for inefficient users that allows for an additional distortion so as to be less attractive to efficient types.

Our second finding refers to the impact of adverse selection on the number of issued licences. Specifically, the licensor is more likely to resort to non-exclusive licensing under asymmetric information than under symmetric information. Under asymmetric information, we distinguish between screening licensing and licensing restricted only to efficient users: if the licensor chooses a screening contract, it is better off granting a single licence. Under symmetric information, in contrast, the licensor obtains a higher expected payoff from non-exclusive licensing, because the rent dissipation effect is offset by the use of a per-unit royalty (Antelo and Sampayo 2017). However, to induce screening, the licensor would need to include a per-unit royalty in the contracts, but this would be so detrimental for its revenues that exclusive licensing is preferred—even though it entails foregoing the advantages of the sampling effect. This result resembles previous findings by Schmitz (2002) for a slightly different context.

[4] The no distortion at the top property (Mussa and Rosen 1978) does not only appears in the licensing literature, but also in many other areas of industrial organization, e.g., vertical differentiation. This would corroborate the general argument boiling down to designing a screening mechanism across a population of individuals who are knocking at the door of a firm in seemingly different contexts.

Regarding welfare, we find that, under symmetric information but not under asymmetric information, non-exclusive licensing as compared to exclusive licensing is unequivocally welfare-enhancing. Under asymmetric information, expected welfare is higher with non-exclusive licensing, except when the licensor screens users, as with screening, social welfare under exclusive licensing is greater even though consumers are better off with non-exclusive licensing. The implication is that, under screening, the loss in consumer surplus caused by a switch to exclusive licensing is more than offset by the increase in the licensor's payoff. This reinforces our second finding by underlining the strength of a screening contract's negative effect on the licensor's payoff when combined with non-exclusive licensing.

Fourth, when the technology is simultaneously licensed to more than two users, a screening contract for each licensee is less likely. Indeed, the licensor would not be tempted to use such a contract when issuing three licences. The rational is as follows: The competition that arises between users makes that the induction of disclosure and the attempt for the triopoly to replicate the monopoly outcome would lead the licensor to distort the contracts designed for each user type in such a way that they would not be profitable. Overall, and compared to issuing a single licence, the licensor prefers to grant three licences under almost identical circumstances that make it prefer to grant two licences.

Finally, we try to convey how the results would be in a scenario in which an ad-valorem royalty is feasible in licensing deals. In this case, we show that, regardless of whether the licensor issues one or two licences, it does not resort to any contract featuring an ad-valorem royalty, and the optimal contractual scheme is therefore the same as when royalties are restricted to a fixed quantity per unit produced. Hence, in our framework we may conclude that a new technology is licensed through contracts based on a fixed payment alone or a fixed payment combined with a per-unit royalty, but not with an ad-valorem royalty.

The remainder of the paper is structured as follows. Section 2 reviews the relevant literature on the subject and Sect. 3 describes our model setup. Section 4 determines the equilibrium of the licensing game when both one licence and two licences are granted and Sect. 5 analyses the welfare impact of the licensor's behaviour. Section 6 focuses on licensing to three potential users and Sect. 7 considers contracts involving an ad-valorem royalty, rather than a per-unit royalty. Finally, Sect. 8 concludes. An appendix contains the proofs of the results obtained.

2 Literature review

The licensing literature has largely explored how an external patent holder with an innovation maximizes its licensing revenues in various contexts. In particular, if the information about the innovation's value is complete, preferred is a two-part tariff (2PT) contract, whereby the licensee only receives the reservation profit (Erutku and Richelle 2007; Sen and Tauman 2007). However, under asymmetric information, the licence can serve as a signalling or screening device, and hence, the contractual structure can be very diverse. For example, if the licensee is better informed about the innovation's value than the licensor, it offers the licensor a single proposal with a per-unit royalty, and a separating equilibrium that allows a more efficient outcome than a fixed-fee contract is possible (Beggs 1992). Contrariwise, when the licensor is better informed about the innovation's value, there may be a conflict between the signalling aim and rent extraction, as shown by Wu et al. (2021). Since a (fixed-fee) contract by an efficient innovator that extracts too much rent invites mimicking by the inefficient innovator, the efficient innovator may switch the fixed-fee contract to a pure royalty contract to discourage the mimicking. Moreover, when potential innovation users configure a sufficiently competitive market, the pure royalty contract will eventually win over the 2PT contract based on a fixed fee combined with a per-unit royalty, because it more effectively signals the efficient user.

In the context of a licensor that is less informed about the innovation's value than its potential users, we may open the lens to study the licensing scheme when the licensor simultaneously chooses the payment terms and the number of issued licences. Although decision-making regarding the number of licences tends to be overlooked by research, it is crucial to exploring its role in the contractual structure (Antelo and Sampayo 2017; Wu et al. 2021). In addition, the possibility of granting one or several licences is a real-world phenomenon, because licensors typically must not only determine contractual terms, but also the number of licensees (Li and Wang 2010).[5]

The number of licensing contracts as a mechanism to minimize informational rents by an uninformed patent holder was first studied by Schmitz (2002). He considered a research lab owning a new patented technology, that cannot itself manufacture a marketable final product, and so the technology is licensed to two downstream firms that might successfully develop a saleable product. If a downstream firm's profits from being the sole licensee is private information, the research lab may sell two licences, even though under complete information it would sell only one. This contrasts sharply with the standard result that, when private information holds, a monopolist will sometimes serve fewer, but never more, buyers. With this remarkable finding, Schmitz (2002) can be considered a seminal work that spurred a lot of subsequent work in different but related contexts.

The number and form of licensing contracts combined have also deserved attention in various contexts. Li and Wang (2010) found that the licensor's preference for granting a single licence depends on both the licensing contract and the novelty of the innovation, and also that non-exclusive licensing may be welfare-reducing. Doganoglu and Inceoglu (2014), considering an upstream innovator, with a novel technology that enables the production of differentiated goods, that must decide whether or not to enter the downstream market, as well as the number of licences and contractual terms, show that a monopolist remaining outside the industry can replicate multiproduct monopoly profits by means of 2PT contracts. Antelo and Sampayo (2017) propose a two-period licensing game in which a non-producing and uninformed innovator licenses a new technology that lasts for two periods to either

[5] A 2001 survey by the Association of University Technology Managers (AUTM) found a nearly even split between the number of exclusive versus non-exclusive licences (Caballero-Sanz et al. 2005). In the same vein, Li and Wang (2010) document several cases that illustrate the convention of using both exclusive and non-exclusive contracts.

one or two users. They show that, when the first-period output of each user signals its technology-related production cost, the innovator is more likely to grant two licences than would be the case under symmetric information. This highlights the benefits of granting several licences and making users compete as a way to reduce information rents. Likewise, Wu et al. (2021) find that, when all firms in an oligopolistic industry must be licensed, a licensor with privileged information about the value of its technology uses a higher but still suboptimal royalty to signal that value. Moreover, if the number of granted licences is endogenously determined, the licensor should use the number of licences—rather than a higher royalty rate—to signal the innovation; thus, in the separating equilibrium that maximizes the licensor's payoff, an efficient licensor grants fewer licences than an inefficient licensor.

Taking Schmitz (2002) as reference, we build a model that differs in three aspects. First, in Schmitz (2002), the issuing of a single licence (exclusive licensing) leads the user to develop a marketable final product with a certain probability, while the concession of two licences (non-exclusive licensing) leads both users to develop the final product and obtain zero profits due to (Bertrand) competition in the marketplace. However, in our model, exclusive licensing guarantees the final product—whenever both user types agree to produce—whereas non-exclusive licensing allows users to obtain a profit due to Cournot competition in the marketplace. Second, in Schmitz (2002), contractual payments are fixed fees, while we allow for a fixed fee alone or combined with a royalty. A third difference that deserves to be highlighted is that we, unlike Schmitz (2002), study the welfare effects of licensing. Therefore, we can understand our work as an inquiry about the robustness of the Schmitz's results to those differences in the environment in which the licensing contract takes place.

Other papers mentioned above have pursued a similar spirit to ours, but also differ from our framework. Li and Wang (2010) and Doganoglu and Inceoglu (2014) assume perfect information while in our model information is asymmetric. Furthermore, rather than a signalling device as occurs in Antelo and Sampayo (2017), in this case the contractual structure may serve as a screening device to elicit hidden information. Likewise, our model departs from the setup in Wu et al. (2021), as it considers a screening game in which each technology user is more informed about its technology-related marginal cost of production than any other player, and also allows users to have different production costs.

Our research is also related to that of Heywood et al. (2014) and Fan et al. (2018), with the crucial difference that we consider an outside rather than an inside licensor. Our model can thus be seen as complementary to both. One shared goal is to understand how both the contractual structure and number of licences (the degree of competition in the created market) can be used to countervail the licensor's costs on eliciting hidden information regarding the innovation's value. As in Fan et al. (2018), we find that the optimal contract depends on the efficiency gap between user types and the probability of having an efficient user. In both Heywood et al. (2014) and Fan et al. (2018), it is found that, although royalties are useful to compensate the licensor for screening and competition costs, these costs may be so large as to lead to a single user. However, while Fan et al. (2018) investigate the interaction between an inside licensor and a competing licensee, we consider a context in which there is an outside licensor and a single or several (downstream) licensees.

Finally, a certain similarity can also be traced between our model and those of Savva and Taneri (2015), Lin et al. (2022) and Tsao et al. (2023). Savva and Taneri (2015), in a study of a licensing deal between a less informed university technology transfer office (TTO) and more informed spin-offs, conclude that royalties can act as a screening device for the TTO to extract information on the project quality, thereby lowering the probability of the spin-offs generating valuable products by extracting full information rents at the expense of the university. However, unlike the model of Savva and Taneri (2015), we allow the licensor to determine not only the contractual terms, but also the number of contracts to be offered. Lin et al. (2022) consider vertical licensing by one (downstream) firm in a differentiated duopoly when the final good is obtained from two complementary inputs. One firm licenses the production technology for one input to a licensee that becomes the (upstream) supplier of that input, whereas the other input is provided by an exclusive supplier. When products are quite similar, vertical licensing improves welfare, but when they are more differentiated, the wholesale price is set above the supplier's marginal cost through licensing, leading to double marginalization and reduced welfare. Likewise, Tsao et al. (2023) investigate two-tier licensing in a vertically related market. In a successive monopoly, with one upstream firm and one downstream firm monopolizing the input and output markets, respectively, both firms may acquire a new technology from their foreign patentees that can reduce the licensee's marginal cost. Thus, two-tier foreign technology licensing may be socially undesirable relative to one-tier foreign technology licensing, i.e., more foreign licensing agreements in a vertically related market may worsen domestic welfare. However, both Lin et al. (2022) and Tsao et al. (2023) are complete information models, and they investigate, not the optimal number of licences, but whether licensing is welfare-enhancing or welfare-reducing; our model, in contrast, features asymmetric information and focuses on the form and number of issued licences.

3 The model

We consider an outside licensor with a new and patented technology that enables the production of a saleable good. The technology, whose development cost is already sunk, lasts for one period, after which it becomes obsolete and so has no economic value. The licensor transfers it either to a single user (exclusive licensing) or several users (non-exclusive licensing)[6] by means of 2PT contracts involving a fixed fee f, $f \geq 0$, in combination with a per-unit royalty r, $r \geq 0$.[7] Each user, when producing the final good with the technology, will incur a low marginal production cost

[6] We are interested in the role of competition in ameliorating information rents in a screening-licensing context. For an alternative licensing scheme with Cournot competition in the marketplace and an endogenous number of users, see Sen and Tauman (2007), where, under symmetric information, an auction is used to determine the number of users and the resulting tariff.

[7] In Sect. 7, we consider an ad-valorem royalty rather than a per-unit royalty.

(an efficient user) or a high marginal cost (an inefficient user). Each user privately knows its cost, whereas other players only know the ex-ante probabilities:

$$\widetilde{c} = \begin{cases} 0, & \text{with Prob. } \mu \\ c > 0, & \text{with Prob. } 1-\mu \end{cases} \tag{1}$$

where $\widetilde{c}$ denotes the random variable representing the marginal production cost incurred by each user when using the new technology, and $0 < \mu < 1$. When issuing each licence, the licensor incurs in a fixed cost S, $S > 0$.[8] All players are risk-neutral and the inverse demand function for the product manufactured with the new technology is given by $p(Q) = \max\{1-Q, 0\}$, where Q denotes the total quantity produced. Finally, the good produced by all users is homogeneous.

To ensure that when the technology is licensed to two or more users, each one produces a positive quantity even when it employs the technology inefficiently and competes with efficient rivals, an upper bound in the poor realization of marginal cost, c, is assumed as follows:

(A1) *The marginal cost borne by an inefficient user of the technology is such that*

$0 < c < \frac{1}{2}$.

In what follows, we consider a three-stage screening game with the following timing. In the first stage (number-of-contracts stage), the licensor chooses to license the technology to one or more users. In the second stage (form-of-contracts stage), the licensor offers the licence on a take-it-or-leave basis to each user in exchange for a contract that combines a fixed payment with a per-unit royalty; when more than one licence is issued, the contracts are offered simultaneously, and each user knows that at least one other licence is being granted. In the third stage (market-competition stage), the final good is produced with the technology (by a monopolist when exclusive licensing holds, or by Cournot competitors when two or more licences are granted).

4 Solving the game

In this section, we determine the solution to the licensing game that arises when the new technology is licensed to a single firm (exclusive licensing) and when it is simultaneously licensed to two firms (non-exclusive licensing).[9]

4.1 Exclusive licensing

When issuing a single licence of the technology the innovator has three options: to offer a contract that is only accepted by an efficient user type; to offer a contract that

[8] This may be seen as the cost of producing an input needed to exploit the new technology.

[9] In Sect. 6 we extend the model by considering simultaneous licensing of the technology to three producing firms.

the licensee accepts, regardless of whether it is efficient or inefficient, and as result, the licensor's beliefs regarding innovation's value continue to be its prior beliefs; and finally, to offer a menu of two contracts—one geared to the efficient user type and the other to the inefficient user type—that leads each one to accept the offer intentionally designed for it and allowing the licensor to elicit hidden information concerning the true value of the technology. Lemma 1, where the subscripts 0 and c denote the user types (efficient and inefficient, respectively), summarizes the feasible contracts and shows how they depend on c, the marginal production cost of an inefficient user, and μ, the licensor's probability of dealing with an efficient user.

Lemma 1 *Let* $\mu_1'(c) = \frac{1-c}{c}$ *and* $\mu_1''(c) = 1 - c$, *with* $0 < \mu_1'(c) < 1$, $0 < \mu_1''(c) < 1$, *and* $\mu_1'(c) < \mu_1''(c)$, *for all c. If the new technology is licensed to a single user, the licensor can offer:*

(i) *The (excluding) fixed-fee contract* $f_0 = \frac{1}{4}$, *which is only accepted if the user is efficient.*
(ii) *The (non-excluding) 2PT contract* $(f, r) = \left(\frac{(1-(1+\mu)c)^2}{4}, \mu c\right)$, *which the user accepts, irrespective of whether it is efficient or inefficient whenever* $\mu < \mu_1'(c)$.
(iii) *The menu of (separating) screening contracts given by* $\{f_0, (f_c, r_c)\} = \left\{\frac{1}{4} - \frac{(2(1-\mu)-(1+\mu)c)c}{4(1-\mu)}, \left(\left(\frac{1-\mu-c}{2(1-\mu)}\right)^2, \frac{\mu}{1-\mu}c\right)\right\}$, *with the fixed-fee contract intended for the efficient user and the 2PT contract intended for the inefficient user, and leading both user types to produce whenever* $\mu < \mu_1''(c)$.

Proof See Appendix.

Parts (i) and (ii) of Lemma 1 reflect the feasible contracts without considering incentive-compatibility constraints. In particular, part (i) states the non-distortionary contract that is acceptable to the licensee when it employs the technology efficiently. In this case, the licensor does not need to grant any information rents, but it renounces exploitation of the technology by an inefficient user, whereby it will only be commercialized with probability μ.

Likewise, part (ii) describes a distortionary 2PT contract acceptable to both the efficient and inefficient users, in which case the technology will be implemented for sure. This contract includes a per-unit royalty amounting to $\mu \times 0 + (1 - \mu) \times \frac{\mu}{1-\mu}c$, i.e., the average of the royalties charged in the menu of 2PT separating contracts of part (iii). This royalty is aimed at distorting the production of both user types and leads the contract to increase the licensor's expected revenues by $\frac{\mu^2 c^2}{4}$ as compared to a non-distortionary fixed-fee contract as $f_c = \frac{(1-c)^2}{4}$ also acceptable to both the efficient and inefficient users.

Finally, part (iii) establishes the feasible menu of separating contracts that leads both user types to produce a positive quantity while inducing them to reveal private information. In this menu, the contract intended for an efficient user degenerates into a fixed-fee contract, leading to no distortion at the top (Mussa and Rosen 1978),[10] whereas the contract intended for an inefficient user is a 2PT contract featuring a positive royalty whose amount depends on both the proportion of efficient users and their level of inefficiency. The purpose of this royalty is to distort the inefficient user's production (distortion at the bottom), which, in turn, makes this contract unattractive to the efficient user and minimizes its information rents. This finding parallels previous results for screening reported in the licensing literature (Macho-Stadler et al. 1996; Poddar and Sinha 2008; Savva and Taneri 2015).

Having examined the feasible contracts for technology transfer to a single user, we can now state the optimal contract for the licensor.

Proposition 1 *There is a cut-off value for the licensor's belief of dealing with an efficient user of the new technology,* $\mu_1(c) = 1 - \frac{2c+c\sqrt{c(4-3c)}+c^2}{2(1+c^2)}$, *with* $0 < \mu_1(c) < 1$, *for all c, such that the technology is transferred to a single user as follows:*

(i) *If* $\mu < \mu_1(c)$, *by means of the (distortionary) menu of separating screening contracts stated in part (iii) of Lemma 1, and the technology is always marketed.*
(ii) *If* $\mu > \mu_1(c)$, *by means of the (non-distortionary) fixed-fee contract stated in part (i) of Lemma 1, and the technology is only marketed if the user is efficient.*

Proof See Appendix.

Part (i) of Proposition 1 states that, given a poor realization of c, when the licensor believes that the user is unlikely to be efficient, its best option is to ensure the new technology is commercialized for sure, thereby making the user reveal the (low or high) value it attaches to the technology. The licence therefore consists of a separating menu of two contracts that distorts the inefficient user's production (distortion at the bottom) while not conceding excessive information rents to the efficient user. What determines a better outcome for this contract than for the contract stated in part (ii) is the fact that the expected information rents are not substantial when the probability of dealing with an efficient user is low.

Contrariwise, if the probability of finding an efficient user for the technology is sufficiently high, as stated in part (ii), the distortion at the bottom required in a separating contract, as stated in part (i), would reduce the licensor's expected revenues from an inefficient user to such an extent that it would no longer compensate for revenues extracted from an efficient user. Thus, the licensor prefers to transfer the technology by means of a sufficiently high non-distortionary fixed-fee payment that leads the inefficient user to reject it. In this case, while the licensor renounces to

[10] The no distortion at the top property holds in all adverse selection models in which the single-crossing property is satisfied.

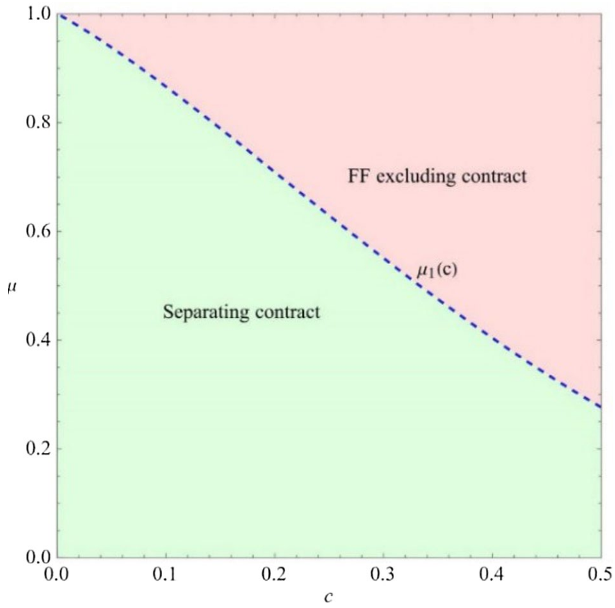

Fig. 1 Optimal exclusive licensing contract for a new technology under asymmetric information

extract revenues from an inefficient user, in exchange it prevents an efficient user from keeping information rents.

Figure 1, where FF denotes fixed fee, illustrates in the (c, μ)-space the content of Proposition 1. As can be seen, it is most likely that the licence will be granted through a menu of separating contracts that allows production by both user types, while simultaneously extracting users' private information on the value of the technology. It is only when the probability of encountering an efficient user is sufficiently high (given the size of the efficiency gap between the efficient and inefficient users) that the licensor prefers to offer an excluding licence that only allows the efficient user to produce, rather than screen the user. Regarding the non-excluding 2PT contract stated in part (ii) of Lemma 1, it is not optimal to use it to grant a single licence, even though it allows the inefficient user's profits to be extracted, because it would leave more information rents to the efficient user than the menu of separating contracts stated in part (iii).

Note that $\mu_1(c)$, i.e., the value of the probability of dealing with an efficient user that leads the licensor to be indifferent between offering the excluding contract and the menu of separating screening contracts, increases as c decreases, i.e., as the marginal cost of the efficient and inefficient users are more similar. When c is low, the foregone profits from an inefficient user in the excluding contract are greater, while the loss caused by productive distortion due to the separating contract is not that large, unless μ is sufficiently high. This is why, for low values of c, a very high value of μ is needed to compensate for the distortions caused by the separating contract so as to offset the benefits of eliciting hidden information.

4.2 Non-exclusive licensing

If the new technology is simultaneously transferred to two users, the licensing game proceeds much as in the single-user case. In the first stage, the licensor offers each user either a menu of two contracts—with one contract intended to attract the efficient types and the other intended to attract the inefficient types—or a single contract. In turn, this single contract may be either a contract that allows each user (efficient or inefficient) to produce, or an excluding contract that allows only the efficient user to produce. In any case, the contracts are all assumed to be simultaneously offered to both users before their realized marginal costs are publicly known. In the second stage, the users accept the licensor's offer. Finally, in the third stage, the licensed technology is used to produce a homogeneous good in a Cournot environment.[11]

For this non-exclusive licensing context, Lemma 2 characterizes the feasible contracts that the licensor can offer. As before, the subscripts 0 and c denote the efficient and inefficient user, respectively.

Lemma 2 *Let* $\mu_2'(c) = \frac{1-c}{2c}$ *and* $\mu_2''(c) = \frac{1-c}{1+c}$*, with* $0 < \mu_2'(c) < 1$*,* $0 < \mu_2''(c) < 1$*, and* $\mu_2'(c) > \mu_2''(c)$*, for all c. If the new technology is licensed to two users, the licensor can offer each user* $i, i = 1, 2$:

(i) *The (excluding) 2PT contract* $\left(f_0, r_0\right) = \left(\frac{1}{4(1+\mu)^2}, \frac{\mu}{2(1+\mu)}\right)$*, which it only accepts if it is efficient.*

(ii) *The (non-excluding) 2PT contract* $(f, r) = \left(\left(\frac{1-(1+2\mu)c}{4}\right)^2, \frac{1-(1-4\mu)c}{4}\right)$*, which is accepted by the user, whether efficient or inefficient, whenever* $\mu < \mu_2'(c)$*.*

(iii) *The menu of 2PT (separating) screening contracts given by* $\left\{\left(f_0, r_0\right), \left(f_c, r_c\right)\right\} = \left\{\left(\frac{1}{16} - \frac{2(1-\mu)c-(1+7\mu)c^2}{16(1-\mu)}, \frac{1-c}{4}\right), \left(\left(\frac{1-\mu-(1+\mu)c}{4(1-\mu)}\right)^2, \frac{1-\mu-(1-5\mu)c}{4(1-\mu)}\right)\right\}$*, with the first contract intended for the efficient type and the second contract intended for the inefficient type, leading both types to produce whenever* $\mu < \mu_2''(c)$*.*

Proof See Appendix.

The feasible contractual scheme that allows the licensor to issue two licences differs sharply from that allowing a single licence. First, according to part (i) of Lemma 2, the excluding contract that, under non-exclusive licensing, leads only the efficient user to exploit the technology is a distortionary 2PT contract rather

[11] Since a user may know, better than not only the licensor, but also any other user of the new technology, how it will be adapted to its production circumstances, there is vertical and horizontal asymmetric information.

than a non-distortionary fixed-fee contract, as occurs under exclusive licensing. This holds because, when the licensor wishes the technology to be only exploited by one or two efficient users, the problem of asymmetric information becomes irrelevant. However, unlike when the licensor grants a single licence, the aim when issuing two licences is to increase market collusion. This is achieved by including a per-unit royalty in the contract, and then reaping the increase in industry profits by means of the fixed part of the contract. In fact, thanks to the sampling effect, this contract allows the licensor to obtain more gross revenues (not taking the cost of granting licences into account) with two licences than with just one.

Second, part (ii) shows that, just as under exclusive licensing, the single contract that allows both types of each user to produce is a 2PT contract. However, in order to increase market collusion, the per-unit royalty is larger than in the analogous contract under exclusive licensing, $\frac{1-(1-4\mu)c}{4} > \mu c$. Ultimately, the contract stated in part (ii) will lead the inefficient users not to produce when $\mu > \mu_2'(c)$ and, as a result, the contract offered in this case becomes that stated in part (i).

Third, in contrast with the exclusive licensing scenario, part (iii) of Lemma 2 indicates that a per-unit royalty is included in both contracts within the menu of separating contracts, regardless of whether the contract is intended to attract efficient or inefficient users. Thus, there is not only distortion at the bottom, as occurs with the separating menu in the single-licence case, but also distortion at the top. The rationale is that, in contrast with exclusive licensing, where contracts are designed to elicit information revelation, in this case the licensor seeks to elicit revelation as well as increase collusion in the product market so as to reproduce the monopoly outcome. Therefore, the contracts intended for efficient users, and not only the contracts intended for inefficient users, includes a per-unit royalty. Moreover, $\frac{1-\mu-(1-5\mu)c}{4(1-\mu)} > \frac{1-c}{4}$, i.e., the royalty involved in the contracts intended for inefficient users is higher than the royalty involved in the contracts intended for efficient users. The reason is that the royalty intended for inefficient users derives from a first-order effect, whereas that intended for efficient users derives from a second-order effect. Likewise, the royalty intended for each inefficient user is higher than the royalty in the (similar) contract intended for the inefficient user in the single-licence case, $\frac{1-\mu-(1-5\mu)c}{4(1-\mu)} > \frac{\mu c}{1-\mu}$. This means that the contracts for inefficient users in a duopoly market are more distorted—and this extra distortion reduces these contracts' attractiveness to efficient users, because the contracts for efficient users are also distorted. Finally, as in part (ii), when $\mu > \mu_2''(c)$, it holds that the contracts stated in part (iii) lead the inefficient users not to produce, and hence the contracts offered will again be those described in part (i).

From the expected revenues provided by the feasible contracts stated in Lemma 2, we can now establish the optimal way to issue two licences for the new technology under asymmetric information.

Proposition 2 *There is a cut-off value for the licensor's belief of dealing with an efficient user of the new technology,* $\mu_2(c)$*, where* $\mu_2(c)$ *is the solution in* $(0,1)$ *to the equation* $c - \frac{1-\mu}{1-\mu+2\mu^2}\left(1 - \sqrt{\frac{2\mu(1-\mu)}{1+\mu}}\right) = 0$*, such that the new technology is non-exclusively licensed as follows:*

(i) *If* $\mu < \mu_2(c)$*, by means of the separating menu of 2PT screening contracts stated in part (iii) of Lemma 2, leading each user, whether efficient or inefficient, to produce.*
(ii) *If* $\mu > \mu_2(c)$*, by means of the (excluding) 2PT contract stated in part (i) of Lemma 2, leading only the efficient users to produce.*

Proof See Appendix.

As in the single-licence case, the licensor now also discards the option of conceding two licences by means of a single contract for each user that allows both the efficient and inefficient users to produce. Such a contract is a poorer option for the licensor than either an excluding contract to each user or a menu of two different contracts, one for each user type. Moreover, when the probability of an efficient user is small, the licensor prefers the technology to be exploited for sure, and so it offers a menu of two contracts, each intended for each user type, rather than offer an excluding contract to both user types that leads the technology to be exploited with a probability of less than one. This is because the menu of separating contracts benefits from the sampling effect, while compensating for the rent dissipation effect, by allowing the inefficient users to produce with higher profits extracted from the efficient users due to the lower information rents.

Contrariwise, if the probability of the efficient user is high, as indicated in part (ii), the licensor offers an (excluding) contract to each user, which will be rejected by inefficient ones. In this case, no information rents are conceded to the efficient users, but the new technology remains unused when each user becomes inefficient. Furthermore, the fact that there will be two efficient firms using the technology rather than only one leads this contract to be a distorted 2PT contract rather than a fixed-fee contract, as in the single-licence case. This is because the licensor, in order to increase its revenues when efficient users exploit the technology, needs to reduce competition in the marketplace by including a per-unit royalty in each licensing contract.

Figure 2 plots the optimal contract for the licensor under non-exclusive licensing in the (c, μ)-space. In the region below the $\mu_2(c)$ locus, the licensor prefers to offer a menu of separating contracts to each user that allows the efficient and inefficient users to produce. Contrariwise, in the region above the $\mu_2(c)$ locus, it prefers to offer a unique contract to each user that leads it to produce only if it is efficient and hence an efficient monopoly or an efficient duopoly may arise in the product market.

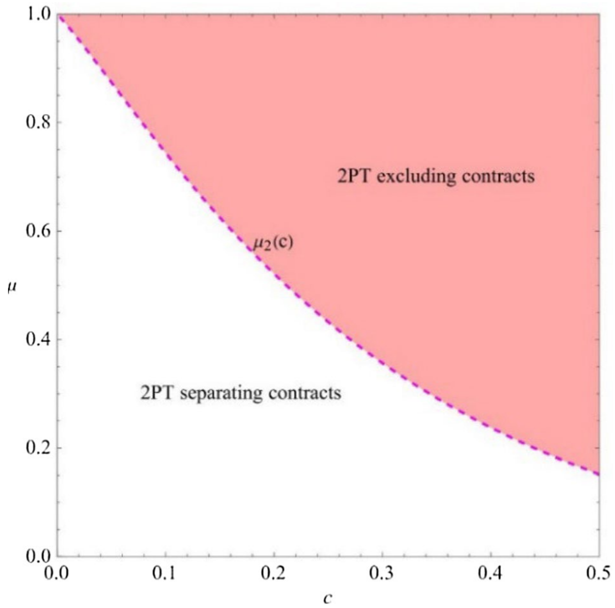

Fig. 2 Optimal non-exclusive licensing contract for a new technology under asymmetric information

When comparing two-licence vs. single-licence contexts, we find that the licensor is less likely to scrutinize technology users in the former case. Put differently, as the number of licences increases, it becomes more likely that they will be granted through excluding contracts that only allow the efficient users to produce. The explanation is as follows: As the number of licences increases, interaction between the rent dissipation effect, the sampling effect, and the information rents conceded to the efficient users for eliciting hidden information make screening costlier. Therefore, the excluding contracts become more profitable with the number of licences. In fact, as we will see below, if the licensor grants three licences and seeks for each user to produce, whether efficient or inefficient, it will never offer a menu of separating contracts to each user. This is because the resulting production distortions, when three firms simultaneously use the new technology, would not offset, in profit terms, the positive effect of information disclosure.

Finally, the optimal number of licences granted under asymmetric information can be compared with those granted under symmetric information. This allows us to determine whether the presence of asymmetric information leads the licensor to resort to more or less competition between users than under symmetric information.

Proposition 3 *Let* $\mu_3(c) = 1 - \frac{\sqrt{8c-3c^2}-c}{2(2-c)}$, $\mu_4(c)$ *the solution in* $(0, 1)$ *to the equation* $c - \frac{1-2\mu+2\mu^2-\mu^3}{1-2\mu+3\mu^2-\mu^3} - (1-\mu)\sqrt{\frac{\mu\left[1-\mu(1-\mu)\left(2+\mu^2\right)\right]}{(1+\mu)\left(1-2\mu+3\mu^2-\mu^3\right)}} = 0$, *and* $\mu_5(c) = \frac{(1-c)^2}{(2-c)c}$, *with* $0 < \mu_k(c) < 1$, $k = 3, 4, 5$, *for all c. And let* $S^*(c, \mu) = \frac{\mu(1-\mu)(2-c)c}{4}$ *be the per-licence issuing cost for which the licensor is indifferent between granting one or two* *licences under symmetric information. Thus, under asymmetric information and at cost $S^{*}(c,\mu)$, the licensor prefers to issue two licences, rather than only one, in the region defined by $\Gamma_1 \cup \Gamma_2 \cup \Gamma_3$, where* $\Gamma_1 = \left\{ (c,\mu) \in \left(0,\frac{1}{2}\right) \times (0,1) \middle| \mu_3(c) \leq \mu \leq \mu_2(c) \right\}$, $\Gamma_2 = \left\{ (c,\mu) \in \left(0,\frac{1}{2}\right) \times (0,1) \middle| \max\{\mu_4(c),\mu_2(c)\} \leq \mu \leq \mu_1(c) \right\}$, *and* $\Gamma_3 = \left\{ (c,\mu) \in \left(0,\frac{1}{2}\right) \times (0,1) \middle| \mu_1(c) \leq \mu \leq \mu_5(c) \right\}$.

Proof See Appendix.

Proposition 3 states that under asymmetric information the new technology is more likely to be licensed to several producers than under symmetric information. This means that the concession of two licences, i.e., the introduction of competition in the marketplace, does not reduce, but may even increase, the licensor's payoff as compared to the concession of a single licence. This result resembles that obtained by Schmitz (2002) for a different setup. In our context, the granting of several licences makes more likely to have an efficient user of the technology (the sampling effect), and this becomes increasingly relevant as uncertainty increases. Moreover, when two licences are granted, the royalty featured in the contract for each user increases market collusion and so allows the licensor to extract more revenues. Thus, when both c and μ are high, granting two licences through excluding contracts is of no interest to the licensor, because the sampling effect tends to decrease and the issuing costs tend to grow. Likewise, when both c and μ are low, the licensor prefers to resort to separating contracts for one or two licences; this is because it is obliged to concede information rents to the efficient users and so cannot replicate the monopoly outcome (when granting two licences). Moreover, since the sampling effect is also weak for low values of μ, the licensor prefers to grant a single licence despite the low issuing cost of granting licences in those circumstances.[12]

The result of Proposition 3 is illustrated in Fig. 3. While under symmetric information the licensor would be indifferent between granting one of two licences in the entire (c,μ)-region, under asymmetric information it strictly prefers to grant two licences in the red coloured region. Thus, the introduction of competition between users may serve as a device for the licensor to countervail the detrimental effects of a lack of information regarding the innovation's value.[13] Note that the area where two licences are preferred excludes nearly the entire region where a separating contract would be chosen in a non-exclusive licensing scenario, plus a large part of the region where a separating contract would be chosen in an exclusive licensing scenario, i.e., the area under the $\mu_2(c)$ locus and the area under the $\mu_1(c)$ locus, respectively.

[12] This result applies to a per-licence issuing cost that renders the licensor indifferent between granting one or two licences under symmetric information. This is not necessarily true when the licence issuing cost is lower, as, in this case, the licensor would prefer to grant two licences under symmetric information.

[13] This is in sharp contrast to the result obtained in a signalling framework, where, for all values of parameters μ and c, issuing two licences is always preferred to issuing one (Antelo and Sampayo 2017).

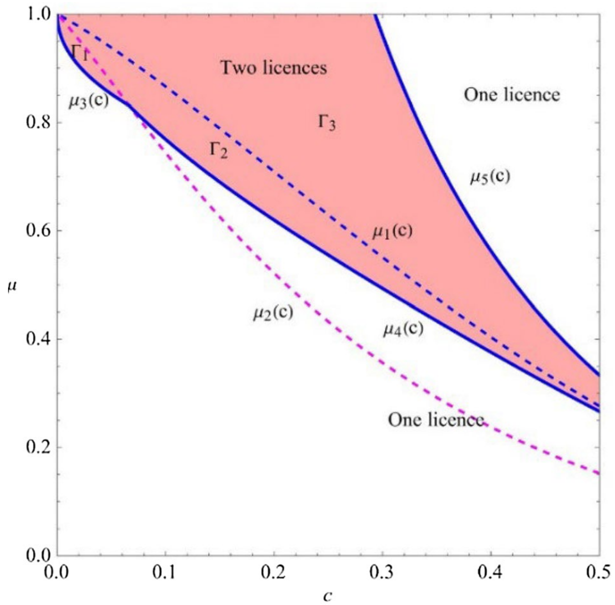

Fig. 3 Region in the (c, μ)-space where, under asymmetric information, the licensor prefers to grant two licences, whereas under symmetric information, it is indifferent between granting one or two licences

Finally, outside the red coloured area in Fig. 3, the licensor continues to be indifferent between granting one or two licences under symmetric information, but prefers to grant a single licence under asymmetric information. Hence, in this region it may happen that the number of licences issued may not be affected or may be reduced under asymmetric information as compared to under symmetric information.

5 Is it socially better to grant one or two licences?

To examine the welfare impact of the licensor's behaviour in deciding the number and contractual terms of licences, under either symmetric or asymmetric information, we define expected aggregate welfare as the unweighted sum of the expected consumer surplus plus the expected gross profits from producing with the new technology. We also assume that a social planner would decide the number of licences (one or two) that maximizes expected welfare, while allowing the licensor to set contractual terms and the technology users to choose their production strategies (as in the equilibria described in previous sections). Put differently, we assume, first, that the licensor takes the social planner-determined number of licences as given and then chooses the optimal contracts, and second, that there is either a single user producing as a monopolist or two users producing as Cournot duopolists.

As a starting point, we consider the per-licence issuing cost $S^*(c, \mu)$ at which, under symmetric information, the licensor is indifferent between granting one or two licences. We also consider $\widetilde{S}(c, \mu) = \mu(1 - \mu)(2 - c)c/8$ as the per-licence issuing cost that, again under symmetric information, yields the same aggregate welfare, regardless of whether one or two licences are granted. In this context, when comparing the socially optimal number of licences with the optimal number of licences from the licensor's viewpoint, we find that the (c, μ)-space, where, in equilibrium, the licensor chooses a socially suboptimal number of licences, is much smaller under asymmetric information than under symmetric information.

Proposition 4 *Let $S^*(c, \mu)$ be the per-licence issuing cost for which, under symmetric information, the licensor is indifferent between granting one or two licences, and let $\widetilde{S}(c, \mu)$ be the per-licence issuing cost for which, under symmetric information, aggregate welfare with one licence is the same as with two licences, with $S^*(c, \mu) < \widetilde{S}(c, \mu)$. Thus, for values of the issuing cost S verifying $S^*(c, \mu) \leq S \leq \widetilde{S}(c, \mu)$, the following holds:*

(i) *The number of issued licences under symmetric information is socially suboptimal.*
(ii) *The number of issued licences under asymmetric information may be socially optimal.*

Proof See Appendix.

Two main conclusions arise from Proposition 4. For the considered range of issuing costs, according to part (i), when symmetric information prevails, non-exclusive licensing is always socially better than exclusive licensing, and hence, the diffusion of the new technology is socially insufficient. However, according to part (ii), the diffusion may be socially appropriate when asymmetric information holds, as only for a sufficiently high probability of having efficient users would asymmetric information result in a suboptimal welfare outcome. Note that this holds even though the expected consumer surplus is always greater with two licences than with only one because a higher quantity of the good at a lower price is available in the marketplace. Hence, when the probability of dealing with an efficient user is sufficiently low, the reduction in consumer surplus that would result in a shift from granting two licences to granting only one is outweighed by the subsequent increase in users' profits. In short, the combination of higher prices and lower consumption—as would be the case for a reduced number of licences—does not result in an aggregate welfare loss.

A second finding, based on part (ii) of Proposition 4, follows from comparing the optimal number of licences from the licensor's perspective with the optimal number from the social viewpoint. When information is asymmetric, it is more

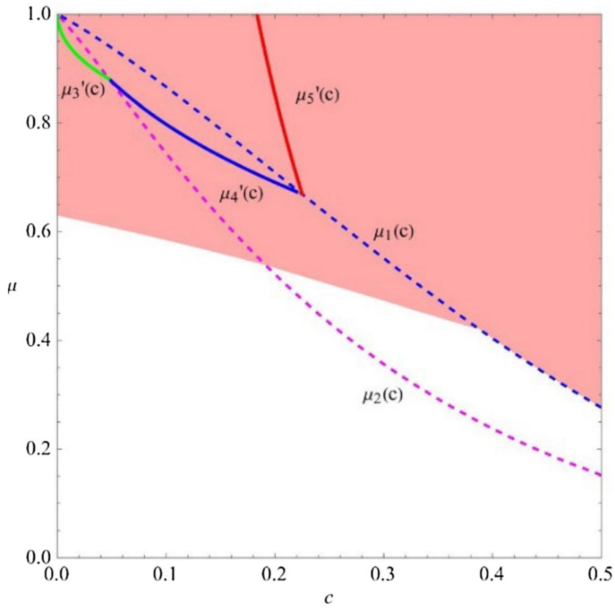

Fig. 4 Region in the (c, μ)-space where, under the issuing cost $\widetilde{S}(c, \mu)$, non-exclusive licensing is welfare enhancing (red region) and where exclusive licensing is welfare enhancing (white region)

likely that the number of licences in equilibrium will equal that which a social planner would choose to maximize aggregate surplus. To emphasize the role of asymmetric information in improving the welfare efficiency of the number of licences, we have proven Proposition 4 for the case of an issuing cost per licence at which the number of licences under symmetric information is inefficient. If $S < S^*(c, \mu)$, we would have non-exclusive licensing under symmetric information, whereas if $S > \widetilde{S}(c, \mu)$ then, under asymmetric information, the higher the number of issued licences, the greater the aggregate welfare in equilibrium. This effect, however, is offset as the cost per issued licence increases, because the number of licences in equilibrium will be more negatively affected by a higher issuing cost than positively affected by the effect stipulated in part (ii) of Proposition 4.

The content of Proposition 4 is illustrated in Fig. 4, where the loci $\mu_3'(c)$, $\mu_4'(c)$, and $\mu_5'(c)$ are defined by the respective solutions in μ as given by $c_3'(\mu)$, $c_4'(\mu)$, and $c_5'(\mu)$ in Eq. (26) in the Appendix.

Furthermore, for an issuing cost of $\widetilde{S}(c, \mu)$, in the area delimited by the loci $\mu_3'(c)$, $\mu_4'(c)$, and $\mu_5'(c)$, the licensor prefers to grant two licences rather than one, and hence, private incentives are aligned with social incentives. As result, the number of granted licences is socially optimal.

6 Non-exclusive licensing through three licences

In this section, to investigate the extent to which our findings are robust for an increased number of issued licences, we consider the granting of three licences rather than two. The resulting licensing game runs as in Sect. 4.2. In the first stage, the licensor may offer each triopolistic user a pair of separating screening contracts — one intended to attract the efficient type and the other to attract the inefficient type. Alternatively, it may offer a single contract to both user types: either a contract that allows both types to produce, or an excluding contract that allows only the efficient type to produce. In the second stage, each user accepts the licensor's offer. In the third stage, the final good is produced using the technology and shipped to market.

Lemma 3 summarizes the feasible contracts the licensor may offer to each of the three users.

Lemma 3 *Let* $\mu_6'(c) = \frac{1-c}{3c}$ *and* $\mu_6''(c) = \frac{1-c}{1+2c}$, *with* $0 < \mu_6'(c) < 1$, $0 < \mu_6''(c) < 1$ *and* $\mu_6'(c) > \mu_6''(c)$, *for all c. If the new technology is licensed to three users, the licensor can offer each user i,* $i = 1, 2, 3$*:*

(i) *The (excluding) 2PT contract* $(f_0, r_0) = \left(\frac{1}{4(1+2\mu)^2}, \frac{\mu}{1+2\mu}\right)$, *which it only accepts if it is efficient.*

(ii) *The (non-excluding) 2PT contract* $(f, r) = \left(\left(\frac{1-(1+3\mu)c}{6}\right)^2, \frac{1-(1-3\mu)c}{3}\right)$, *which the user accepts, whether efficient or inefficient, whenever* $\mu < \mu_6'(c)$.

(iii) *The menu of 2PT screening contracts, as given by* $\{(f_0, r_0), (f_c, r_c)\} = \left\{\left(\frac{1}{36} - \frac{(2(1-\mu)-(1+17\mu)c)c}{36(1-\mu)}, \frac{1-c}{3}\right), \left(\left(\frac{1-\mu-(1+2\mu)c}{6(1-\mu)}\right)^2, \frac{1-\mu-(1-4\mu)c}{3(1-\mu)}\right)\right\}$, *with the first contract intended for the efficient type and the second contract intended for the inefficient type, leading both user types to produce whenever* $\mu < \mu_6''(c)$.

Proof See Appendix.

The excluding contracts stated in part (i) are also 2PT contracts, like the excluding contracts with two licences. However, the increased number of licences leads to an increase in the royalty component and a decrease in the fixed part of the tariff. This is because more users make competition in the marketplace fiercer, and this offsets the impact of the sampling effect. Hence, to countervail this dissipation effect, the licensor seeks to increase market collusion by means of a higher per-unit royalty. The same applies to the contracts stated in parts (ii) and (iii) when compared with the contracts stated in parts (ii) and (iii) of Lemma 2.

On the other hand, with three licences rather than two licences, the parameter region in which each user, efficient or inefficient, accepts the contract stated in part (ii) is lower. The same happens for the separating contract stated in part (iii). Hence, we conclude that the increased number of granted licences favours licensing by means of excluding contracts to the detriment of contracts that allow scrutiny of the technology users.

Having described the feasible contracts to transfer the new technology to three users, we can now summarize the licensor's contract preferences.

Proposition 5 *There is a cut-off value for the licensor's belief of dealing with an efficient user, $\mu^*(c)$, where $\mu^*(c)$ is the solution in $(0,1)$ to the equation $\frac{1-\mu}{1+2\mu} - 2c + \left(1+3\mu^2\right)c^2 = 0$, such that the new technology is optimally licensed to three users as follows:*

(i) *If $\mu < \mu^*(c)$, by offering each user the 2PT contract stated in part (ii) of Lemma 3.*

(ii) *If $\mu > \mu^*(c)$, by offering each user the (excluding) 2PT contract stated in part (i) of Lemma 3.*

Proof See Appendix.

Proposition 5 shows that, when the licensor transfers the new technology to three users with the aspiration that each one, whether efficient or inefficient, will produce, it chooses not to offer a menu of separating contracts. Rather, it prefers to offer the same contract to all users. Thus, an increased number of licences favours the use of excluding contracts, to the detriment of screening contracts to the point where these contracts are obviated when the technology is licensed to three firms. The explanation is as follows: The beneficial impact of information disclosure on the licensor's expected revenues does not outweigh the negative impact of the distortions resulting from the menu of contracts for both users' types. In general, therefore, it is more profitable for the licensor to offer the same contract allowing both users' types to produce but not allowing the extraction of private information from each type, than to offer a menu of separating contracts allowing production by both users' types and also the extraction of private information from each type.[14] Finally, comparing the licensor's payoff from this 2PT non-revealing contract with the payoff from an excluding contract that only allows production by efficient users, we find that when μ, the probability of dealing with an efficient user, is small (large), the former contract is better (worse) than the latter. Figure 5 shows the regions where it is optimal to grant three licences via the 2PT excluding contract and via the 2PT non-excluding contract.

Paralleling the content of Proposition 3, in Proposition 6 we state the conditions in which the licensor would prefer to grant three licences as defined in Proposition 5, two licences as defined in Proposition 2, or a single licence as defined in Proposition 1.

[14] This is true except in a very small region in the (c, μ)-space, in which the optimal contract that the licensor offers to each user is, in any case, the excluding contract.

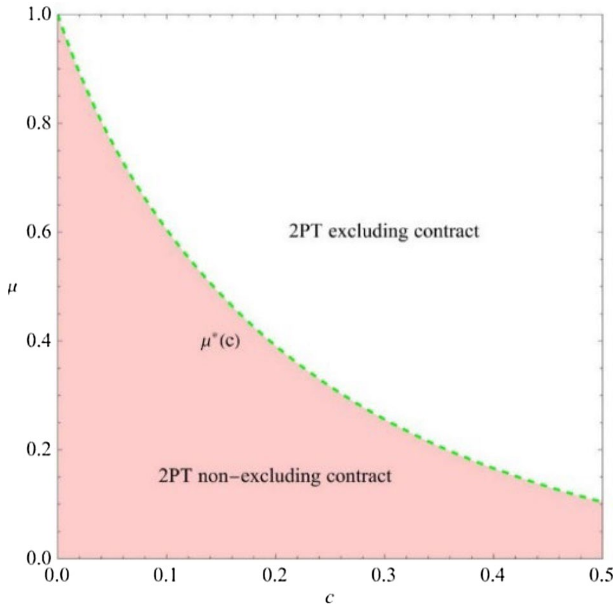

Fig. 5 Regions in the (c, μ)-space where the licensor prefers either a pair of 2PT non-excluding contracts or a pair of 2PT excluding contracts

Proposition 6 *Regarding the number of licenses, the following occurs:*

(i) *Let* $c_1(\mu) = \frac{1-3\mu+5\mu^2-4\mu^3+\mu^4}{1-3\mu+6\mu^2-4\mu^3+\mu^4} - \sqrt{\frac{\mu(1-6\mu+19\mu^2-40\mu^3+62\mu^4-67\mu^5+44\mu^6-15\mu^7+2\mu^8)}{(1+2\mu)(1-3\mu+6\mu^2-4\mu^3+\mu^4)^2}}$ *and* $c_2(\mu) = 1 - \sqrt{\frac{\mu(3-2\mu)}{2+3\mu-2\mu^2}}$, with $0 \leq c_1(\mu) \leq c_2(\mu) \leq \frac{1}{2}$. *And let* $S^{**}(c, \mu) = \frac{\mu(1-\mu)(2-\mu)(2-c)c}{8}$ *be the per-licence issuing cost at which the licensor is indifferent between granting one or three licences under symmetric information. Thus, under asymmetric information and at cost* S^{**}*, the licensor prefers to issue three licences rather than one in the region of parameters defined by* $\left\{(c, \mu) \in \left(0, \frac{1}{2}\right) \times (0, 1) \middle| c_1(\mu) \leq c \leq c_2(\mu)\right\}$ *and one licence rather than three, otherwise.*

(ii) *Let* $c_3(\mu) = \frac{1-2\mu+3\mu^2-3\mu^3+\mu^4}{1-2\mu+5\mu^2-3\mu^3+\mu^4} - \frac{\sqrt{\mu(2-8\mu+17\mu^2-25\mu^3+29\mu^4-30\mu^5+24\mu^6-11\mu^7+2\mu^8)}}{(1+2\mu)[1-2\mu+5\mu^2-3\mu^3+\mu^4]^2}$ *and* $c_4(\mu) = 1 - \sqrt{\frac{\mu(2-2\mu-2\mu^2)}{1+2\mu-\mu^2-2\mu^3}}$, *with* $0 \leq c_3(\mu) < c_4(\mu) \leq \frac{1}{2}$. *And let* $S^{***}(c, \mu) = \frac{\mu(1-\mu)(2-\mu)(2-c)c}{4}$ *be the per-licence issuing cost at which the licensor is indifferent between granting two or three licences under symmetric information. Thus, under asymmetric information and at cost* S^{***}*, the licensor prefers to issue*

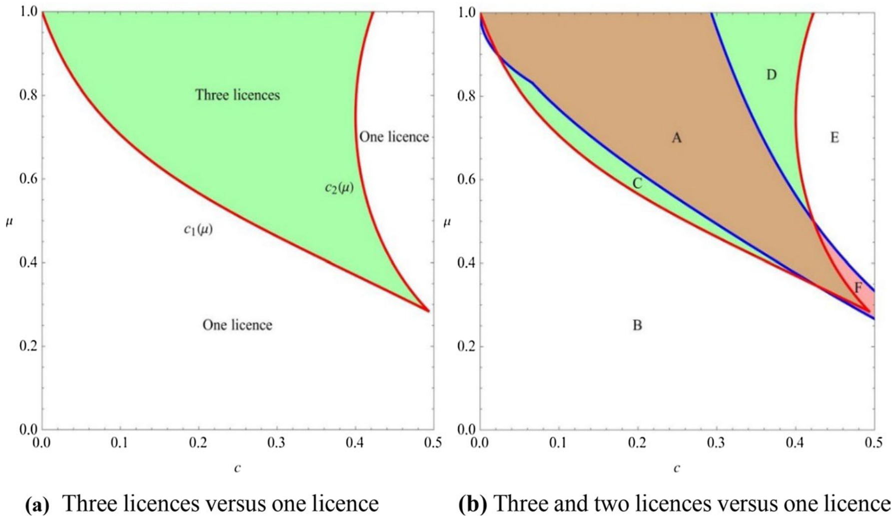

Fig. 6 **a** Regions in the (c, μ) space where the licensor prefers to grant three licences (green) and one licence (white). **b** Regions in the (c, μ) space where the licensor, rather than grant one licence, prefers to grant three licences (areas A, C and D) and two licences (areas A and F)

three licences rather than two in the region of parameters defined by $\left\{ (c, \mu) \in \left(0, \frac{1}{2}\right) \times (0, 1) \middle| c_3(\mu) \leq c \leq c_4(\mu) \right\}$ *and two licences rather than three, otherwise.*

Proof See Appendix.

Part (i) of Proposition 6 shows that, when the licensor can choose between granting one or three licences to exploit the new technology, there are certain conditions under which it is more likely that three (rather than one) licences will be granted under asymmetric information than under symmetric information. This result parallels that of Proposition 3 in the sense that the introduction of greater competition in the product market is more likely to occur when information about the technology's value is asymmetric.

Figure 6a illustrates the parameter region where—with the licensor, under symmetric information, indifferent between granting three licences or one licence—granting three licences is better than granting one when the licensor is less informed than any potential licensee; also illustrated is the region corresponding to a single licence. In turn, Fig. 6a and Fig. 3 are combined in Fig. 6b, which depicts the parameter region where the licensor prefers to grant more than one licence. In an asymmetric information scenario, a large overlap is evident between the regions reflecting the licensor's preference for three licences over one licence, and two licences over

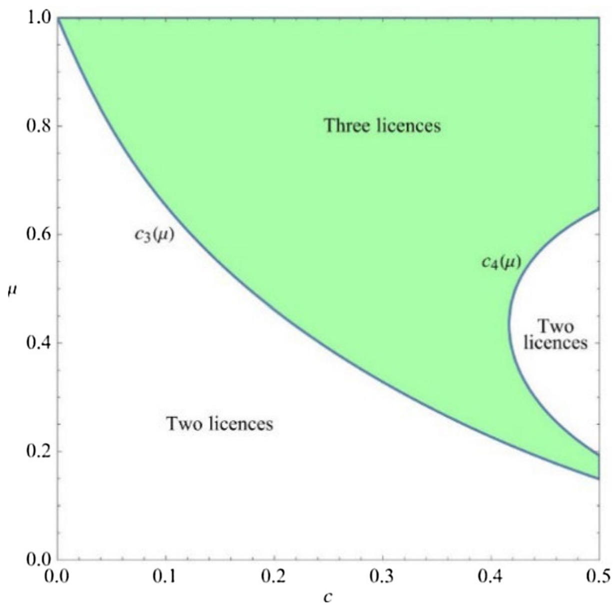

Fig. 7 Regions in the (c, μ) space where the licensor prefers to grant three licences and two licences when information is asymmetric

one licence. If anything, the region in which three licences are preferred over one licence is larger than the region in which two licences are preferred over one licence. Thus, in Fig. 6b, preferable to granting one licence is granting two licences in areas A and F, and granting three licences in areas A, C, and D. This suggests that the greater the number of licences issued, the greater the region in which the licensor prefers to grant as many licences as possible.

Likewise, part (ii) of Proposition 6 asserts that when there are three possible users of the technology instead of two, both under symmetric and asymmetric information, the licensor is more likely to grant three licences when information is asymmetric. This would suggest that, in our context, asymmetric information may lead to more users of the technology, i.e., more competition in the product market, than symmetric information. Figure 7 illustrates this result: While under symmetric information the licensor is indifferent between granting two or three licenses in the entire parameter region, if the information is asymmetric and excluding contracts are used to grant two and three licences, then there are conditions that lead the licensor to strictly prefer three licenses rather than to two. Therefore, it is confirmed once again that there is more likely to be more competition with asymmetric information than with symmetric information.

7 Licensing through contracts featuring an ad-valorem royalty

In this section, we examine the context in which the new technology is licensed, to one or two users, by means of contracts based on a fixed fee alone or on a fixed fee combined with an ad-valorem royalty, v, $0 < v \leq 1$, rather than a per-unit royalty as before. The aim is to examine at which extent the optimal licensing changes with the type of royalty used.

7.1 Exclusive licensing

If the new technology is licensed on an exclusive basis by means of a contract that, if featuring a royalty, it is ad-valorem, rather than per unit produced, then the feasible contracts for the licensor can be summarized as in Lemma 4.

Lemma 4 *Let $\mu_6(c) = \frac{2(1-c)}{2-c}$, with $0 < \mu_6(c) < 1$, for all c. To license the new technology to a single user by means of a contract with ad-valorem royalty, the licensor can offer:*

(i) *The (excluding) contract (f_0, v_0) such that $f_0 + \frac{1}{4}v_0 = \frac{1}{4}$, which the user only accepts if it is efficient.*

(ii) *The (non-excluding) 2PT contract $(f, v) = \left(\frac{[2(1-\mu)-(2-\mu)c]^2}{8(1-\mu)(2-\mu)}, 1 - \frac{2(1-\mu)}{2-\mu}\right)$, which is accepted by the user, regardless of whether it is an efficient or inefficient type, whenever $\mu < \mu_6(c)$.*

(iii) *The menu of (separating) screening contracts $\left\{\left(f_0, v_0\right), \left(f_c, v_c\right)\right\}$, where $\left(f_0, v_0\right)$ such that $f_0 + \left(\frac{1}{4} - \frac{(2-\mu)^2c^2}{16(1-\mu)^2}\right)v_0 = \frac{1}{4} - \frac{4(1-\mu)c-(2-\mu)c^2}{8(1-\mu)}$ is the contract intended for the efficient type of user, whereas $(f_c, v_c) = \left(\frac{[2(1-\mu)-(2-\mu)c]^2}{8(1-\mu)(2-\mu)}, 1 - \frac{2(1-\mu)}{2-\mu}\right)$ is the contract intended for the inefficient type, leading both types to produce whenever $\mu < \mu_6(c)$.*

Proof See Appendix.

Part (i) describes the set of excluding contracts, which includes a pure fixed-fee contract, a pure ad-valorem royalty contract, or any combination of both, that allows to collect the profit of an efficient user. Part (ii) states that, if the licence consists of a (non-excluding) 2PT single contract offered to both user types, then the ad-valorem royalty will be larger (smaller) than the average of the ad-valorem royalty included in the separating screening contract given in part (iii) when small (large) ad-valorem royalties are included in the contract intended for the efficient user. Moreover, the ad-valorem royalty included in this 2PT single contract distorts (reduces) the inefficient user's production from $q_c = \frac{1-c}{2}$ to $q_c = \frac{1-c}{2} - \frac{\mu c}{4(1-\mu)}$, but does not distort the

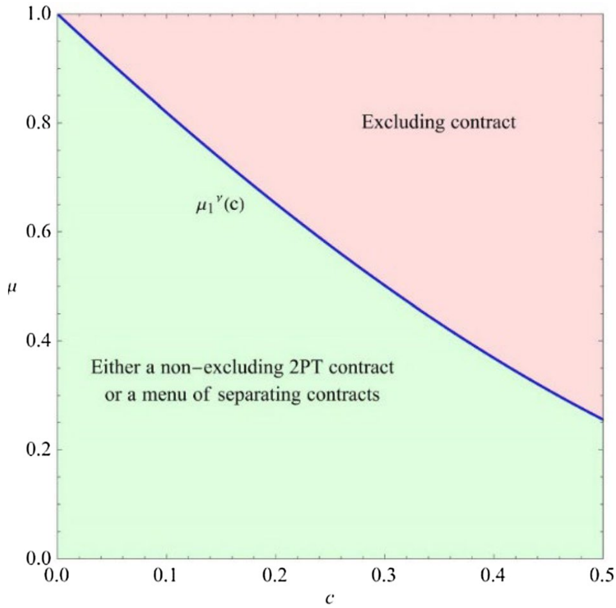

Fig. 8 Optimal exclusive licensing contract for a new technology when the royalty is ad-valorem

efficient user's production, which continues to be $q_0 = \frac{1}{2}$, due to our assumption of zero marginal cost for this type.[15]

Part (iii) indicates that a screened user will produce the same quantities as under the (non-excluding) 2PT contract stated in part (ii).[16] Hence, when the royalty is ad-valorem, as occurs when the royalty is per-unit, the user is screened by distortion, not at the top, but at the bottom. However, since the distortion at the bottom is, in this case, lower than the distortion at the bottom with a per-unit royalty, we can conclude that screening contracts are less distortionary with an ad-valorem royalty than with a per-unit royalty. In other words, for an ad-valorem royalty the information rents granted to the efficient user, $\frac{4(1-\mu)c-(2-\mu)c^2}{8(1-\mu)}$, are smaller. Finally, and unlike what happens with a per-unit royalty, the (ad-valorem) royalty in the contract intended for the inefficient user is the same as in the non-excluding contract offered to both user types in part (ii). Furthermore, those royalties depend exclusively on the probability that the technology will be used efficiently and not on the inefficient user's cost.

[15] However, if the efficient type had a positive marginal cost c_L such that $0 < c_L < c_H$, the equivalence of this single 2PT contract and the menu of separating contracts would not hold, since production for an efficient type induced by the single contract would be different from production induced by the menu of separating contracts.

[16] Note that $q_c > 0$ whenever $\mu < \mu^v(c) \equiv 2(1-c)/(2-c)$ or, alternatively, $c < 2(1-\mu)/(2-\mu)$. Otherwise, the inefficient type of user would not produce. Note also that $\mu^v(c) > \mu_1''(c)$, and thus a menu of separating contracts featuring a per-unit royalty does not exist if $\mu_1''(c) < \mu < \mu^v(c)$, but does exist with an ad-valorem royalty.

Taking into account the licensor's expected revenues from the feasible contracts stated in Lemma 4, we can now state the optimal exclusive licensing when the involved royalty is ad-valorem.

Proposition 7 *When the contractual royalty is ad-valorem, there is a cut-off value for the licensor's belief of dealing with an efficient user,* $\mu_1^v(c) = 1 - \frac{4c+2c\sqrt{3-2c}-c^2}{4+c^2}$, *with* $0 < \mu_1^v(c) < 1$, *for all c, such that exclusive licensing of the new technology is as follows:*

(i) *If* $\mu > \mu_1^v(c)$, *through (excluding) non-distortionary contracts stated in part (i) of Lemma 4.*
(ii) *If* $\mu < \mu_1^v(c)$, *through either (non-excluding) 2PT contracts or the menu of separating contracts stated in part (ii) and part (iii), respectively, of Lemma 4.*

Proof See Appendix.

Part (i) indicates that, for a sufficiently high probability of an efficient user, the licensor prefers the technology to be only exploited by the efficient user. Part (ii) shows that when the probability of an efficient user is low, the licensor is indifferent between licensing by means of a menu of two separating screening contracts or by means of a single 2PT non-screening contract like those described in part (ii) of Lemma 4. In both cases, the technology is implemented for sure and the licensor's revenues are the same because the marginal cost of the efficient user is assumed to be zero, and consequently, the ad-valorem royalty does not distort the efficient user's production level, neither when licensed under a 2PT contract nor when licensed under a separating menu, as stated in part (ii) and part (iii), respectively, in Lemma 4. Thus, information rents for the efficient user amount to $\frac{[4(1-\mu)-(2-\mu)c]c}{8(1-\mu)}$ under either of these two contracts. The content of Proposition 7 is illustrated in Fig. 8.

Finally, comparing licensing through contracts that include per-unit versus ad-valorem royalties, Figs. 1 and 8 allow us to conclude that, in the case of an ad-valorem royalty, the use of the contract with which to scrutinize licensees and resulting in exploitation of the technology for sure, is somewhat less frequent. Therefore, an ad-valorem royalty is more likely to ensure that a new technology reaches the market, but provided the user is efficient, which implies less diffusion.

7.2 Non-exclusive licensing

When the new technology is simultaneously licensed to two users rather than only one and the royalty, if used, is ad-valorem, the set of available contracts can be summarized as in Lemma 5.

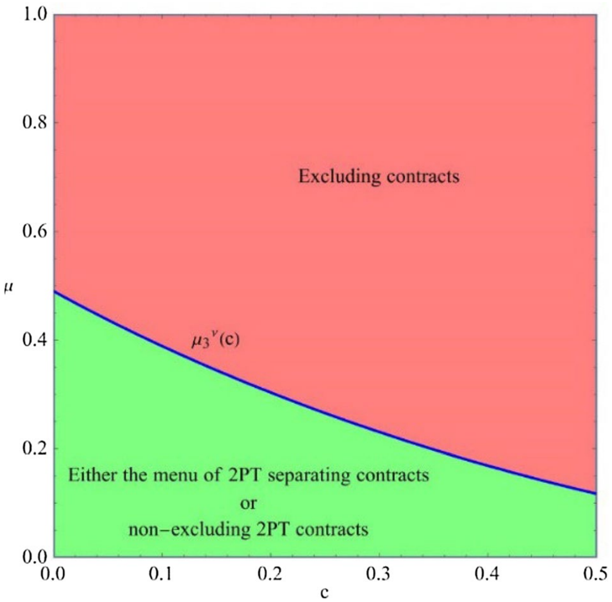

Fig. 9 Optimal granting of two licences by means of contracts featuring an ad-valorem royalty

Lemma 5 *Let* $\mu_2^v(c) = 1 - \frac{\sqrt{16+9c^2}-4}{c}$, *with* $0 < \mu_2^v(c) < 1$, *for all c. To license the new technology to two users when royalties are ad-valorem, the licensor can offer each user* i, $i = 1, 2$*:*

(i) *The (excluding) contract* $v_0 = 1$ *or, equivalently, the fixed-fee contract* $f_0 = \frac{1}{(2+\mu)^2}$, *which the user only accepts if it is efficient.*

(ii) *The (non-excluding) contract given by*

$$(f, v) = \begin{cases} \left(\frac{[8(1-\mu)-(4-\mu)(2+\mu)c]^2 c}{8(1-\mu)(8+\mu)[4(1-\mu)+3(4-\mu)c]}, 1 - \frac{2(8-7\mu-\mu^2)c}{4(1-\mu)+3(4-\mu)c} \right) & \text{if } \mu < \mu_2^v(c) \\ \left(0, 1 - \frac{(2+\mu)c}{2} \right) & \text{if } \mu > \mu_2^v(c) \end{cases}$$

which allows each user to produce, regardless of whether it is an efficient or inefficient type.

(iii) *The menu of screening contracts* $\left\{ (f_0, v_0), (f_c, v_c) \right\}$, *where*

$$(f_0, v_0) = \begin{cases} \left(\frac{(1-v_0)[2(1-v_c)+(1-\mu)c]^2+3(1-v_c)[(1+2\mu)c-4(1-v_c)]c}{36(1-v_c)^2}, \left[0, 1 - \frac{2(8+\mu)[20(1-\mu)-(4+7\mu-2\mu^2)c]c}{(1-\mu)[12+(4-\mu)c]^2}\right] \right) \\ \text{if } \mu < \mu_2^v(c) \\ \left(\frac{(1-v_0)[2(1-v_c)+(1-\mu)c]^2+3(1-v_c)[(1+2\mu)c-4(1-v_c)]c}{36(1-v_c)^2}, \left[0, 1 - \frac{(2+\mu)c}{2}\right] \right) \\ \text{if } \mu > \mu_2^v(c) \end{cases}$$

is the contract intended for the efficient type, and

$$\left(f_c, v_c\right) = \begin{cases} \left(\frac{[8(1-\mu)-(4-\mu)(2+\mu)c]^2 c}{8(1-\mu)(8+\mu)[4(1-\mu)+3(4-\mu)c]}, 1 - \frac{2(8-7\mu-\mu^2)c}{4(1-\mu)+3(4-\mu)c}\right) & \text{if } \mu < \mu_2^v(c) \\ \left(0, 1 - \frac{(2+\mu)c}{2}\right) & \text{if } \mu > \mu_2^v(c) \end{cases}$$

is the contract intended for the inefficient type.

Proof See the Appendix.

As stated in part (i), the amount of the excluding contract offered to the efficient type of each user i, varies between $1/4$ when the rival j is an inefficient type and, consequently, only user i is efficient, and $1/9$ when both users, i and j, are efficient. On the other hand, and as in the case of a single licence, both the non-excluding 2PT contract (f, v) stated in part (ii) and the 2PT contract $\left(f_c, v_c\right)$ intended for the inefficient user in the screening menu defined in part (iii) are the same. However, both contracts involve an ad-valorem royalty whose amount depends on both the μ and $c$parameters, rather than only on μ as when the licensor grants a single licence.

Having defined the feasible contracts that allow two licences for the technology, those that are optimal for the licensor are now summarized in Proposition 8 and illustrated in Fig. 9.

Proposition 8 *Given the cut-off value $\mu_3^v(c)$ that solves in $(0, 1)$ the equation $c = \frac{4(1-\mu)}{(4-\mu)^2}\left(4 + \mu - \frac{8+\mu}{2+\mu}\sqrt{\frac{\mu(16+4\mu+11\mu^2)}{8-7\mu-\mu^2}}\right)$, the new technology is optimally licensed to two users as follows:*

(i) *If $\mu < \mu_3^v(c)$, either through the contracts stated in part (ii) of Lemma 5 or those stated in part (iii) of Lemma 5.*
(ii) *If $\mu > \mu_3^v(c)$, by means of the contracts stated in part (i) of Lemma 5.*

Proof See the Appendix.

The new technology is optimally exploited by efficient or inefficient users when the licensor's probability of dealing with efficient users is sufficiently small, and so the licensor stands to benefit most from a contract with all types of users. In this case, and unlike what happens with a per-unit royalty, the licensor is indifferent between offering contracts that screen or do not screen users. In contrast, when that probability is sufficiently large, the licensor prefers to offer each user a contract that leads it to produce only if efficient, and so, as a result, the technology will be marketed only by efficient users.

Of the licensor's two options, the most frequently used, in view of Fig. 9, is the excluding contract to each user. This contrasts with the case of a single licence, where (as illustrated in Fig. 8) either the (non-excluding) 2PT non-screening contract or the menu of screening contracts are most frequently used. Hence, when the royalty, if applied, is ad-valorem, the use of excluding contracts increases as the number of granted licences increases from one to two. Finally, comparing the granting of two licences through contracts with a per-unit versus an ad-valorem royalty, we may conclude that the use of separating screening contracts is less frequent with an ad-valorem royalty.

Finally, comparison of the results stated in Propositions 1 and 7, on the one hand, and Propositions 2 and 8, on the other hand, allows us to state Corollary 1.[17]

Corollary 1 *An external licensor with a new technology and uninformed as to its economic value does not use an ad-valorem royalty in any optimal contract, regardless of whether it grants one or two licences. If any royalty is used, it is per unit of production.*

This result shows that an external licensor, less informed about the technology's value than its potential users, will license it by means of contracts that, if any, feature a per-unit rather than an ad-valorem royalty, given that configuring the contractual terms with a sales royalty adds nothing compared to the use of a per-unit royalty. Hence, from the standard result that an external licensor, fully informed as to the value of its technology, finds it optimal to transfer it through fixed-fee contracts (Kamien and Tauman 1986; Kamien et al. 1992), we can now go one step further and describe the scenario when the external licensor is less informed than the potential users of the technology. The optimal contract in this case may be based on either a fixed fee alone or a fixed fee combined with a royalty, depending on the probability of encountering an efficient user of the technology and the difference in exploitation efficiency is large or small. Nonetheless, regardless of whether one or two licences are granted, the royalty is always per-unit, never ad-valorem.

8 Conclusions

We have analysed the licensing behaviour of an external licensor owning a new and patented technology but not knowing its value. To benefit from the technology, the licensor has to choose both the number of licences to be granted and the contractual terms of each licence, in a context in which users hold privileged information on the technology's value.

Our first finding is that the option to choose the number of licences affects the payment structure of the contracts. Under exclusive licensing, only the contract intended for an inefficient user features a per-unit royalty (distortion at the bottom), whereas when two licences are granted, a per-unit royalty is included not only in the contract intended for the inefficient user, but also in the contract intended for

[17] Recall that we assume zero marginal cost for the user when it employs the technology efficiently.

the efficient user (distortion at the top). This result is partly driven by the licensor's incentive to replicate a monopoly outcome even when granting two licences, which, in turn, explains the use of royalties in contracts intended for both types of users. In other words, when granting two licences the licensor approximates the "created" market as much as possible to a monopoly, minimizing the concession of information rents to efficient users. Although the preference for non-exclusive licensing with the distortion-at-the-top effect parallels previous studies regarding licensing under symmetric information (see, for instance, Sen 2005; Sen and Tauman 2007), we find that, just as under exclusive licensing, a per-unit royalty also serves the licensor's purpose of making contracts intended for inefficient users less attractive to efficient users.

We also find that two licences are more likely to be issued under asymmetric information than under symmetric information. For this scenario, however, we distinguish screening from the granting of licences to only efficient users. A licensor that screens users is better off under exclusive than non-exclusive licensing. However, if information is symmetric, then, ceteris paribus, non-exclusive licensing will benefit the licensor more than exclusive licensing; this is because the negative effect of competition on its profits will be offset by royalty payments. We find that, when both competition and asymmetric information apply, the licensor needs to include a per-unit royalty in the contract as a screening device. However, using royalties for screening purposes reduces the licensor's expected profits to such an extent that exclusive licensing becomes preferable to the sampling advantage of non-exclusive licensing.

We also show how the number of licences granted affects aggregate welfare, taking as given both the contracts subsequently offered by the licensor and the users' behaviour. If the cost of awarding the technology to several users is such that aggregate welfare under exclusive licensing is greater when information is symmetric rather than asymmetric, then non-exclusive licensing can yield greater welfare under asymmetric information, except when the licensor uses a screening contract. With a screening contract, societal welfare is greater under exclusive licensing, whereas consumer surplus is greater under non-exclusive licensing. Hence, under screening, in shifting from non-exclusive to exclusive licensing, consumer surplus losses are more than offset by increased licensor profits. This result reinforces our second finding, in revealing the negative effects on licensor profits of screening combined with non-exclusive licensing, with the resulting Cournot competition.

If the licensor issues three licences by means of contracts featuring a per-unit royalty, its licensing behaviour differs from when two licences are issued. First, the per-unit royalty included in the different contracts is higher (and the fixed payment is lower) than in analogous contracts involving two licences. Second, the optimal concession of three licences leads the licensor not to screen potential users and to use only (excluding or non-excluding) non-separating contracts. Third, when we compare the use of three licences versus one licence, the parameter region where three licences are preferred over one licence is very similar to the region where two licences are preferred over one licence. This would suggest that the licensor's incentive to introduce competition in the marketplace does not much depend on the number of licences offered. Hence, a valuable extension of our model would be to consider n potential users (each capable, as here, of producing a final good with the new technology) and derive the optimal number of licences to be granted under symmetric and asymmetric information conditions.

These findings contribute to the licensing literature by adding to the scant research into licensor behaviour when it chooses both the number of users to transfer the new technology to and the optimal form of licensing contracts to be offered to each user. We believe that the results reported here can serve as a useful benchmark for a more general case than three users.

Finally, we have also shown that, even though feasible, an ad-valorem royalty is never incorporated in licensing contracts. From the licensor's perspective, and regardless of whether one or two licences are granted, contracts involving an ad-valorem royalty perform worse than contracts featuring a per-unit royalty. From here, it would be interesting to analyse the welfare impact of both types of royalty contract. If the use of an ad-valorem royalty is welfare enhancing, then the licensor's incentives would no longer be aligned with social incentives and the possibility arises that licensing deals may be regulated, e.g., by discouraging the use of a per-unit royalty and encouraging the use of an ad-valorem royalty. This is left for future research.

Appendix

Proof of Lemma 1 To offer the menu of screening contracts to the user the innovator solves the problem:

$$\left.\begin{array}{l} \max\limits_{\{(f_0,r_0),(f_c,r_c)\}} \mu\left(f_0 + r_0 q_0\right) + (1-\mu)\left(f_c + r_c q_c\right) \\ s.t. \begin{cases} \frac{(1-r_0)^2}{4} - f_0 \geq 0 & \left(\mathrm{PC}_0\right) \\ \frac{(1-c-r_c)^2}{4} - f_c \geq 0 & \left(\mathrm{PC}_c\right) \\ \frac{(1-r_0)^2}{4} - f_0 \geq \frac{(1-r_c)^2}{4} - f_c & \left(\mathrm{IC}_0\right) \\ \frac{(1-c-r_c)^2}{4} - f_c \geq \frac{(1-c-r_0)^2}{4} - f_0 & (\mathrm{IC}_c) \end{cases} \end{array}\right\} \tag{2}$$

where the subscripts 0 and c denote, respectively, the efficient and inefficient user, whereas $q_0 = \left(1 - r_0\right)/2$ and $q_c = \left(1 - c - r_c\right)/2$ are their profit-maximizing quantities as a naïve monopolist. In (2), $\left(\mathrm{PC}_0\right)$ and $\left(\mathrm{PC}_c\right)$ refer the participation constraints for the efficient and inefficient user, respectively, whereas $\left(\mathrm{IC}_0\right)$ and (IC_c) are the incentive compatibility conditions also for the efficient and inefficient user, respectively.

(i) If problem (2) is solved while ignoring $\left(\mathrm{IC}_0\right)$ and $\left(\mathrm{IC}_c\right)$, the licensor cannot screen the user's type and so will offer a single contract (f, r) to both types. However, it must still assess the profits (if any) from allowing an inefficient type to use the technology. If f is such that only the efficient user produces, then $f = (1-r)^2/4$, i.e., it equals the profits of a monopolist producing $q_0 = (1-r)/2$. Therefore, from:

$$\max_r \left\{ \mu \left(\frac{(1-r)^2}{4} + r\frac{1-r}{2} \right) \right\}$$

the licensor's optimal per-unit royalty is $r = 0$. Hence, $f = 1/4$.

(ii) If the fixed fee required by the licensor allows the user to produce according to its type, the PC conditions imply that $f = (1-c-r)^2/4$, which is the profit of an inefficient user producing $q_c = (1-c-r)/2$. However, because the efficient user also receives the same contract, then $q_0 = (1-r)/2$. Thus, from:

$$\max_r \left\{ \mu \left(\frac{(1-c-r)^2}{4} + r\frac{1-r}{2} \right) + (1-\mu)\left(\frac{(1-c-r)^2}{4} + r\frac{1-c-r}{2} \right) \right\}$$

the optimal per-unit royalty is $r = \mu c$, and thus $f = [1-(1+\mu)c]^2/4$. Production levels of the efficient and inefficient user are, therefore, $q_0 = (1-\mu c)/2$ and $q_c = [1-(1+\mu)c]/2$, respectively, and both q_0, q_c and f are positive by virtue of Assumption 1 and the fact that $0 < \mu < 1$.

(iii) First, note that, in problem (2), the RHS of $\left(\mathrm{IC}_0\right)$ is higher than the LHS of $\left(\mathrm{PC}_c\right)$, while the LHS of $\left(\mathrm{IC}_0\right)$ is equal to the LHS of $\left(\mathrm{PC}_0\right)$. Therefore, if $\left(\mathrm{PC}_c\right)$ holds, then $\left(\mathrm{PC}_0\right)$ also holds and hence can be ignored. Second, we prove that the solution verifies $\left(\mathrm{IC}_0\right)$ with equality and (IC_c) with strict inequality. If we assume, for the moment, that instead we solve the problem ignoring both ICs, it can be seen that the solution would verify both PCs with equality. However, the efficient user would then be better off misrepresenting the technology as of low value: if it tells the truth, its profit will be zero; if it lies, its profits will be $\frac{(1-r_c)^2}{4} - \frac{(1-c-r_c)^2}{4} > 0$. But given that $f_c = \left[1-c-r_c\right]^2/4$, those profits are equal to the RHS of $\left(\mathrm{IC}_0\right)$, and so the inequality would not hold. In contrast, the inefficient user has no incentive to misrepresent the technology's perceived value: if it tells the truth, its profits are zero; but if it lies, its profits amount to $\frac{(1-r_0-c)^2}{4} - \frac{(1-r_0)^2}{4} < 0$. Therefore, it is safe to ignore $\left(\mathrm{IC}_c\right)$.

We can now prove that both $\left(\mathrm{IC}_0\right)$ and $\left(\mathrm{PC}_c\right)$ in (2) will be verified as equalities in the solution. If $\left(\mathrm{IC}_0\right)$ were not verified as such, the licensor could always raise f_0 and thereby increase its revenues, since this would not affect $\left(\mathrm{PC}_c\right)$. Condition $\left(\mathrm{PC}_c\right)$ will also be verified as an equality, because, if not, the licensor could raise f_c to increase its revenues. Such an increase would have the additional effect of diminishing the RHS of $\left(\mathrm{IC}_0\right)$, which, in turn, would allow the licensor to further increase f_0. Hence $f_c = \left[1-c-r_c\right]^2/4$, from which:

$$f_0 = \frac{\left(1-r_0\right)^2}{4} - \frac{\left(1-r_c\right)^2}{4} + f_c = \frac{\left(1-r_0\right)^2}{4} - \frac{\left(1-r_c\right)^2}{4} + \frac{\left(1-c-r_c\right)^2}{4} \quad (3)$$

Taking into account (3), the licensor's problem given in (2) can be rewritten as:

$$\max_{\{(f_0,r_0),(f_c,r_c)\}} \left\{ \mu\left(f_0 + r_0\frac{1-r_0}{2}\right) + (1-\mu)\left(f_c + r_c\frac{1-r_c-c}{2}\right) \right\} = \max_{r_0,r_c} \left\{ \frac{1}{4}(1-c)^2 + \frac{c}{2}\mu r_c - \frac{1}{4}(1-\mu)r_c^2 - \frac{1}{2}\mu r_0^2 \right\}$$

which yields $r_0 = 0$, $r_c = \frac{\mu c}{1-\mu}$, $f_c = \frac{(1-\mu-c)^2}{4(1-\mu)^2}$ and $f_0 = \frac{1-\mu-2(1-\mu)c+(1+\mu)c^2}{4(1-\mu)}$, and the user produces $q_0 = 1/2$ (if it is efficient) and $q_c = (1-\mu-c)/2(1-\mu)$ (if it is inefficient), where $q_c > 0$ whenever $\mu < 1-c$. □

Proof of Proposition 1 The (excluding) contract stated in part (i) of Lemma 1 yields the licensor's expected revenues:

$$E[\pi_0^{L1}] = \frac{\mu}{4} \quad (4)$$

where the subscript 0 denotes that only the efficient user produces, and the superscript $L1$ stands for a single licence. Likewise, the optimal (non-excluding) non-screening contract in part (ii) of Lemma 1 yields the licensor's expected revenues:

$$E\left[\pi_{0+c}^{L1}\right] = \frac{(1-c)^2}{4} + \frac{\mu^2 c^2}{4} \quad (5)$$

where the subscript $0+c$ indicates that both the efficient and inefficient user produce. Finally, the optimal screening contract in part (iii) of Lemma 1 renders the expected revenues:

$$E\left[\pi_S^{L1}\right] = \frac{(1-c)^2}{4} + \frac{\mu^2 c^2}{4(1-\mu)} \quad (6)$$

where the subscript S reflects a screening scenario. From (5) and (6), it follows that $E\left[\pi_S^{L1}\right] > E\left[\pi_{0+c}^{L1}\right]$, and hence, the comparison reduces to (4) and (6). This leads to $E\left[\pi_0^{L1}\right] > E\left[\pi_S^{L1}\right]$ iff:

$$c_0(\mu) \equiv \frac{(1-\mu)(1-\sqrt{\mu(1-\mu)})}{1-\mu+\mu^2} < c < \frac{(1-\mu)(1+\sqrt{\mu(1-\mu)})}{1-\mu+\mu^2} \equiv c_0'(\mu) \quad (7)$$

but the fact that $c < 1/2$ leads the upper bound on c in (7) to be irrelevant. Finally, the condition $c > \frac{(1-\mu)(1-\sqrt{\mu(1-\mu)})}{1-\mu+\mu^2}$ is equivalent to $\mu > 1 - \frac{\left(2+c+\sqrt{(4-3c)c}\right)c}{2(1+c^2)}$, as stated in part (ii). Part (i) immediately follows. □

Proof of Lemma 2

(i) If the licensor offers the same contract (f, r) to each user, regardless of its type, it may offer a contract that will be only accepted by the efficient types, or a contract that will be accepted by both the efficient and inefficient types. In the first case, the licensor solves the problem:

$$\max_{(f_0,r_0)} 2\left[\mu^2 + \mu(1-\mu)\right]\left(f + rq_0^i\right), \text{s. t} : f \leq E\left[\pi_0^i\right] \tag{8}$$

because there may be two efficient users or only one. In (8), q_0^i denotes the production level of efficient user i, $i = 1, 2$, when only efficient users produce, and $E\left[\pi_0^i\right]$ denotes its expected profit. To compute $E\left[\pi_0^i\right]$, each user i, taking user j's production as given, solves:

$$\max_{q_0^i}\left(1 - r - q_0^i - \mu q^j\right)q_0^i, j \neq i$$

which yields $q_0^i = (1-r)/(2+\mu)$ (when $\mu = 0$, the rival j is inefficient and thus i produces as a monopolist; when $\mu = 1$, the rival j is efficient and thus i produces $(1-r)/3$). Efficient user i's expected profit is then $E\left[\pi_0^i\right] = \left(\frac{1-r}{2+\mu}\right)^2$ and, as result, (8) can be rewritten as:

$$\max_{r_0}\left\{2\mu\left[\left(\frac{1-r}{2+\mu}\right)^2 + r\frac{1-r}{2+\mu}\right]\right\}$$

and its solution is $r_0 = \mu/2(1+\mu)$.

(ii) Offering each user i a contract that allows it to produce, regardless of whether it is efficient or inefficient, and provided that $E\left[\pi_c^i\right] < E\left[\pi_0^i\right]$, leads the licensor to solve:

$$\begin{aligned}&\max_{(f_c,r_c)}\left\{2\mu^2\left(f_c + rq_0^i\right) + 2\mu(1-\mu)\left[2f_c + r\left(q_0^i + q_c^i\right)\right] + 2(1-\mu)^2\left(f_c + rq_c^i\right)\right\},\\ &\text{s.t} : \ f_c \leq E\left[\pi_c^i\right]\end{aligned} \tag{9}$$

where $q_{\widetilde{c}}^i$, $\widetilde{c} = \{0, c\}$, is the result of solving:

$$\max_{q_{\widetilde{c}}^i}(1 - \widetilde{c} - r - q_{\widetilde{c}}^i - \mu q_0^j - (1-\mu)q_c^j)q_{\widetilde{c}}^i$$

Therefore, the production level for each efficient and inefficient user i are $q_0^i = [2 + (1-\mu)c - 2r]/6$ and $q_c^i = [2 - (2+\mu)c - 2r]/6$, respectively, and thus $E\left[\pi_c^i\right] = [2 - (2+\mu)c - 2r]^2/36$. This allows us to rewrite the problem given in (9) as:

$$\max_{r_c}\frac{4(1 + r - 2r^2) - 4(2 + \mu + (1-4\mu)r)c + (2+\mu)^2c^2}{18}$$

yielding $r_c = [1 - (1-4\mu)c]/4$, which applies whenever $c < c_2'(\mu) = \min\left\{\frac{1}{2}, \frac{1}{1+2\mu}\right\}$ or, equivalently, $\mu < \frac{1-c}{2c}$; otherwise, the inefficient users do not produce.

(iii) Offering the menu of screening contracts $\{(f_0, r_0), (f_c, r_c)\}$ to each user i leads the licensor to solve the problem:

$$\left.\begin{array}{l}\max_{\{(f_0,r_0),(f_c,r_c)\}} \left\{2\mu^2\left(f_0 + r_0q_0\right) + 2\mu(1-\mu)\left(f_0 + r_0q_0 + f_c + r_cq_c\right) + 2(1-\mu)^2\left(f_c + r_cq_c\right)\right\} \\ \qquad s.t.\begin{cases} E[\pi_0^i] - f_0 \geq 0 & (\mathrm{PC}_0) \\ E[\pi_c^i] - f_c \geq 0 & (\mathrm{PC}_c) \\ E[\pi_0^i] - f_0 \geq E[\pi_{0|c}^i] - f_c & (\mathrm{IC}_0) \\ E[\pi_c^i] - f_c \geq E[\pi_{c|0}^i] - f_0 & (\mathrm{IC_c}) \end{cases}\end{array}\right\} \tag{10}$$

where $E[\pi_{\widetilde{c}}^i]$ for $\widetilde{c} = \{0, c\}$ denotes user i's expected profit when it is of type $\widetilde{c}$ and is perceived as such by other players, $E[\pi_{0|c}^i]$ denotes user i's expected profit when it is efficient but is perceived by the other players as inefficient, and $E[\pi_{c|0}^i]$ denotes user i's expected profit when it is inefficient but is perceived by the other players as efficient, in both cases assuming that user j tells the truth about its type. Expected profits $E[\pi_0^i]$ and $E[\pi_c^i]$ are obtained from:

$$\max_{q_{\tilde{c}}^i} \left(1 - \tilde{c} - r_{\tilde{c}} - q_{\tilde{c}}^i - \mu q_0^j - (1-\mu)q_c^j\right)q_{\tilde{c}}^i, j \in \{1, 2\}, j \neq i$$

and, solving the four FOCs, $1 - r_{\widetilde{c}} - \widetilde{c} - 2q_{\widetilde{c}}^i - \mu q_0^j - (1-\mu)q_c^j = 0$, yields:

$$q_0^i = \frac{2 + (1-\mu)c - (3-\mu)r_0 + (1-\mu)r_c}{6} \text{ and } q_c^i = \frac{2 - (2+\mu)c - (2+\mu)r_c + \mu r_0}{6} \tag{11}$$

whereby user i's expected profits amount to $E[\pi_0^i] = \left[2 + (1-\mu)c - (3-\mu)r_0 + (1-\mu)r_{\mathrm{c}}\right]^2/36$ and $E[\pi_c^i] = \left[2 - (2+\mu)c - (2+\mu)r_c + \mu r_0\right]^2/36$. On the other hand, user i's expected profits $E[\pi_{0|c}^i]$ and $E[\pi_{c|0}^i]$ are derived from:

$$\max_{q_{\widetilde{c}|\widetilde{c}^M}^i} \left(1 - \widetilde{c} - r_{\widetilde{c}^M} - q_{\widetilde{c}|\widetilde{c}^M}^i - \mu q_0^j - (1-\mu)q_c^j\right)q_{\widetilde{c}|\widetilde{c}^M}^i \tag{12}$$

where the superscript M denotes that the user i's type is misperceived, whereby $\widetilde{c}|\widetilde{c}^M \in \{0|c, c|0\}$, and both q_0^j and q_c^j ($j \in \{1, 2\}$, $j \neq i$) are defined by (11). The solution to problem given in (12) is $q_{0|c}^i = \left[2 + (1-\mu)c + \mu r_0 - (2+\mu)r_c\right]/6$ and $q_{c|0}^i = \left[2 - (2+\mu)c + (1-\mu)r_c - (3-\mu)r_0\right]/6$, from which the user i's expected profits amount to $E\left[\pi_{0|c}^i\right] = \left[2 + (1-\mu)c + \mu r_0 - (2+\mu)r_c\right]^2/36$ and $E[\pi_{c|0}^i] = \left[2 - (2+\mu)c + (1-\mu)r_c - (3-\mu)r_0\right]^2/36$. Hence, the licensor's problem defined in (10) can be rewritten as:

$$\left.\begin{array}{l}\max\limits_{\{(f_0,r_0),(f_c,r_c)\}}\left\{2\mu^2\left(f_0+r_0\frac{2+(1-\mu)c-(3-\mu)r_0+(1-\mu)r_c}{6}\right)\right.\\ +2\mu(1-\mu)\left(f_0+r_0\frac{2+(1-\mu)c-(3-\mu)r_0+(1-\mu)r_c}{6}+f_c+r_c\frac{2-(2+\mu)c-(2+\mu)r_c+\mu r_0}{6}\right)\\ \left.+2(1-\mu)^2\left(f_c+r_c\frac{2-(2+\mu)c-(2+\mu)r_c+\mu r_0}{6}\right)\right\}\\ s.t.\begin{cases}\frac{\left[2+(1-\mu)c-(3-\mu)r_0+(1-\mu)r_c\right]^2}{36}-f_0\geq 0 & (\mathrm{PC}_0)\\ \frac{\left[2-(2+\mu)c-(2+\mu)r_c+\mu r_0\right]^2}{36}-f_c\geq 0 & (\mathrm{PC}_c)\\ \frac{\left[2+(1-\mu)c-(3-\mu)r_0+(1-\mu)r_c\right]^2}{36}-f_0\geq \frac{\left[2+(1-\mu)c+\mu r_0-(2+\mu)r_c\right]^2}{36}-f_c & (\mathrm{IC}_0)\\ \frac{\left[2-(2+\mu)c-(2+\mu)r_c+\mu r_0\right]^2}{36}-f_c\geq \frac{\left[2-(2+\mu)c+(1-\mu)r_c-(3-\mu)r_0\right]^2}{36}-f_0 & (\mathrm{IC}_c)\end{cases}\end{array}\right\} \tag{13}$$

which is solved by taking into account that restrictions (PC_0) and $\left(\mathrm{IC}_c\right)$ are not binding (and they can be ignored), whereas restrictions $\left(\mathrm{PC}_c\right)$ and $\left(\mathrm{IC}_0\right)$ hold with equality. Thus, taking $\left(\mathrm{PC}_c\right)$ and $\left(\mathrm{IC}_0\right)$ as equalities allows us to rewrite the problem (13) as:

$$\max_{\{r_0,r_c\}}\left\{\frac{\mu(\mu-9)r_0^2}{18}+\frac{\left(7\mu+\mu^2-8\right)r_c^2}{18}+\frac{\mu(2-c(2+\mu))r_0}{9}\right.$$
$$\left.+\frac{\left(2-2c-2\mu+10c\mu+c\mu^2\right)r_c}{9}+\frac{\mu(1-\mu)r_0r_c}{9}+\frac{\left(4-8c+4c^2-4c\mu+4c^2\mu+c^2\mu^2\right)}{18}\right\}$$

and its solution, provided that $c\leq\min\left\{\frac{1}{2},\frac{1-\mu}{1+\mu}\right\}\equiv c_2''(\mu)$, or, equivalently, $\mu\leq\frac{1-c}{1+c}$, is given by $r_0=\frac{1-c}{4}$ and $r_c=\frac{1-\mu-(1-5\mu)c}{4(1-\mu)}$. Finally, substituting r_0 and r_c into constraints $\left(\mathrm{PC}_0\right)$ and $\left(\mathrm{IC}_c\right)$ of (13)—when written as equalities—yields $f_c=\left(\frac{1-\mu-(1+\mu)c}{4(1-\mu)}\right)^2$ and $f_0=\frac{1-\mu-2(1-\mu)c+(1+7\mu)c^2}{16(1-\mu)}$. □

Proof of Proposition 2 If the licensor offers each user an excluding 2PT contract that only allows the efficient users to produce, its expected revenues are:

$$E\left[\pi_0^{L2}\right]=\frac{\mu}{2(1+\mu)} \tag{14}$$

where the superscript $L2$ denotes the concession of two licences. Likewise, if the licensor offers each user a non-excluding contract that lead it to always produce, its expected revenues amount to:

$$E[\pi_{0+c}^{L2}]=\frac{(1-c)^2}{4}+\frac{\mu^2c^2}{2} \tag{15}$$

Finally, when offering a menu of 2PT separating contracts to each user, the expected revenues are:

$$E\left[\pi_S^{L2}\right] = \frac{(1-c)^2}{4} + \frac{\mu^2 c^2}{2(1-\mu)} \tag{16}$$

Since the comparison of revenues stated in (15) and (16) yields $E\left[\pi_S^{L2}\right] > E[\pi_{0+c}^{L2}]$, the optimal decision reduces to compare the revenues in (14) and (16). This yields $E\left[\pi_0^{L2}\right] < E[\pi_S^{L2}]$ if $c \leq \min\left\{\frac{1}{2}, \frac{1-\mu}{1-\mu+2\mu^2}\left(1-\sqrt{\frac{2\mu(1-\mu)}{1+\mu}}\right)\right\} \equiv c_2(\mu)$ and thus a menu of 2PT separating contracts to each user is optimal, but an excluding 2PT contract if $c \geq c_2(\mu)$. □

Proof of Proposition 3 According to Proposition 1, the licensor's expected payoff when it grants a single licence amounts to:

$$E[\pi^{L1}] = \begin{cases} \frac{1-\mu-2(1-\mu)c-\left(1+\mu-\mu^2\right)c^2}{4(1-\mu)} & \text{if } 0 < c < \frac{(1-\mu)(1-\sqrt{\mu(1-\mu)})}{1-\mu+\mu^2} \equiv c_1(\mu) \\ \frac{\mu}{4} & \text{if } c_1(\mu) < c < \frac{1}{2} \end{cases}$$

and according to Proposition 2, that from granting two licences is:

$$E[\pi^{L2}] = \begin{cases} \frac{1-\mu-2(1-\mu)c+\left(1-\mu+2\mu^2\right)c^2}{4(1-\mu)} & \text{if } 0 < c < \frac{1-\mu}{1-\mu+2\mu^2}\left(1-\sqrt{\frac{2\mu(1-\mu)}{1+\mu}}\right) \equiv c_2(\mu) \\ \frac{\mu}{2(1+\mu)} & \text{if } c_2(\mu) < c < \frac{1}{2} \end{cases}$$

Comparing $E\left[\pi^{L1}\right] - S^*(c,\mu)$ with $E\left[\pi^{L2}\right] - 2S^*(c,\mu)$ yields the stated result. □

Proof of Proposition 4 Granting one licence (denoted by the superscript 1) yields the expected consumer surplus:

$$E\left[CS^1\right] = \frac{\mu q_0^2 + (1-\mu)q_c^2}{2} \tag{17}$$

and the expected user surplus:

$$E[PS^1] = \mu q_0^2 + (1-\mu)\left(1-c-q_c\right)q_c \tag{18}$$

Similarly, granting two licences (denoted by the superscript 2) yields the expected consumer surplus:

$$E\left[CS^2\right] = \frac{4\mu^2 q_0^2 + 2\mu(1-\mu)\left(q_0+q_c\right)^2 + 4(1-\mu)^2 q_c^2}{2} \tag{19}$$

and the expected user surplus:

$$E\left[PS^2\right] = \mu^2\left(1-2q_0\right)2q_0 + 2\mu(1-\mu)\left(1-\left(q_0+q_c\right)\right)\left(q_0+q_c\right) + (1-\mu)^2\left(1-2q_c\right)2q_c$$

(i) From (17), the expected consumer surplus under symmetric information is:

$$E[CS_{SI}^{1}] = \frac{1 - 2(1-\mu)c + (1-\mu)c^2}{8} \tag{20}$$

where the subscript *SI* stands for symmetric information, whereas (18) leads the user's expected surplus to be:

$$E[PS_{SI}^{1}] = \frac{1 - 2(1-\mu)c + (1-\mu)c^2}{4} \tag{21}$$

Likewise, the expected consumer surplus under non-exclusive licensing amounts to:

$$E[CS_{SI}^{2}] = \frac{1 - 2\left(1 - 2\mu + \mu^2\right)c + \left(1 - 2\mu + \mu^2\right)c^2}{8} \tag{22}$$

and the expected user net surplus is:

$$E[PS_{SI}^{2}] = \frac{1 - 2\left(1 - 2\mu + \mu^2\right)c + \left(1 - 2\mu + \mu^2\right)c^2}{4} \tag{23}$$

Taking into account (20)–(23), the total surplus rendered by non-exclusive licensing is greater than that rendered by exclusive licensing iff $S \leq \widetilde{S}(c,\mu)$, where $\widetilde{S}(c,\mu) = 3\mu(1-\mu)(2-c)c/8$. It is easy to show that $\widetilde{S}(c,\mu) > S^{*}(c,\mu)$ for all $(c,\mu) \in \left(0, \frac{1}{2}\right) \times (0,1)$, and that the licensor chooses exclusive licensing if $S^{*}(c,\mu) \leq S \leq \widetilde{S}(c,\mu)$.

(ii) Under asymmetric information (denoted by the subscript *AI*) the expected consumer surplus from exclusive licensing is:

$$E[CS_{AI}^{1}] = \begin{cases} \frac{(1-c)^2 - \mu(1-2c)}{8(1-\mu)} & \text{if } c < c_1(\mu) \\ \frac{\mu}{8} & \text{if } c > c_1(\mu) \end{cases} \tag{24}$$

Note that no surplus is defined when $c = c_1(\mu)$ because production in that case is not continuous. The expected consumer surplus stated in (24) is obtained from (17), taking into account that output levels q_0 and q_c are the monopoly quantities produced by the efficient and inefficient user, respectively. According to Proposition 1, a screening contract results in $q_0 = 1/2$ and $q_c = (1-\mu-c)/2(1-\mu)$, whereas an excluding contract yields $q_0 = 1/2$ and $q_c = 0$. Furthermore, Proposition 1 shows that the licensor will choose the former contract if $c \leq c_1(\mu)$, and the latter contract, otherwise. Substituting these values into (17) affords the consumer surplus given in (24). On the other hand, the user's expected surplus (before issuing costs) with exclusive licensing is:

$$E[PS_{AI}^{1}] = \begin{cases} \frac{1-\mu-2(1-\mu)^2 c + (1-2\mu)c^2}{4(1-\mu)} & \text{if } c < c_1(\mu) \\ \frac{\mu}{4} & \text{if } c > c_1(\mu) \end{cases}$$

whereby expected welfare, defined as the unweighted sum of the consumer and user surpluses, amounts to:

$$E[W_{AI}^1] = \begin{cases} \frac{3(1-\mu)-2(3-5\mu+2\mu^2)c+(3-4\mu)c^2}{8(1-\mu)} & \text{if } c < c_1(\mu) \\ \frac{3\mu}{8} & \text{if } c > c_1(\mu) \end{cases}$$

Likewise, when two licences are granted, we can use (19) and the results of Proposition 2 to obtain the expected consumer surplus:

$$E[CS_{AI}^2] = \begin{cases} \frac{1-\mu+(c+\mu(2+c)-2)c}{8(1-\mu)} & \text{if } c < c_2(\mu) \\ \frac{1}{4(1+\mu)} & \text{if } c > c_2(\mu) \end{cases}$$

and the expected user surplus:

$$E[PS_{AI}^2] = \begin{cases} \frac{1-\mu-(3-6\mu+2\mu^2)c+(2-\mu-2\mu^2)c^2}{8(1-\mu)} & \text{if } c < c_2(\mu) \\ \frac{\mu(1+2\mu)}{2(1+\mu)^2} & \text{if } c > c_2(\mu) \end{cases}$$

by which expected aggregate welfare amounts to:

$$E[W_{AI}^2] = \begin{cases} \frac{3(1-\mu)-2(4-7\mu+2\mu^2)c+(1-\mu)(5+4\mu)c^2}{8(1-\mu)} & \text{if } c < c_2(\mu) \\ \frac{\mu(3+5\mu)}{4(1+\mu)^2} & \text{if } c > c_2(\mu) \end{cases}$$

From an issuing cost $S^*(\mu, c)$ for which the licensor is indifferent between granting one or two licences under symmetric information, it is verified that $E[W_{AI}^2] - E[W_{AI}^1] \geq S^*(\mu, c)$ if parameters c and μ are:

$$\begin{cases} c \geq \min\left\{\frac{1}{2}, c_2(\mu)\right\} & \text{if } 0 < \mu \leq 0.29 \\ c \geq \frac{3-7\mu+6\mu^2-2\mu^3}{3-6\mu+4\mu^2-2\mu^3} - \sqrt{\frac{\mu(3+\mu-13\mu^2+21\mu^3-34\mu^4+34\mu^5-16\mu^6+4\mu^7)}{(1+\mu)^2(3-6\mu+4\mu^2-2\mu^3)^2}} & \text{if } 0.29 \leq \mu \leq 0.47 \\ \frac{2-8\mu^2+4\mu^3}{2+5\mu-8\mu^2+2\mu^3} \leq c \leq c_5(\mu) & \text{if } 0.47 \leq \mu \leq 0.49 \\ c \geq \max\left\{\frac{2-8\mu^2+4\mu^3}{2+5\mu-8\mu^2+2\mu^3}, 0\right\} & \text{if } 0.49 \leq \mu < 1 \end{cases} \tag{25}$$

Comparing the region defined in (25) with the region in which two licences are granted (plotted in Fig. 3), the latter is nearly — but not completely — subsumed by the former. This implies that, under asymmetric information, if the issuing cost amounts to $S^*(c, \mu)$, the number of granted licences (either one or two) is socially efficient for a wide range of parameters.

If we now assume that the issuing cost is given instead by $\widetilde{S}(c, \mu)$, the parameter region in which the licensor chooses non-exclusive licensing is defined as in Proposition 3, but using $\widetilde{S}(c, \mu)$ rather than $S^*(c, \mu)$, and replacing $c_3(\mu)$, $c_4(\mu)$ and $c_5(\mu)$ by:

$$c_3'(\mu)=\frac{2\left(3-6\mu+3\mu^2\right)}{3-4\mu+3\mu^2},$$
$$c_4'(\mu)=\frac{\mu\left(5-6\mu+3\mu^2\right)}{-2+5\mu-8\mu^2+3\mu^3}-\sqrt{\frac{\mu\left(2-11\mu+25\mu^2-34\mu^3+36\mu^4-27\mu^5+9\mu^6\right)}{(1+\mu)\left(2-5\mu+8\mu^2-3\mu^3\right)^2}},$$
and $c_5'(\mu)=1-\sqrt{\frac{1+3\mu}{3(1+\mu)}}$
$$\tag{26}$$

respectively. Non-exclusive licensing leads to a higher aggregate welfare whenever:

$$c\geq\begin{cases}\min\left\{\frac{1}{2},c_1(\mu)\right\} & \text{if } 0<\mu\leq 0.42\\ \frac{3-8\mu+8\mu^2-3\mu^3}{3-7\mu+6\mu^2-3\mu^3}-\frac{\mu\sqrt{7-13\mu+14\mu^2-33\mu^3+46\mu^4-30\mu^5+9\mu^6}}{(1+\mu)\left(3-7\mu+6\mu^2-3\mu^3\right)} & \text{if } 0.42\leq\mu\leq 0.54\\ \max\left\{\frac{2+2\mu-12\mu^2+6\mu^3}{2+6\mu-10\mu^2+3\mu^3},0\right\} & \text{if } 0.54\leq\mu<1\end{cases}\tag{27}$$

Notice that the licensor chooses non-exclusive licensing in the region defined by (26), contained, however, within the region defined by (27)—where the aggregate surplus (under non-exclusive licensing) is higher (see Fig. 4). □

Proof of Lemma 3

(i) The (excluding) 2PT contract (f,r) that leads each user i, $i=1,2,3$, to only produce if it is efficient is that which solves:

$$\max_{(f,r)} 3\mu\left(f+rq_0^i\right),\ \text{s.t}: f\leq E\left[\pi_0^i\right]\tag{28}$$

where q_0^i is the production level of each efficient user in a Cournot triopoly, and $E\left[\pi_0^i\right]$ is its expected profit, when only efficient users operate. This production level is derived from:

$$\max_{q^i}\left(1-r-q^i-\mu(q^j+q^k)\right)q^i$$

which yields $q^i=(1-r)/2(1+\mu)$. Hence, $f=(1-r)^2/4(1+\mu)^2$ and (28) can be rewritten as:

$$\max_r\left(\frac{1-r}{2+2\mu}\right)^2+r\frac{1-r}{2(1+\mu)}$$

yielding $r=\mu/(1+2\mu)$. Thus, $f=1/4(1+2\mu)^2$.

(ii) If the same 2PT contract is offered to, and accepted by, all users' types, then each user i solves, for $\widetilde{c}=\{0,c\}$:

$$\max_{q^i_{\widetilde{c}}}\Big[1 - r - \widetilde{c} - q^i_{\widetilde{c}} - \mu^2\Big(q^j_0 + q^k_0\Big) - \mu(1-\mu)\Big(q^j_0 + q^k_c\Big)$$
$$-(1-\mu)\mu\big(q^j_c + q^k_0\big) - (1-\mu)^2\big(q^j_c + q^k_c\big)\Big]q^i_{\widetilde{c}}$$

which yields $q^i_0 = [1 + (1-\mu)c - r]/4$ if it is efficient and $q^i_c = [1 - (1+\mu)c - r]/4$ if inefficient. Hence, $f = [1 - (1+\mu)c - r]^2/16$, and the licensor solves:

$$\max_r\big\{3\mu^3\big(f + rq^i_0\big) + 3\mu^2(1-\mu)\big[3f + r\big(2q^i_0 + q^i_c\big)\big]$$
$$+3\mu(1-\mu)^2\big[3f + r\big(q^i_0 + 2q^i_c\big)\big] + 3(1-\mu)^3\big(f + rq^i_c\big)\big\}$$

yielding $r = [1 - (1 - 3\mu)c]/3$. This leads $q^i_0 > q^i_c > 0$ to hold as long as $\mu < (1-c)/3c \equiv \mu_6'(c)$. Thus, $f = [1 - (1+3\mu)c]^2/36$.

(iii) If the licensor offers a menu of separating contracts as $\big\{\big(f_0, r_0\big), \big(f_c, r_c\big)\big\}$ to each user i, it faces the problem:

$$\left.\begin{array}{l}\max\limits_{\{(f_0,r_0),(f_c,r_c)\}} 3\{\mu^3\big(f_0 + r_0q_0\big) + \mu^2(1-\mu)\big[2\big(f_0 + r_0q_0\big) + f_c + r_cq_c\big] + \\ \quad +\mu(1-\mu)^2\big[f_0 + r_0q_0 + 2\big(f_c + r_cq_c\big)\big] + (1-\mu)^3\big(f_c + r_cq_c\big)\} \\ \\ s.t.\begin{cases} E[\pi^i_0] - f_0 \geq 0 & (\mathrm{PC}_0) \\ E[\pi^i_c] - f_c \geq 0 & (\mathrm{PC}_c) \\ E[\pi^i_0] - f_0 \geq E[\pi^i_{0|c}] - f_c & (\mathrm{IC}_0) \\ E[\pi^i_c] - f_c \geq E[\pi^i_{c|0}] - f_0 & (\mathrm{IC_c}) \end{cases}\end{array}\right\} \tag{29}$$

which is solved by referring to the proof of Lemma 2, due to the same notation and reasoning, with the only difference here that there are three users instead of two. To obtain $E[\pi^i_{\widetilde{c}}]$ for $\widetilde{c} = \{0, c\}$, each user i solves:

$$\max_{q^i_{\widetilde{c}}}\Big[1 - \widetilde{c} - r_{\widetilde{c}} - q^i_{\widetilde{c}} - \mu^2\Big(q^j_0 + q^k_0\Big) - 2\mu(1-\mu)\Big(q^j_0 + q^k_c\Big) - (1-\mu)^2\big(q^j_c + q^k_c\big)\Big]q^i_{\widetilde{c}}$$

$(i,j,k = 1,2,3;\ \ i \neq j \neq k)$, yielding the output levels $q^i_0 = [1 + (1-\mu)c - (2-\mu)r_0 + (1-\mu)r_c\big]/4$ and $q^i_c = \big[1 - (1+\mu)c - (1+\mu)r_c + \mu r_0\big]/4$, and the expected profits:

$$E\big[\pi^i_0\big] = \frac{\big(1 + (1-\mu)\big(c + r_c\big) - (2-\mu)r_0\big)^2}{16} \text{ and } E[\pi^i_c] = \frac{\big(1 - (1+\mu)\big(c + r_c\big) + \mu r_0\big)^2}{16} \tag{30}$$

The problem solved by user i affords $q^i_{0|c} = \big[1 + (1-\mu)c + \mu r_0 - (1+\mu)r_c\big]/4$ as the output level of an efficient type when the rest of players believe it to be inefficient, and $q^i_{c|0} = \big[1 - (1+\mu)c + (1-\mu)r_c - (2-\mu)r_0\big]/4$ as the output level of an inefficient type when the rest of players believe it to be efficient. Hence, the corresponding profits are:

$$E\left[\pi^i_{0|c}\right] = \frac{\left(1+(1-\mu)c+\mu r_0-(1+\mu)r_c\right)^2}{16} \text{ and}$$
$$E\left[\pi^i_{c|0}\right] = \frac{\left(1-(1-\mu)c+(1+\mu)r_c-(2+\mu)r_0\right)^2}{16} \tag{31}$$

Using (30) and (31), the problem given in (29) can be written as:

$$\left.\begin{array}{l}\max\limits_{\{(f_0,r_0),(f_c,r_c)\}} 3\{\mu^3\left(f_0+r_0q_0\right)+\mu^2(1-\mu)\left[2\left(f_0+r_0q_0\right)+f_c+r_cq_c\right]+ \\ \quad +\mu(1-\mu)^2\left[f_0+r_0q_0+2\left(f_c+r_cq_c\right)\right]+(1-\mu)^3\left(f_c+r_cq_c\right)\} \\ s.t.\begin{cases}\frac{\left[1+(1-\mu)\left(c+r_c\right)-(2-\mu)r_0\right]^2}{16}-f_0\geq 0 & (\mathrm{PC}_0) \\ \frac{\left[1-(1+\mu)\left(c+r_c\right)+\mu r_0\right]^2}{16}-f_c\geq 0 & (\mathrm{PC}_c) \\ \frac{\left[1+(1-\mu)\left(c+r_c\right)-(2-\mu)r_0\right]^2}{16}-f_0\geq \frac{\left[1+(1-\mu)c+\mu r_0-(1+\mu)r_c\right]^2}{16}-f_c & (\mathrm{IC}_0) \\ \frac{\left[1-(1+\mu)\left(c+r_c\right)+\mu r_0\right]^2}{16}-f_c\geq \frac{\left[1-(1-\mu)c+(1+\mu)r_c-(2+\mu)r_0\right]^2}{16}-f_0 & (\mathrm{IC_c})\end{cases}\end{array}\right\}$$

where, as occurs in Lemmas 1 and 2, restrictions $\left(\mathrm{PC}_0\right)$ and $\left(\mathrm{IC}_c\right)$ hold with strict inequality (and so they can be ignored), whereas restrictions $\left(\mathrm{PC}_c\right)$ and $\left(\mathrm{IC}_0\right)$ hold with equality. The solution, therefore, is $r_0 = (1-c)/3, r_c = [1-\mu-(1-4\mu)c]/3(1-\mu)$, $f_0 = \left[1-\mu-2(1-\mu)c+(1+17\mu)c^2\right]/36(1-\mu)$, and $f_c = [1-2\mu-2\mu c]^2/36(1-\mu)^2$. Finally, this menu of contracts allows all users to produce a positive output as long $\mu < (1-c)/(1+2c) \equiv \mu_6''(c)$. □

Proof of Proposition 5 Using the solution to (28) in the objective function, the licensor's expected revenues from an (excluding) 2PT contract for each user amount to:

$$E[\pi_0^{L3}] = \frac{3\mu}{4(1+2\mu)} \tag{32}$$

where the superscript $L3$ indicates the concession of three licences. On the other hand, the licensor's expected revenues from a (non-excluding) 2PT contract for each user that produces, whether efficient or inefficient, is:

$$E\left[\pi_{0+c}^{L3}\right] = \frac{(1-c)^2}{4} + \frac{3\mu^2c^2}{4} \tag{33}$$

whereas that from a screening menu of 2PT contracts, whenever $\mu < \mu_3''(c)$, amounts to:

$$E\left[\pi_S^{L3}\right] = \frac{(1-c)^2}{12} + \frac{\mu^2 c^2}{4(1-\mu)} \tag{34}$$

Finally, comparing (32)–(34) affords the stated result. □

Proof of Proposition 6

(i) We need to determine the per-licence issuing cost that makes the licensor indifferent between offering one or three licences under symmetric information. Its expected payoff from a single licence is:

$$E\pi_M^{PH} = \frac{\mu + (1-\mu)(1-c)^2}{4} - S \tag{35}$$

where S is the per-licence issuing cost (see Proposition 2 in Antelo and Sampayo 2017). On the other hand, with three licences each user produces $q_i = \left(1 - 3\widetilde{c}_i + \widetilde{c}_j + \widetilde{c}_k - 3r_i + r_j + r_k\right)/4$, where $i,j,k = 1,2,3$; $i \neq j \neq k$; $\widetilde{c}_i, \widetilde{c}_j, \widetilde{c}_k \in \{0, c\}$, and the licensor faces the problem:

$$\left.\begin{array}{l} \max\limits_{\{f_i \geq 0, r_i \geq 0\}_{i=1,2,3}} \sum_{i=1,2,3} f_i + r_i q_i = \max\limits_{r_i \geq 0} \sum_{\substack{i,j,k=1,2,3 \\ i \neq j \neq k}} \left(1 - r_i - \widetilde{c}_i - q_i - q_j - q_k\right) q_i + r_i q_i \\ s.t:\ q_i = \frac{1-3\widetilde{c}_i+\widetilde{c}_j+\widetilde{c}_k-3r_i+r_j+r_k}{4} \geq 0 \left(i,j,k = 1,2,3; i \neq j \neq k; \widetilde{c}_i, \widetilde{c}_j, \widetilde{c}_k = \{0, c\}\right) \end{array}\right\} \tag{36}$$

which solution is not unique for some users. In particular, we have:

$$\left\{f_i, r_i\right\} = \begin{cases} \left\{\frac{(1-3r_i+r_j+r_k)^2}{16}, 1 - r_j - r_k\right\} & if\ \widetilde{c}_i = 0, \widetilde{c}_j = \widetilde{c}_k = 0 \\ \left\{\frac{(1+c-3r_i+r_j+r_k)^2}{16}, \frac{1}{2} - r_j\right\} & if\ \widetilde{c}_i = 0, \widetilde{c}_j = 0, \widetilde{c}_k = c \\ \left\{\frac{1}{4}, 0\right\} & if\ \widetilde{c}_i = 0, \widetilde{c}_j = \widetilde{c}_k = c \\ \left\{0, \frac{1-2c}{2}\right\} & if\ \widetilde{c}_i = c, \widetilde{c}_j = \widetilde{c}_k = 0 \\ \left\{0, \frac{1-2c}{2}\right\} & if\ \widetilde{c}_i = c, \widetilde{c}_j = 0, \widetilde{c}_k = c \\ \left\{\frac{(1-c-3r_i+r_j+r_k)^2}{16}, 1 - r_j - r_k\right\} & if\ \widetilde{c}_i = c, \widetilde{c}_j = \widetilde{c}_k = c \end{cases}$$

but, in the symmetric solution:

$$\{f_i, r_i\} = \begin{cases} \left\{\frac{1}{36}, \frac{1}{3}\right\} & \text{if } \widetilde{c}_i = 0, \widetilde{c}_j = \widetilde{c}_k = 0 \\ \left\{\frac{1}{16}, \frac{1}{4}\right\} & \text{if } \widetilde{c}_i = 0, \widetilde{c}_j = 0, \widetilde{c}_k = c \\ \left\{\frac{1}{4}, 0\right\} & \text{if } \widetilde{c}_i = 0, \widetilde{c}_j = \widetilde{c}_k = c \\ \left\{0, \frac{1-2c}{2}\right\} & \text{if } \widetilde{c}_i = c, \widetilde{c}_j = \widetilde{c}_k = 0 \\ \left\{0, \frac{1-2c}{2}\right\} & \text{if } \widetilde{c}_i = c, \widetilde{c}_j = 0, \widetilde{c}_k = c \\ \left\{\frac{(1-c)^2}{36}, \frac{1-c}{3}\right\} & \text{if } \widetilde{c}_i = c, \widetilde{c}_j = \widetilde{c}_k = c \end{cases} \tag{37}$$

the quantities produced by each user i are:

$$q_i = \begin{cases} \frac{1}{6} & \text{if } \widetilde{c}_i = 0, \widetilde{c}_j = \widetilde{c}_k = 0 \\ \frac{1}{4} & \text{if } \widetilde{c}_i = 0, \widetilde{c}_j = 0, \widetilde{c}_k = c \\ \frac{1}{2} & \text{if } \widetilde{c}_i = 0, \widetilde{c}_j = \widetilde{c}_k = c \\ 0 & \text{if } \widetilde{c}_i = c, \widetilde{c}_j = \widetilde{c}_k = 0 \\ 0 & \text{if } \widetilde{c}_i = c, \widetilde{c}_j = 0, \widetilde{c}_k = c \\ \frac{1-c}{6} & \text{if } \widetilde{c}_i = c, \widetilde{c}_j = \widetilde{c}_k = c \end{cases} \tag{38}$$

Finally, substituting (37) and (38) into the objective function of problem (36) yields the licensor's net expected revenues under symmetric information:

$$E[\pi_{SI}^{L3}] = \frac{1 - (1-\mu)^3(2-c)c}{4} - 3S \tag{39}$$

Likewise, from (35) and (39), the per-licence issuing cost that leads the licensor to be indifferent between granting one or three licences under symmetric information amounts to $S^{**}(c, \mu) = \mu(1-\mu)(2-\mu)(2-c)c/8$. On the other hand, the licensor's expected payoff from granting one licence under asymmetric information are:

$$E[\pi_{AI}^{L1}] = \begin{cases} \frac{1-\mu-2(1-\mu)c-\left(1+\mu-\mu^2\right)c^2}{4(1-\mu)} & \text{if } 0 < c < \frac{(1-\mu)(1-\sqrt{\mu(1-\mu)})}{1-\mu+\mu^2} \\ \frac{\mu}{4} & \text{if } \frac{(1-\mu)(1-\sqrt{\mu(1-\mu)})}{1-\mu+\mu^2} < c < \frac{1}{2} \end{cases}$$

and that from granting three licences, also under asymmetric information, is:

$$E\left[\pi_{AI}^{L3}\right] = \begin{cases} \frac{1-2c+\left(1+3\mu^2\right)c^2}{4} & \text{if } \mu \le \mu^{**} \\ \frac{3\mu}{4(1+2\mu)} & \text{if } \mu \ge \mu^{**} \end{cases}$$

Finally, comparison of $E\left[\pi_{AI}^{L1}\right] - S^{**}(c, \mu)$ with $E\left[\pi_{AI}^{L3}\right] - 3S^{**}(c, \mu)$ affords $E\left[\pi^{L3}\right] - 3S^{**}(c, \mu) \ge E[\pi^{L1}] - S^{**}(c, \mu)$ iff $(\mu) \le c \le c_2(\mu)$, where

$c_1(\mu) = \frac{1-3\mu+5\mu^2-4\mu^3+\mu^4}{1-3\mu+6\mu^2-4\mu^3+\mu^4} - \sqrt{\frac{\mu\left(1-6\mu+19\mu^2-40\mu^3+62\mu^4-67\mu^5+44\mu^6-15\mu^7+2\mu^8\right)}{(1+2\mu)\left(1-3\mu+6\mu^2-4\mu^3+\mu^4\right)^2}}$ and $c_2(\mu) = 1 - \sqrt{\frac{\mu(3-2\mu)}{2+3\mu-2\mu^2}}$, with $0 \le c_1(\mu) \le c_2(\mu) \le 1/2$.

(ii) The licensor's expected revenues from offering two licences under symmetric information (see Proposition 2 in Antelo and Sampayo 2017) are $E\pi_{SI}^{L2} = \frac{\mu(2-\mu)+(1-\mu)^2(1-c)^2}{4} - 2S$ and the per-licence issuing cost that makes the licensor indifferent between granting two or three licences is $S^{***}(c,\mu) = \mu(1-\mu)(2-\mu)(2-c)c/4$. Likewise, Proposition 3 allows us to obtain the licensor's expected revenues from selling two licences under asymmetric information:

$$E[\pi_{AI}^{L2}] = \begin{cases} \frac{1-\mu-2(1-\mu)c+(1-\mu+2\mu^2)c^2}{4(1-\mu)} & \textit{if } 0 < c < \frac{1-\mu}{1-\mu+2\mu^2}\left(1 - \sqrt{\frac{2\mu(1-\mu)}{1+\mu}}\right) \equiv c_2(\mu) \\ \frac{\mu}{2(1+\mu)} & \textit{if } c_2(\mu) < c < \frac{1}{2} \end{cases}$$

Hence, $E\left[\pi_{AI}^{L3}\right] - 3S^{***}(c,\mu) \ge E[\pi_{AI}^{L2}]] - S^{***}(c,\mu)$ iff $c_3(\mu) \le c \le c_4(\mu)$, where $c_3(\mu) = \frac{1-2\mu+3\mu^2-3\mu^3+\mu^4}{1-2\mu+5\mu^2-3\mu^3+\mu^4} - \frac{\sqrt{\mu\left(2-8\mu+17\mu^2-25\mu^3+29\mu^4-30\mu^5+24\mu^6-11\mu^7+2\mu^8\right)}}{(1+2\mu)\left[1-2\mu+5\mu^2-3\mu^3+\mu^4\right]^2}$ and $c_4(\mu) = 1 - \sqrt{\frac{\mu(2-2\mu-2\mu^2)}{1+2\mu-\mu^2-2\mu^3}}$, being $0 \le c_3(\mu) < c_4(\mu) \le \frac{1}{2}$. □

Proof of Lemma 4

(i) If the licence consists of an excluding contract, the licensor solves the problem:

$$\max_{v} \mu\left(\frac{v}{4} + \frac{1-v}{4}\right)$$

which yields $v_0 \in [0,1]$. As result, any contract (f_0, v_0) such that $f_0 + \frac{1}{4}v_0 = \frac{1}{4}$ does not distort the efficient user's production and allows the licensor to extract the profits of a user producing $q_0 = 1/2$.

(ii) If the licence consists of a contract (f, v) for both user types and allows them to produce, the fixed payment is $f = (1-c-v)^2/4(1-v)$, the profit of an inefficient user producing $q_c = (1-c-v)/2(1-v)$. From here, and given that the efficient type also pays the same fixed fee f and that the ad-valorem royalty v does not distort its production from $q_0 = 1/2$, the licensor solves:

$$\max_{v}\left\{\mu\left[\frac{(1-c-v)^2}{4(1-v)} + v\left(1-\frac{1}{2}\right)\frac{1}{2}\right] + (1-\mu)\left[\frac{(1-c-v)^2}{4(1-v)} + v\left(1-\frac{1-c-v}{2(1-v)}\right)\frac{1-c-v}{2(1-v)}\right]\right\}$$

which affords $v = \mu/(2-\mu)$, and, as result, $f = [2(1-\mu)-(2-\mu)c]^2/8(1-\mu)(2-\mu)$. Production levels for the efficient and inefficient types of each user are $q_0 = 1/2$ and

$q_c = [2(1-\mu) - (2-\mu)c]/4(1-\mu)$, respectively, being $q_c > 0$ if $c < 2(1-\mu)/(2-\mu)$ or, equivalently, if $\mu < 2(1-c)/(2-c)$.

(iii) If the licence consists of the menu of contracts $\{(f_0, v_0), (f_c, v_c)\}$, with $f_0 \geq 0$, $0 \leq v_0 \leq 1$, $f_c \geq 0$ and $0 \leq v_c \leq 1$, where the contract (f_0, v_0) is intended for the efficient type and the contract (f_c, v_c) is intended for the inefficient type, the licensor's problem is:

$$\left.\begin{array}{l} \max\limits_{\{(f_0,v_0),(f_c,v_c)\}} \left\{\mu\left[f_0 + v_0(1-q_0)q_0\right] + (1-\mu)\left[f_c + v_c(1-q_c)q_c\right]\right\} \\ s.t.\begin{cases} \frac{1-v_0}{4(1-v_0)} - f_0 \geq 0 & (\text{PC}_0) \\ \frac{(1-c-v_c)^2}{4(1-v_c)} - f_c \geq 0 & (\text{PC}_c) \\ \frac{1-v_0}{4} - f_0 \geq \frac{1-v_c}{4} - f_c & (\text{IC}_0) \\ \frac{(1-c-v_c)^2}{4(1-v_c)} - f_c \geq \frac{(1-c-v_0)^2}{4(1-v_0)} - f_0 & (\text{IC}_c) \end{cases} \end{array}\right\} \tag{40}$$

Bearing in mind that if (PC_c) holds, then (PC_0) must also hold and so can be ignored; that since the solution of (40) verifies (IC_0) with equality and (IC_c) with strict inequality, it is safe to ignore (IC_c); that both (IC_0) and (PC_c) restrictions are verified with equalities in the solution; and that $q_0 = 1/2$ and $q_c = (1-c-v_c)/2(1-v_c)$, it follows that $f_c = (1-c-v_c)^2/4(1-v_c)$ and, as result, $f_0 = \frac{1-v_0}{4} - \frac{1-v_c}{4} + \frac{(1-c-v_c)^2}{4(1-v_c)}$ by (IC_0). Thus, the licensor's problem (40) becomes:

$$\max_{(v_0,v_c)} \left\{ \mu\left[\frac{1-v_0}{4} - \frac{1-v_c}{4} + \frac{(1-c-v_c)^2}{4(1-v_c)} + v_0\left(1-\frac{1}{2}\right)\frac{1}{2}\right] + (1-\mu)\left[\frac{(1-c-v_c)^2}{4(1-v_c)} + v_c\left(1 - \frac{1-c-v_c}{2(1-v_c)}\right)\frac{1-c-v_c}{2(1-v_c)}\right]\right\}$$

yielding $(v_0, v_c) = \left([0,1], 1 - \frac{2(1-\mu)}{2-\mu}\right)$. □

Proof of Proposition 7 From the contracts described in Lemma 4, the licensor's expected revenues are $E[\pi_0^{L1}] = \mu/4$ when only the efficient type accepts to produce, and $E\left[\pi_{0+c}^{L1}\right] = E\left[\pi_S^{L1}\right] = \frac{1}{16}\left(4 - 8c + \frac{(2-\mu)^2}{1-\mu}c^2\right) = \frac{(1-c)^2}{4} + \frac{c^2\mu^2}{16(1-\mu)}$ under either a non-screening contract that allows both user types to produce or a menu of screening contracts that also allows both user types to produce. From here, $E\left[\pi_0^{L1}\right] > E\left[\pi_{0+c}^{L1}\right] = E\left[\pi_S^{L1}\right]$ if $\frac{2(1-\mu)\left(2-\sqrt{2+\mu}\right)}{2-\mu} < c < \frac{2(1-\mu)\left(2+\sqrt{2+\mu}\right)}{2-\mu}$, where the upper bound on c is irrelevant because $\frac{2(1-\mu)\left(2+\sqrt{2+\mu}\right)}{2-\mu} > \frac{2(1-\mu)}{2-\mu}$ for all $\mu \in (0,1)$. Finally, taking into account that the condition $c > \frac{2(1-\mu)\left(2+\sqrt{2+\mu}\right)}{2-\mu}$ is equivalent to the condition $\mu > 1 - \frac{\left(8+\sqrt{48-16c+c^2}\right)c-c^2}{8}$, the result follows. ☐

Proof of Lemma 5

(i) If no user is screened, the licensor can offer an excluding or a non-excluding contract to each user. In the former case, the licensor solves:

$$\max_{(f,v)} 2\left(\mu^2 + \mu(1-\mu)\right)\left[f + v\left(1 - q^i - q^j\right)q^i\right] \; s.t.: \; E\left[\pi_0^i\right] \geq f \tag{41}$$

where $E[\pi_0^i]$ is derived taking into account that each user i, taking user j's production as given, solves:

$$\max_{q^i} (1-v)\left(1 - \left(q^i + \mu q^j\right)\right)q^i, j \neq i$$

This yields $q^i = 1/(2+\mu)$ and, consequently $E\left[\pi_0^i\right] = (1-v)/(2+\mu)^2$. Hence, in the solution to the licensor's problem (41), the fixed fee is $f = (1-v)/(2+\mu)^2$ and, as result, (41) can be rewritten as:

$$\max_v \frac{2\mu(1+v\mu)}{(2+\mu)^2}, s.t.: \; 0 \leq v \leq 1$$

yielding $v_0 = 1$.

(ii) To allow both user types to produce, the licensor solves:

$$\left.\begin{array}{l} \max_{(f,v)}\left\{2\mu^2\left(f + v\left(1-2q_0\right)q_0\right) + 2\mu(1-\mu)\left(2f + v\left(1-\left(q_0+q_c\right)\right)\left(q_0+q_c\right)\right) + 2(1-\mu)^2\left(f + v\left(1-2q_c\right)q_c\right)\right\} \\ s.t.\begin{cases} E[\pi_0^i] \geq f \\ E[\pi_c^i] \geq f \\ 0 \leq v \leq 1 \\ q_c \geq 0, q_0 \geq 0, f \geq 0 \end{cases}\end{array}\right\} \tag{42}$$

where $E[\pi_{\tilde{c}}^i]$, $i \in \{1,2\}$, $\tilde{c} \in \{0, c\}$, is derived from:

$$\max_{q_{\tilde{c}}^i} (1-v)\left(1 - q_{\tilde{c}}^i - \mu q_0^j - (1-\mu)q_c^j\right)q_{\tilde{c}}^i - \tilde{c}q_{\tilde{c}}^i$$

which yields:

$$q_0 = \frac{2(1-v)+(1-\mu)c}{6(1-v)} \text{ and } q_c = \frac{2(1-v)-(2+\mu)c}{6(1-v)} \tag{43}$$

Using the quantities given in (43) to compute $E[\pi_{\tilde{c}}^i]$ in problem (42), and considering in (43) that $q_c > 0$ leads to $E[\pi_0^i] = \frac{[2(1-v)+(1-\mu)c]^2}{36(1-v)} > \frac{[2(1-v)-(2+\mu)c]^2}{36(1-v)} = E[\pi_c^i]$, the solution to (42) must verify $f = E[\pi_c^i] = [2(1-v) - (2+\mu)c]^2/36(1-v)$ and (42) can therefore be rewritten as:

$$\max_v \left\{ \frac{2}{9}\left(1 - \frac{(2-3v+\mu)c}{1-v}\right) + \frac{\left[(2+\mu)^2 - 3(4-\mu)v\right]c^2}{18(1-v)^2} \right\}$$

whose solution is:

$$v = \begin{cases} 1 - \frac{2(8-7\mu-\mu^2)c}{4(1-\mu)+3(4-\mu)c} & \text{if } \mu \le \mu_2^v(c) \\ 1 - \frac{(2+\mu)c}{2} & \text{if } \mu \ge \mu_2^v(c) \end{cases}$$

where $\mu_2^v(c) = 1 - \frac{\sqrt{16+9c^2}-4}{c}$ is the value of μ that solves $c = \frac{8(1-\mu)}{8+\mu(2-\mu)}$. Consequently, the fixed fee is:

$$f = \begin{cases} \frac{[8(1-\mu)-(4-\mu)(2+\mu)c]^2 c}{8(1-\mu)(8+\mu)[4(1-\mu)+3(4-\mu)c]} & \text{if } \mu \le \mu_2^v(c) \\ 0 & \text{if } \mu \ge \mu_2^v(c) \end{cases}$$

(iii) Offering the menu of screening contracts $\{(f_0, v_0), (f_c, v_c)\}$ to each user i leads the licensor to solve:

$$\left.\begin{array}{l} \max\limits_{\{(f_0,v_0),(f_c,v_c)\}} \left\{ \begin{array}{c} 2\mu^2\left(f_0 + v_0\left(1-2q_0\right)q_0\right) + 2\mu(1-\mu)\left(f_0 + f_c + v_0\left(1-\left(q_0+q_c\right)\right)q_0 + v_c\left(1-\left(q_0+q_c\right)\right)q_c\right) + \\ +2(1-\mu)^2\left(f_c + v_c\left(1-2q_c\right)q_c\right) \end{array} \right\} \\ \qquad s.t. \left\{ \begin{array}{ll} 0 \le v_0, v_1 \le 1; q_c, q_0 \ge 0; f_c, f_0 \ge 0 & \\ E[\pi_0^i] - f_0 \ge 0 & (\mathrm{PC}_0) \\ E[\pi_c^i] - f_c \ge 0 & (\mathrm{PC}_c) \\ E[\pi_0^i] - f_0 \ge E[\pi_{0|c}^i] - f_c & (\mathrm{IC}_0) \\ E[\pi_c^i] - f_c \ge E[\pi_{c|0}^i] - f_0 & (\mathrm{IC}_c) \end{array} \right. \end{array}\right\} \tag{44}$$

Here we also refer to the proof of Lemma 2, given that we are using the same notation and reasoning. To obtain $E[\pi_{\widetilde{c}}^i]$ for $\widetilde{c} = \{0, c\}$, each user i solves:

$$\max_{q_{\tilde{c}}^i} \left(1 - v_{\tilde{c}}\right)\left(1 - q_{\tilde{c}}^i - \mu q_0^j - (1-\mu)q_c^j\right)q_{\tilde{c}}^i - \tilde{c}q_{\tilde{c}}^i, j \in \{1,2\}, j \neq i$$

whose FOC is:

$$\left(1 - v_{\widetilde{c}}\right)\left(1 - q_{\widetilde{c}}^i - \mu q_0^j - (1-\mu)q_c^j\right) - \widetilde{c} = 0 \tag{45}$$

Solving, for $i \in \{1, 2\}$, the four equations defined by (45) yields:

$$q_0^i = \frac{2\left(1-v_c\right) + (1-\mu)c}{6\left(1-v_c\right)} \text{ and } q_c^i = \frac{2\left(1-v_c\right) - (2+\mu)c}{6\left(1-v_c\right)} \tag{46}$$

Hence:

$$E\left[\pi_0^i\right]=\frac{\left(1-v_0\right)\left[2\left(1-v_c\right)+(1-\mu)c\right]^2}{36\left(1-v_c\right)^2} \text{ and } E\left[\pi_c^i\right]=\frac{\left[2\left(1-v_c\right)-(2+\mu)c\right]^2}{36\left(1-v_c\right)} \tag{47}$$

Off-equilibrium expected profits $E[\pi^i_{0|c}]$ and $E[\pi^i_{c|0}]$ for each user $i\in\{1,2\}$ are derived from:

$$\max_{q^i_{\tilde{c}|\tilde{c}^M}}\left(1-v_{\tilde{c}^M}\right)\left(1-q^i_{\tilde{c}|\tilde{c}^M}-\mu q^j_0-(1-\mu)q^j_c\right)-\tilde{c}q^i_{\tilde{c}|\tilde{c}^M}$$

where q^j_0 and q^j_c, $j\neq i$, are given by (46) and $\tilde{c}|\tilde{c}^M\in\{0|c,c|0\}$, with the superscript M denoting that the user i's type is misperceived. The solution is given by $q^i_{0|c}=\frac{2(1-v_c)+(1-\mu)c}{6(1-v_c)}$ and $q^i_{c|0}=\frac{1}{3}+\frac{c(3v_c-v_0(1-\mu)-2-\mu)}{6(1-v_0)(1-v_c)}$, from which $E[\pi^i_{0|c}]=\frac{\left[2(1-v_c)+(1-\mu)c\right]^2}{36(1-v_c)}$ and $E[\pi^i_{c|0}]=\frac{2\left(v_0(1-v_c)+v_c-1\right)-c\left(3v_c-v_0(1-\mu)-2-\mu\right)^2}{36(1-v_0)(1-v_c)^2}$. As we know from similar problems solved before, in the solution to (44) the inequalities $\left(\mathrm{PC}_0\right)$ and $\left(\mathrm{IC}_c\right)$ are strict, whereas $\left(\mathrm{PC}_c\right)$ and $\left(\mathrm{IC}_0\right)$ are verified as equalities. Then, we have:

$$f_0=\frac{\left(1-v_0\right)\left[2\left(1-v_c\right)+c(1-\mu)\right]^2+3c\left(1-v_c\right)\left[c(1+2\mu)-4\left(1-v_c\right)\right]}{36\left(1-v_c\right)^2} \text{ and } f_c=\frac{\left[2\left(1-v_c\right)-(2+\mu)c\right]^2}{36\left(1-v_c\right)} \tag{48}$$

Since the constraints $f_c\geq 0$ and $q_c\geq 0$ hold as long as $v_c\leq 1-\frac{(2+\mu)c}{2}$, and $q_0>q_c$, we can ignore the non-negativity constraint on q_0, and the problem (44) can be reformulated as:

$$\left.\begin{array}{c}\max\limits_{\{v_0,v_c\}}\left\{2\mu^2\left(f_0+v_0\left(1-2q_0\right)q_0\right)+2\mu(1-\mu)\left(f_0+f_c+v_0\left(1-\left(q_0+q_c\right)\right)q_0+v_c\left(1-\left(q_0+q_c\right)\right)q_c\right)\right.\\ \left.+2(1-\mu)^2\left(f_c+v_c\left(1-2q_c\right)q_c\right)\right\}\\ s.t.:\ (46),(48),f_0\geq 0,0\leq v_0\leq 1,0\leq v_c\leq 1-\frac{(2+\mu)c}{2}\end{array}\right\}$$

which, after substituting the equality constraints, becomes:

$$\max_{\{v_c\}}\left\{\frac{2}{9}\left(1-\frac{\left(2-3v_c+\mu\right)c}{1-v_c}\right)+\frac{\left[(2+\mu)^2-3v_c(4-\mu)\right]c^2}{18\left(1-v_c\right)^2}\right\}, s.t.:\ 0\leq v_c\leq 1-\frac{(2+\mu)c}{2} \tag{49}$$

and its solution is:

$$v_c=\begin{cases}1-\frac{2(8-7\mu-\mu^2)c}{4(1-\mu)+3(4-\mu)c} & \text{if } \mu<\mu_2^v(c)\\ 1-\frac{(2+\mu)c}{2} & \text{if } \mu>\mu_2^v(c)\end{cases} \tag{50}$$

Substituting (50) into (48) yields:

$$f_c=\begin{cases}\frac{\left[8(1-\mu)-(4-\mu)(2+\mu)c\right]^2c}{8(1-\mu)(8+\mu)\left[4(1-\mu)+3(4-\mu)c\right]}, & \text{if } \mu\leq\mu_2^v(c)\\ 0, & \text{if } \mu>\mu_2^v(c)\end{cases}$$

Finally, we know that v_0 is indeterminate in the interval $[0, 1]$. However, since $f_0 \geq 0$, (48) joint with the value obtained for v_c implies that:

$$0 \leq v_0 \leq \begin{cases} 1 - \frac{2(8+\mu)\left[20(1-\mu)-\left(4+7\mu-2\mu^2\right)c\right]c}{(1-\mu)[12+(4-\mu)c]^2} & \text{if } \mu < \mu_2^v(c) \\ 1 - \frac{(2+\mu)c}{2} & \text{if } \mu > \mu_2^v(c) \end{cases}$$

and f_0 is given by (48). □

***Proof of Proposition* 8** The licensor's expected revenues, when only efficient users produce, amounts to:

$$E[\pi_0^{L2}] = \frac{2\mu(1+\mu)}{(2+\mu)^2} \tag{51}$$

whereas the licensor's expected revenues, when both efficient and inefficient users produce and provided they accept the same contract offered to all, or self-select by choosing the specific contract intended for it in the screening menu, amounts to:

$$E\left[\pi_{0+c}^{L2}\right] = E\left[\pi_S^{L2}\right] = \begin{cases} \frac{c^2}{8(1-\mu)} + \frac{2(1+c)^2}{8+\mu} - \frac{(8+c)c}{8} = \frac{(1-c)^2}{4} - \frac{2\mu(1-\mu)+4\mu(1-\mu)c-3\mu(2+\mu)c^2}{8(1-\mu)(8+\mu)} & \text{if } \mu < \mu_2^v(c) \\ \frac{\mu[2-(2+\mu)c]}{(2+\mu)^2} = \frac{(1-c)^2}{4} - \frac{(2-\mu)^2-2\left(4-\mu^2\right)c+(2+\mu)^2c^2}{4(2+\mu)^2} & \text{if } \mu > \mu_2^v(c) \end{cases} \tag{52}$$

From (51) and (52), the licensor prefers to grant two licences by means of excluding contracts if $c > \frac{4(1-\mu)}{(4-\mu)^2}\left(4+\mu-\frac{8+\mu}{2+\mu}\sqrt{\frac{\mu\left(16+4\mu+11\mu^2\right)}{8-7\mu-\mu^2}}\right)$ and either through a non-screening contract or a menu of screening contracts otherwise. □

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Declarations

Conflict of interest The authors declare no affiliations with or involvement in any organization or entity with any financial or non-financial interest in the subject matter or materials discussed in this research.

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