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Sustaining collusion in markets with entry driven by balanced growth

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Abstract This paper studies the sustainability of collusion in markets where growth is not restricted to occur at a constant rate and may trigger future entry. Entry typically occurs later along the punishment path than along the collusive path (since profits are lower in the former case), and may not even occur along the punishment path. The possibility of delaying or even deterring entry may, therefore, constitute an additional incentive for deviating just before entry is supposed to occur along the collusive path. If firms set quantities and revert to Cournot equilibrium after a deviation, this incentive more than compensates for the fact that there are more firms after entry, making collusion harder to sustain before entry than after entry. If, instead, firms set prices or use optimal penal codes, deterring entry by breaking the cartel is not profitable, and thus collusion is harder to sustain after entry than before entry. The proposed model encompasses and explains conflicting results derived in the extant literature under more restrictive settings, and derives some novel results.

Keywords Collusion · Entry · Market growth

JEL Classification L11 · L13 · K21

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1 Introduction

The impact of market growth on the sustainability of collusion is not straightforward. On the one hand, market growth increases future profitability, which reduces the incentives for firms to disrupt the collusive agreement in the present. On the other hand, market growth may foster the entry of new firms, and the prospect of entry makes collusion harder to sustain because it reduces the future benefits of complying with the collusive agreement, without affecting the present benefits of deviating.

While in previous works the number of firms was assumed to remain constant over time, recent theoretical contributions have studied these countervailing effects of market growth on collusion, by taking into account the fact that market growth may induce the entry of a new firm (Capuano 2002; Vasconcelos 2008; Brandão et al. 2014; Correia-da-Silva et al. 2015).[1] These contributions aim at investigating under which conditions is collusion sustainable, and whether collusion is harder to sustain before or after entry. Both issues are relevant for regulators and competition authorities. Knowledge of the environmental factors that favor or hinder collusion allows them to identify markets where collusion is most likely and to design policies that diminish the scope for collusion. In addition, understanding how the incentives to collude change over time allows regulators to increase the effectiveness of their investigations by concentrating their monitoring efforts on the periods that are more conducive to cooperation.

In antitrust practice, the analysis of joint dominance or coordinated effects (i.e., of the impact of a merger on the sustainability of collusive agreements) is traditionally at the core of merger policy in the United States and, since the *Nestlé/Perrier*[2] and *Kali & Salz*[3] cases, also in the European Union. Subsequent decisions on various cases such as *Gencor/Lonrho*[4], *Price Waterhouse/Coopers & Lybrand*[5], *Airtours/First Choice*[6] and *Time Warner/EMI*[7] have confirmed that analysis of joint dominance or coordinated effects became a cornerstone of merger policy in the EU. The perception that entry is perhaps the major threat to cartel stability was expressed by Osborne (1976). The line of research to which the present paper contributes is relevant for policy makers because it clarifies the conditions in which and the extent to which potential entry can mitigate joint dominance in the long run (after entry) and also in the short run (before entry).

Whether collusion is easier to sustain before or after entry crucially depends on the type of punishment strategies adopted by cartel members. Capuano (2002), Vasconcelos (2008) and Brandão et al. (2014) concluded that, if firms permanently revert to

[1] The work of Vasconcelos (2008) underpins the contributions of Brandão et al. (2014) and Correia-da-Silva et al. (2015). Brandão et al. (2014) considered asymmetric cartel members, while Correia-da-Silva et al. (2015) studied the impact of considering alternative punishment strategies and reactions to entry.

[2] *Nestlé SA/Source Perrier SA* (Case IV/M24), [1992] OJ C53 L356.

[3] *Kali & Salz/Mdk/Treuhand* (Case IV/M.308), [1994] OJ L186, [1998] OJ C196 C275.

[4] *Gencor/Lonrho* (Case IV/M.619), [1995] OJ C314 C347, [1997] OJ L11.

[5] *Price Waterhouse/Coopers & Lybrand* (Case IV/M.1016), [1997] OJ C376, [1999] OJ L50.

[6] *Airtours/First Choice* (Case IV/M.1524), [1999] OJ C124 C162, [2000] OJ L93.

[7] *Time Warner/EMI* (Case COMP/M.1852), [2000] OJ C136.

Cournot equilibrium after a deviation, collusion is typically more difficult to sustain before entry than after entry. This surprising result (in stationary markets, collusion typically becomes more difficult to sustain as the number of firms increases) is driven by two features of their frameworks: (i) entry occurs later under Cournot competition than under collusion; and (ii) the Cournot profit before entry is higher than the collusive profit after entry. As a result, delaying entry constitutes an additional incentive to break the collusive agreement just before entry is expected to occur along the collusive path, which outweighs the importance of the number of firms.

In contrast, Capuano (2002) and Correia-da-Silva et al. (2015) found that, if firms adopt optimal penal codes, collusion is more difficult to sustain after entry than before entry. The incentive to deviate before entry in order to delay entry disappears because the punishment scheme is so severe that absorbs all the potential gain from delaying entry.[8] These works, therefore, contributed to clarify the conditions under which breaking the cartel is an effective strategic barrier to entry (in addition to limit pricing, advertising expenditures and capacity investments, which are beyond the scope of this contribution).

Despite providing important conclusions, all these works rely on quite specific assumptions: linear demand and costs, and constant rate of market growth. In this paper, our aim is to relax these assumptions and provide a more general framework to study the sustainability of collusion in industries with endogenous entry driven by market growth. More precisely, we extend the existing literature in two directions. First, instead of considering a linear demand function and constant marginal costs, we only require market growth to be *balanced*, i.e., to have the same relative impact on profits in all market regimes (collusion, unilateral deviation and punishment).[9] Second, instead of assuming a constant market growth rate, we only restrict the evolution of market size to be quasi-concave. Apart from encompassing the existing models as special cases, our framework also enables us to obtain novel results.

The basic setup is similar to that of Capuano (2002) and Vasconcelos (2008). There are two incumbents and one potential entrant producing homogeneous goods and supporting symmetric production costs.[10] Firms interact during an infinite number of periods, with the objective of maximizing the discounted sum of their profits. At the beginning of the game, the incumbents combine to maximize the industry profit in

[8] It suffices for this conclusion that the continuation value of the deviator is null. It does not matter whether other firms are also hurt, as when firms permanently revert to a symmetric zero-profit equilibrium, or benefit from punishing the deviator, as in the penal code proposed by Aramendia (2008), where the deviator shuts down temporarily while the other firms revert to an equilibrium with $n-1$ active firms. The penal code of Aramendia (2008) provides a continuation value after a deviation that is greater than zero, but smaller than the continuation value associated with permanent reversion to Cournot equilibrium.

[9] This property holds in all the mentioned literature, and also in the model of Bagwell and Staiger (1997).

[10] The number of incumbents is not crucial for our results and can be easily relaxed. Multiple potential entrants can also be allowed, as long as only one can effectively enter. In this case, entry occurs when the value of entering the market becomes positive instead of maximal (Capuano 2002). In contrast, the consideration of more than one actual entrant would complicate the analysis (Correia-da-Silva et al. 2015). One justification for considering a single potential entrant is the possible existence of structural barriers to entry that effectively limit the number of firms in the industry. These barriers may be legal, like in the mobile phone industry, where licenses issued by the government are required; or economic, like in the bottled water industry, where ownership of a water source (which is a scarce and indivisible input) is indispensable.

all periods, and accommodate the entrant in a more inclusive agreement immediately after entry.[11] Firms abide by the collusive agreement as long as the others do the same; and irreversibly revert to a punishment equilibrium after a deviation.

When deciding when to enter the market, the entrant faces the following trade-off: on the one hand, it wants to enter as soon as possible, to start receiving profits; on the other hand, delaying entry decreases the discounted value of the entry cost. Assuming that the entrant chooses the entry period that maximizes the discounted value of its flow of profits, we find that entry occurs when the profit level hits a certain threshold (independent of past and future profits). Typically, entry occurs later along the punishment path than along the collusive path (because profits are lower). However, it may happen that the entry periods coincide, or that entry is not profitable along the punishment path.

The incentives for firms to comply with the collusive agreement, which result from the comparison between the short-run gain from deviating (difference between deviation profits and collusive profits) and the long-run losses (difference between the continuation value of profits along the collusive path and along the punishment path), depend on whether entry has already occurred or is yet to occur.

After entry, in any given period, market growth only affects the condition for the sustainability of collusion through an adjustment term that is multiplied to the discount factor. This term is the ratio between the value of the market in the next period and the value of the market in the current period (i.e., the gross rate of growth of the value of the market).[12] Therefore, the relevant adjustment term for the sustainability of collusion after entry is the infimum of this ratio across all periods after entry. The period wherein the value of the market grows the least (or declines the most) is the most critical for collusion sustainability after entry. If the value of the market is non-decreasing, collusion is sustainable after entry as long as the discount factor is sufficiently high. If, at some point, the value of the market decreases by more than a certain factor, collusion cannot possibly be sustained.

Before entry, under a mild condition (weaker than sustainability of collusion without entry), we conclude that collusion is the most difficult to sustain in the period that immediately precedes entry. In all periods before entry, the incumbents are tempted to defect in order to enjoy the deviation profit and delay entry (by lowering the profits of the entrant from the collusive level to the punishment level). However, deviating in the period that immediately precedes entry allows an incumbent to receive these benefits without forgoing the collusive profits in the previous periods. We derive a necessary and sufficient condition for collusion to be sustainable before entry, and show that collusion becomes harder to sustain as the entry delay (that would result from a cartel breakdown) increases.

We obtain more clear-cut results in two extreme cases for the entry delay (null or infinite). When the cartel breakdown does not delay entry and the market grows at a

[11] Assuming that firms maximize industry profits does not seem overly restrictive because this is the most profitable cartel behavior (whenever it is sustainable). Lower pre-entry prices would not be advantageous in deterring entry, because what determines the timing of entry is post-entry behavior. For a discussion on alternative reactions to entry, see Correia-da-Silva et al. (2015) and the references therein.

[12] By value of the market, we mean the discounted sum of present and future profits.

non-decreasing rate, collusion is harder to sustain after entry (than before entry) if the single-period deviation gain is greater after entry. At the other extreme case, when the cartel breakdown permanently deters entry, collusion is not sustainable if the collusive profit after entry is lower than the punishment profit before entry.

If firms adopt optimal penal codes (Abreu 1986) that drive punishment profits to zero, the cartel breakdown permanently deters entry. However, this does not constitute an additional incentive to deviate before entry because the continuation value after a deviation is null. In this scenario, we conclude that collusion is more difficult to sustain after entry than before entry if and only if the single-period deviation gain is greater with three than with two firms.

In general, it is not possible to state whether the incentives to disrupt the collusive agreement are stronger before or after entry. For this reason, we focus on two particular cases: (i) firms set prices; (ii) firms set quantities and adopt grim trigger strategies.

The analysis is much simpler when firms set prices (rather than quantities) because punishment profits are null and, unless there are diseconomies of scale, deviation profits coincide with monopoly profits. In this case, entry makes collusion harder to sustain, because the present value of collusive profits is lower with entry, while deviation profits are the same with or without entry. We find that, if the market grows at a non-decreasing rate, collusion is more difficult to sustain after entry than before entry. In addition, the sustainability of the collusive agreement is influenced by the convexity of the cost function and by the long-run trend (lower bound) of the market growth rate.

When, instead, firms set quantities, support no production costs, and face linear demand, entry also makes collusion more difficult to sustain.[13] In this case, if the cartel breakdown deters entry, collusion is not sustainable (because the incumbents profit more competing against each other than colluding with the entrant). If, in the opposite case, the cartel breakdown has no impact on the entry period, collusion is harder to sustain after entry than before entry.

To deepen our study of collusion in linear markets under the threat of Cournot competition, we assume two different specifications for the evolution of the market over time and analyze whether the results are qualitatively similar. In the benchmark case of constant market growth rate (Capuano 2002; Vasconcelos 2008), we find that collusion is harder to sustain before entry if and only if the entry delay is strictly positive. Assuming that the market grows at a decreasing rate, we conclude that if a deviation delays entry by more than two periods, collusion is surely more difficult to sustain before entry. Hence, the comparison of collusion sustainability before and after entry depends on the magnitude of the entry cost and on the speed at which the market growth rate converges to its long-term trend. These two variables determine whether entry occurs at an early stage, when the market is growing faster; or at a later stage, when the market is growing slowly. The faster is market growth at the entry period, the lower is the entry delay generated by the cartel breakdown (because demand takes less time to go from the threshold for entry along the collusive path to the threshold for entry along the punishment path). Hence, if the entry cost is sufficiently low, collusion

[13] This scenario captures the models of Capuano (2002) and Vasconcelos (2008) as particular cases and also goes beyond their assumption of constant rate of demand growth.

may be more difficult to sustain after entry; while, if it is sufficiently high, collusion may be harder to sustain before entry.

We also find that the sustainability of collusion may vary non-monotonically with the rate of market growth. It is possible that: if market growth is slow, there is no entry (under collusion or punishment) and a collusive agreement involving the two incumbents is sustainable; if market growth is fast, entry is profitable (under collusion and punishment) and collusion is sustainable; for intermediate values of the rate of market growth, collusion is precluded by the fact that a cartel breakdown deters entry.

The remainder of the paper is organized as follows. Section 2 briefly relates our work to the existing literature. Section 3 presents the general model, derives conditions for entry to be profitable (under collusion and along the punishment path), and characterizes the optimal entry periods. Section 4 studies the sustainability of collusion before and after entry. Section 5 considers the case in which firms are price-setters. Section 6 addresses the case of linear demand and quantity-setting firms. Section 7 offers some concluding remarks. The Appendix contains most proofs and some auxiliary calculations.

2 Related literature

In the existing theoretical literature on the sustainability of collusion in non-stationary markets, the specifications for the evolution of profits differ in terms of: (i) the deterministic or stochastic nature of growth; and (ii) the absence or presence of fluctuations around a constant trend.[14] We will briefly describe the main contributions.

Tirole (1988), Motta (2004) and Ivaldi et al. (2007) analyzed the impact of constant market growth on the sustainability of collusion. They concluded that a positive (negative) growth rate makes collusion easier (harder) to sustain.[15] Assuming that demand is subject to deterministic cyclical fluctuations, Haltiwanger and Harrington (1991) found that, although deviation gains are highest at the peak of the cycle, collusion is the most difficult to sustain when demand is declining. The reason is that the punishment is less severe if deviations take place during recessions, because profits are lower in the subsequent periods.

The contribution of Rotemberg and Saloner (1986) is seminal. If demand is stochastic (subject to observable i.i.d. shocks), collusion is harder to sustain in periods of high demand. The reason is the following: as the punishment does not depend on the level of demand in the deviation period, firms are the most tempted to deviate when the

[14] In the present work, we consider that the market evolves deterministically and without fluctuations. More precisely, the profitability of the market may increase forever or have a single peak (i.e., starts by increasing, reaches the peak and then declines).

[15] Empirical studies diverge on their conclusions about the impact of market growth on the sustainability of collusion. Dick (1996) concluded that Webb-Pomerene cartels are more frequent in growing industries. Contrariwise, Asch and Seneca (1975) found that collusion is more frequent when the growth of sales is slow than when it is fast. Symeonidis (2003) found a non-monotonic relation between growth and the likelihood of collusion: collusion is easier to sustain when the market grows at a moderate rate than when it declines or grows at a fast rate. Somewhat puzzling was the result of the experiment conducted by Abbink and Brandts (2009): collusion is more easily established when demand is shrinking than when it is expanding.

deviation gain is the highest, i.e., when demand is high.[16] Hence, the optimal collusive mechanism implies lower prices when demand is high than when it is low.

In the related model of Bagwell and Staiger (1997), demand grows according to a Markov process with two states: fast growth and slow growth. Most-collusive prices are higher when growth is fast (slow) if growth rates are positively (negatively) correlated along time, and the amplitude of collusive pricing cycle is larger if booms are short and recessions are long.

In all the works mentioned above, the number of active firms in the industry was assumed to remain constant over time. However, it is expected that favorable market conditions encourage entry, while adverse market conditions induce exit.[17] In addition, the number of firms in the market is usually perceived as a key determinant of the likelihood of collusion (Ivaldi et al. 2007). Thus, it is restrictive to assume a fixed number of firms in markets that grow or decline over time.

While most studies of collusion in growing markets have assumed a fixed number of firms over time, most studies of collusion in markets with entry have assumed constant demand.[18] Capuano (2002) and Vasconcelos (2008) were the first to unify these literatures, by building models to study the sustainability of collusion in markets where demand growth triggers entry.[19] We generalize their framework, by not relying on a specific demand function or on a constant market growth rate. We also allow for different punishment mechanisms, capturing the standard grim trigger strategies and optimal penal codes as particular cases. Finally, we do not restrict marginal costs to be null. Although relaxing these assumptions, we are able to recover their main results and put forward some novel findings.

3 Model

Consider a market with two incumbents (firms 1 and 2) and one potential entrant (firm 3) that produce homogeneous goods and have the same cost function. The objective of each firm, $i \in \{1, 2, 3\}$, is to maximize the discounted value of its flow of profits, $V_i \equiv \sum_{t=0}^{+\infty} \delta^t \pi_{it}$, where $\delta \in (0, 1)$ is the common discount factor and π_{it} denotes the profit of firm i in period $t \in \{0, 1, \ldots\}$. The profit function of firm i in period t is given by:

$$\pi_{it}(q_{it}) = P_t(Q_t)q_{it} - C_t(q_{it}), \tag{1}$$

[16] Ensuing contributors tried to understand whether the main conclusions of Rotemberg and Saloner (1986) remain valid in the presence of serial correlation between demand shocks or capacity constraints. See, for example, Haltiwanger and Harrington (1991), Kandori (1991), Staiger and Wolak (1992), Fabra (2006), Knittel and Lepore (2010) and Montero and Guzman (2010).

[17] In their survey, Siegfried and Evans (1994) reported several empirical studies finding that market growth (measured by the past growth rate of industry sales revenue) positively effects entry.

[18] The latter have focused on the comparison between various reactions to entry by a cartel (Harrington 1989; Stenbacka 1990; Friedman and Thisse 1994; Vasconcelos 2004). This is not the focus of our contribution. We suppose that there is a single entrant, which is accommodated in the collusive agreement immediately after entry.

[19] Brandão et al. (2014) and Correia-da-Silva et al. (2015) have analyzed whether the main results of Vasconcelos (2008) are robust, respectively: to asymmetries in production costs between incumbents and entrant; and to alternative punishment strategies or cartel reactions to entry.

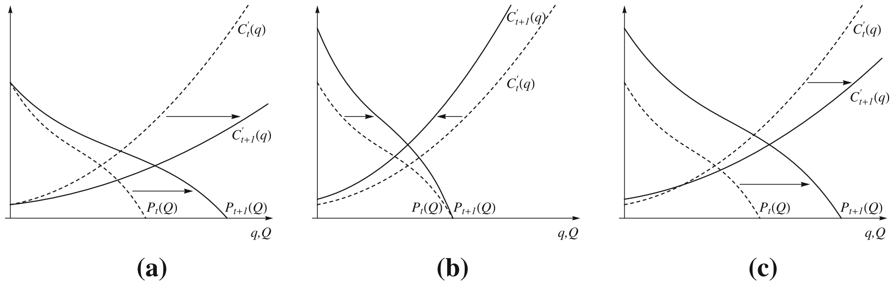

Fig. 1 Allowed forms of market growth

where $P_t(\cdot)$ is the inverse demand function, $C_t(\cdot)$ is the cost function, q_{it} is the quantity produced by firm i, and $Q_t = \sum_{j=1}^{n} q_{jt}$ is the total output of the industry. Assume that the inverse demand function and the cost function can be written as:[20]

$$P_t(Q_t) = P\left(\frac{Q_t}{g_t}\right) h_t \quad \text{and} \quad C_t(q_{it}) = C\left(\frac{q_{it}}{g_t}\right) g_t h_t, \tag{2}$$

where g_t and h_t are strictly positive parameters that describe different types of market growth, and $P(\cdot)$ and $C(\cdot)$ are functions that do not change over time. The normalized inverse demand function, $P : \mathbb{R}_+ \to \mathbb{R}_+$, is assumed to be such that: $P(0) = \bar{P}$; $P'(Q)$ is negative and continuous, $\forall Q \in \left(0, \bar{Q}\right)$; and $P(Q) = 0$, $\forall Q \geq \bar{Q}$. The normalized cost function, $C : \mathbb{R}_+ \to \mathbb{R}_+$, is assumed to be continuously differentiable and such that $C(0) = 0$. The effect of variations in g and/or h is illustrated in Fig. 1.[21]

An increase in g can be designated as *extensive* growth, in the sense that each price and cost level becomes associated with a proportionally greater output level (Fig. 1a). It seems natural to expect population growth to have such an effect on inverse demand, and it may also have an analogous effect on marginal cost if labor is the only factor of production and the increasing marginal cost is due to differences in productivity across workers.

An increase in h can be designated as *intensive* growth, in the sense that each output level becomes associated with proportionally higher price and cost levels. This kind of market growth can result from a proportional increase of the prices of all goods and factors of production. Varying h can also be a form of describing a non-constant discount rate.[22]

[20] It may seem very restrictive to assume that the market size parameters simultaneously impact the demand and cost functions. However, this specification captures the existing formulations in the literature as particular cases. More precisely, Capuano (2002), Vasconcelos (2008) and Correia-da-Silva et al. (2015) consider null production costs and constant market growth rate. Except for the asymmetry among firms, Brandão et al. (2014) is also captured by our formulation.

[21] We allow for positive or negative market growth, i.e., expansion or contraction of the market.

[22] To see this, notice that we can also write the objective function of the firm as $V_i = \sum_{t=0}^{+\infty} \delta^t h_t \pi_i(q_{it}, Q_t)$, where $\pi_i(q_{it}, Q_t) \equiv h_t^{-1} \pi_{it}(q_{it}, Q_t)$ is a function that does not depend on t. This means that $\delta^t h_t$ corresponds to the discount factor from t to the present.

Under this formulation, market growth corresponds to a transformation of scale.[23] Growth is said to be *balanced*, in the sense that it has the same proportional impact on profits in the three possible market regimes: collusion (m), unilateral deviation from the collusive agreement (d), and continuation equilibrium after a deviation (c).[24] In period t, the profit of firm i can be written as:

$$\pi_{it}^{jn} = \pi^{jn} f_t,$$

where $j \in \{m, d, c\}$ denotes the market regime, $n \in \{2, 3\}$ is the number of active firms, and $f_t \equiv g_t h_t$.[25] Profits in the different market regimes are assumed to satisfy the usual ordering: $\pi^{cn} \leq \pi^{mn} \leq \pi^{dn}$. The evolution of market profitability over time, $f : \mathbb{N}_0 \rightarrow \mathbb{R}_+$, is assumed to be quasi-concave and summable: $\sum_{t=0}^{+\infty} \delta^t f_t < +\infty$. It is useful to define $F : \mathbb{N}_0 \rightarrow \mathbb{R}_+$ as $F_t \equiv \sum_{s=t}^{+\infty} f_s \delta^{s-t}$. While f_t is an index of current period profitability, F_t is an index of the profitability of the market from t onwards, i.e., of the value of the market.

Our framework generalizes the models of Capuano (2002) and Vasconcelos (2008), where profits grow at a constant rate (i.e., $f_t = \beta^t$ with $\beta > 1$) and single-period profits (π^{cn}, π^{mn} and π^{dn}) are derived for quantity-setting firms that face a linear market demand.

To enter the market, firm 3 must support a fixed entry cost, $K > 0$. Exiting the market is assumed to entail no costs or revenues. As a result, given that $C(0) = 0$, it is never profitable to exit. Firms remain in the market (even if inactive) forever after entering.

In each period, if entry did not occur before: first, the potential entrant decides whether to enter or not and this decision is observed by the incumbents; then, the active firms simultaneously set prices or quantities.

Since we are assuming the existence of a single potential entrant, entry occurs in the period that maximizes the discounted value of the entrant's flow of profits:

$$V_3(T) = \sum_{t=T}^{+\infty} \delta^{t-T} \pi_{3t} - \delta^T K.$$

The following result describes if and when entry occurs.

Lemma 1 *If firm* 3 *expects the post-entry market regime to be* $j \in \{m, c\}$*, it enters the market if and only if the entry cost is sufficiently low:*

[23] Johnson and Myatt (2006) studied transformations of demand, focusing on rotations (transformations that change in opposite directions the willingness-to-pay of consumers with high willingness-to-pay and the willingness-to-pay of consumers with low willingness-to-pay). A particular kind of rotation can be obtained by varying simultaneously g and h (Fig. 1c). Transformations of demand that can be described by varying only g or only h are shifts, and never rotations, in the sense of Johnson and Myatt (2006).

[24] This is shown in Appendix A for the scenario in which firms set quantities.

[25] The fact that collusive, deviation and continuation profits vary in the same proportion greatly simplifies the study of collusion sustainability, because (as we will see) it implies that the incentive compatibility constraints in the different periods differ by a term that only depends on the shape of market growth.

$$K \leq \pi^{j3} \sup_t \{F_t\}. \tag{3}$$

If the above condition is satisfied, entry occurs at the lowest T^j such that:[26]

$$f_{T^j} \geq \frac{(1-\delta)K}{\pi^{j3}}. \tag{4}$$

Proof See Appendix B. □

We conclude that entry occurs earlier under collusion than along the punishment path ($T^m \leq T^c$); and that entry may not occur along the punishment path (even if it occurs under collusion). Furthermore, if entry also occurs along the punishment path, the faster is the market growth in the periods that immediately follow the entry period under collusion (i.e., in the periods after T^m), the shorter is the entry delay associated with the cartel breakdown ($T^c - T^m$).

To be able to study the sustainability of collusion before entry, we assume that the entry cost is sufficiently high for firm 3 not to enter in period $t = 0$ (even if firms are colluding), i.e., $f_0 < \frac{(1-\delta)K}{\pi^{m3}}$.

4 Collusion

Suppose that, before period $t = 0$, the incumbents agree to maximize the industry profit (perfect collusion) in all periods. They also agree to accommodate the entrant in a more inclusive agreement immediately after it enters the market (full collusion).[27]

Firms adopt grim trigger strategies: if there is a deviation from the collusive agreement, firms permanently revert to a punishment equilibrium.

4.1 Without entry

As a benchmark, we start by considering the case in which there is no entry. This may be due to an exogenous entry barrier or to a prohibitively high entry cost. As we have shown in Lemma 1, if $K > \pi^{m3} \sup_t \{F_t\}$, entry is never profitable.

In the absence of entry, the incumbents abide by the collusive agreement if the following incentive compatibility condition (ICC) is satisfied:

$$\sum_{s=t}^{+\infty} \delta^{s-t} \pi_{is}^{m2} \geq \pi_{it}^{d2} + \sum_{s=t+1}^{+\infty} \delta^{s-t} \pi_{is}^{c2}, \quad \forall t \geq 0. \tag{5}$$

[26] The timing of entry, given by (4), results from the comparison between the current period's profit with the gain associated with supporting the entry cost one period later.

[27] The case in which the entrant is not incorporated in the collusive agreement can be addressed simply by letting π^{m3} in the incentive compatibility conditions below denote the post-entry profit of an incumbent in this scenario. Alternative reactions to entry have been studied by Harrington (1989), Stenbacka (1990), Friedman and Thisse (1994), Vasconcelos (2004), and Correia-da-Silva et al. (2015).

Proposition 1 *Collusion is sustainable in the absence of entry if and only if:*

$$\delta \geq \frac{\pi^{d2} - \pi^{m2}}{\pi^{d2} - \pi^{c2}} \sup_{t \geq 0} \left\{ \frac{F_t}{F_{t+1}} \right\}. \tag{6}$$

Proof Manipulating the ICC (5), we obtain:

$$\pi^{m2} \sum_{s=t}^{+\infty} \delta^{s-t} f_s \geq \pi^{d2} f_t + \pi^{c2} \delta \sum_{s=t+1}^{+\infty} \delta^{s-(t+1)} f_s$$

$$\Leftrightarrow \pi^{m2} F_t \geq \pi^{d2} F_t - \pi^{d2} \delta F_{t+1} + \pi^{c2} \delta F_{t+1} \Leftrightarrow \delta \geq \frac{\left(\pi^{d2} - \pi^{m2}\right) F_t}{\left(\pi^{d2} - \pi^{c2}\right) F_{t+1}}.$$

□

Hence, we have obtained the standard critical discount factor, $\frac{\pi^{d2}-\pi^{m2}}{\pi^{d2}-\pi^{c2}}$, multiplied by a constant that only depends on how the market evolves over time, $\sup_{t \geq 0} \left\{ \frac{F_t}{F_{t+1}} \right\}$. This allows us to conclude that, in the absence of entry, the most critical moment for collusion to be sustainable is the period in which the value of the market, measured by F_t, increases the least (or decreases the most).

4.2 After entry

Suppose, now, that the entry cost is sufficiently low for entry to be profitable, at least under collusion, i.e., that $K \leq \pi^{m3} \sup_t \{F_t\}$.

The incentives to collude before entry are typically different from those after entry. We start by deriving the conditions for collusion to be sustainable after entry. Firms abide by the collusive agreement after entry if the following ICC is satisfied:

$$\sum_{s=t}^{+\infty} \delta^{s-t} \pi_{is}^{m3} \geq \pi_{it}^{d3} + \sum_{s=t+1}^{+\infty} \delta^{s-t} \pi_{is}^{c3}, \quad \forall t \geq T^m.$$

Proposition 2 *Collusion is sustainable after entry if and only if:*

$$\delta \geq \frac{\pi^{d3} - \pi^{m3}}{\pi^{d3} - \pi^{c3}} \sup_{t \geq T^m} \left\{ \frac{F_t}{F_{t+1}} \right\}.$$

Proof Analogous to the proof of Proposition 1. □

4.3 Before entry

The ICC that must be satisfied for collusion to be sustainable before entry is more complex than after entry.[28] Before entry, there are two distinct phases along the collu-

[28] With a slight abuse of language: we say that collusion is sustainable in period t when it is sustainable in period t conditionally on being sustainable in all subsequent periods; and we say that collusion is sustainable

sive and the punishment paths (corresponding to periods before and after entry). The incumbents comply with the collusive agreement in period $t < T^m$ (before entry) if and only if:

$$\sum_{s=t}^{T^m-1} \delta^{s-t} \pi_{is}^{m2} + \sum_{s=T^m}^{+\infty} \delta^{s-t} \pi_{is}^{m3} \geq \pi_{it}^{d2} + \sum_{s=t+1}^{T^c-1} \delta^{s-t} \pi_{is}^{c2} + \sum_{s=T^c}^{+\infty} \delta^{s-t} \pi_{is}^{c3},$$

which can be rewritten as:

$$-\left(\pi^{d2} - \pi^{m2}\right) F_t + \left(\pi^{d2} - \pi^{c2}\right) \delta F_{t+1} - \left(\pi^{m2} - \pi^{m3}\right) \delta^{T^m-t} F_{T^m} + \left(\pi^{c2} - \pi^{c3}\right) \delta^{T^c-t} F_{T^c} \geq 0. \tag{7}$$

The following assumption will be shown to imply that the critical period for the sustainability of collusion before entry is the period that immediately precedes entry.

Assumption 1 Firms are not too impatient:

$$\delta \geq \frac{\pi^{d2} - \pi^{m2}}{\pi^{d2} - \pi^{c2}} \sup_{t \leq T^m-1} \left\{ \frac{f_{t-1}}{f_t} \right\}.$$

Note that Assumption 1 is weaker than the condition for the sustainability of collusion in a duopoly without market growth, $\delta \geq \frac{\pi^{d2}-\pi^{m2}}{\pi^{d2}-\pi^{c2}}$, because $f_{t-1} \leq f_t$ in all periods before entry. In the particular case of constant growth rate $\left(\frac{f_t}{f_{t-1}} = \frac{F_t}{F_{t-1}}\right)$, it coincides with the condition for the sustainability of collusion in a duopoly without entry, given in (6).

Lemma 2 *Under Assumption 1, if the ICC (7) is satisfied in the period that immediately precedes entry, it is satisfied in all previous periods.*

Proof See Appendix B. □

According to Lemma 2, if Assumption 1 is satisfied, we only need to check the ICC (7) in period $t = T^m - 1$.

Proposition 3 *Under Assumption 1, collusion is sustainable before entry if and only if:*

$$\left(\pi^{m2} - \pi^{d2}\right) F_{T^m-1} + \left(\pi^{d2} - \pi^{c2} - \pi^{m2} + \pi^{m3}\right) \delta F_{T^m} + \left(\pi^{c2} - \pi^{c3}\right) \delta^{T^c-T^m+1} F_{T^c} \geq 0. \tag{8}$$

Proof Using Lemma 2, write condition (7) at $t = T^m - 1$. □

Footnote 28 continued
before entry when it is sustainable before entry conditionally on being sustainable after entry. In rigor, if collusion is not sustainable in some period, it is never sustainable in earlier periods.

From condition (8), we conclude that the greater is the entry delay that results from a cartel breakdown before entry, $T^c - T^m$, the greater are the incentives for the incumbents to deviate from the collusive agreement before entry.

Lemma 3 *Under Assumption 1, for given T^m, collusion before entry becomes more difficult to sustain as T^c increases. This effect is strict if and only if $\pi^{c2} > \pi^{c3}$.*

Proof Consider a given T^m. The left-hand side of (8) with $T^c + 1$ instead of T^c is lower than the left-hand side of (8) with T^c if and only if:

$$\begin{aligned}\left(\pi^{c2} - \pi^{c3}\right) \delta^{T^c+1-T^m+1} F_{T^c+1} &\le \left(\pi^{c2} - \pi^{c3}\right) \delta^{T^c-T^m+1} F_{T^c} \\ &\Leftrightarrow \delta F_{T^c+1} \le F_{T^c} \Leftrightarrow 0 \le f_{T^c},\end{aligned}$$

which is always true. □

To better understand the importance of the entry delay as an incentive to defect, we briefly consider two extreme scenarios: (i) cartel breakdown does not delay entry; and (ii) cartel breakdown deters entry (i.e., entry is not profitable along the punishment path).

4.3.1 Cartel breakdown does not delay entry

Suppose that entry is profitable along the punishment path, i.e., $K \le \pi^{c3} \sup_t \{F_t\}$. Using Lemma 1, we know that the cartel breakdown does not delay entry (i.e., $T^c = T^m$) if and only if the thresholds (4) under collusion and punishment are attained in the same period. This is the case if there exists a period T such that:[29]

$$\pi^{m3} f_{T-1} < (1 - \delta)K \le \pi^{c3} f_T.$$

Proposition 4 *Under Assumption 1, if entry occurs at the same period along the collusive and punishment paths ($T^m = T^c$), collusion is sustainable before entry if and only if:*

$$\delta \ge \frac{\left(\pi^{d2} - \pi^{m2}\right) F_{T^m-1}}{\left(\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c3}\right) F_{T^m}}. \tag{9}$$

Proof From Lemma 2, we know that the binding ICC for collusion to be sustainable before entry is in period $T^m - 1$. Substituting $T^c = T^m$ in ICC (8), we obtain:

$$\begin{aligned}&\left(\pi^{m2} - \pi^{d2}\right) F_{T^m-1} + \left(\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c3}\right) \delta F_{T^m} \ge 0 \\ &\Leftrightarrow \delta \ge \frac{\left(\pi^{d2} - \pi^{m2}\right) F_{T^m-1}}{\left(\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c3}\right) F_{T^m}}.\end{aligned}$$

□

[29] On the one hand, f is quasi-concave, which implies that it is strictly increasing until T^c (i.e., $f_{T-1} < f_T$). On the other hand, the collusive profit is greater than the punishment profit (i.e., $\pi^{m3} > \pi^{c3}$). Thus, the interval $\left(\pi^{m3} f_{T-1}, \pi^{c3} f_T\right]$ may be empty or not.

We will sometimes focus on cases wherein the market growth rate, $\frac{f_{t+1}}{f_t}$, is non-increasing over time (Assumption 2). This implies that the growth rate of the value of the market, $\frac{F_{t+1}}{F_t}$, is also non-increasing over time. Furthermore, the two growth rates converge to the same limit, which is also their infimum, denoted by:

$$\beta \equiv \lim_{t\to\infty}\left\{\frac{f_{t+1}}{f_t}\right\} = \lim_{t\to\infty}\left\{\frac{F_{t+1}}{F_t}\right\} = \inf_t\left\{\frac{f_{t+1}}{f_t}\right\} = \inf_t\left\{\frac{F_{t+1}}{F_t}\right\}.$$

Assumption 2 The size of the market grows at a non-increasing rate:

$$\frac{f_{t+2}}{f_{t+1}} \leq \frac{f_{t+1}}{f_t}, \ \forall t \geq 0.$$

Under Assumption 2, even in the extreme case in which the cartel breakdown (before entry) does not delay entry, collusion may be harder to sustain before entry or after entry.

Result 4.1 *Under Assumptions 1 and 2, if entry occurs at the same period along the collusive and punishment paths ($T^m = T^c$), collusion is harder to sustain after entry than before entry if the single-period deviation gain is greater with three firms than with two firms ($\pi^{d3} - \pi^{m3} \geq \pi^{d2} - \pi^{m2}$). If the growth rate is constant, this condition is necessary and sufficient.*

Proof See Appendix B. □

4.3.2 Cartel breakdown deters entry

The other extreme case, in which the cartel breakdown deters entry, occurs if and only if $K \in \left(\pi^{c3}\sup_t\{F_t\}, \pi^{m3}\sup_t\{F_t\}\right]$.

Proposition 5 *Under Assumption 1, if the incumbents deter entry by disrupting the collusive agreement before entry, collusion is sustainable before entry if and only if:*

$$\pi^{c2} < \pi^{m3} \quad \textit{and} \quad \delta \geq \frac{\left(\pi^{d2} - \pi^{m2}\right) F_{T^m-1}}{\left(\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c2}\right) F_{T^m}}.$$

Proof See Appendix B. □

If the cartel breakdown deters entry, the comparison between the sustainability of collusion before entry and after entry reduces to an expression that is independent of market growth.

Result 4.2 *Under Assumptions 1 and 2, if cartel breakdown before entry deters entry and:*

$$\frac{\pi^{d3} - \pi^{m3}}{\pi^{m3} - \pi^{c3}} \geq \frac{\pi^{d2} - \pi^{m2}}{\pi^{m3} - \pi^{c2}}, \tag{10}$$

collusion is harder to sustain after entry (than before entry).

Proof See Appendix B. □

To understand the intuition behind Result 4.2, keep in mind that the fact that the market grows at a non-increasing rate (Assumption 2) tends to make collusion harder to sustain as time passes. Therefore, a condition that is sufficient for collusion to be harder to sustain after entry with a constant growth rate is also sufficient if the growth rate is non-increasing.

Focusing on the case of constant growth rate, notice that left-hand side of (10) is the ratio between the gain from a deviation after entry ($\pi^{d3} - \pi^{m3}$) and the loss from the punishment after entry ($\pi^{m3} - \pi^{c3}$). The right-hand side is the same ratio for a scenario in which the deviation occurs before entry (the gain is $\pi^{d2} - \pi^{m2}$) but the punishment is the difference between the collusive profit after entry, because entry would occur in the absence of a deviation, and the punishment profit before entry, because entry would be deterred in the event of a deviation (the loss is $\pi^{m3} - \pi^{c2}$).

4.3.3 Security level penal codes

The consideration of security-level optimal penal codes that drive punishment profits to zero significantly simplifies the analysis.[30] It is as if firms permanently revert to a zero profit equilibrium ($\pi^{c2} = \pi^{c3} = 0$). Of course, in this scenario, the cartel breakdown permanently deters entry.

For the results to be more clear-cut, suppose that the market growth rate is non-increasing (Assumption 2). Without entry, collusion is sustainable if and only if (Proposition 1):

$$\beta\delta \geq \frac{\pi^{d2} - \pi^{m2}}{\pi^{d2}}. \tag{11}$$

After entry, collusion is sustainable if and only if (Proposition 2):

$$\beta\delta \geq \frac{\pi^{d3} - \pi^{m3}}{\pi^{d3}}. \tag{12}$$

Result 4.3 *Under Assumption 2, with security-level optimal penal codes, collusion is harder to sustain after entry than without entry if and only if the single-period deviation gain (in relative terms) is greater with three firms than with two firms:* $\frac{\pi^{d3}}{\pi^{m3}} > \frac{\pi^{d2}}{\pi^{m2}}$.

Proof The proof is direct from conditions (11) and (12). □

The ICC for collusion sustainability is satisfied in period $T^m - 1$ if and only if:

$$\delta \geq \frac{\left(\pi^{d2} - \pi^{m2}\right) F_{T^m-1}}{\left(\pi^{d2} - \pi^{m2} + \pi^{m3}\right) F_{T^m}}. \tag{13}$$

[30] In the case of Bertrand competition with homogeneous goods, this extreme punishment is what results from permanent reversion to the single-period equilibrium (Kaplan and Wettstein 2000). In other models, this kind of punishment is less natural but may also be sustainable in equilibrium. For example, in the Cournot model with $P(Q) = a - bQ$ and $C(q) = 0$, all firms producing $q = \frac{a}{b}$ is a zero-profit equilibrium.

A sufficient condition for collusion to be harder to sustain after entry than before entry is that single-period deviation gains are greater with three firms than with two firms.[31]

Result 4.4 *Under Assumptions 1 and 2, with security-level optimal penal codes, collusion is harder to sustain after entry than before entry if the single-period deviation gain (in absolute terms) is greater with three firms than with two firms:* $\pi^{d3} - \pi^{m3} > \pi^{d2} - \pi^{m2}$. *If the growth rate is constant, this condition is necessary and sufficient.*

Proof The proof is direct from conditions (12) and (13). □

It is straightforward to verify that the conditions of Results 4.3 and 4.4 are verified in the Cournot model with linear demand and constant unit costs. This means that it is harder to sustain collusion after entry ($\beta\delta \geq \frac{1}{4}$) than without entry ($\beta\delta \geq \frac{1}{9}$), and that sustainability of collusion after entry implies sustainability of collusion before entry.

5 Bertrand markets

In this Section, we consider the case in which firms simultaneously set prices. With collusion taking place under the threat of Bertrand competition, profits along the punishment path are null ($\pi^{c2} = \pi^{c3} = 0$).

If marginal costs are constant, cartel profits coincide with monopoly profits ($\pi^{m2} = \frac{\pi^{m1}}{2}$ and $\pi^{m3} = \frac{\pi^{m1}}{3}$) and, furthermore, deviation profits also coincide with monopoly profits ($\pi^{d2} = \pi^{d3} = \pi^{m1}$). This provides sufficient structure for the critical discount factors for the sustainability of collusion to be determined.

Result 5.1 *If firms set prices and have constant marginal costs, under Assumption 2, collusion is sustainable without entry, after entry and before entry if and only if, respectively:*

$$\beta\delta \geq \frac{1}{2}, \qquad \beta\delta \geq \frac{2}{3} \quad \textit{and} \quad \frac{F_{T^m}}{F_{T^m-1}}\delta \geq \frac{3}{5}.$$

Proof Apply Propositions 1, 2 and 5, respectively. □

Figure 2 illustrates this result. It is after entry that collusion is most difficult to sustain. In particular, if the market growth rate is decreasing, it is the tendency to deviate in the distant future that threatens the sustainability of collusion.

If marginal costs are increasing, it is neither true that deviation profits coincide with monopoly profits nor that cartel profits coincide with monopoly profits. Nevertheless, the fact that punishment profits are null is sufficient for the results of Sect. 4.3.3 to be applied.

[31] Observe that Result 4.4 is the exact analog of Result 4.1.

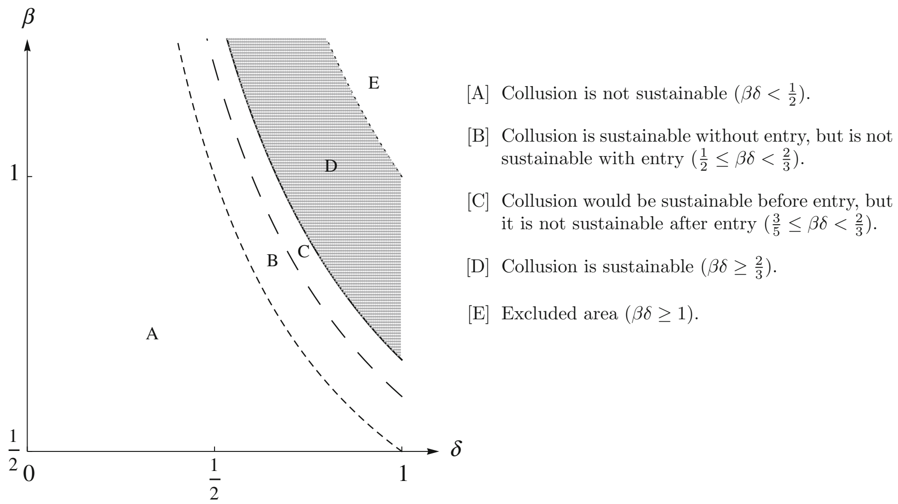

Fig. 2 Sustainability of collusion if firms set prices and have constant marginal costs

If marginal costs are decreasing, it is still true that deviation profits coincide with monopoly profits ($\pi^{d2} = \pi^{d3} = \pi^{m1}$).[32] However, cartel profits no longer coincide with monopoly profits ($\pi^{m1} > 2\pi^{m2} > 3\pi^{m3}$).

In this case, we can still conclude that entry makes collusion harder to sustain. This stems from the fact that, in any period (before or after entry), the present value of profits along the collusive path is lower when compared to the case in which entry is not possible, while deviation profits do not depend on past or future entry.

Result 5.2 *If firms set prices and marginal costs are non-increasing, the possibility of entry makes collusion harder to sustain.*

Proof See Appendix B. □

We now obtain the critical discount factors for collusion to be sustainable when the market grows at a non-decreasing rate (Assumption 2).

Without entry, the collusive agreement is sustainable if and only if (Proposition 1):[33]

$$\beta\delta \geq \frac{\pi^{m1} - \pi^{m2}}{\pi^{m1}}.$$

After entry, collusion is sustainable if and only if (Proposition 2):

$$\beta\delta \geq \frac{\pi^{m1} - \pi^{m3}}{\pi^{m1}}.$$

32 More precisely, what is necessary and sufficient for deviation profits to coincide with monopoly profits is that the cartel price is not lower than the monopoly price.

33 Recall that $\beta \equiv \lim_{t\to\infty}\left\{\frac{f_{t+1}}{f_t}\right\} = \lim_{t\to\infty}\left\{\frac{F_{t+1}}{F_t}\right\} = \inf_t\left\{\frac{f_{t+1}}{f_t}\right\} = \inf_t\left\{\frac{F_{t+1}}{F_t}\right\}$.

The ICC for the sustainability of collusion before entry is simplified by the fact that there is no entry along the punishment path (because profits are null). Under Assumption 1, it is necessary and sufficient that (Proposition 5):

$$\frac{F_{T^m}}{F_{T^m-1}}\delta \geq \frac{\pi^{m1} - \pi^{m2}}{\pi^{m1} - \pi^{m2} + \pi^{m3}}. \tag{14}$$

Result 5.3 *Under Assumptions 1 and 2, if firms set prices and marginal costs are non-increasing, collusion is harder to sustain after entry than before entry.*

Proof See Appendix B. □

The possibility of deterring entry could seem to be a strong incentive for deviating before entry. However, as profits are null along the punishment path, the potential gain from deterring entry is completely absorbed by the punishment. The comparison of the incentives to deviate before and after entry only concerns: the single-period gain from deviating (which is greater after entry); the severity of punishment (which is lower after entry); and the growth rate of the value of the market (which is lower or equal after entry, under Assumption 2). Clearly, the three effects go in the same direction, making it harder to sustain collusion after entry than before entry.

When firms set prices and the market grows at a non-increasing rate, the sustainability of collusion depends on the long-run market growth rate and on the existence of economies of scale. By increasing the future gains from colluding, long-term market growth ($\beta > 1$) makes collusion easier to sustain, relatively to a stationary market; while long-run market decline ($\beta < 1$) hinders collusion. Economies of scale make collusion harder to sustain, because deviation profits become greater than cartel profits.

6 Linear Cournot markets

In this Section, we study the case in which firms set quantities and permanently revert to Cournot equilibrium if there is a deviation from the collusive agreement.

To go beyond the results obtained in Sect. 4, we assume that firms have no production costs and that the inverse demand function in period $t \in \{0, 1, \ldots\}$ is given by:

$$P_t = \left(1 - \frac{Q_t}{g_t}\right) h_t. \tag{15}$$

Recall that variations of g_t and h_t correspond to extensive and intensive growth, respectively, and that the evolution of the size of the market is described by $f_t \equiv g_t h_t$. Henceforth, we refer to a market that conforms to the above assumptions as a *linear* Cournot market.

The profits that are relevant for our analysis are the collusive profits, the deviation profits and the Cournot profits:[34]

[34] The expressions for π^{mn}, π^{dn} and π^{cn} are obtained in Appendix C.

$$\pi^{mn} = \frac{1}{4n}, \qquad \pi^{dn} = \left(\frac{n+1}{4n}\right)^2 \quad \text{and} \quad \pi^{cn} = \frac{1}{(n+1)^2}.$$

Using these expressions, we obtain, from Proposition 1, the condition for collusion to be sustainable in the absence of entry:

$$\delta \geq \frac{9}{17} \sup_{t \geq 0} \left\{ \frac{F_t}{F_{t+1}} \right\}.$$

Similarly, from Proposition 2, collusion is sustainable after entry if and only if:

$$\delta \geq \frac{4}{7} \sup_{t \geq T^m} \left\{ \frac{F_t}{F_{t+1}} \right\}$$

Finally, from Proposition 3 and under Assumption 1, collusion is sustainable before entry if and only if:

$$-9F_{T^m-1} - 7\delta F_{T^m} + 28\delta^{T^c - T^m + 1} F_{T^c} \geq 0.$$

Result 6.1 *In linear Cournot markets, under Assumption 1, if a deviation from the collusive agreement deters entry, collusion is not sustainable before entry.*

Proof Observe that $\pi^{c2} > \pi^{m3}$. From Proposition 5, this implies that collusion cannot be sustained before entry. □

The incumbents profit more by competing against each other than by colluding with the entrant. Thus, the incumbents prefer to deter entry by disrupting the collusive agreement (before entry) rather than proceeding along the collusive path.[35]

Result 6.2 *In linear Cournot markets, under Assumptions 1 and 2, if a deviation from the collusive agreement does not delay entry, collusion is harder to sustain after entry than before entry.*

Proof This is a corollary of Result 4.1, since $\frac{1}{36} = \pi^{d3} - \pi^{m3} > \pi^{d2} - \pi^{m2} = \frac{1}{64}$. □

6.1 Constant rate of market growth

As a benchmark, it is instructive to consider the case in which the market grows at a constant rate, as assumed by Capuano (2002), Vasconcelos (2008), and the subsequent literature:[36]

[35] As explained by Correia-da-Silva et al. (2015), the incumbents could try to establish collusive agreements that are more advantageous for them (than competing since the beginning of the game). For example, in this case, if they can credibly threat to revert to competition if firm 3 enters the market, they can sustain a collusive agreement involving just the two of them forever. We leave to future research the analysis of alternative cartel reactions to entry, as well as the consideration of imperfect collusion.

[36] The case in which $\beta \leq 1$ is not interesting because entry would either occur at $t = 0$ or never.

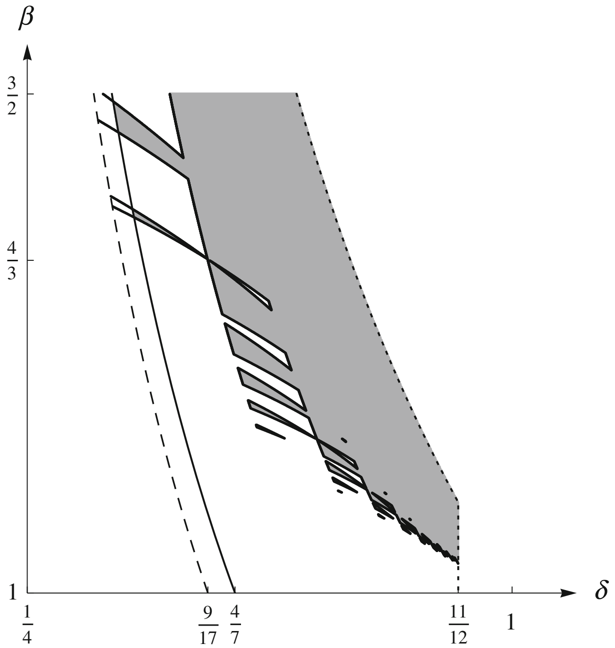

Fig. 3 Sustainability of collusion with a constant rate of market growth ($K = 1$)

$$f_t = \beta^t, \quad \text{with} \quad 1 < \beta < \delta^{-1}.$$

Since $\lim_{t\to\infty} F_t = +\infty$, firm 3 always enters the market, regardless of the market regime (collusion or competition). Observe also that the ratio $\frac{F_t}{F_{t+1}}$ is constant and equal to β^{-1}.

Therefore, from Proposition 1, the collusive agreement is sustainable in the absence of entry if and only if:

$$\beta\delta \geq \frac{9}{17}.$$

From Proposition 2, collusion is sustainable after entry if and only if:

$$\beta\delta \geq \frac{4}{7}.$$

From Proposition 3, under the hypothesis that $\beta\delta \geq \frac{9}{17}$ (Assumption 1), collusion is sustainable before entry if and only if:

$$-9 - 7(\beta\delta) + 28(\beta\delta)^{T^c - T^m + 1} \geq 0. \tag{16}$$

In Fig. 3, we plot the conditions for collusion to be sustainable before and after entry (assuming $K = 1$).[37] More precisely: (i) the dashed line represents the ICC for

[37] Similar configurations are obtained for other values of the entry cost.

collusion to be sustainable in the absence of entry, $\beta\delta = \frac{9}{17}$; (ii) the thin solid line represents the ICC for collusion to be sustainable after entry, $\beta\delta = \frac{4}{7}$; and (iii) the thicker and erratic solid line represents the ICC for collusion to be sustainable before entry, given by (16). Finally, the dotted line delimits the domain of our analysis: we exclude the region in which $\beta\delta \geq 1$, for the discounted sum of profits to be finite; and the region in which entry occurs at $t = 0$, which corresponds to $\delta \geq \frac{11}{12}$ (with $K = 1$). The painted area represents the combinations of parameters for which collusion is globally sustainable.

In particular, Fig. 3 illustrates that: (i) collusion is harder to sustain after entry than in the absence of entry; and (ii) collusion may be harder to sustain before or after entry, depending on the magnitudes of the discount rate and the market growth rate.

Result 6.3 *In linear Cournot markets with constant growth, if $\beta\delta \geq \frac{9}{17}$, collusion is harder to sustain before entry than after entry if and only if the cartel breakdown delays entry ($T^c - T^m \geq 1$). A sufficient condition for that is $\beta \leq \frac{4}{3}$.*

Proof See Appendix C. □

We conclude that, if the cartel breakdown strictly delays entry, collusion is harder to sustain before entry than after entry. As explained by Vasconcelos (2008), before entry, there are additional incentives to defect, because the cartel breakdown delays entry, and competition between two firms is more profitable than collusion among three firms.

6.2 Decreasing rate of market growth

Consider, now, that demand grows at a decreasing and convergent rate, according to:

$$f_t = (1 - \alpha^{-t})\beta^t, \quad \alpha > 1, \quad 0 < \beta < \delta^{-1}. \tag{17}$$

The value of α determines the speed of convergence to the long-run rate, β. A constant growth rate, $f_t = \beta^t$, is the pointwise limit of (17) when $\alpha \to +\infty$ (except at $t = 0$).[38]

The ratios $\frac{f_t}{f_{t+1}}$ and $\frac{F_t}{F_{t+1}}$ are increasing over time and converge to β^{-1}. Therefore, in the absence of entry, collusion is sustainable if and only if (Proposition 1):

$$\beta\delta \geq \frac{9}{17}.$$

If $\beta > 1$, entry is surely profitable along the punishment path (Lemma 1). In Appendix C, we obtain conditions for entry to be profitable (along the collusive and punishment paths) if $\beta \leq 1$.

[38] The dashed line represents the long-term tendency, β^t, while the dots represent $f_t = (1 - \alpha^{-t})\beta^t$, $t \in \mathbb{N}_0$.

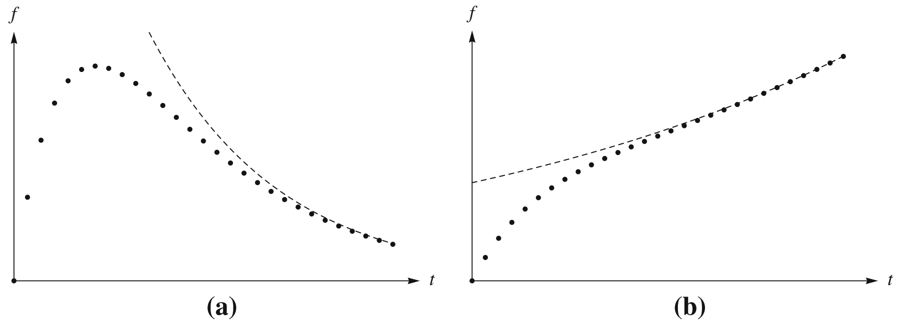

Fig. 4 Evolution of the market according to $f_t = (1 - \alpha^{-t})\beta^t$

After entry, collusion is sustainable if and only if (Proposition 2):

$$\beta\delta \geq \frac{4}{7}.$$

Notice that the critical discount factors for collusion to be sustainable without entry and after entry are independent of α. This occurs because, as $\frac{F_t}{F_{t+1}}$ increases over time, what is relevant for the sustainability of collusion is the value of this ratio in the long-run. When $t \to +\infty$, market demand is approximately given by $f_t = \beta^t$. We obtain, therefore, exactly the same conditions as in the case of constant growth rate (Fig. 4).

Analyzing the sustainability of collusion before entry is more complicated. Under Assumption 1, which coincides with the condition for collusion to be sustainable without entry $\left(\beta\delta \geq \frac{9}{17}\right)$, collusion is sustainable before entry if and only if (Proposition 3):

$$\begin{aligned} &- 9\left[\alpha - \beta\delta - \alpha^{2-T^m}(1-\beta\delta)\right] - 7\beta\delta\left[\alpha - \beta\delta - \alpha^{1-T^m}(1-\beta\delta)\right] \\ &+ 28\,(\beta\delta)^{T^c - T^m + 1}\left[\alpha - \beta\delta - \alpha^{1-T^c}(1-\beta\delta)\right] \geq 0. \end{aligned} \tag{18}$$

Result 6.4 *In linear Cournot markets, with* $f_t = (1-\alpha^{-t})\beta^t$ *and* $\beta\delta \geq \frac{9}{17}$*, collusion is harder to sustain before entry than after entry when:* $T^c - T^m \geq 3$*;* $T^c - T^m = 2 \wedge T^m \geq 2$*; or* $T^c - T^m = 1 \wedge T^m \geq 4$*.*

Proof See Appendix C. □

The results obtained with this form of market evolution are qualitatively similar to those obtained with a constant market growth rate. Result 6.4 is only more mitigate than Result 6.3 because it addresses a scenario in which the market is relatively smaller in early periods, implying that the short-term gain from deviating is lower. This is why collusion may be easier to sustain before entry than after entry even if the entry delay is not null ($T^c - T^m \geq 1$). However, this can only be the case if entry occurs at a very early stage ($T^m \leq 3$).

Figure 5 illustrates the conditions involved in the analysis of collusion sustainability in markets that grow and then decline: (i) the solid line corresponds to the ICC before

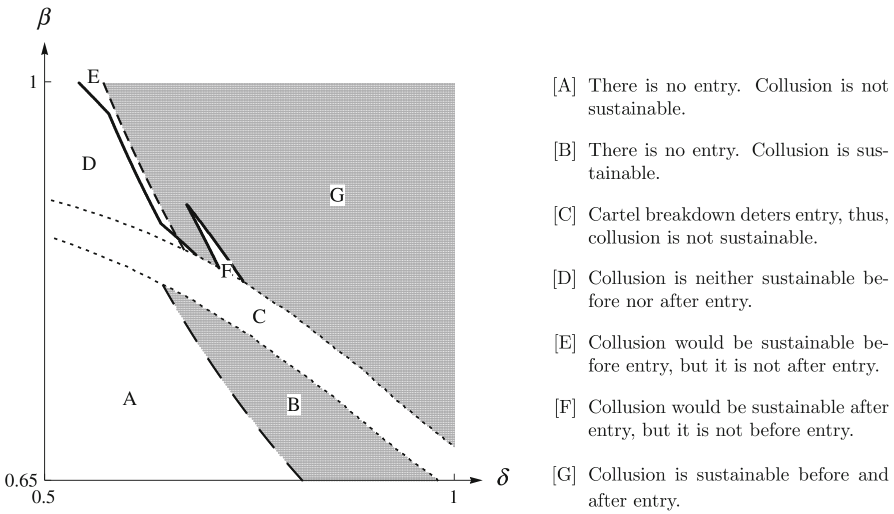

Fig. 5 Sustainability of collusion in markets that grow and then decline, with $f_t = (1 - \alpha^{-t})\beta^t$, $\alpha = 1.01$ and $K = 0.004$

entry (whose shape is explained in Appendix C); (ii) the dashed line between regions E and G corresponds to the ICC after entry; (iii) the dashed line between regions A and B represents the ICC without entry; and (iv) the dotted lines correspond to the conditions for entry to be profitable (under collusion and under competition). The painted area is the parameter region wherein collusion is globally sustainable.

There are two painted regions in Fig. 5, labeled B and G. In region B, there is no entry and collusion (between the two incumbents) is sustainable. In region G, there is entry and collusion is sustainable (before and after entry). In the white region between regions B and G, labeled C, entry only occurs under collusion, i.e., the cartel breakdown before entry permanently deters entry. In this region, according to Result 6.1, collusion is not sustainable (before entry). We conclude, therefore, that the sustainability of collusion may depend on the long-term growth rate in a non-monotonic way. This provides a theoretical rationale for the empirical finding of Symeonidis (2003), who concluded that: *"while a moderate growth rate is more conducive to stable collusion than a stagnant or declining demand, fast growth hinders collusion."* Among other factors, Symeonidis (2003) pointed the likelihood of entry as a possible explanation for his finding: *"fast growth may lead to significant new entry and hinder the attempts of firms to coordinate on a collusive price or set of prices."*

In this scenario of temporary growth and then decline, collusion may be harder to sustain before entry or after entry. In fact, the entry cost may decisive for this comparison (whereas it is irrelevant if the growth rate is constant). A low entry cost implies that entry occurs early, when the market is growing fast. In this case, the entry delay caused by the cartel breakdown is small, implying that the ICC after entry is binding. Contrariwise, a higher entry cost implies that entry occurs later, when the market is growing slower and, therefore, the entry delay caused by the cartel breakdown is greater. In this case, it may be the ICC before entry that is binding.

7 Conclusions

We have built a general framework to study the sustainability of collusion in markets where growth may trigger the entry of a new firm. Our working assumption is that market growth is balanced, i.e., has the same proportional impact in the three market regimes (collusion, deviation and punishment). We find that the entry delay that may result from the cartel breakdown before entry is a key determinant of the sustainability of collusion. In particular, collusion is impossible to sustain if the cartel breakdown permanently deters entry and the collusive profit after entry is lower than the punishment profit before entry.

After entry, firms feel the strongest temptation to deviate from the collusive agreement when the value of the market (i.e., the discounted sum of present and future profits) grows the least or declines the most. Market growth only affects the critical discount factor for the sustainability of collusion after entry through an adjustment term that is equal to the infimal growth rate of the value of the market. Before entry, the sustainability of collusion is also impacted by the timing of entry. The slower is growth when entry occurs, the greater is the entry delay that results from a cartel breakdown before entry, and, therefore, the greater is the incentive for deviating before entry.

If firms set prices, punishment profits are null. This simultaneously implies that the cartel breakdown permanently deters entry, and that this deterrence is irrelevant (since profits are null anyway). In this scenario, if there aren't economies of scale, the possibility of entry always hinders collusion. In addition, if the market grows at a non-decreasing rate, collusion is more difficult to sustain after entry than before entry. We also conclude that, in any context in which punishment profits are null, whether collusion is harder to sustain before or after entry only depends on the whether the one-shot deviation gain increases or decreases with the number of firms.

Our work generalizes the models and conclusions of Capuano (2002) and Vasconcelos (2008), by relaxing the assumptions of: linear demand, constant rate of market growth, and constant marginal production costs. Moreover, in a scenario in which the market starts by expanding and then declines, we obtain results that are qualitatively different from those obtained by Capuano (2002) and Vasconcelos (2008). The reason is the following. After reversion to Cournot competition, entry is surely profitable if the rate of market growth is constant, but it may not be profitable if the market initially grows and then declines. In the latter case, if Cournot profits with two firms are greater than collusive profits with three firms, collusion will not be sustainable because the incumbents prefer to deviate from the collusive agreement in order to deter entry. In any case, we find that the sustainability of collusion depends crucially on the long-run trend of the market evolution. This relation may be non-monotonic: if, in the limit, the market declines sufficiently fast or sufficiently slow, collusion is facilitated by the evolution of the market; for intermediate values of the long-run trend, collusion can be impossible to sustain.

This paper therefore contributes with theoretical grounds for competition authorities and regulators to make better decisions when assessing the likelihood of joint dominance (or coordinated effects) resulting from mergers in contexts wherein market growth may trigger future entry. As our analysis highlights, subtle market characteristics may have a decisive impact on the sustainability of collusive agreements. In particular, the importance of potential competition may be magnified or diminished by the rate of market growth expected to prevail at the moment of entry, the rate of market growth in the long-run, and the shape of market growth in general.

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Appendix

Appendix A: Profits are proportional to market size parameters

Observe that, combining (1) and (2), the profit function of firm $i \in \{1, \ldots, n\}$ in period $t \in \{0, 1, \ldots\}$ can be written as:

$$\pi_{it}(q_{it}) = \left[P\left(\sum_k \frac{q_{kt}}{g_t} \right) \frac{q_{it}}{g_t} - C\left(\frac{q_{it}}{g_t} \right) \right] g_t h_t.$$

Normalizing payoff units by dividing profits by $g_t h_t$ and normalizing choice units by dividing quantities by g_t, we obtain an equivalent objective function:

$$\pi_i\left(\frac{q_{it}}{g_t}\right) \equiv \frac{\pi_{it}(q_{it})}{g_t h_t} = P\left(\sum_k \frac{q_{kt}}{g_t} \right) \frac{q_{it}}{g_t} - C\left(\frac{q_{it}}{g_t} \right).$$

A setting in which firms choose ratios $\frac{q_{it}}{g_t}$ (instead of q_{it}) with the objective of maximizing functions π_i (instead of π_{it}) has resulting payoffs that are invariant with g_t and h_t.

The corresponding profits of firms coincide with these normalized equilibrium payoffs when $g_t h_t = 1$. When $g_t h_t \neq 1$, profits are proportional to normalized payoffs: $\pi_{it} = \pi_i \, g_t h_t$.

Appendix B: Proofs of Lemmas and Propositions

Proof of Lemma 1 In market regime $j \in \{m, c\}$, the discounted value of the profits of firm 3, entering at T^j, is:

$$V_3^j(T^j) = \pi_3^{j3} \sum_{s=T^j}^{+\infty} \delta^s f(s) - K\delta^{T^j} = \delta^{T^j}\left(\pi_3^{j3} F_{T^j} - K\right).$$

Thus, it is profitable for firm 3 to enter the market at T^j if and only if $K < \pi_3^{j3} F_{T^j}$.

If entry is profitable, it occurs in the earliest period that satisfies the following condition:

$$V_3^j(T^j) \geq V_3^j(T^j+1) \;\Leftrightarrow \pi_3^{j3}\left(f_{T^j} + \delta F_{T^j+1}\right) - K \geq \delta\left(\pi_3^{j3} F_{T^j+1} - K\right)$$
$$\Leftrightarrow f_{T^j} \geq \frac{(1-\delta)\,K}{\pi_3^{j3}}.$$

□

Proof of Lemma 2 We want to show that if the ICC (7) is satisfied in period $t+1$, it is also satisfied in period t, for $t \in \{1, \ldots, T^m - 2\}$.

If condition (7) is satisfied in period $t+1$, the following variable is positive:

$$\Delta \equiv -\left(\pi^{d2} - \pi^{m2}\right) F_{t+1} + \left(\pi^{d2} - \pi^{c2}\right) \delta F_{t+2} - \left(\pi^{m2} - \pi^{m3}\right) \delta^{T^m - t - 1} F_{T^m}$$
$$+ \left(\pi^{c2} - \pi^{c3}\right) \delta^{T^c - t - 1} F_{T^c} \geq 0.$$

Condition (7) in period t can be written as:

$$-\left(\pi^{d2} - \pi^{m2}\right)\left(f_t + \delta F_{t+1}\right) + \left(\pi^{d2} - \pi^{c2}\right) \delta\left(f_{t+1} + \delta F_{t+2}\right)$$
$$-\left(\pi^{m2} - \pi^{m3}\right) \delta^{T^m - t} F_{T^m} + \left(\pi^{c2} - \pi^{c3}\right) \delta^{T^c - t} F_{T^c} \geq 0$$
$$\Leftrightarrow \delta\Delta - \left(\pi^{d2} - \pi^{m2}\right) f_t + \left(\pi^{d2} - \pi^{c2}\right) \delta f_{t+1} \geq 0.$$

When $\Delta \geq 0$, for (7) to be satisfied in period t, it is sufficient that:

$$\delta \geq \frac{\left(\pi^{d2} - \pi^{m2}\right) f_t}{\left(\pi^{d2} - \pi^{c2}\right) f_{t+1}},$$

which is true, under Assumption 1. □

Proof of Result 4.1 Using Proposition 2 and Assumption 2, collusion is sustainable after entry if and only if:

$$\beta\delta \geq \frac{\pi^{d3} - \pi^{m3}}{\pi^{d3} - \pi^{c3}}.$$

Observe the following equivalence:

$$\frac{\pi^{d3} - \pi^{m3}}{\pi^{d3} - \pi^{c3}} \geq \frac{\pi^{d2} - \pi^{m2}}{\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c3}}$$
$$\Leftrightarrow 1 - \frac{\pi^{m3} - \pi^{c3}}{\pi^{d3} - \pi^{c3}}$$

$$\begin{aligned}
&\geq 1 - \frac{\pi^{m3} - \pi^{c3}}{\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c3}} \\
&\Leftrightarrow \frac{\pi^{m3} - \pi^{c3}}{\pi^{d3} - \pi^{c3}} \leq \frac{\pi^{m3} - \pi^{c3}}{\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c3}} \\
&\Leftrightarrow \pi^{d3} - \pi^{c3} \geq \pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c3} \\
&\Leftrightarrow \pi^{d3} - \pi^{m3} \geq \pi^{d2} - \pi^{m2}.
\end{aligned}$$

If $\pi^{d3} - \pi^{m3} \geq \pi^{d2} - \pi^{m2}$, sustainability of collusion after entry implies that:

$$\beta\delta \geq \frac{\pi^{d2} - \pi^{m2}}{\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c3}},$$

which, under Assumption 2, implies condition (9). □

Proof of Proposition 5 If firm 3 does not enter the market if the incumbents deviate from the collusive agreement before entry, the ICC for collusion to be sustainable before entry, given by (8), becomes:

$$\begin{aligned}
&-\left(\pi^{d2} - \pi^{m2}\right)\left(f_{T_m-1} + \delta F_{T^m}\right) + \left(\pi^{d2} - \pi^{c2} - \pi^{m2} + \pi^{m3}\right)\delta F_{T^m} \geq 0 \\
&\Leftrightarrow -\left(\pi^{c2} - \pi^{m3}\right)\delta F_{T^m} \geq \left(\pi^{d2} - \pi^{m2}\right) f_{T^m-1},
\end{aligned}$$

which cannot be satisfied if $\pi^{c2} \geq \pi^{m3}$. For $\pi^{c2} < \pi^{m3}$, the ICC is equivalent to:

$$\delta F_{T^m} \geq \frac{\pi^{d2} - \pi^{m2}}{\pi^{m3} - \pi^{c2}}\left(F_{T^m-1} - \delta F_{T^m}\right) \Leftrightarrow \delta \geq \frac{\left(\pi^{d2} - \pi^{m2}\right) F_{T^m-1}}{\left(\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c2}\right) F_{T^m}}.$$

□

Proof of Result 4.2 Under Assumption 2, collusion is sustainable after entry if and only if (Proposition 2):

$$\beta\delta \geq \frac{\pi^{d3} - \pi^{m3}}{\pi^{d3} - \pi^{c3}}.$$

Observe the following equivalence (which holds if $\pi^{m3} \geq \pi^{c2}$):

$$\begin{aligned}
&\frac{\pi^{d3} - \pi^{m3}}{\pi^{d3} - \pi^{c3}} \geq \frac{\pi^{d2} - \pi^{m2}}{\pi^{d2} - \pi^{m2} + \pi^{m3} - \pi^{c2}} \\
&\quad\Leftrightarrow \pi^{d3}\pi^{m3} - \pi^{d3}\pi^{c2} - \pi^{m3}\pi^{d2} + \pi^{m3}\pi^{m2} - \pi^{m3}\pi^{m3} + \pi^{m3}\pi^{c2} \\
&\qquad \geq -\pi^{c3}\pi^{d2} + \pi^{c3}\pi^{m2} \\
&\quad\Leftrightarrow (\pi^{d3} - \pi^{m3})(\pi^{m3} - \pi^{c2}) \geq (\pi^{m3} - \pi^{c3})(\pi^{d2} - \pi^{m2}) \\
&\quad\Leftrightarrow \frac{\pi^{d2} - \pi^{m2}}{\pi^{d3} - \pi^{m3}} \leq \frac{\pi^{m3} - \pi^{c2}}{\pi^{m3} - \pi^{c3}}.
\end{aligned}$$

Thus, if $\frac{\pi^{d2}-\pi^{m2}}{\pi^{d3}-\pi^{m3}} \leq \frac{\pi^{m3}-\pi^{c2}}{\pi^{m3}-\pi^{c3}}$, the sustainability of collusion after entry implies that:

$$\beta\delta \geq \frac{\pi^{d2}-\pi^{m2}}{\pi^{d2}-\pi^{m2}+\pi^{m3}-\pi^{c2}}.$$

Under Assumption 2, this implies sustainability of collusion before entry (Proposition 5). □

Proof of Result 5.2 If firms set prices and there is no entry, the incumbents abide by the collusive agreement in period t if and only if:

$$\sum_{s=t}^{+\infty} \delta^{s-t}\pi_{is}^{m2} \geq \pi_{it}^{m1}. \tag{19}$$

If there is entry, collusion is sustainable in any period after entry, $t \geq T^m$, if and only if:

$$\sum_{s=t}^{+\infty} \delta^{s-t}\pi_{is}^{m3} \geq \pi_{it}^{m1},$$

which is more restrictive than (19).

At any period $t < T^m$, the ICC before entry is:

$$\sum_{s=t}^{T^m-1} \delta^{s-t}\pi_{is}^{m2} + \sum_{s=T^m}^{+\infty} \delta^{s-t}\pi_{is}^{m3} \geq \pi_{it}^{m1},$$

which is also more restrictive than (19). □

Proof of Result 5.3 Under Assumption 2, we have $\frac{F_{T^m}}{F_{T^m-1}} \geq \beta$. Thus, if collusion is sustainable after entry, $\beta\delta \geq 1 - \frac{\pi^{m3}}{\pi^{m1}}$, we obtain:

$$\frac{F_{T^m}}{F_{T^m-1}}\delta \geq 1 - \frac{\pi^{m3}}{\pi^{m1}} \geq 1 - \frac{\pi^{m3}}{\pi^{m1}-\pi^{m2}+\pi^{m3}} = \frac{\pi^{m1}-\pi^{m2}}{\pi^{m1}-\pi^{m2}+\pi^{m3}},$$

which coincides with condition (14). □

Appendix C: Linear Cournot markets

Collusive profits

Suppose that, in period t, the $n \in \{2, 3\}$ active firms maximize their joint profit:

$$\pi_t^{mn}(Q_t) = \left(1 - \frac{Q_t}{f_t}\right) Q_t.$$

Using the first-order condition for profit-maximization, and assuming that firms divide the industry profit in equal parts, we obtain the collusive output and profit of firm i:

$$q_{it}^{mn} = \frac{f_t}{2n} \quad \text{and} \quad \pi_{it}^{mn} = \frac{f_t}{4n}.$$

Deviation profits

Suppose that firm i deviates from the collusive agreement in period t and there are $n \in \{2, 3\}$ firms in the market. This firm produces the quantity that maximizes its individual profit, assuming that the rival firms produce the collusive quantities:

$$\pi_{it}^{dn}(q_{it}) = \left(1 - \frac{n-1}{2n} - \frac{q_{it}}{f_t}\right) q_{it}.$$

Solving the corresponding first-order condition, we obtain:

$$q_{it}^{dn} = \frac{n+1}{4n} f_t \quad \text{and} \quad \pi_{it}^{dn} = \left(\frac{n+1}{4n}\right)^2 f_t.$$

Punishment profits

When the $n \in \{2, 3\}$ active firms compete *à la* Cournot, each firm i produces the quantity that maximizes its individual profit:

$$\pi_{it}^{cn}(q_{it}) = \left(1 - \frac{Q_{-it}}{f_t} - \frac{q_{it}}{f_t}\right) q_{it},$$

where Q_{-it} denotes the quantity produced by the rivals of firm i. Solving the first-order condition for profit-maximization, we obtain the output and profit of each firm:

$$q_{it}^{cn} = \frac{f_t}{n+1} \quad \text{and} \quad \pi_{it}^{cn} = \frac{f_t}{(n+1)^2}.$$

Timing of entry when the rate of market growth is constant

Let $f_t = \beta^t$, with $1 < \beta < \delta^{-1}$. Then, $F_t = \frac{\beta^t}{1-\beta\delta}$, which is strictly increasing in t, and $\lim_{t \to \infty} F_t = +\infty$. Hence, firm 3 always enters the market (even under competition). The optimal entry periods under competition and collusion are, respectively:

$$T^c = int\left\{\frac{\ln[16(1-\delta)K]}{\ln\beta}\right\} + 1 \quad \text{and} \quad T^m = int\left\{\frac{\ln[12(1-\delta)K]}{\ln\beta}\right\} + 1,$$

where $int\,\{x\}$ denotes the integer part of x. The entry delay that results from breaking the cartel (before entry) can be:[39]

$$T^c - T^m = int\left[\frac{\ln(4/3)}{\ln\beta}\right] \quad \text{or} \quad T^c - T^m = int\left[\frac{\ln(4/3)}{\ln\beta}\right] + 1. \tag{20}$$

Proof of Result 6.3 Replacing the critical (adjusted) discount factor for collusion to be sustainable after entry ($\beta\delta = \frac{4}{7}$) in the ICC for collusion sustainability before entry, (16), we obtain:

$$T^c - T^m \geq \frac{\ln(13/28)}{\ln(4/7)} - 1 \Leftrightarrow T^c - T^m \geq 1.$$

Using the expression for the lowest possible entry delay, given in (20), it is clear that $T^c - T^m \geq 1$ is implied by $\beta \leq \frac{4}{3}$. □

The following result is instrumental for the proof of Result 6.4.

Result *Let $\beta\delta = \frac{4}{7}$. If collusion before entry is not sustainable for given T^m and T^c, with $T^c > T^m$, it is also not sustainable if T^m and T^c increase by the same amount.*

Proof The left-hand side of (18) is lower with $T^m + 1$ and $T^c + 1$ than with T^m and T^c if and only if:

$$\begin{aligned} & -9\left(-\alpha^{-T^m} + \alpha^{1-T^m}\right)(1-\beta\delta)(\alpha-\beta\delta) - 16\,\beta\delta\left(-\alpha^{-T^m} + \alpha^{1-T^m}\right)(1-\beta\delta) \\ & \quad + 28\,(\beta\delta)^{T^c-T^m+1}\left(-\alpha^{-T^c} + \alpha^{1-T^c}\right)(1-\beta\delta) \leq 0 \\ & \quad \Leftrightarrow -9 - 7\left(\frac{\beta\delta}{\alpha}\right) + 28\left(\frac{\beta\delta}{\alpha}\right)^{T^c-T^m+1} \leq 0. \end{aligned} \tag{21}$$

The worst case for (21) to hold is when $T^c - T^m = 1$. In that case, it becomes:

$$-9 - 7\left(\frac{\beta\delta}{\alpha}\right) + 28\left(\frac{\beta\delta}{\alpha}\right)^2 \leq 0,$$

which is satisfied for $\frac{\beta\delta}{\alpha} \leq 0.7056$ (approximately). Since $\alpha > 1$, it holds for $\beta\delta = \frac{4}{7}$. □

[39] To obtain the two possible values for the entry delay, note that $\frac{\ln[16(1-\delta)K]}{\ln\beta} - \frac{\ln[12(1-\delta)K]}{\ln\beta} = \frac{\ln(4/3)}{\ln\beta}$. This ratio corresponds to the number of periods that are necessary for the market to grow from the threshold for entry under collusion, $f_t = 12(1-\delta)K$, to that for entry under competition, $f_t = 16(1-\delta)K$.

Profitability of entry when $f_t = (1-\alpha^{-t})\beta^t$

Recall that entry is profitable if and only if condition (3) is satisfied. To check whether it is satisfied for $\beta < 1$, we need to obtain $\sup_t\{F_t\}$. Since $F_t = \beta^t \frac{\alpha-\beta\delta-\alpha^{1-t}(1-\beta\delta)}{(1-\beta\delta)(\alpha-\beta\delta)}$:

$$\begin{aligned} F_t < F_{t+1} &\Leftrightarrow \frac{\alpha-\beta\delta-\alpha^{1-t}(1-\beta\delta)}{\beta\left[\alpha-\beta\delta-\alpha^{-t}(1-\beta\delta)\right]} < 1 \\ &\Leftrightarrow (\alpha-\beta\delta)(1-\beta)-(1-\beta\delta)(\alpha-\beta)\alpha^{-t} < 0 \\ &\Leftrightarrow t < \frac{1}{ln\alpha} ln\left[\frac{(1-\beta\delta)(\alpha-\beta)}{(\alpha-\beta\delta)(1-\beta)}\right]. \end{aligned}$$

Thus, there is entry under market regime $j \in \{m, c\}$ if:

$$K < \pi_{j3} F_{\hat{t}}, \quad \text{where } \hat{t} = int\left\{\frac{1}{ln\alpha} ln\left[\frac{(1-\beta\delta)(\alpha-\beta)}{(\alpha-\beta\delta)(1-\beta)}\right]\right\} + 1. \tag{22}$$

Proof of Result 6.4 The strategy of the proof is to replace the critical discount factor for collusion to be sustainable after entry, $\beta\delta = \frac{4}{7}$, in the ICC (18) for collusion sustainability before entry, and check whether it is satisfied for: (i) $T^c - T^m = 1$; (ii) $T^c - T^m = 2$; (iii) $T^c - T^m = 3$.

Replacing $\beta\delta = \frac{4}{7}$ in the ICC (18), we obtain:

$$\begin{aligned} &-27\left(1-\alpha^{1-T^m}\right)\left(\alpha-\frac{4}{7}\right)-64\left(\alpha-\frac{4}{7}-\frac{3}{7}\alpha^{1-T^m}\right) \\ &+112\left(\frac{4}{7}\right)^{T^c-T^m}\left(\alpha-\frac{4}{7}-\frac{3}{7}\alpha^{1-T^c}\right) \geq 0. \end{aligned} \tag{23}$$

(i) $[\mathbf{T^c - T^m = 1}]$ Replacing $T^c = T^m + 1$ in (23), we obtain:

$$\begin{aligned} &-63\left(1-\alpha^{1-T^m}\right)\left(\alpha-\frac{4}{7}\right)+64\left(\alpha^{1-T^m}-\alpha^{-T^m}\right) \geq 0 \\ &\Leftrightarrow -63\alpha^{T^m+1}+36\alpha^{T^m}+63\alpha^2+28\alpha-64 \geq 0. \end{aligned} \tag{24}$$

The derivative of the left-hand side of (24) with respect to T^m is:

$$-63\ln(\alpha)\alpha^{T^m+1}+36\ln(\alpha)\alpha^{T^m},$$

which is always negative because $\alpha > 1$. Hence, the most favorable case for the ICC (24) to be satisfied is when $T^m = 1$. Replacing $T^m = 1$, we obtain: $64\alpha - 64 \geq 0$, which is true.

The second most favorable case is $T^m = 2$. Replacing $T^m = 2$, we obtain:

$$-63\alpha^3+99\alpha^2+28\alpha-64 \geq 0,$$

which is satisfied for $\alpha \leq \frac{4}{3}$.

Replacing $T^m = 3$, we obtain:

$$-63\alpha^4 + 36\alpha^3 + 63\alpha^2 + 28\alpha - 64 \geq 0,$$

which is satisfied for $\alpha \leq 1.046$ (approximately).
Replacing $T^m = 4$, and dividing by $\alpha - 1$, we obtain:

$$-63\alpha^4 - 27\alpha^3 - 27\alpha^2 + 36\alpha + 64 \geq 0,$$

which is never satisfied. It is not satisfied, therefore, for any $T^m \geq 4$.

(ii) $[\mathbf{T^c - T^m = 2}]$ Replacing $T^c - T^m = 2$ in (23) and expanding, we get:

$$-889\alpha^{2+T^m} + 508\alpha^{1+T^m} + 441\alpha^3 + 196\alpha^2 - 256 \geq 0,$$

whose left-hand side is decreasing in T^m. The most favorable case is when $T^m = 1$:

$$-448\alpha^3 + 704\alpha^2 - 256 \geq 0,$$

which is satisfied for $\alpha \leq 1.094$ (approximately).
When $T^m = 2$, the ICC becomes:

$$\begin{aligned} &- 889\alpha^4 + 949\alpha^3 + 196\alpha^2 - 256 \geq 0 \\ &\Leftrightarrow (\alpha - 1)(-889\alpha^3 + 60\alpha^2 + 256\alpha + 256) \geq 0, \end{aligned}$$

which is always false. Therefore, the ICC is not satisfied for any $T^m \geq 2$.

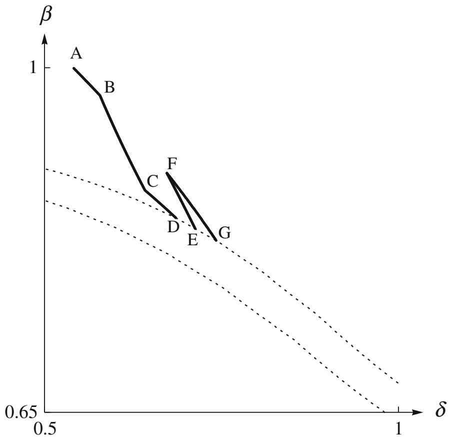
(iii) $[\mathbf{T^c - T^m = 3}]$ Replacing $T^m = 1$ and $T^c = 4$ in (23), we obtain:

$$\begin{aligned} &-3136\,(\alpha - 1) + 1024\left(\alpha - \frac{4}{7} - \frac{3}{7}\alpha^{-3}\right) \geq 0 \\ &\Leftrightarrow -14784\alpha + 17856 - 3072\alpha^{-3} \geq 0, \end{aligned}$$

which holds (in equality) for $\alpha = 1$. The derivative of the left-hand side with respect to α is:

$$-14784 + 9216\alpha^{-4},$$

which is always negative.
As $\alpha > 1$, the ICC is never satisfied if $T^c - T^m = 3$. Finally, using Lemma 3, we conclude that the ICC is never satisfied if $T^c - T^m \geq 3$. □



[AB] T^c jumps from 3 (to the right of the line segment) to 4 (to the left).

[BC] ICC before entry with $T^c = 4$ and $T^m = 3$ (satisfied in equality).

[CD] T^c jumps from 4 (below) to 5 (above).

[EF] T^m jumps from 2 (below) to 3 (above).

[FG] T^c jumps from 3 (to the right of the line segment) to 4 (to the left).

Fig. 6 ICC for collusion sustainability before entry (*solid line*) and conditions for entry profitability under collusion and competition (*dashed lines*), with $\alpha = 1.01$ and $K = 0.004$

ICC before entry when $f_t = (1 - \alpha^{-t})\beta^t$

In Fig. 6, the dashed lines correspond to the conditions for entry to be profitable (under collusion and under competition), given by (22). The solid line corresponds to the ICC for collusion to be sustainable before entry, given by (18).

The erratic shape of the ICC before entry is due to the discrete nature of time. Small changes in parameters can make T^c or T^m jump (by 1 period), leading to kinks in the ICC.

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