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Customer poaching and coupon trading

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Abstract The price discrimination literature typically makes the assumption of no consumer arbitrage. This assumption is increasingly violated in the digital economy, where coupons are traded with increased frequency online. In this paper, we analyze the welfare impacts of coupon trading using a modified Hotelling model where firms send coupons to poach each other's loyal customers. The possibility of coupon trading renders this important instrument for price discrimination less effective. Moreover, coupon distribution has unintended consequences when coupon traders sell coupons

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back to a firm's loyal customers. Consequently, coupon trading may reduce firms' incentive to distribute coupons, leading to higher prices and profits. We find that, an increase in coupon distribution cost lowers promotion frequency but raises promotion depth, and an increase in the fraction of coupon traders lowers both promotion frequency and promotion depth.

Keywords Customer poaching · Coupon trading · Consumer arbitrage

JEL Classification D43 · L13 · M31

1 Introduction

In the price discrimination literature, with the exception of a few studies, consumers cannot engage in arbitrage. In traditional markets, it is indeed costly for someone to locate another potential buyer of the same product and trade. In that setting, the no arbitrage assumption may not be unrealistic. The evolution of the digital economy has gradually undermined the validity of this assumption. It is easier now to buy and resell products for a profit online, since the direct "consumer-to-consumer" markets are more developed and information can be easily exchanged on the internet. Moreover, coupons, one of the commonly used means to achieve price discrimination, can be easily traded. In online shopping, all that consumers need is a coupon code. If you go on ebay.com and search for "coupons" you can find over 20,000 listings for sale (not counting multiple coupons in one listing).[1]

In this paper, we develop a location model of oligopolistic third-degree price discrimination. Firms can select shoppers with specific characteristics and send targeted coupons, due to the availability of more and more information on actual consumer transactions and better technology to utilize such information. In particular, we assume that firms can differentiate between their own and their rivals' loyal customers and price discriminate between them by sending targeted coupons. A popular form of targeted coupon is an *offensive coupon* (also called a poaching coupon) which firms use to poach rival firms' loyal customers, i.e., those who will purchase from the rival firm if prices are the same.[2] Since coupon trading may impede the effectiveness of couponing strategies, firms can personalize targeted coupons to be used by a specific person or someone using a specific email address. Firms may also restrict that only their competitors' clientele can use the coupons, given that they have the ability to distinguish their own vs. their rivals' loyal customers. However, such restriction has its disadvantages as well. First, it costs more for firms to send and redeem personalized

1 These are only listings of coupons auctioned, and not all of them are actually sold. To get a sense of how many are sold, we searched for a specific coupon (Staples coupon), and checked the 10 listings with the earliest expiration time. We found that 6 of them had bids submitted.

2 This is somewhat similar to "reciprocal dumping" in the trade literature (e.g. Brander and Krugman 1983; Deltas et al. 2012). In both settings, each firm has disadvantage in one market, whether it is due to weaker preferences of consumers in that market (our case) or higher transportation cost to serve consumers in that market (the reciprocal dumping case). Firms poach each other's strong markets, leading to lower profits for both firms and a *prisoner's dilemma* game.

coupon physically and electronically. Second, personalized coupons may increase consumers' hassle costs of use, thus reducing the effectiveness of this strategy. Third, such restrictions prevent the coupon receivers from sharing coupons with their families and friends, who might have been potential targets of a firm. Taking these disadvantages into consideration, firms may prefer anonymous coupons. While our paper builds on the price discrimination literature, we depart by allowing coupons to be traded across consumers and analyzing the welfare impacts of coupon trading.

In our model, some consumers have low hassle cost of selling or buying coupons, and we call them coupon traders.[3] Others have prohibitively high cost of trading coupons and are called non-traders. We find that each firm's optimal strategy is to send offensive coupons. When these coupons reach non-traders, some of them use the coupons and switch firms, improving the coupon issuing firms' profits unilaterally. However, these coupons may also reach coupon traders who in general value the coupons less than their face value. Therefore, they have an incentive to sell these coupons to those who value them higher. In this case, not only are these coupons ineffective in inducing consumers to switch, but they also have unintended consequences as they will end up in the hands of the issuing firms' own loyal customers. The introduction of coupon trading therefore reduces the attractiveness of couponing. If the fraction of coupon traders increases, firms would respond by lowering both promotion frequency and promotion depth and raising prices, leading to higher industry profits.[4] We also analyze the impact of coupon distribution cost. Similar to the results in Bester and Petrakis, we find that an increase in coupon distribution cost lowers promotion frequency but raises promotion depth. This is different from an increase in the fraction of coupon traders which lowers both promotion frequency and promotion depth. We then extend our model in several directions where we introduce coupon non-users, include continuous hassle cost of selling coupons and allow coupons to be non-transferrable. Our qualitative results continue to hold in these extensions.

The rest of the paper is organized as follows. We review the literature in Sect. 2. Our model is presented in Sect. 3. Section 4 contains the analysis of the benchmark model. We introduce several extensions in Sect. 5 and conclude in Sect. 6. The proof of Proposition 1 can be found in the Appendix.

2 Literature review

Depending on the method of distribution, coupons can be divided into two types: *mass media coupons* and *targeted coupons*. Mass media coupons are distributed randomly by the firms. Consumers, based on their characteristics, decide whether or not to collect and use these coupons (Narasimhan 1984). Early studies on couponing strategies focus on consumers' self-selection and consider mass media coupons. In Narasimhan, couponing enables price discrimination, providing a lower price to a particular segment

[3] For example, some consumers (one of the authors included) may be familiar with eBay and may have various accounts already set up for transactions, so the incremental transaction cost of trading coupons on eBay is minimal.

[4] Promotion frequency is the probability that a random consumer will receive a firm's coupon. Promotion depth is the dollar off value when a consumer gets and uses a coupon. More details are provided in Sect. 2.

of consumers, while keeping the price high for others. With the availability of more information on consumers and better technology to utilize such information, firms can rely less on consumer self-selection and more on targeted coupons (see Shaffer and Zhang 1995 for examples of such practices).[5] Targeted coupons are mostly modeled in the literature as offensive coupons (Shaffer and Zhang 1995; Bester and Petrakis 1996; Chen 1997). Our paper builds on this literature but deviates from them by allowing consumer arbitrage through coupon trading.

Bester and Petrakis (1996) and Armstrong (2006) are two studies closely related to ours. Bester and Petrakis consider a duopoly model where firms can send out coupons to poach each other's loyal customers. Couponing intensifies competition between firms, generating lower equilibrium prices and profits. Correspondingly, an increase in the cost of coupon distribution reduces couponing intensity, and benefits firms at the cost of consumers. Armstrong (2006) considers a variant of the Bester and Petrakis model where firms have private information about brand preferences. In particular, the firm can draw a signal for each customer suggesting whether this customer prefers its product. The signal is correct with probability $\alpha \geq \frac{1}{2}$.

As in Bester and Petrakis (1996) and Armstrong (2006), we assume that firms have private information about consumers' brand preferences, which they can obtain, for example, from marketing companies. Armstrong (2006) differs from Bester and Petrakis and our paper in that this information (signal) may be inaccurate. In contrast, in our paper, the information is always accurate, but coupon may reach the wrong type of consumers (i.e., coupon traders).[6] Bester and Petrakis focus on the cost of coupon distribution, while we focus on coupon trading which alters firms' optimal couponing strategies.[7] Intuitively coupon distribution cost and coupon trading have different mechanisms in affecting firms' promotion intensities. When the cost of distributing coupons increases, firms respond by sending fewer coupons of larger face value. However, when the fraction of coupon traders increases, firms send fewer coupons and reduce their face value.

Our paper is also related to studies on behavior-based price discrimination.[8] Fudenberg and Tirole (2000) analyze a two-period game. In the second period, firms can distinguish among consumers who bought their own product in period 1 from those who did not. Consequently, each firm can poach the customers of their competitor by sending them coupons to induce them to switch. They find that poaching leads to lower

[5] Coupons can also enable firms to reward repeat purchase customers. In particular, firms can issue coupons to consumers buying from them, and these coupons offer discounts when these consumers buy from the same firms later. See Fong and Liu (2011) for details.

[6] In our model, firms do not have information to differentiate between coupon traders and non-traders. It is intuitive that firms would have unilateral incentive to acquire such information which would then allow them to send coupons to non-traders only.

[7] Signal accuracy in Armstrong (2006) is related in some sense to coupon trading in our model. However, in Armstrong (2006), firms have access to private signals for all consumers, and can reach all consumers without cost.

[8] Behavior-based price discrimination is based on past purchases rather than on observable and exogenous consumer characteristics. Results in this literature have somewhat similar flavor as those in the standard third-degree price discrimination but due to its dynamic nature there are other considerations (e.g., the use of long-term contracts). See Fudenberg and Villas-Boas (2006) for a survey of this literature.

prices. Chen (2008) considers a model with more than two periods and asymmetric firms. He finds that price discrimination can either benefit or hurt consumers.

In the previous papers, a poaching firm sends the same coupons to all of its rival's customers. However, coupons differ by customer in Liu and Serfes (2004) and Shaffer and Zhang (1995, 2002). In Liu and Serfes, firms have detailed information which enables them to segment consumers into various groups and send coupons of different face value. In Shaffer and Zhang (1995), each firm offers only one type of coupons, but it can choose to send coupons to only a portion of the customers, since each firm has the ability to identify and target each individual consumer. This leads to different couponing strategies compared to our model. First, firms will not send coupons to consumers whose preference for either firm is sufficiently strong since they will not switch firms even with coupons. Second, firms may send defensive coupons. In contrast, in our model, each firm's coupons are the same for all consumers in a segment and the inability to target makes the use of defensive coupons suboptimal.[9]

There have been few studies analyzing resale or consumer arbitrage, and they typically consider monopolies.[10] In Anderson and Ginsburgh (1999), consumers differ in two dimensions: willingness to pay and arbitrage cost. In their setup, a monopolist can sell its product in two countries. It may sell in a second country even if there is no local demand. The goal is to discriminate across consumers with different arbitrage costs in the first country. Gans and King (2007) consider a monopolist serving a market with a finite number of consumers. They find that price discrimination may be optimal and potentially profitable even if consumer arbitrage is costless.[11] Our paper differs from this literature in that we consider duopoly competition. In fact, offensive coupon is defined as an attempt to poach the rival's loyal customers.

3 The description of the model

Two firms—1 and 2—produce competing goods at a constant marginal cost which we normalize to zero. Each consumer buys at most one unit of the good and is willing to pay V. We assume that V is sufficiently high and therefore the market is always covered. Consumers are heterogeneous with respect to the premium they are willing to pay for their favorite brand. This heterogeneity is captured by a parameter l, which represents the consumer's degree of loyalty. Specifically, a consumer located at l is indifferent between buying from the two firms if and only if $l = p_2 - p_1$. We assume

[9] Our analysis is more applicable to markets where firms only have crude information on consumers and thus cannot identify and target individual customers. Such limitation on information may due to technology restrictions or it may be too costly to acquire more refined information.

[10] An exception is Aguirre and Espinosa (2004), who analyze a different type of consumer arbitrage in a duopoly setting (Hotelling model). The auction literature also examined how resale affects bidding. See Haile (2003) and relatedly, Calzolari and Pavan (2006).

[11] Gans and King (2007) assume that consumer types are public information. If on the other hand, consumer types are private information, then perfect arbitrage would prevent a firm to exercise price discrimination (see Alger 1999).

that l is uniformly distributed in the interval $[-L, L]$ with density 1.[12] When two firms charge the same prices, consumers located at $l > 0$ will buy from firm 2 and are called firm 2's loyal customers. Similarly, customers with $l < 0$ are firm 1's loyal customers.

Accordingly, the interval $[-L, L]$ is partitioned into two segments: $[-L, 0]$ (segment 1) and $[0, L]$ (segment 2), corresponding to firm 1's and firm 2's loyal customers respectively. Firms know which segment each consumer is located at, but not the exact location within the segment. For example, for someone located at $L/2$, firms know that she is located at $[0, L]$, but not that she is located at $L/2$.[13] There are two types of pricing strategies that a firm can adopt in our context:

Uniform pricing

Each consumer on the interval $[-L, L]$ receives the same price. This price is also called the regular price. One can view this price as the price listed in store or on the web, which consumers would pay without coupons. Let p_2 and p_1 denote the regular prices of the two firms. Without loss of generality, assume that $p_2 \geq p_1$.

Segment couponing

Firms can send coupons to the rival firm's loyal consumers as in the existing literature; such coupons are called offensive coupons. Let (λ_i, r_i) denote firm i's couponing intensity. This means that every loyal consumers of firm $j (\neq i)$ has an equal probability λ_i of receiving coupons of face value r_i from firm i. λ_i is called the promotion frequency (the fraction of consumers receiving coupons) and r_i is the promotion depth. Following the literature, we consider dollars-off coupons instead of percentage-off coupons. When consumers do not use coupons, they will pay either p_1 or p_2, depending on which firm they buy from. Consumers who use coupons, will pay $p_i - r_i$ when they buy from firm i.[14]

Next, we introduce the cost of distributing coupons. We assume that the coupon distribution cost is increasing and convex in the promotion effort and the size of the segment. In particular, it takes the form of $k(\lambda_i L)^2$ for firm i's promotion effort

[12] A similar model with $l = p_1 - p_2$ has been used in Shaffer and Zhang (2002) and Liu and Serfes (2006). We define marginal consumer by $l = p_2 - p_1$ so consumers on the left (right) like firm 1's (2's) product more. We do not conduct comparative statics with respect to L. In our model, an increase in L means higher level of product differentiation and larger market size. Alternatively, one can assume that the density of consumers is $\frac{1}{2L}$ so the total measure of consumers will be fixed at 1. In this case, an increase in L implies higher product differentiation only.

[13] One can think of a dynamic model where firms choose uniform price in the first period. Consumers' purchasing decisions then reveal their preferences and our assumed information structure can be obtained after the first period. Considering such a dynamic game explicitly will complicate our analysis a great deal. For simplicity, we assume that such information structure is exogenous. This information structure is also similar to that in Bester and Petrakis (1996) and Armstrong (2006).

[14] In our setting, firms have only crude information and can identify only two groups of consumers, not individual consumers. If a defensive coupon (say with face value d) is offered, the same coupon will be offered to all customers in the firm's own turf. Such a promotion strategy is dominated by (i) lowering both regular price and the value of the offensive coupon by d and then (ii) getting rid of defensive coupon. A more detailed analysis of defensive couponing is available upon request.

at segment j.[15] Consumers incur no cost when using coupons but differ in their willingness to trade (buy/sell) coupons.[16] A fraction α of them have zero cost of trading coupons. We call them *coupon traders*. If these consumers receive coupons, they will either use them or sell them to other consumers who value coupons more. They may also buy coupons from other customers. The remaining $1 - \alpha$ fraction of consumers have infinite cost of trading coupons and are called *coupon non-traders*.[17]

Our models are related to those in Bester and Petrakis, and Fudenberg and Tirole. If we set $\alpha = 0$, our model becomes the one in Bester and Petrakis (with uniform distribution). If we set $\alpha = 0$ and $k = 0$, our model becomes the second period model in Fudenberg and Tirole (short-term contracts with uniform distribution).

The game we study can be described as follows.

- Stage 1. Firms, simultaneously and independently, decide on their regular prices (p_i), promotion frequency (λ_i) and depth (r_i), $i = 1, 2$.[18] Note that not sending any coupon is equivalent to setting $\lambda_i = 0$ for firm $i = 1, 2$.
- Stage 2. Coupons are distributed and coupon trading takes place.
- Stage 3. Consumers make purchasing decisions. If they use coupons, they will pay a regular price minus the coupon face value.

We assume that firms are risk neutral and maximize expected profits. So firm i's problem is to choose $p_i, \lambda_i, r_i, i = 1, 2$, to maximize its profit,

$$\max_{p_i,\lambda_i,r_i} \pi_i(p_i, \lambda_i, r_i, p_{-i}, \lambda_{-i}, r_{-i}), \quad i = 1, 2.$$

4 Analysis

Consumers can be segmented into the following groups, depending on whether they are coupon traders and whether they receive coupons.

Type (a): Non-traders without coupon;
Type (b): Non-traders with coupon;
Type (c): Traders with or without coupon.

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15 Alternatively, coupon distribution cost may be increasing but concave in L, the size of the market. Also, one can introduce a fixed cost of couponing so that firms may have an incentive not to distribute any coupon in equilibrium under certain conditions.

16 In Sect. 5, we introduce coupon non-users, i.e., those who incur prohibitively high cost when using coupons. The results do not change qualitatively.

17 For tractability, we assume that α is exogenous. Alternatively, one can endogenize coupon trading choices by introducing a smooth distribution of coupon trading costs among consumers. That is, everything else held the same, consumers with lower trading costs would be more willing to trade coupons. We analyze this setup in an extension, and find that the results are qualitatively the same as in our main model.

18 Similar to Bester and Petrakis, we model the price and promotion strategies as a simultaneous game. An alternative way of modeling is a sequential-move game where firms chooses one strategy (say price) before they choose the other strategy (say promotion strategy). However, it is unclear to us whether firms should choose price strategy or promotion strategy first. On the one hand, it is often viewed that regular price is a higher level managerial decision and is relatively slow to adjust in practice than promotions. On the other hand, we often observe that regular price changes while promotion strategy (e.g. coupon face value) is relatively stable over time.

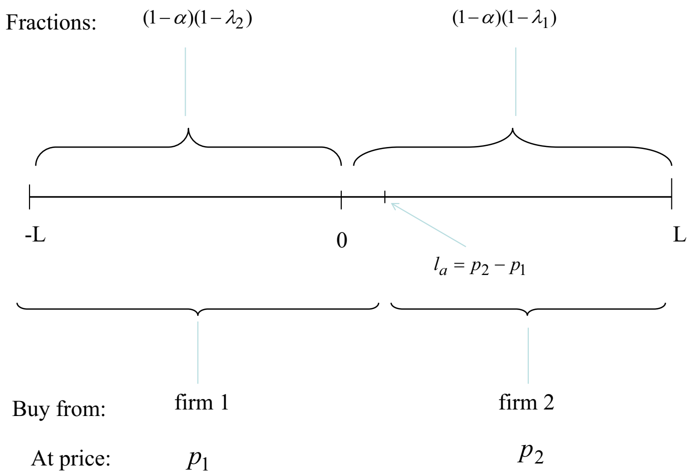

Fig. 1 Type (a): non-traders without coupon

Based on firm i's promotion effort λ_i $(i = 1, 2)$, each firm's loyal consumer has an equal probability, λ_i, of receiving a coupon of face value r_i from its rival firm i.

Next, we calculate firms' demand and profits from each type of consumers, starting with type (a).

Type (a): non-traders without coupon

Type (a) consumers are depicted in Fig. 1. Densities of type (a) consumers differ across the two segments: $[-L, 0]$ and in $[0, L]$. First consider consumers in the segment $[-L, 0]$. The fraction of non-traders is $1 - \alpha$. The probability of not receiving firm 2's coupon is $1 - \lambda_2$. Overall, the probability of being a non-trader and not receiving firm 2's coupon is $(1 - \alpha)(1 - \lambda_2)$. Similarly the fraction of non-traders without firm 1's coupon in the segment $[0, L]$ is $(1 - \alpha)(1 - \lambda_1)$.

Let l_a denote the location of the marginal consumer, which is defined by $l_a = p_2 - p_1 \geq 0$, since $p_2 \geq p_1$. Every consumer to the left of l_a will buy from firm 1 at price p_1, and those to the right will buy from firm 2 at price p_2. Therefore, firms 1 and 2 make sales of

$$\begin{aligned} d_{1a} &= (1-\alpha)(1-\lambda_2)L + (1-\alpha)(1-\lambda_1)l_a \\ &= (1-\alpha)(1-\lambda_2)L + (1-\alpha)(1-\lambda_1)(p_2 - p_1), \\ d_{2a} &= (1-\alpha)(1-\lambda_1)(L - l_a) \\ &= (1-\alpha)(1-\lambda_1)(L - p_2 + p_1). \end{aligned}$$

Their profits from type (a) consumers are

$$\pi_{1a} = p_1 d_{1a} = p_1(1-\alpha)(1-\lambda_2)L + p_1(1-\alpha)(1-\lambda_1)(p_2 - p_1),$$

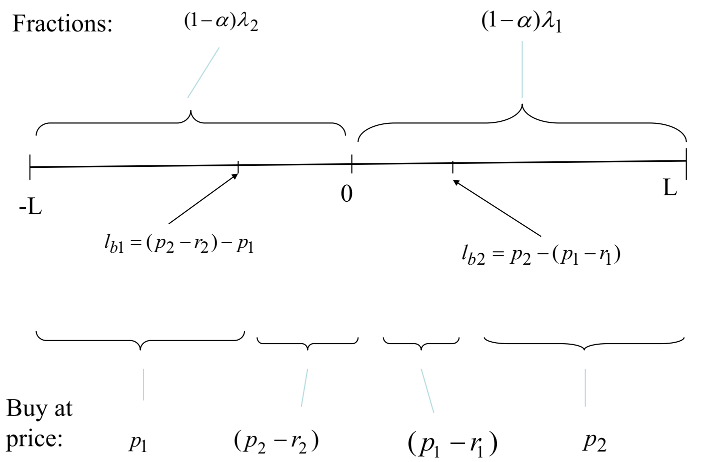

Fig. 2 Type (b): non-traders with coupon

$$\pi_{2a} = p_2 d_{2a} = p_2(1-\alpha)(1-\lambda_1)(L - p_2 + p_1).$$

Type (b): non-traders with coupons

These consumers are depicted in Fig. 2. Let's start with consumers on the interval $[-L, 0]$. The probability of being a non-trader, and receiving firm 2's coupons is $(1-\alpha)\lambda_2$. Firms' roles are reversed on the interval $[0, L]$. The density of consumers receiving coupons there is $(1-\alpha)\lambda_1$.

Let l_{b1} and l_{b2} denote the marginal consumer in segment 1 and 2 respectively. The left marginal consumer, located at l_{b1}, is indifferent between buying from firm 2 with a coupon (thus paying $p_2 - r_2$) and buying from firm 1 without a coupon (thus paying p_1).[19] Similarly, the right marginal consumer (located at l_{b2}) is indifferent between buying from firm 2 at p_2 and buying from firm 1 at $p_1 - r_1$. The exact locations of these two marginal consumers are

$$l_{b1} = (p_2 - r_2) - p_1, \quad l_{b2} = p_2 - (p_1 - r_1).$$

It's easy to see that $l_{b1} < 0$ and $l_{b2} > 0$. Otherwise, these coupons are not attracting any extra customers for the firms and they would be better off not to send any coupons. Consumers located in the interval $[-L, l_{b1}]$ receive coupons from firm 2, but the face value of firm 2's coupon is not enough to compensate for their strong preferences for firm 1 's product. As a result, they will buy from firm 1 at p_1. Since they are non-traders, they will not sell firm 2's coupons. However, consumers located in $(l_{b1}, 0]$ only have weak preferences for firm 1's product. Having firm 2's coupons, they will choose to

[19] We assume that firms do not match each other's coupons. If coupons are matched, then firms would have no incentive to send poaching coupons unless some consumers do not request coupon-matching.

buy from firm 2 and pay $p_2 - r_2$. Similarly, consumers located in $[0, l_{b2})$ will buy from firm 1 at a price of $p_1 - r_1$, and consumers in $[l_{b2}, L]$ will buy from firm 2 at the price p_2. Consumers whose purchasing decisions are affected by couponing (switchers), are non-traders with coupons between $[l_{b1}, l_{b2}]$. Consequently, firms' profits from type (b) consumers are:

$$\begin{aligned}
\pi_{1b} &= p_1(1-\alpha)\lambda_2(l_{b1}+L) + (p_1 - r_1)(1-\alpha)\lambda_1 l_{b2} \\
&= p_1(1-\alpha)\lambda_2[(p_2 - r_2) - p_1 + L] \\
&\quad +(p_1 - r_1)(1-\alpha)\lambda_1[p_2 - (p_1 - r_1)], \\
\pi_{2b} &= (p_2 - r_2)(1-\alpha)\lambda_2(0 - l_{b1}) + p_2(1-\alpha)\lambda_1(L - l_{b2}) \\
&= (p_2 - r_2)(1-\alpha)\lambda_2[(p_1 - (p_2 - r_2)] \\
&\quad +p_2(1-\alpha)\lambda_1[L - p_2 + (p_1 - r_1)].
\end{aligned}$$

Type (c): traders with or without coupons

The last type of consumers are traders with or without coupons.[20] Their density is α in both segments. If coupons are auctioned off online, coupon sales will result in an efficient or nearly efficient allocation depending on the market environment considered. These coupons could be sold at auctions using one of many standard formats. In an ascending first price auction, for example, or in a second price auction similar to eBay's environment, the price will be determined in competition among buyers and sellers somewhere between zero and the coupon face value depending on how thin participation is in an given auction.[21] In such environments, coupon trading is efficient and coupons being traded will be bought by those who value them the most.[22] Next, we describe the coupon trading incentives and outcomes in detail.

[20] Notice that, non-traders may make different purchasing decisions depending on whether they receive coupons or not. Traders, on the other hand, will never use offensive coupons. If they receive such coupons from a firm, they will trade them away to others who were the firm's loyal customers in the first place. Therefore, receiving coupons affects how well-off they will be but not their purchasing decisions. As such, demand functions are the same for traders whether they receive poaching coupons or not.

[21] Alternatively, one can think of a double auction environment where all coupons are sold at once and the price is determined by a linear combination of the bid and ask prices that clears the market. In a buyer's bid double auction, at the one extreme, the price, depending on the level of participation, will be again between zero and the coupon face value. Assuming that valuations are independent and private (IPV), the coupon distribution problem is nearly efficient (converging to efficiency at a rate of $O(n/m^2)$ where m is the number of buyers and n is the number of sellers. See Zacharias and Williams 2001). In an ask market when the sellers highest ask is determining the uniform price, price would be close to the coupon face value. These different forms vary in terms of the transaction prices of the coupons, but the outcomes are all the same. Poaching coupons received by traders will be traded back to the coupon issuing firms' loyal customers—who value the coupons the highest (at their face values).

[22] Different consumers may value the same coupon differently. For example, suppose that $p_1 = p_2$ and consider the two consumers located on $[-L, 0)$ and $(0, L]$ respectively. If the consumer on $[-L, 0)$ receives a coupon from firm 2 with face value $r < L$, she will still buy from firm 1 so her valuation of firm 2's coupon is 0. On the other hand, the consumer located at $(0, L]$ will buy from firm 2 and value this coupon at its face value r. Therefore, the consumer at $[-L, 0)$ will have an incentive to sell this coupon to the consumer at $(0, L]$. For our purpose, it does not matter at what price this transaction occurs, but rather that *it occurs.*

Coupon trading incentives and outcomes

Let us first consider segment 1 ($[-L, 0]$), where firm 1's loyal customers are located. These customers may receive firm 2's coupons. Since $p_2 \geq p_1$, they will buy from firm 1 in the absence of coupons. Therefore, they value firm 2's coupons at less than the face value. If they receive firm 2's coupons, they will sell these coupons to those who value the coupons higher. The intended objective of coupons is to poach a rival firm's loyal customers, but since these poached customers generally value coupons less than the face value, those coupons reaching traders will end up in the hands of the coupon-issuing firm's loyal customers.

Next, consider segment 2 ($[0, L]$), where firm 2's loyal customers are located. These customers will buy from firm 2 in the absence of coupons, except those located close to zero if $p_2 > p_1$. Specifically, anyone located to the right of $l_c = p_2 - p_1$ will buy from firm 2 in the absence of coupons. They will sell firm 1's coupons (if they receive such coupons) to firm 1's loyal customers, i.e., those located on $[-L, 0)$. The rest of the consumers $[-L, l_c]$ will buy from firm 1 in the absence of coupons. Therefore, if they receive firm 2's coupons, such coupons will be traded to firm 2's loyal customers.

Intuitively, when α is too large, no firm will distribute coupons. We assume throughout the paper that $\alpha < \frac{1}{2}$, i.e., there are fewer coupon traders than non-traders. After coupon trading takes place, all coupons reaching traders will be sold to consumers who would buy from the coupon-issuing firm with or without its coupons (thus they value the coupons at their face value—the maximum value).[23] Therefore, after the coupon distribution and trading, there is a marginal consumer l_c near the middle who does not have a coupon and is indifferent between both products at their regular prices,[24]

$$l_c = p_2 - p_1.$$

To the left of l_c, all consumers buy from firm 1. Those close to $-L$ will buy and then use firm 1's coupon. Those to the right of l_c all buy from firm 2, with consumers close to L buying/using firm 2's coupon. Consumers in the neighborhood of l_c will not have coupons to use, since there is more demand than supply for coupons.

Firms' profits from the traders are,

$$\begin{aligned}
\pi_{1c} &= p_1\alpha(l_c + L) - r_1\alpha\lambda_1 L \\
&= p_1\alpha(L + p_2 - p_1) - r_1\alpha\lambda_1 L, \\
\pi_{2c} &= p_2\alpha(L - l_c) - r_2\alpha\lambda_2 L \\
&= p_2\alpha(L - p_2 + p_1) - r_2\alpha\lambda_2 L.
\end{aligned}$$

[23] For this to happen it is important that coupon traders have zero hassle cost of trading coupons. Coupon sellers in general value coupons less than face value while coupon buyers value these coupons at their face value. Since the demand of coupons from traders is higher than the supply, together with zero hassle cost, all coupons reaching traders will be traded. This means that for a seller who values the coupon arbitrarily close to the face value, the selling price must be arbitrarily close to the face value as well. We consider the case of positive hassle costs of trading coupons in Sect. 5.

[24] This requires that there is more demand than supply for each firm's coupons, and the consumers in the neighborhood of l_c will not have coupons. Intuitively this holds if distributing coupons is sufficiently costly (k is large) so that λ_{ij} is significantly less than 1.

Aggregating firms' profits over all types of consumers, and subtracting the cost of distributing coupons, we can obtain firm i's overall profit

$$\pi_i = \sum_{j=a}^{c} \pi_{ij} - k(\lambda_i L)^2, \quad i = 1, 2,$$

with j being the segment.

Firm i's problem is

$$\max_{p_i, \lambda_i, r_i} \pi_i(p_i, \lambda_i, r_i, p_{-i}, \lambda_{-i}, r_{-i}), \quad i = 1, 2.$$

Solving the first order conditions, we obtain equilibrium prices, promotion frequencies and depths. Note that distributing no coupons cannot be optimal given the quadratic coupon distribution cost. The following proposition characterizes the symmetric pure strategy equilibrium.

Proposition 1 *For any loyalty parameter (L), there exists a symmetric pure strategy equilibrium in which firms send coupons, when the cost of coupon distribution (k) is not too low.*[25] *In this equilibrium:*

(i) *The regular prices are*

$$p_1 = p_2 = p^* = \frac{\left(\frac{2}{3}A - \frac{3}{2}\frac{-\frac{4}{9}\alpha^2 + \frac{16}{3}k}{A} - \frac{1}{3}\alpha\right)L}{-1+\alpha}, \tag{1}$$

where

$$A = \left(\alpha^3 + 36k\alpha - 27k + 3\sqrt{12\alpha^4 k + 96\alpha^2 k^2 + 192k^3 - 6k\alpha^3 - 216k^2\alpha + 81k^2}\right)^{\frac{1}{3}}.$$

(ii) *Promotion depths are*

$$r_1 = r_2 = r^* = \frac{1}{2}\left(p^* - \frac{\alpha L}{1-\alpha}\right). \tag{2}$$

(iii) *Promotion frequencies are*

$$\lambda_1 = \lambda_2 = \lambda^* = \frac{(\alpha L - p^* + \alpha p^*)^2}{8(1-\alpha)kL^2}. \tag{3}$$

(iv) *Firms' equilibrium profits are*

$$\pi_1 = \pi_2 = \pi^*$$

[25] When k is sufficiently small, firms may have an incentive to deviate. See Proof of Proposition 1 in the Appendix for details.

$$= \frac{\alpha^4 L^4 + 64(1-\alpha)^2 k L^3 p^* - 6(1-\alpha)^2 \alpha^2 L^2 (p^*)^2 + 8(1-\alpha)^3 \alpha L (p^*)^3 - 3(1-\alpha)^4 (p^*)^4}{64(1-\alpha)^2 k L^2}. \tag{4}$$

Proof See the Appendix. □

From Eqs. (1), (2), (3) and (4), we can see that p^* and r^* are linear in L , π^* is quadratic in L, and λ^* is independent of L. Out of all consumer types, only type (b) consumers may switch. In particular, only non-traders in $[l_{b1}, l_{b2}]$ switch to buy from the firm further away from them. The fraction of switchers, denoted by s, is given by

$$s = \frac{1}{2}\lambda^*(1-\alpha)(l_{b2} - l_{b1}).$$

A numerical example

Without loss of generality we normalize $L = 1$. We choose $k = 1/2$ and further set $\alpha = 1/5$, so that 20 % of the consumers become traders. Then the equilibrium is

$$p^* = 0.9525, \quad r^* = 0.3513, \quad \lambda^* = 0.0987, \quad \pi^* = 0.9310, \quad s = 0.0277.$$

Recall that the coupon distribution cost is $k(\lambda L)^2$. Substituting the value of k, λ and L, this cost is about 0.005, or 0.5% of the regular price.

Prisoners′ dilemma : The model without coupons is essentially a standard Hotelling model (with the measure of consumers being 2 instead of 1). It can be easily verified that the equilibrium price is $p = 1$. Each firm attracts half of the market and enjoys a profit of $\pi = 1$. In a model with coupons, firms choose their couponing strategies in terms of couponing frequency and depth simultaneously.[26] Sending poaching coupons to the rival's consumers reduces regular prices ($p^* < 1$) as each firm attempts to retain some of its loyal customers. Lower regular prices lead to lower profits. The discounts which some consumers get by using coupons and the coupon distribution cost will lower firms' profits further ($\pi^* < p^*$).[27]

4.1 Comparative statics

Proposition 1 provides expressions for the equilibrium price, promotion variables and profit functions (p, λ, r and π). If we normalize $L = 1$, these variables remain only

[26] Sending no coupons is equivalent to choosing $\lambda = 0$, and can never be optimal given the quadratic coupon distribution cost and $\alpha < \frac{1}{2}$.

[27] Poaching coupons, as a tool for third-degree price discrimination, only intensify competition (best-response asymmetry, Corts 1998) and hurt firms' profits. Therefore, when an increase in α or k reduces firms' couponing intensity, firms become better off. On the other hand, if firms have the ability to target consumers based either on their willingness to pay when the market is not covered or on their unit transport cost t which differs across consumers groups (called "choosiness" in Armstrong 2006), then couponing may actually improve firms' profits.

functions of α and k.[28] A natural question then is how they change with α and k. The results are summarized in the next Proposition.

Proposition 2 *In the symmetric pure strategy equilibrium,*

(i) *Promotion frequency (λ) decreases with the fraction of coupon traders (α) and the coupon distribution cost (k).*
(ii) *Promotion depth (r) decreases with the fraction of coupon traders (α) but increases with the coupon distribution cost (k).*
(iii) *Regular price (p) and profit (π) increase with the fraction of coupon traders (α) and the coupon distribution cost (k).*
(iv) *The fraction of switchers (s) decreases with the fraction of coupon traders (α) and the coupon distribution cost (k).*

Proof Using the expressions in Proposition 1, we take partial derivatives with respect to α or k. Checking the signs of these partial derivatives gives us the comparative statics results above. □

The expressions for the relevant partial derivatives are very lengthy for reporting.[29] We discuss these results and offer some intuition below.

Fix k and vary α

When the fraction of coupon traders (α) increases, firms promote less frequently ($\lambda \downarrow$) and with lower promotion depth ($r \downarrow$). They charge higher prices and their profits increase. The intuition is as follows. The optimal promotion effort balances the benefit of couponing against the loss of couponing and the coupon distribution cost. In particular:

$$\begin{aligned}\text{benefit of couponing} &= (1-\alpha)\lambda r(p-r),\\ \text{loss of couponing} &= \alpha\lambda L r,\\ \text{coupon distribution cost} &= k(\lambda L)^2.\end{aligned}$$

The term $(1-\alpha)\lambda r$ measures the extra consumers (non-traders) the firm can attract at the discounted price of $(p-r)$. However, a loss of $\alpha\lambda L r$ is realized when the coupons reach traders. $\alpha\lambda L$ represents the proportion of affected consumers, and r is the loss of revenue from each of these consumers. An increase in α lowers the benefit and increases the loss of couponing. To re-balance the benefit, loss, and distribution cost, λ needs to decrease. This is because, the benefit and loss are linear in λ, while the cost of distributing coupons is quadratic in λ.

28 The qualitative results remain the same if $L \neq 1$ is chosen.

29 The Maple file which contains all the expressions is available upon request. In the Maple file, we also fix the value of either α or k and plot the equilibrium price, promotion intensity and profit against the other parameter.

Now, let's examine why an increase in α puts downward pressure on coupon face value r. When α increases, the benefit decreases and the loss increases. To re-balance the benefit and loss, r needs to decrease. While r does not enter into the term reflecting distribution costs, there is an *indirect tradeoff effect* between promotion frequency and depth. That is, a firm can poach more of a rival's customers by either sending more coupons of the same face value or sending the same number of coupons of a larger face value. This indirect effect implies that, when a firm reduces its promotion frequency, it increases its promotion depth. Our result suggests that, this indirect tradeoff effect is dominated by the direct effect of downward pressure on promotion depth. With fewer poaching coupons of less value there is less competition; thus price and profit increase. Obviously, consumers become worse off.

In a model with covered market and inelastic demand like ours, welfare analysis is not very informative. Nevertheless, we would like to illustrate how coupon trading can improve efficiency. First, customer poaching leads to inefficient brand switching and coupon trading among traders eliminates brand switching among traders, improving efficiency. Moreover, with coupon trading, firms reduce their promotion intensity. So there will be fewer coupons and of lesser value reaching non-traders, reducing inefficient brand switching as well. The fraction of traders (s) thus decreases with the fraction of coupon traders.

Fix α and vary k

Our results suggest that when k increases, firms respond by promoting less frequently ($\lambda \downarrow$) but with higher promotion depth ($r \uparrow$). Prices (even net of coupon face value) and profits go up with k. These results are comparable to those in Bester and Petrakis, and are quite similar to the results for fixed k and variable α, and so is the intuition. Both coupon trading (α) and distribution costs (k) work against sending coupons, and firms have fewer incentives to promote. However, the implications on promotion depth are different. When firms promote less frequently due to larger costs of distributing coupons, they respond by increasing the promotion depth (*tradeoff effect*). This is because, while an increase in α applies a direct downward pressure on r, an increase in k does not directly affect the benefit and loss of promotion, but only indirectly through λ and r. Thus, when k increases, only the indirect tradeoff effect (higher promotion depth to go with lower promotion frequency) exists. Consequently, promotion depth increases with k. Since sending coupons constitutes a prisoners' dilemma game, less promotion reduces the intensity of competition, which leads to higher prices (including prices net of coupons). There are two opposing effects on profits from the increase in k. First, the cost of distributing coupons increases, affecting profits negatively. Second, when k increases, competition is less intense which improves profits. Our results show that the second effect dominates the first as in Bester and Petrakis.

We next look at how the number of switcher (s) varies with k. As k increases, non-traders have a smaller chance (λ) to get a coupon, but with higher face value (r). This means fewer consumers will get coupon, but once they have one, they are more likely to switch. These two effects works against each other, and the decrease in λ dominates the rise in r, and the number of switchers decreases with k.

5 Extensions

In the benchmark model, we have assumed that (i) all consumers have zero cost of using coupons; (ii) coupons are transferrable and (iii) consumers' hassle costs of trading coupons take extreme values: either zero or prohibitively large. In this section, we relax these assumptions, one at a time and show that our qualitative results continue to hold.[30]

5.1 Introducing coupon non-users

Different from the main model, we allow some consumers to be coupon non-users. In particular, a fraction, $1-\gamma$, of the consumers has prohibitively high cost of using or trading coupons. The rest of the consumers are coupon users as in the main model. Coupon users and non-users are all uniformly distributed on the interval $[-L, L]$, but we allow them to have different price sensitivity. Our results show that, equilibrium prices and profits go up after the introduction of coupon non-users, but the main qualitative result that coupon trading raises prices and profits continue to hold.

5.2 Continuous hassle costs of selling coupons

In the main model, we have assumed that hassle costs of trading coupons are either zero or sufficiently high. This simplifying assumption ensures a clear and exogenous distinction between coupon traders and non-traders, and the coupon trading mechanism is greatly simplified. In this section, we relax this assumption and introduce a more realistic coupon trading mechanism. In particular, we assume that the cost of selling coupons is a random draw from a continuous distribution while the cost of buying coupons is kept at zero. The key difference between this extension and the main model is the following. In the main model, when a consumer receives a coupon, whether this coupon will be traded or not is independent of the consumer's location. In contrast, here, due to positive hassle cost, whether a consumer chooses to sell the coupon or not will depend on his/her location. In particular, those who value the coupon close to its face value (i.e., those close to the middle) are likely to keep the coupon and switch firms. Taking this into account, we find that firms respond by reducing coupon face value (relative to that in the main model). This does two things at the same time. First, consumers who switch firms are more likely in the middle of the range, and for those consumers lower coupon face value is needed to induce switching. Second, coupons of larger face value are likely to be traded back to the issuing firm's own loyal customers, something the firm wants to avoid. Other than this difference on promotion depth (coupon face value), the results here are quite comparable to those in the main model.

5.3 Non-transferable coupons

We have assumed that all coupons are transferrable in the main model. This is in line with what we observe in practice where coupons generally do not carry restrictions

[30] More details are provided in a separate online Appendix available upon request.

on the identity of those who can or cannot use them. In this section, we allow firms to make choices on whether or not to allow their coupons to be transferrable.[31] If coupons are all transferrable, the game becomes the same as in the main model. However, if a firm's coupons are not transferrable, then its coupons cannot be traded. Depending on parameter values, we find that firms want to mimic each other's behavior so both offering transferrable coupons and both offering non-transferrable coupons can be supported as equilibria. However, both firms offering transferrable coupons leads to higher profits, making it more likely from firms' perspective.[32]

6 Conclusion

There is a large literature on price discrimination, which has typically maintained the assumption that consumer arbitrage is infeasible. This assumption is increasingly violated when price discrimination is achieved through coupons which are traded at ever-higher rates online. We relax the no-arbitrage assumption by allowing coupons to be traded across consumers. When firms' poaching coupons reach non-traders, some of them use the coupons and switch firms, benefiting the issuing coupon firms unilaterally. However, coupon traders who receive coupons never switch firms so coupons are ineffective. Moreover, coupons backfire in this case. These coupon traders in general value the coupons less than the face value and they want to sell these coupons to traders who value them higher, i.e., the coupon issuing firms' own loyal customers. This leads to a strict profit loss for the issuing firms. Correspondingly, we find that when the fraction of coupon traders increases, firms respond by promoting less frequently (sending fewer coupons to consumers) and reducing the face value of coupons. These actions reduce competition and lead to higher equilibrium prices and profits. On the other hand, when the cost of distributing coupons increases, firms promote less frequently but at higher face value. Once again prices and profits increase. Consumers become worse off due to higher prices. Our main results continue to hold in several extensions where we include coupon non-users, introduce continuous hassle cost of selling coupons and allow coupons to be non-transferrable.

Appendix

Proof of Proposition 1 We divide this proof into two parts.[33] In part 1, we derive the optimal prices and couponing strategies. This is the equilibrium candidate. Then in part 2, we show that neither firm has an incentive to deviate unilaterally.

[31] Alternatively, whether coupons are transferrable or not may be a matter of specificities of the market. In this case, only symmetric configurations may be realistic, i.e., either both firms' coupons are transferrable or both are non-transferrable.

[32] The case where both firms choose non-transferrable coupons is equivalent to $\alpha = 0$ in our model, i.e., no coupon traders.

[33] A companion Maple file for the proof is available for download at http://faculty-staff.ou.edu/L/Qihong.Liu-1/research.html.

Part 1: equilibrium candidate

Firms' profit functions are

$$\pi_1 = p_1(1-\alpha)(1-\lambda_2)L + p_1(1-\alpha)(1-\lambda_1)(p_2-p_1) + p_1(1-\alpha)\lambda_2(L + p_2 - r_2 - p_1) + (p_1 - r_1)(1-\alpha)\lambda_1(p_2 - p_1 + r_1) + p_1\alpha(L + p_2 - p_1) - r_1\alpha\lambda_1 L - k(\lambda_1 L)^2, \quad (5)$$

$$\pi_2 = p_2(1-\alpha)(1-\lambda_1)(L - p_2 + p_1) + p_2(1-\alpha)\lambda_1(L - p_2 + p_1 - r_1) + (p_2 - r_2)(1-\alpha)\lambda_2(p_1 - p_2 + r_2) + p_2\alpha(L - p_2 + p_1) - r_2\alpha\lambda_2 L - k(\lambda_2 L)^2. \quad (6)$$

Taking derivative of π_2 with respect to p_2, r_2 and λ_2 respectively, then imposing the symmetry conditions ($p_1 = p_2, r_1 = r_2$ and $\lambda_1 = \lambda_2$), we can obtain

$$\frac{\partial \pi_2}{\partial r_2} = -\lambda_2(\alpha L - p_2 - 2r_2\alpha + 2r_2 + \alpha p_2) = 0, \quad (7)$$

$$\frac{\partial \pi_2}{\partial \lambda_2} = -2k\lambda_2 L^2 - r_2\alpha L - \alpha p_2 r_2 + r_2^2\alpha + p_2 r_2 - r_2^2 = 0, \quad (8)$$

$$\frac{\partial \pi_2}{\partial p_2} = L - p_2 - \lambda_2 r_2\alpha + \lambda_2\alpha p_2 + r_2\lambda_2 - \lambda_2 p_2 = 0. \quad (9)$$

Since the cost of coupon distribution is quadratic in λ, and the rest is roughly linear in λ, it must be that the optimal $\lambda_2 > 0$. Then, Eq. (7) implies,

$$r_2 = \frac{\alpha L - p_2 + \alpha p_2}{2(\alpha - 1)} = \frac{1}{2}\left(p_2 - \frac{\alpha L}{1-\alpha}\right). \quad (10)$$

From this expression, we can see that $p_2 > r_2$.

Next, we substitute the expression for r_2 into Eq. (8) and solve for λ_2. We obtain

$$\lambda_2 = \frac{(\alpha L - p_2 + \alpha p_2)^2}{8(1-\alpha)kL^2}. \quad (11)$$

Using r_2 and λ_2 in Eq. (9), we can solve for the equilibrium price[34]

$$p_2 = \frac{\left(\frac{2}{3}A - \frac{3}{2}\frac{-\frac{4}{9}\alpha^2 + \frac{16}{3}k}{A} - \frac{1}{3}\alpha\right)L}{-1+\alpha},$$

where

$$A = \left(\alpha^3 + 36k\alpha - 27k + 3\sqrt{12\alpha^4 k + 96\alpha^2 k^2 + 192k^3 - 6k\alpha^3 - 216k^2\alpha + 81k^2}\right)^{\frac{1}{3}}.$$

[34] There are three solutions. We pick the one that is real and positive.

We can then substitute p_2 back into the expressions for r_2 and λ_2. The final expressions are too lengthy to report.

So far, we have used first-order conditions (FOCs) to solve for the optimal choices of prices and promotion intensities. However, FOCs are necessary but not sufficient. We need to make sure that the solution we obtained indeed constitutes an equilibrium. Instead of checking whether the Hessian matrix is negative semidefinite (which is quite messy), we show that neither firm has an incentive to unilaterally deviate from this pair of strategies (Bester and Petrakis use a similar method). Without loss of generality, we fix firm 1's price and promotion strategies as given in Proposition 1, and show that firm 2 to has no incentive to deviate.

Part 2: firm 2 has no incentive to deviate

Note that, the demand/profit functions depend on the locations of marginal consumers and there are two cases. In the first case, $p_2 \geq p_1$ still holds and thus $l_a \geq 0$. In the second case, $p_2 < p_1$. In both cases, we assume that $l_{b1} < 0$ and $l_{b2} > 0$.[35]

Start with case 1 where $p_2 \geq p_1$. Firm 2's deviation profit is given by equation (6), with $p_1 = p^*$, $r_1 = r^*$ and $\lambda_1 = \lambda^*$. We normalize $L = 1$. The optimal choice requires that

$$\frac{\partial \pi_2^{dev}}{\partial r_2} = \frac{\partial \pi_2^{dev}}{\partial \lambda_2} = 0.$$

Solving the first order conditions, we obtain

$$r_2^{dev} = \frac{2(1-\alpha)p_2 - (1-\alpha)p^* - \alpha}{2(1-\alpha)},$$

$$\lambda_2^{dev} = \frac{\alpha^2 + \alpha^2(p^*)^2 + 4\alpha^2 p_2 - 2p^*\alpha^2 + 2\alpha p^* - 4\alpha p_2 - 2\alpha(p^*)^2 + (p^*)^2}{(1-\alpha)k}.$$

The first order conditions are necessary and sufficient. Note that, we do not substitute p^* in these expressions as they would be too lengthy to report. Notice that, firm 2's deviation profit depends only on p_2^{dev}, α and k. We want to check whether firm 2 can increase its profit by choosing $p_2^{dev} \neq p^*$, i.e., to have

$$\pi_2^{dev}(p_2^{dev}) > \pi^*, \quad \forall\, \alpha, k.$$

We tried various combinations of α and k, and we found that firm 2 can never increase its profit by choosing a price different than p^* . Therefore, firm 2 has no incentive to deviate. We then proceeded to the case of $l_a < 0$ (i.e. $p_2 < p_1$). The steps

35 If $l_{b2} \leq 0$, then our formula of d_{2b} would be exaggerated. This is because the relevant demand is capped at L while our formula leads to $d_{2b} \geq L$. Since we show that firm 2 has no incentive to deviate under the exaggerated demand function, it surely has no incentive to deviate under the correct demand function. Thus we ignore the case of $l_{b2} < 0$. Note that $l_{b1} < 0$ must hold. This is because, as the deviating firm, firm 2 must be able to sell to some of firm 1's loyal customers, i.e., $l_{b1} < 0$.

are similar and we found that firm 2 has no incentive to deviate if k is not too small relative to L.[36] □

References

Aguirre I, Espinosa M (2004) Product differentiation with consumer arbitrage. Int J Ind Organ 22:219–239
Alger I (1999) Consumer strategies limiting the monopolist's power: multiple and joint purchases. Rand J Econ 30:736–758
Anderson S, Ginsburgh V (1999) International pricing with costly consumer arbitrage. Rev Int Econ 7:126–139
Armstrong M (2006) Recent developments in the economics of price discrimination. In: Advances in economics and econometrics: theory and applications: Ninth World Congress, eds. Blundell, Newey and Persson, Cambridge University Press
Bester H, Petrakis E (1996) Coupons and oligopolistic price discrimination. Int J Ind Organ 14:227–242
Brander J, Krugman P (1983) "A reciprocal dumping" model of international trade. J Int Econ 15:313–321
Calzolari G, Pavan A (2006) Monopoly with resale. RAND J Econ 37:362–375
Chen Y (1997) Paying customers to switch. J Econ Manage Strategy 6(4):877–897
Chen Y (2008) Dynamic price discrimination with asymmetric firms. J Ind Econ 56:729–751
Corts K (1998) Third-degree price discrimination in oligopoly. RAND J Econ 29:306–323
Deltas G, Salvo A, Vasconcelos H (2012) Consumer-surplus-enhancing collusion and trade. RAND J Econ 43(2):315–328
Fong Y, Liu Q (2011) Loyalty rewards facilitate tacit collusion. J Econ Manage Strategy 20:739–775
Fudenberg D, Tirole J (2000) Customer poaching and brand switching. RAND J Econ 31:634–657
Fudenberg D, Villas-Boas M (2006) Behavior-based price discrimination and customer recognition. In: Hendershott TJ (ed) Handbook on economics and information systems, pp 377–436
Gans J, King S (2007) Perfect price discrimination with costless arbitrage. Int J Ind Organ 25:431–440
Haile P (2003) Auctions with private uncertainty and resale. J Econ Theor 108:72–100
Liu Q, Serfes K (2004) Quality of information and oligopolistic price discrimination. J Econ Manage Strategy 13:671–702
Liu Q, Serfes K (2006) Customer information sharing among rival firms. Eur Econ Rev 50:1571–1600
Narasimhan C (1984) A price discrimination theory of coupons. Market Sci 3:128–147
Shaffer G, Zhang ZJ (1995) Competitive coupon targeting. Market Sci 14:395–415
Shaffer G, Zhang ZJ (2002) Competitive one-to-one promotions. Manage Sci 48:1143–1160
Zacharias E, Williams SR (2001) EX post efficiency in the buyer's bid double auction when demand can be arbitrarily larger than supply. J Econ Theor 97:175–190

[36] Details are available in the companion Maple file. For $L = 1$ and $\alpha = 0$, the threshold value for k is around $k = 0.159$. Technically, there is additional constraint on k, namely, it should not be too small so that λ^* does not exceed 1 but this constraint is never binding.