Technology licensing under optimal tax policy

Arijit Mukherjee · Yingyi Tsai

Received: 16 March 2011 / Accepted: 9 March 2012 / Published online: 19 April 2012 © Springer-Verlag 2012

Abstract This paper examines the role of government policy in technology licensing decision. We show that both the outside and the inside innovators license a new product (or drastic process innovation) to all potential licensees in the presence of tax/subsidy policies. An implication of our analysis is that a monopolist producer may prefer technology licensing in a homogeneous goods industry. Our results also provide a rationale for franchising to multiple sellers.

Keywords Licensing · Tax · Knowledge diffusion

JEL Classification L13 · L24 · L40 · H25 · D43

1 Introduction

The seminal works by Kamien and Tauman [\(1984,](#page-15-0) [1986](#page-15-1)) show that "outside innovators"1 prefer fixed-fee licensing and auction to royalty licensing, regardless of the industry size and/or magnitude of the innovation. Nevertheless, the wide prevalence of output royalty in the licensing contracts (see, e.g., [Rostoker 1984\)](#page-15-2) remains a puzzle,

A. Mukherjee University of York, Heslington, York, UK

A. Mukherjee CESifo, Munich, Germany

Y. Tsai (\boxtimes) Department of Applied Economics, National University of Kaohsiung, Kaohsiung 811, Taiwan e-mail: yytsai@nuk.edu.tw

¹ Outside (inside) innovator refers to the situation were the innovator is not (is) a product-market competitor of the licensees.

and has drawn significant interest in analysing the implications of technology licens-ing.^{[2](#page-1-0)} Recent work of Sen and Tauman [\(2007](#page-16-0), p. 164) points out that the technology licensing literature "has been restrictive by not allowing the innovator to realise the full potential of the innovation: either the licensing policies are confined to pure up-front fee or pure royalty, or the number of firms is considered to be very large (perfect competition) or very small (monopoly or duopoly), thus limiting the extent of strategic interaction."

Using a general licensing contract with non-negative fixed-fee and output royalty,^{[3](#page-1-1)} [Sen and Tauman](#page-16-0) [\(2007\)](#page-16-0) explore the implications of licensing for both the outside and the inside innovator.^{[4](#page-1-2)} They show, for both the outside and the inside innovators, that full knowledge diffusion generally occurs for any non-drastic innovation⁵ in which the opportunity costs of the licensees are positive; yet full knowledge diffusion does not occur under drastic innovations where the opportunity costs of the licensees are zero, and that the outside innovator auctions only one license $⁶$ and the inside innovator</sup> [does](#page-16-0) [not](#page-16-0) [license.](#page-16-0)

Sen and Tauman [\(2007](#page-16-0)) have provided important insights to the subject; nonetheless, similar to other works on technology licensing, their contribution is restrictive by not considering the role of government policies. In fact, government, in imperfectly competitive product markets, can use tax/subsidy policies to improve welfare by reducing the distortion due to product market imperfection [\(Myles 1996;](#page-15-3) [Hamilton](#page-15-4) [1999\)](#page-15-4). Hence, the study of government policies and its implications for technology licensing would enhance our understanding of optimal licensing contracts.

In this paper, we consider licensing of a new product by both the outside and the inside innovators with the presence of tax/subsidy policies. Licensing of a new product (or drastic process innovation) is used here as a simplest way to capture the zero opportunity costs facing the licensees. In contrast to [Sen and Tauman](#page-16-0) [\(2007](#page-16-0)),

² The literature of technology licensing has been well-developed with rich insights. [Kamien](#page-15-5) [\(1992\)](#page-15-5) provides a survey of the earlier literature on technology licensing. Some of the issues considered in the technology licensing literature are the implications of informational asymmetry [\(Gallini and Wright 1990;](#page-15-6) [Beggs 1992;](#page-14-0) [Macho-Stadler et al. 1996](#page-15-7);[Choi 2001](#page-14-1); [Schmitz 2002](#page-15-8); [Sen 2005b\)](#page-16-1), bargaining [\(Katz and Shapiro](#page-15-9) [1987;](#page-15-9) [Sempere Monerris and Vannetelbosch 2001](#page-16-2)), relicensing [\(Muto 1987](#page-15-10)), quality of licensed technology [\(Rockett 1990](#page-15-11)), product differentiation [\(Muto 1993;](#page-15-12) [Wang and Yang 1999;](#page-16-3) [Mukherjee and Balasubramanian](#page-15-13) [2001;](#page-15-13) [Caballero-Sanz et al. 2002;](#page-14-2) [Faulí-Oller and Sandonís 2002](#page-14-3); [Poddar and Sinha 2004](#page-15-14)), risk aversion [\(Bousquet et al. 1998\)](#page-14-4), the decision of incumbent innovators [\(Shapiro 1985;](#page-16-4) [Marjit 1990;](#page-15-15) [Wang 1998](#page-16-5), [2002;](#page-16-6) [Kamien and Tauman 2002;](#page-15-16) [Sen 2002](#page-16-7)), leadership structure [\(Kabiraj 2004,](#page-15-17) [2005](#page-15-18); [Filippini 2005](#page-14-5)), innovation [\(Gallini and Winter 1985](#page-14-6); [Mukherjee 2005\)](#page-15-19), strategic managerial delegation [\(Mukherjee 2001;](#page-15-20) [Saracho 2002](#page-15-21)), trade costs [\(Kabiraj and Marjit 2003](#page-15-22); [Mukherjee and Pennings 2006](#page-15-23); [Mukherjee 2007](#page-15-24)), integer constraint [\(Sen 2005a\)](#page-16-8), the role of input market [\(Mukherjee 2010](#page-15-25)), the implications of returns to scale [\(Sen and Stamatopoulos 2009b](#page-16-9); [Mukherjee 2011\)](#page-15-26) and new product invention [\(Kamien et al. 1988](#page-15-27)).

³ See the references in [Sen and Tauman](#page-16-0) [\(2007\)](#page-16-0) for the empirical evidences on the licensing contracts with fixed-fee and royalty.

⁴ In duopoly markets with inside innovators, two-part tariff licensing contracts with fixed-fee and royalty have been analysed in [Rockett](#page-15-11) [\(1990](#page-15-11)), [Mukherjee and Balasubramanian](#page-15-13) [\(2001](#page-15-13)), [Faulí-Oller and Sandonís](#page-14-3) [\(2002](#page-14-3)), [Mukherjee](#page-15-24) [\(2007](#page-15-24)), [Fosfuri and Roca](#page-14-7) [\(2004](#page-14-7)) and [Poddar and Sinha](#page-15-28) [\(2010](#page-15-28)).

⁵ See [Arrow](#page-14-8) [\(1962](#page-14-8)) for discussions on drastic and non-drastic innovations.

⁶ [Sen and Stamatopoulos](#page-16-10) [\(2009a](#page-16-10)) show that an outside innovator earns the same profit from selling one license or multiple licenses of a drastic innovation. However, a cost of licensing the technology will make selling one license as the innovator's optimal policy.

we show that full knowledge diffusion can occur even if the opportunity costs of the licensees are zero, and that such diffusion happens for both the outside and the inside innovators. Our results also show that the outside innovator's incentive for innovation is greater than that of the inside innovator. An implication of our analysis suggests that a monopolist final goods producer has the incentive for technology licensing in a homogenous goods industry in the presence of tax/subsidy policies even though such licensing creates competition in the product market.

Our paper is closely related to recent literature studying the effects of government policies on technology licensing [\(Kabiraj and Marjit 2003](#page-15-22); [Mukherjee and Pennings](#page-15-23) 2006 .^{[7](#page-2-0)} Nonetheless, this paper differs from these earlier contributions in some distinct ways. First, previous papers study duopoly markets with one licenser and one licensee and, thus, ignore the issue of optimal number of licenses that we discuss here. Second, those papers examine technology licensing in the context of open economy, where, except for the case of licensing to a foreign licensee, the foreign licenser alone faces tariff or import tax. In contrast, both the licenser and the licensee(s) face tax/subsidy in the present analysis. Finally, those papers do not take into account the case of outside innovator, and we study both situations of the inside and the outside innovator.

An important alternative interpretation of our analysis deserves attention. If we interpret the royalty rate as whole sale price and the fixed fee as franchise fee, our analysis characterises the scenario in which a manufacturer (which may or may not be a retailer) sells its products through several retailers. Accepting this interpretation, our analysis of licensing by an outside innovator can be related to the strategies of food and drink suppliers such as Coca-Cola, Heinz and Walkers, which sell their products through several retail outlets, while licensing by an inside innovator can be related to the strategy by Apple, which invents iPod, iPhone and iPad, and sells through both its own retail and the outlets of the competing retailers. Similarly, Samsung sells its Galaxy Tab not only from its own retail outlet but also from the outlets of the competing retailers. Hence, our analysis has a broader appeal than a mere investigation of technology licensing.

Before proceeding to the formal analysis, it is worth explaining the rationale for investigating licensing of a new product (or drastic process innovation) where the opportunity costs of the licensees are zero. First, a motivation of the present study originates from [Sen and Tauman](#page-16-0) [\(2007\)](#page-16-0), who show that technology licensing is an effective way of diffusing knowledge under non-drastic innovation with positive opportunity costs of the licensees. This has an interesting implication. It is generally believed that, while patent protection increases the incentive for innovation by reducing knowledge spillover, it prevents the non-innovators to benefit from the invented technology of the innovator. In other words, patent protection tends to create a negative effect on the society by increasing technological difference between the producers, thus reducing the extent of product-market competition. The result of [Sen and Tauman](#page-16-0) [\(2007](#page-16-0)) suggests that technology licensing reduces this negative effect of patent protection under

⁷ Other reasons for licensing by a monopolist producer is the input market imperfection [\(Mukherjee et al.](#page-15-29) [2008\)](#page-15-29), product differentiation [\(Wang and Yang 1999](#page-16-3); [Mukherjee and Balasubramanian 2001](#page-15-13); [Wang 2002;](#page-16-6) [Faulí-Oller and Sandonís 2002\)](#page-14-3) and network externalities [\(Economides 1993\)](#page-14-9).

non-drastic innovation. We show that technology licensing creates similar benefit even under drastic innovation in the presence of tax/subsidy policies.

Second, a closer observation into the reality of several new inventions such as steam engine, microprocessor, compact disc and laser printer would suggest that the assumption of zero opportunity costs facing by the licensees do not seem too unrealistic [\(Greenhalgh and Rogers 2010,](#page-15-30) p. 9).

The remainder of the paper is organised as follows. Section [2](#page-3-0) describes the model. Section [3](#page-4-0) provides the results on optimal licensing contracts for both the outside and the inside innovators. Section [4](#page-12-0) compares an outside innovator's incentive for innovation to that of an inside innovator. Section [5](#page-14-10) concludes.

2 The model

Consider an environment with an innovator, denoted by *I*, who has invented a new product. There are $n \geq 1$ symmetric potential licensees, each can produce the product if it obtains license from *I*. To capture zero opportunity costs of the licensees in the simplest way, we assume that the potential licensees have no existing technologies for the product invented by *I*. Similar to [Sen and Tauman](#page-16-0) [\(2007\)](#page-16-0), we assume that *I* licenses its technology to the potential licensees through "auction plus royalty" where *I* determines the number of licenses to auction (possibly with a minimum bid) and also announces the royalty rate so that the up-front fixed-fee that a licensee pays is its winning bid. 8 We assume that licensing is costless.

We consider two scenarios in the following analysis. First, we consider, in Sect. [3.1,](#page-4-1) the case of an outside innovator, where I is not a producer of the product. We then consider, in Sect. [3.2,](#page-8-0) the case of an inside innovator, where *I* is a producer.

We assume the *i*th licensee, $i = 1, 2, \ldots, n$, can produce the commodity at a marginal cost *c* provided that it is granted a license. Otherwise, a licensee cannot produce the product. All producers pay for their outputs a per-unit sales or output tax, *t* (subsidy, if *t* is negative).

An alternative interpretation of the tax rate considered in the present model can be provided as follows. Consider that production requires an input and if the producers purchase the input from the competitive world market at a price *c*, the tax rate considered here can be termed as tariff imposed on the per-unit imported input.

The outputs of the licensees are perfect substitutes, and the inverse market demand function for the product is

$$
P = a - Q,\tag{1}
$$

where *P* is price and *Q* is the total output. For simplicity, we normalise $(a - c)$ to 1 in the subsequent analysis.

We study the following game. At stage 1, innovator *I* announces *k* number of licenses to auction through a sealed bid English auction, where $1 \leq k \leq n$. At stage 2,

⁸ Intuitively, auction creates more competition than does the up-front fixed-fee scheme set by the innovator. In fact, [Katz and Shapiro](#page-15-31) [\(1985](#page-15-31)) showed the superiority of auction over up-front fixed-fee without taking into account of royalty. However, the same intuition holds in the presence of royalty.

innovator *I* determines the output royalty, r , that a licensee needs to pay if it purchases the technology. At stage 3, the licensees simultaneously and independently decide whether or not to purchase a license, and how much to bid under auction. The highest bidder obtains the license. The ties are resolved by *I*. [9](#page-4-2) At stage 4, the government sets the welfare maximising per-unit sales or output tax. At stage 5, the potential licensees, which purchase the technology, choose their outputs simultaneously, and the profits are realised. If only one potential licensee obtains the license, he produces like a monopolist, and the profit is realised. We solve the game through backward induction.

It is worth pointing out that if *I* auctions *n* licenses (which will actually be the case in the following analysis), all licensees are assured of the technology. It follows that each licensee will bid at the lowest price for the license, thus, reducing the effectiveness of bidding in extracting profits from the licensees. Hence, an auction for licensing can help the innovator in extracting entire profits of the licensees *only if* the innovator specifies a minimum bid that is required for obtaining the licensed technology [\(Kamien et al. 1992\)](#page-15-32).

Furthermore, it should be noted that we consider a situation in which the government cannot [commit](#page-15-23) [to](#page-15-23) [the](#page-15-23) [tax](#page-15-23) [policy](#page-15-23) [before](#page-15-23) [licensing.](#page-15-23) [This](#page-15-23) [is](#page-15-23) [in](#page-15-23) [line](#page-15-23) [with](#page-15-23) Mukherjee and Pennings [\(2006](#page-15-23)), where the government policy is announced after technology licensing, and can be motivated by the observation that government policies are often "time inconsistent", implying that governments have an incentive to reverse their preannounced policies [\(Staiger and Tabellini 1987](#page-16-11); [Neary and Leahy 2000\)](#page-15-33).

3 Analysis

3.1 The case of an outside innovator

We first solve for the Cournot–Nash equilibrium outputs. If *I* auctions *k* licenses, where $1 \leq k \leq n$, and charges the per-unit output royalty *r*, the *i*th licensee, $i = 1, 2, \ldots, k$, chooses its output to maximise

$$
\max_{q_i} (1 - Q - r - t)q_i - F_i,\tag{2}
$$

where F_i is the equilibrium bid by the *i*th licensee. It is straightforward to verify that the equilibrium output of the *i*th licensee is $q_i^* = \frac{1-r-t}{k+1}$, where $i = 1, 2, ..., k$ and $\sum_{i=1}^{k} q_i^* = Q^*$. The equilibrium outputs of the licensees are positive for any $r + t < 1$, which is assumed to hold.

⁹ If a licensee obtains the technology, it has to pay the royalty and the amount of its bid. If the licensees were not required to bid for the technology, their net profits (i.e., the profits after paying the royalty) would be positive as long as $r < (1 - t)$ in our analysis. Hence, the requirement for bidding by the licensees for the technology at stage 3 allows the innovator to extract more profits. It is worth noting that, at stage 1, the innovator announces the number of licenses it will auction, but the bidding game occurs at stage 3. Of course, the licensees could refrain from purchasing the technology. However, we assume that the licensees will bid and purchase the technology as long as they are not worse off by purchasing the technology as compared to not purchasing the technology.

Given the equilibrium outputs of the licensees, the symmetric equilibrium profit of each licensee is $\pi_1^* = \pi_2^* = \cdots = \pi_k^* = \frac{(1 - r - t)^2}{(k + 1)^2}$, which suggests the amount F_i of each licensee's maximum willingness to bid. It follows that if I auctions k licenses, where $k < n$, each licensee bids $\frac{(1-r-t)^2}{(k+1)^2}$ and the outside innovator is able to extract the entire profits of any licensee. However, if *I* auctions *k* licenses, where $k = n$, each licensee is assured of the technology, and thus, bids F_i as little as possible. Under such circumstance, the bidding for licenses helps the innovator to extract the entire profits of the licensees *only if* the innovator specifies the minimum bid of $\frac{(1-r-t)^2}{(n+1)^2}$ for acquiring the technology.

We now look into the decision of the government. Social welfare consists of consumer surplus, profits of the innovator and the licensees, and tax revenue. Hence, the government's problem is choosing a tax rate *t* to maximise

$$
W = \underbrace{\frac{1}{2} \left(\sum_{i=1}^{k} q_i \right)^2}_{CS} + \underbrace{\sum_{i=1}^{k} \left[\left(1 - \sum_{i=1}^{k} q_i \right) - t - r \right] q_i + r \sum_{i=1}^{k} q_i}_{profit} + \underbrace{\delta t \sum_{i=1}^{k} q_i}_{tax\text{ revenue}}
$$
\n
$$
= \frac{1}{2} k^2 \left(\frac{1 - t - r}{k + 1} \right)^2 + k \left(\frac{1 - t + kr}{k + 1} \right) \left(\frac{1 - t - r}{k + 1} \right)
$$
\n
$$
+ \delta t k \left(\frac{1 - t - r}{k + 1} \right),
$$
\n(3)

where δ measures the importance to the government of tax revenue, relative to consumer surplus and firm profit. A higher value of δ suggests that the government concerns more about tax revenue than it does to consumer welfare and firm profit. The weight parameter $\delta \geq 1$ accounts for this Leviathan motive [\(Brennan and Buchanan](#page-14-11) [1980;](#page-14-11) [Mueller 1989](#page-15-34)) and the limiting case $\delta = 1$ corresponds to a benevolent government. Intuitively, even though tax revenue will be reimbursed to consumers and firms, there are situations in which government (or politicians) may appreciate tax revenue more than consumer welfare and/or profit, since higher tax revenue increases the discretion of the government (or politicians) about what and whom to subsidize.

Furthermore, if we interpret δ as the shadow cost of public funding for research in advanced technology, e.g., environmental protection, the Leviathan motive suggests that the government's tax revenue generating from the industry is more valuable to the society than the surplus generated to the producers and the consumers, *inter alia*, when such revenue is used to finance public spending.

Solving for Eq. [\(3\)](#page-5-0), we can establish the equilibrium tax rate as

$$
t^* = \frac{[(\delta - 1)(k+1) - 1] - r[(\delta - 1)(k+1) + k]}{2(\delta - 1)(k+1) + k}.
$$
 (4)

It follows from [\(4\)](#page-5-1) that, if $\delta = 1$, the equilibrium tax rate is $t^* = \frac{-(1+rk)}{k} < 0$. This result suggests that if the government weighs tax revenue equally important to that of consumer surplus and firm's profit, it should subsidise the licensees to eliminate the imperfection created by oligopolistic competition.

We are now in position to determine the royalty rate charged by the innovator. While choosing the royalty rate, the innovator internalises the effect of royalty on the tax rate and the outputs of the licensees. The innovator maximises the following expression to determine *r*:

$$
\Pi_I = \sum_{i=1}^k (F_i + r q_i) = \frac{k[\delta - r(\delta - 1)]^2}{[2(\delta - 1)(k + 1) + k]^2} + \frac{rk[\delta - r(\delta - 1)]}{2(\delta - 1)(k + 1) + k}.
$$
 (5)

Solving [\(5\)](#page-6-0), the equilibrium royalty is found as

$$
r^* = \frac{\delta k (2\delta - 1)}{2(\delta - 1)[(\delta - 1)(k + 1) + \delta k]},
$$
\n(6)

which is less than $(1 - t)$, ensuring that the equilibrium outputs of the licensees are positive.

Given the equilibrium royalty, when *I* auctions *k* licenses, we establish the outside innovator's equilibrium profit as $\pi_I^* = \frac{\delta^2 k}{4(\delta - 1)[(\delta - 1)(k+1) + \delta k]}$. Clearly, the outside innovator's profit rises with the number of licenses, that is, $\frac{\partial \pi_i^*}{\partial k} = \frac{\delta^2}{4[(\delta-1)(k+1)+\delta k]^2} > 0.$ It is evident that the outside innovator auctions *n* licenses in equilibrium. Hence, full knowledge diffusion occurs for the case of an outside innovator.

Proposition [1](#page-6-1) summarizes the above discussion.

Proposition 1 For $\delta > 1$, *n* potential licensees and zero opportunity costs of obtain*ing the license by each licensee, the outside innovator auctions n licenses. The equilibrium bid by each licensee, the equilibrium royalty and the equilibrium profit of the innovator is given by* $F^* = \frac{\delta^2}{4[(\delta-1)(n+1)+\delta n]^2}$, $r^* = \frac{\delta n(2\delta-1)}{2(\delta-1)[(\delta-1)(n+1)+\delta n]}$ and $\pi_I^* = \frac{\delta^2 n}{4(\delta - 1)[(\delta - 1)(n+1) + \delta n]},$ *respectively.*

Intuitively, the tax-saving effects of royalty and multiple licensing are responsible for the above result. As the royalty increases, it reduces the tax rate, $\frac{10}{10}$ and the absolute change in the tax rate due to a higher royalty increases with the number of licenses.¹¹ These effects encourage the innovator to charge positive output royalty and to increase the number of licenses as far as possible. Output royalty also helps to raise the price of the final goods by reducing the outputs of the licensees, thus softening competition between the licensees. Hence, the tax-saving and competition softening effects of royalty and the tax-saving effects of multiple licenses encourage full knowledge diffusion by the outside innovator through technology licensing.

If there is no tax or the tax rate is fixed, the royalty and the number of licenses do not yield any tax-saving advantage. Under such situation, our analysis shows, like the

¹⁰ Using the equilibrium tax rate in Eq. [\(4\)](#page-5-1), we have $\frac{\partial t^*}{\partial r} = -\frac{(\delta + \delta k - 1)}{2(\delta - 1)(k + 1) + k} < 0$. ¹¹ Following the result contained in footnote [\(11\)](#page-9-0), we obtain $\frac{\partial(-\frac{\partial t^*}{\partial r})}{\partial r}$

$$
^{11} \text{ Following the result contained in footnote (11), we obtain } \frac{\partial(-\frac{\partial r}{\partial r})}{\partial k} = \frac{\delta - 1}{[2(\delta - 1)(k + 1) + k]^2} > 0.
$$

previous paper such as [Sen and Tauman](#page-16-0) [\(2007\)](#page-16-0), that the outside innovator will auction one license with zero output royalty.

Let us now discuss why $\delta > 1$ is important for our result. Note that the royalty income is a transfer from the licensees to the innovator, it does not affect the total profit of the innovator directly.^{[12](#page-7-0)} Similarly, since the tax/subsidy payment is a transfer between the licensees and the government, it does not affect the welfare directly for $\delta = 1$.

The tax policy in our analysis helps to eliminate the welfare loss under the imperfectly competitive product market. This motivation may actually induce the government to subsidise the producers. The subsidy payment increases the profits of the producers, which, in turn, increases the profit of the innovator. If $\delta = 1$, the valuation of the tax/subsidy payment is the same to both the licensees and the government. It follows that the government can always choose the tax/subsidy rate in a way so that it can eliminate the welfare loss for any given royalty rate and the number of licenses. Hence, if $\delta = 1$, the government will provide subsidy at the rate $-t^* = \frac{(1+rk)}{k}$ and the equilibrium industry output will always be equal to the competitive output corresponding to the marginal cost of the innovator's technology (which is zero by assumption). Although the equilibrium industry output and the equilibrium price remain the same always, the subsidy rate falls with more licenses. Hence, the innovator has no incentive for licensing the technology to more than one firm in this situation, since its profit $(P - t) \sum_{i=1}^{k} q_i^*$ falls with more licenses.

However, the situation changes if $\delta > 1$, which suggest that, from the welfare point of view, the valuation of the tax/subsidy payment is lower to the licensees than to the government. In this situation, the tax/subsidy payment to the licensees is an imperfect substitute of the tax/subsidy payment to the government. Because of the higher valuation of the tax/subsidy payment to the government than to the licensees, the government does not have the incentive to charge a tax/subsidy to make the equilibrium industry output equal to the competitive output corresponding to the marginal cost of the innovator's technology. Hence, unlike $\delta = 1$, the innovator can influence the equilibrium industry output and the equilibrium price by choosing the licensing strategy (the royalty rate and the number of licensees) suitably for $\delta > 1$. Although more licenses tends to reduce the subsidy rate (or increase the tax rate), it increases the equilibrium industry output. 13 We establish that this output effect dominates the effect on the tax rate, and encourages the innovator to give multiple licenses.

It is worth mentioning that tax incidence on the outputs of the licensees is also important for our results. When the innovator pays tax on royalty, instead of tax on outputs, full knowledge diffusion does not occur. Under such circumstance, tax rate does not enter into the objective functions of the licensees, and therefore, it does not have a direct effect on the equilibrium outputs of the licensees. Clearly, the equilibrium output of the *i*th licensee is $q_i^* = \frac{1-r}{k+1}$, where $i = 1, 2, ..., k$. This, in turn, implies

¹² The total profit of the innovator is $(P - r - t) \sum_{i=1}^{k} q_i^* + r \sum_{i=1}^{k} q_i^* = (P - t) \sum_{i=1}^{k} q_i^*$

¹³ It is evident that $\frac{\partial Q^*}{\partial k} = \frac{2(\delta - 1)[\delta(1 - r^*) + r^*]}{(2 + k - 2\delta(1 + k))^2} > 0$, where r^* is the equilibrium royalty rate.

that neither consumer surplus nor the profits of the licensees is directly affected by the tax rate. Further, it is evident from the welfare function,

$$
W = \underbrace{\frac{1}{2} \left(\sum_{i=1}^{k} q_i \right)^2}_{CS} + \underbrace{\sum_{i=1}^{k} \left[\left(1 - \sum_{i=1}^{k} q_i \right) - r \right] q_i + (1-t)r \sum_{i=1}^{k} q_i}_{profit} + \underbrace{\delta tr \sum_{i=1}^{k} q_i}_{tax\,revenue},
$$

that only the royalty income and tax revenue are directly affected by the tax rate. Hence, for $\delta > 1$, it is now evident that the government chooses $t = 1$ to extract the entire royalty income. Accordingly, the innovator does not benefit at all from output royalty and, thus, auctions only one license since this allows extracting the monopoly profit through licensing. An immediate implication of the alternative setting suggests that when government tax is imposed on royalty payment, an outside innovator always auctions one license with zero output royalty. This result illustrates the importance of tax incidence on outputs for our result.

We now explore the welfare implications of licensing. If the outside innovator auctions *k* licenses, the equilibrium welfare is

$$
W^* = \frac{\delta^2 k [2(\delta - 1)(k + 1) + k]}{8[(\delta - 1)(k + 1) + \delta k]^2}.
$$
 (7)

It is easy to show that $\frac{\partial W^*}{\partial k} = \frac{\delta^2(\delta-1)^2}{4[(\delta-1)(k+1)+\delta k]} > 0$, which implies that welfare rises with more licenses. This suggests that an outside innovator's preference for licensing moves in the same direction to that of the society, in that more licenses reduce tax rate but raise the total outputs. Hence, more licenses create higher total profits and higher welfare.

3.2 The case of an inside innovator

We now turn to a scenario that is similar to Sect. [2](#page-3-0) with the exception that the innovator is an insider, implying that it is a producer of the product. Hence, the innovator is a monopolist under no licensing, and, it competes with the licensees under licensing. Therefore, the innovator needs to internalise the effects of licensing on its own profit along with the revenues from licensing.

If the innovator auctions *k* licenses, where $1 \leq k \leq n$, and charges the per-unit output royalty r, the problems facing the innovator and the *i*th licensee, $i = 1, 2, \ldots, k$, at the production stage are respectively:

$$
\underset{q_I}{Max(1 - Q - t)q_I} + \sum_{i=1}^{k} (F_i + r q_i), \tag{8}
$$

$$
\underset{q_i}{Max}(1 - Q - r - t)q_i - F_i, \tag{9}
$$

 \mathcal{D} Springer

where the bidding game will ensure that F_i is binding and is equal to $(1 - Q - r - t)q_i$ in equilibrium.

The Cournot–Nash equilibrium output of the innovator and the *i*th licensee can be obtained as $q_l^* = \frac{1 - t + kr}{k+2}$ and $q_i^* = \frac{1 - t - 2r}{k+2}$, where $i = 1, 2, ..., k$ and $(q_l^* +$ $\sum_{i=1}^{k} q_i^*$ = Q^* . The outputs of all firms are positive if $r < \frac{1-t}{2}$, which is assumed to hold.

Given the equilibrium outputs of the producers, the profits of *I* and each licensee are $\pi_I^* = \frac{(1 - t + kr)^2}{(k+2)^2}$ and $\pi_I^* = \pi_I^* = \cdots = \pi_I^* = \frac{(1 - t - 2r)^2}{(k+2)^2}$, respectively. Hence, each licensee's maximum willingness to bid is $\frac{(1-t-2r)^2}{(k+2)^2}$. Similar to the case of an outside innovator, if $k = n, I$ can guarantee this equilibrium bid by specifying a minimum bid. However, *I* does not need to specify a minimum bid for $k < n$.

We now look at the decision of the government. The government chooses *t* to maximise:

$$
W = \underbrace{\frac{1}{2} \left(q_I + \sum_{i=1}^k q_i \right)^2}_{CS} + \underbrace{\left[\left(1 - q_I + \sum_{i=1}^k q_i \right) - t \right] \left(q_I + \sum_{i=1}^k q_i \right)}_{profit} + \underbrace{\delta t \left(q_I + \sum_{i=1}^k q_i \right)}_{tax\,revenue}
$$
\n
$$
= \frac{1}{2} \left(\frac{(k+1)(1-t) - kr}{k+2} \right)^2 + \left(\frac{1-t+kr}{k+2} \right) \left(\frac{(k+1)(1-t) - kr}{k+2} \right)
$$
\n
$$
+ \delta t \left(\frac{(k+1)(1-t) - kr}{k+2} \right),
$$
\n(10)

where $\delta > 1$, which has the interpretation similar to the case of an outside innovator.

It is straightforward to show that the equilibrium tax rate is

$$
t^* = \frac{(k+1)[\delta(k+2) - 3 - k] - rk[\delta(k+2) - 1]}{(k+1)[2\delta(k+2) - 3 - k]}.
$$
\n(11)

We now determine the equilibrium royalty rate. The innovator maximises the following expression to determine *r*:

$$
\Pi_{I} = \pi_{I} + \sum_{i=1}^{k} (F_{i} + rq_{i})
$$
\n
$$
= \left(\frac{1 - t^{*} + kr}{k + 2}\right)^{2} + k \left(\frac{1 - t^{*} - 2r}{k + 2}\right)^{2} + rk \left(\frac{1 - t^{*} - 2r}{k + 2}\right), \quad (12)
$$

where t^* is given in [\(11\)](#page-9-0). The royalty rate that maximises [\(12\)](#page-9-1) is $r = \frac{\delta(2\delta-1)(k+1)^2}{\delta(2\delta-1)}$, which is greater than $\frac{1-t^*}{\delta(2\delta)}$. Since the outputs of all firms are $\frac{\delta(2\delta-1)(k+1)^2}{2(\delta-1)k[\delta(2k+3)-(k+2)]}$, which is greater than $\frac{1-t^*}{2}$. Since the outputs of all firms are positive for $r < \frac{1-t}{2}$, it follows that the equilibrium royalty will be

$$
r^* = \frac{1 - t^*}{2} = \frac{\delta(k+1)}{\delta(3k+4) - (2k+3)}.
$$
 (13)

 $\textcircled{2}$ Springer

Given the equilibrium royalty, if innovator *I* auctions *k* licenses, the equilibrium profit of *I* is $\pi_I^* = \frac{\delta^2 (k+1)^2}{[\delta (3k+4)-3-2k]^2}$. Since $\frac{\partial \pi_I^*}{\partial k} = \frac{2(\delta-1)\delta^2 (k+1)}{[\delta (3k+4)-3-2k]^2} > 0$, it follows that the innovator auctions *n* licenses.

Given the royalty rate in (13) and the tax rate in (11) , total welfare is

$$
W^* = \frac{\delta^2(k+1)[2\delta(k+2) - 3 - k]}{2[\delta(3k+4) - 3 - 2k]^2}.
$$
 (14)

It is interesting to note that $\frac{\partial W^*}{\partial k} = -\frac{\delta^2(\delta-1)^2 k}{[\delta(3k+4)-3-2k]^3} < 0$, implying that welfare is higher under no licensing than it is under licensing. The above analysis suggests that the equilibrium output of the licensees are zero and welfare under licensing is lower than that of under no licensing if the innovator charges the royalty rate as in [\(13\)](#page-9-2) and the government maximises (10) , i.e., considers the credible threat of competition from the licensees. Notably, given the royalty rate in [\(13\)](#page-9-2), if the government charges the tax rate as $t^m = \frac{2\delta - 3}{4\delta - 3}$, which is the optimal tax rate under no licensing (i.e., when the innovator produces like a monopolist), the threat of competition from the licensees is not credible.^{[14](#page-10-0)} Hence, this tax rate will allow the government to achieve a welfare level that is otherwise obtained under no licensing, i.e., $W^m = \frac{\delta^2}{8\delta - 6}$. This implies that the behaviour of the government in the above discussion is naive and a rational government should not follow the tax rate in (11) if the royalty rate is given by (13) . Instead, if the royalty rate is given by [\(13\)](#page-9-2), the government should choose the tax as $t^m = \frac{2\delta - 3}{4\delta - 3}$. Indeed, if the innovator intends to induce the government charging the tax rate according to (11) , the innovator should charge the royalty rate satisfying

$$
\frac{[\delta(k+1) - rk(\delta-1)]^2}{2(k+1)[2\delta(k+2) - 3 - k]} \ge \frac{\delta^2}{8\delta - 6},
$$
\n(15)

where left-hand side of (15) shows the welfare for a royalty rate, r, and the tax rate as shown in (11) , and the right-hand side of (15) shows welfare under no licensing. The equilibrium royalty rate satisfying (15) is 15 15 15

$$
r^{**} = \frac{\delta(4\delta - 3)(k+1) - \sqrt{\delta^2(4\delta - 3)(k+1)[2\delta(k+2) - 3 - k]}}{k(\delta - 1)(4\delta - 3)}.
$$
 (16)

Using the result in (16) , we can establish that the corresponding tax rate is $t^{**} = \frac{(k+1)\delta(k+2)-3-k-1-r^{**}k[\delta(k+2)-1]}{(k+1)[2\delta(k+2)-3-k]}$, the equilibrium bid of the *i*th licensee, $i = 1, 2, ..., k$, is

¹⁴ If the innovator produces like a monopolist, the optimal tax is $t^m = \frac{2\delta - 3}{4\delta - 3}$ and the innovator's equilibrium output is $q^m = \frac{\delta}{4\delta - 3}$. If the innovator charges the royalty $r^* = \frac{\delta(k+1)}{\delta(3k+4)-3-2k}$, the government charges the tax $t^m = \frac{(2\delta - 3)}{4\delta - 3}$ and the innovator produces $q^m = \frac{\delta}{4\delta - 3}$, the symmetric equilibrium outputs of the licensees, determined from the first order condition of profit maximisation, are zero.

¹⁵ The equilibrium royalty rate will equate both sides of (15) .

Fig. 1 $\frac{\partial \pi_i^*}{\partial k}$ for any $\delta \in [1.1, 5]$ and $k \in [1, 10]$

$$
F_i = \frac{(1 - t^{**} - 2t^{**})^2}{(k+2)^2},\tag{17}
$$

and the profit of the innovator is

$$
\pi_I^{**} = \left(\frac{1 - t^{**} + kr^{**}}{k+2}\right) \left(\frac{(k+1)(1 - t^{**}) - kr^{**}}{k+2}\right)
$$

=
$$
\frac{\delta[\delta(k+2) - \delta^2(2k+3) + \sqrt{\delta^2(4\delta - 3)(k+1)(2\delta(k+2) - 3 - k)}]}{(\delta - 1)(4\delta - 3)[2\delta(k+2) - 3 - k]}.
$$
 (18)

It is evident, from [\(18\)](#page-11-0), that a simple analytical result for the sign of $\frac{\partial \pi_i^{**}}{\partial k}$ cannot be obtained.^{[16](#page-11-1)} To facilitate our analysis of the licensing decision by the innovator, we use Fig. [1](#page-11-2) to illustrate $\frac{\partial \pi_i^{**}}{\partial k}$ for any *k* ∈ [1, 10] and δ ∈ [1.1, 5]. The vertical axis of Fig. [1](#page-11-2) measures the value of $\frac{\partial \pi_i^{**}}{\partial k}$. Figure 1 shows that $\frac{\partial \pi_i^{**}}{\partial k} > 0$ for any $\delta \in [1, 1, 5]$ and $k \in [1, 10]$, suggesting that the innovator licenses to all the potential licensees.

Although the inside innovator takes into account the effects of licensing on its own profit, the rationale behind the above result is similar to the rationale provided for Proposition 1. It is evident, from (11) , that as the royalty rate increases, it reduces the tax rate, and the absolute change in the tax rate due to a higher royalty increases with the

¹⁶ We use 'Mathematica 7' [\(Wolfram 2010](#page-16-12)) for the Figures of this paper and we have obtained that $\frac{\partial \pi_j^{**}}{\partial k} = \frac{\delta[-\delta(2\delta-1)(1+k)+\sqrt{\delta^2(4\delta-3)(1+k)(2\delta(2+k)-3-k)}}{(4\delta-3)(1+k)(3+k-2\delta(2+k))^2}.$

number of licenses.^{[17](#page-12-1)} Hence, a positive output royalty and more licenses tend to reduce the tax rate, thus providing the monopolist inside innovator the incentive for auctioning *n* licenses with positive royalties. Even if licensing increases product-market competition, a monopolist final goods producer has the incentive for licensing due to the tax-saving benefit.

The above mechanism can be related to [Mukherjee and Pennings](#page-15-23) [\(2006](#page-15-23)), where an exporting monopolist innovator licenses its technology to another exporter in order to reduce the tariff charged by the importing country. Licensing in their paper reduces the tariff and makes licensing profitable. Although the mechanism is related, there are some differences between our paper and theirs. Unlike this paper, the profits of the firms do not enter the welfare functions of [Mukherjee and Pennings](#page-15-23) [\(2006](#page-15-23)). Further, they consider one license and do not look at the optimal number of licenses, which is the main focus of this paper.

It is also easy to understand that, like the case of an outside innovator, tax on the outputs is important for our result. If the tax is imposed on the royalty income of the innovator, instead of the outputs of the producers, it does not affect the outputs directly, and therefore, does not create the incentive for licensing by the innovator. In this situation, the inside innovator prefers no licensing.

Proposition [2](#page-12-2) follows from the above discussion.

Proposition 2 *In the presence of tax/subsidy policies, n potential licensees and zero opportunity costs of the licensees for getting the license, if* δ > 1, *an inside innovator auctions n licenses. The equilibrium royalty, the equilibrium bid by each licensee and the equilibrium profit of the innovator is given by the expressions* [\(16\)](#page-10-3), [\(17\)](#page-11-3) *and* [\(18\)](#page-11-0)*, respectively.*

It is worth mentioning that there is no incentive for licensing by an inside innovator for $\delta = 1$. The implication of $\delta > 1$ in creating the incentive for licensing under inside innovator is similar to that under outside innovator.

Since the royalty rate and the tax rate are such that they equate both sides of [\(15\)](#page-10-1), it is immediate that licensing by the inside innovator does not increase welfare. However, licensing reduces the tax rate and increases the total profit in the industry (as is evident from Fig. [1\)](#page-11-2), increases consumer surplus compared to no licensing (as shown in Fig. [2\)](#page-13-0), but reduces the tax revenue (as shown in Fig. 3).^{[18](#page-12-3)} Hence, licensing makes the firms and the consumers better off at the expense of the government revenue.

4 The incentive for innovation

This section compares an outside innovator's incentive for innovation to that of an inside innovator. Assume that an innovator needs to invest G to invent the technology. An innovator invents in innovation if its profit from the innovated technology is greater

¹⁷ Using Eq. [\(11\)](#page-9-0), it is easy to verify that $\frac{\partial t^*}{\partial r} = -\frac{k[\delta(2+k)-1]}{(k+1)[2\delta(k+2)-3-k]} < 0$, and that $\frac{\partial(-\frac{\partial t^*}{\partial r})}{\partial k}$ = $\frac{1}{2} \left[\frac{1}{(k+1)^2} + \frac{4\delta - 3}{(2\delta(k+2) - 3 - k)^2} \right] > 0.$

¹⁸ Since the innovator licenses to all the potential licensees, we denote the number of licensees in Figs. [2](#page-13-0) and [3](#page-13-1) by *n* instead of *k*.

Fig. 2 Consumer surplus under licensing minus consumer surplus under no licensing for any $\delta \in [1.1, 5]$ and $k \in [1, 10]$

Fig. 3 Tax revenue under licensing minus tax revenue under no licensing for any $\delta \in [1, 1, 5]$ and $k \in [1, 10]$

than the investment *G*. Hence, an outside innovator and an inside innovator invest in innovation, respectively, if

$$
G < \frac{\delta^2 n}{4(\delta - 1)[(\delta - 1)(n + 1) + \delta n]},\tag{19}
$$
\n
$$
G < \frac{\delta[\delta(n + 2) - \delta^2(2n + 3) + \sqrt{\delta^2(4\delta - 3)(n + 1)(2\delta(n + 2) - 3 - n)}]}{(\delta - 1)(4\delta - 3)[2\delta(n + 2) - 3 - n]}.\tag{20}
$$

² Springer

It is straightforward to show that right-hand side of (19) is greater than that of (20) , implying that the outside innovator's incentive for innovation is higher than that of the inside innovator.

The intuition for this result is, for both the inside and the outside innovators, that licensing creates the beneficial tax-saving effects. In the case of an outside innovator, the absence of the licenser as a producer increases the total profit in the industry by reducing the product-market competition. By contrast, in the case of an inside innovator, the presence of the licenser leads to lower industry profit by raising market competition. It follows immediate that the benefits due to innovator's research effort in innovation is more rewarding under outside innovator than it is under the inside innovator. Hence, our analysis into the incentive of innovation suggests that there is greater incentive for the outside innovator in conducting innovation than it is for the inside innovator.

5 Conclusion

We have shown in the presence of tax/subsidy policy that both the outside innovator and the inside innovator license a new product (or drastic process innovation) to all potential licensees. We also showed that the outside innovator's incentive for innovation is greater than that of an inside innovator.

Our results complement those established in [Sen and Tauman](#page-16-0) [\(2007\)](#page-16-0), where full knowledge diffusion through technology licensing occurs only under non-drastic innovation. An implication of our analysis is that, in the presence of tax/subsidy policy, a monopolist producer may license in a homogeneous goods industry even though that creates competition in the product market.

Acknowledgments We thank two anonymous referees of this journal for helpful comments and suggestions. Yingyi Tsai gratefully acknowledges financial support under research grant no. "NSC99-2410- H-390-009-" from the National Science Council, Taiwan.

References

- Arrow K (1962) Economic welfare and the allocation of resources for invention. In: Nelson R (ed) The rate and direction of inventive activity. Princeton University Press, Princeton
- Beggs AW (1992) The licensing of patents under asymmetric information. Int J Ind Organ 10:171–191
- Bousquet A, Cremer H, Ivaldi M, Wolkowicz M (1998) Risk sharing in licensing. Int J Ind Organ 16:535– 554
- Brennan G, Buchanan JM (1980) The power to tax: analytical foundations of a fiscal constitution. Cambridge University Press, Cambridge
- Caballero-Sanz F, Moner-Colonques R, Sempere-Monerris JJ (2002) Optimal licensing in a spatial model. Annals d'économie et de statistique 66:257–279
- Choi JP (2001) Technology transfer with moral hazard. Int J Ind Organ 19:249–266
- Economides N (1993) A monopolist's incentive to invite competitors to enter in telecommunications services. In: Pogorel G (ed) Global telecommunications services and technological changes. Elsevier, Amsterdam
- Faulí-Oller R, Sandonís J (2002) Welfare reducing licensing. Games Econ Behav 41:192–205
- Filippini L (2005) Licensing contract in a Stackelberg model. Manch Sch 73:582–598

Fosfuri A, Roca E (2004) Optimal licensing strategy: royalty or fixed fee? Int J Bus Econ 3:13–19 Gallini NT, Winter RA (1985) Licensing in the theory of innovation. RAND J Econ 16(2):237–252

- Gallini NT, Wright BD (1990) Technology transfer under asymmetric information. RAND J Econ 21:147– 160
- Greenhalgh C, Rogers M (2010) Innovation, intellectual property and economic growth. Princeton University Press, Princeton
- Gruenspecht HK (1988) Export subsidies for differentiated products. J Int Econ 24:331–344
- Hamilton SF (1999) Tax incidence under oligopoly: a comparison of policy approaches. J Public Econ $71.233 - 245$
- Kabiraj T (2004) Patent licensing in a leadership structure. Manch Sch 72:188–205
- Kabiraj T (2005) Technology transfer in a Stackelberg structure: licensing contracts and welfare. Manch Sch 73:1–28
- Kabiraj T, Marjit S (2003) Protecting consumers through protection: the role of tariff-induced technology transfer. Eur Econ Rev 47:113–124
- Kamien MI (1992) Patent licensing. In: Aumann RJ, Hart S (eds) Handbook of game theory. Elsevier, Amsterdam, pp 331–355
- Kamien MI, Tauman Y (1984) The private value of a patent: a game theoretic analysis. J Econ (Zeitschrift für Nationalökonomie) 4(Supplement):93–118
- Kamien MI, Tauman Y (1986) Fees versus royalties and the private value of a patent. Q J Econ 101:471–479 Kamien MI, Tauman Y (2002) Patent licensing: the inside story. Manch Sch 70:7–15
- Kamien MI, Oren SS, Tauman Y (1992) Optimal licensing of cost-reducing innovation. J Math Econ 21:483–508
- Kamien MI, Tauman Y, Zhang I (1988) Optimal license fees for a new product. Math Soc Sci 16:77–106
- Katz ML, Shapiro C (1985) On the licensing of innovations. RAND J Econ 16:504–520
- Katz ML, Shapiro C (1987) R&D rivalry with licensing or imitation. Am Econ Rev 77:402–420
- Laffont J-J, Tirole J (1986) Using cost observation to regulate firms. J Polit Econ 94:614–641
- Laffont J-J, Tirole J (1993) A theory of incentives in procurement and regulation. MIT Press, Cambridge
- Macho-Stadler I, Martinez-Giralt X, Pérez-Castrillo JD (1996) The role of information in licensing contract design. Res Policy 25:25–41
- Marjit S (1990) On a non-cooperative theory of technology transfer. Econ Lett 33:293–298
- Mueller DC (1989) Public choice, vol II. Cambridge University Press, Cambridge
- Mukherjee A (2001) Technology licensing with commitment. Econ Theory 17:345–369
- Mukherjee A (2005) Innovation, licensing and welfare. Manch Sch 73:29–39
- Mukherjee A (2007) Optimal licensing contract in an open economy. Econ Bull 12(3):1–6
- Mukherjee A (2010) Licensing a new product: fee vs. royalty licensing with unionized labor market. Labour Econ 17:735–742
- Mukherjee A (2011) Licensing a new product under convex costs. Mimeo, University of Nottingham
- Mukherjee A, Pennings E (2006) Tariffs, licensing and market structure. Eur Econ Rev 50:1699–1707
- Mukherjee A, Balasubramanian N (2001) Technology transfer in horizontally differentiated product market. Res Econ 55:257–274
- Mukherjee A, Broll U, Mukherjee S (2008) Unionized labor market and licensing by a monopolist. J Econ 93:59–79
- Muto S (1987) Possibility of relicensing and patent protection. Eur Econ Rev 31:927–945
- Muto S (1993) On licensing policies in Bertrand competition. Games Econ Behav 5:257–267
- Myles GD (1996) Imperfect competition and the optimal combination of ad valorem and specific taxation. Int Tax Public Finance 3:29–44
- Neary JP (1994) Cost asymmetries in international subsidy games: should governments help winners or losers? J Int Econ 37:197–218
- Neary JP, Leahy D (2000) Strategic trade and industrial policy towards dynamic oligopolies. Econ J 110:484–508
- Neary JP, Leahy D (2004) Revenue-constrained strategic trade and industrial policy. Econ Lett 82:409–414 Poddar S, Sinha UB (2004) On patent licensing and spatial competition. Econ Rec 80:208–218
- Poddar S, Sinha UB (2010) Patent licensing from high-cost to low-cost firm. Econ Rec 86:384–395
- Rockett K (1990) The quality of licensed technology. Int J Ind Organ 8:559–574
- Rostoker M (1984) A survey of corporate licensing. IDEA: J Law Technol 24:59–92
- Saracho AI (2002) Patent licensing under strategic delegation. J Econ Manag Strat 11:225–251
- Schmitz PW (2002) On monopolistic licensing strategies under asymmetric information. J Econ Theory 106:177–189
- Sempere Monerris JJ, Vannetelbosch VJ (2001) The relevance of bargaining for the licensing of a costreducing innovation. Bull Econ Res 53:101–115
- Sen D (2002) Monopoly profit in a Cournot oligopoly. Econ Bull 4:1–6
- Sen D (2005a) Fee versus royalty reconsidered. Games Econ Behav 53:141–147
- Sen D (2005b) On the coexistence of different licensing schemes. Int Rev Econ Finance 14:393–413
- Sen D, Stamatopoulos G (2009a) Drastic innovations and multiplicity of optimal licensing policies. Econ Lett 105:7–10
- Sen D, Stamatopoulos G (2009b) Technology transfer under returns to scale. Manch Sch 77:337–365
- Sen D, Tauman Y (2007) General licensing scheme for a cost-reducing innovation. Games Econ Behav 59:163–186
- Shapiro C (1985) Patent licensing and R&D rivalry. Am Econ Rev Papers Proc 75:25–30
- Staiger R, Tabellini G (1987) Discretionary trade policy and excessive protection. Am Econ Rev 77:823– 837
- Wang XH (1998) Fee versus royalty licensing in a Cournot duopoly model. Econ Lett 60:55–62
- Wang XH (2002) Fee versus royalty licensing in a differentiated Cournot duopoly. J Econ Bus 54:253–266
- Wang XH, Yang B (1999) On licensing under Bertrand competition. Aust Econ Pap 38:106–119
- Wolfram S (2010) The mathematica book, 7th edn. Wolfram Media, Cambridge University Press