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Optimal privatization in a mixed duopoly with consistent conjectures

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Abstract We show that partially privatizing a public firm alters underlying conjectures, in turn, changing the optimal degree of privatization. The consistent conjectures equilibrium (CCE) generates substantially greater optimal privatization than does any conjecture shared between the firms including the standard Cournot–Nash equilibrium (CNE). Yet, when the private rival is foreign, the CCE generates substantially less privatization than the CNE. The optimal extent of privatization with a domestic rival exceeds that with a foreign rival in the CCE as well as in the CNE.

Keywords Consistent conjectures · Partial privatization · Mixed oligopoly

JEL Classification L1 · L3

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1 Introduction

Partial privatization of public firms can be warranted for several economic reasons but one of the most interesting comes from the literature on mixed oligopolies.[1] When a welfare-maximizing public firm faces increasing marginal cost and competes against private firms, it produces more than its private rivals generating an inefficient cost asymmetry. Matsumura (1998) shows that partial privatization reduces the firm's output and so the inefficiency. His demonstration that there exists a socially optimal degree of partial privatization stands among the best known results in the mixed oligopoly literature.[2]

While researchers since Matsumura examine partial privatization in increasingly varied settings, we emphasize that the very process of partial privatization changes the fundamental strategic assumptions held by firms.[3] As partial privatization causes a previously public firm to give more weight to its own profit, its strategic response to changes in output from its rivals becomes larger. This increase in strategic substitution should be built into the conjectures of firms. Doing so changes equilibrium outputs and the optimal extent of privatization. This point has not been previously made in the literature and has important policy implications. As an illustration, we demonstrate that with linearly increasing marginal costs the optimal extent of privatization in a mixed private duopoly with consistent conjectures is 2/3rds larger than that identified by Matsumura (1998). Moreover, allowing the extent of strategic substitution to change seems warranted as empirical studies find that rivals to a previously public firm face stiffer competition, shorter reaction time and lower profitability following privatization (Altintig et al. 2009; Otchere 2007; Otchere and Chan 2003).

Conjectural variations generalize the Cournot model and are seen as explicitly modeling the beliefs formation process of each firm about the conduct of other. This generalization has been criticized because the models remain fundamentally one-shot with no explicit maximization of a stream of profits and they generate out of equilibrium behavior that can make little sense lacking normal stability properties (Friedman 1983, pp. 109–110). Despite this criticism, they remain in use as a substitute for complete dynamic modeling. Indeed, a substantial literature has embedded static Cournot behavior in a fully dynamic model and isolated the conditions under which the outcomes match that of consistent conjectures (see Cabral 1995 for an early example). As Martin (2002, p. 51) emphasizes, "these results provide a formal justification for using the static conjectural variations model as a short cut to analyze inherently dynamic imperfectly competitive markets." In addition to being a shortcut to full dynamic modeling, the conjectural variations framework is routinely used in the literature on private

[1] Alternative reasons include the managerial discipline generated by even minority private stock ownership (Gupta 2005), generating a solution to an incomplete contracting problem (Schmitz 2000), political economy motivations including the raising of governmental revenue (Maw 2002) and creating heterogeneous ownership to limit the ability of public sector managers to collude (Bhaskar et al. 2006).

[2] Indeed, Google Scholar lists 148 citations to Matsumura (1998) as of April 2009.

[3] The more varied settings include the presence of externalities (Ohori 2006), an R&D rivalry (Ishibashik and Matsumura 2006; Heywood and Ye 2009), strategic trade models (Fujiwara 2007; Chao and Yu 2006; Lee and Hwang 2003) and foreign direct investment (Mukherjee and Suetrong 2009).

provision of public goods (Itaya and Shimomura 2001) and has found practical applications both in bidding strategies for electric power (Song et al. 2003) and in estimating market power (Perloff et al. 2007).[4]

Yet, our examination of partial privatization is important not only for its new setting of conjectural variations but for its policy relevance. Many countries retain large sectors of publically owned firms (Bortolotti et al. 2004) and these firms are frequently partially privatized. Among numerous examples, Germany sold approximately a quarter of the Deutsche Bahn to private investors and, through the state of Lower Saxony, retains partial ownership in Volkswagen. Controversy currently surrounds the UK government's intention to privatize approximately one-third of the Royal Mail. In 2002 China sold the majority of the high tech firm TCL but retains a 40 percent share. More generally, Gupta (2005) emphasizes that the extensive privatization in India has been largely limited to the sale of minority stakes while Maw (2002) makes clear that privatization in many transition economies is routinely partial. In a single industry, Doganis (2001) identifies 44 partially privatized airlines in both developed and transition economies. In 2008, Indonesian lawmakers added to this number by privatizing 30 percent of the flag carrier Indonesian Gurda[5] In these cases, and many others, partial privatization can change the underlying conjectures of the firms influencing the optimal extent of privatization and the resulting equilibrium.

In what follows, the next section describes our model of a mixed duopoly and solves for the consistent conjectures equilibrium assuming no privatization. The third section imagines an earlier stage in which the government adopts the welfare maximizing extent of partial privatization. We contrast this new equilibrium and the traditional one based on Cournot conjectures highlighting that the optimal extent of privatization is greater in the new equilibrium. The fourth section conducts sensitivity tests showing that the degree of privatization under consistent conjectures exceeds not only than that in Cournot but in all cases of identical conjectures. Moreover, while allowing the costs of the public firm to be greater than those of the private firm increases the optimal extent of privatization, it does not change the conclusion that the optimal extent of privatization is greater in the consistent conjectures equilibrium. This section also explores expanding the number of private sector firms. The fifth section changes the model by assuming that the private firm is foreign rather than domestic. Again, we highlight how partial privatization changes conjectures but show that in the presence of the foreign firm accounting for these changes reduces the optimal extent of privatization in the CCE compared to an analogous CNE. The final section concludes and draws policy implications.

2 Model and initial equilibrium

Consider a public firm and a private firm producing homogenous goods in a domestic market. Let q_0 be the output of the public firm and q_1 be the output of the private

[4] We nonetheless recognize the failure to replicate consistent conjectural equilibriums in laboratory experiments (see Holt 1985 for an early inquiry).

[5] See Dadpay and Heywood (2006) for a theoretical model of mixed oligopolies in the international airline industry.

firm. Following a dominant strand in the mixed oligopoly literature, assume quadratic costs, $C(q_i) = \varphi + \frac{1}{2}kq_i^2, i = 0, 1$, where k is the rate of increase in marginal cost.[6] As we ignore entry issues, we set $\varphi = 0$ without loss of generality. Adopting demand, $P(Q) = a - Q$, gives consumer surplus $CS = \int_0^Q P(x)dx - QP(Q)$ where $Q = q_0 + q_1$ and profit is $\pi_i = Pq_i - C(q_i), i = 0, 1$. The private firm maximizes its profit and the public firm maximizes social welfare:

$$W = CS + \sum_{i=0}^{1} \pi_i \tag{1}$$

The first order conditions from the simultaneous quantity game generate best response functions in terms of the conjectures:

$$\begin{aligned} Br_0 : q_0 &= \frac{a(\gamma_0 + 1) - (\gamma_0 + k\gamma_0 + 1)q_1}{\gamma_0 + k + 1} \\ Br_1 : q_1 &= \frac{a - q_0}{\gamma_1 + k + 2} \end{aligned} \tag{2}$$

where $\gamma_i = \frac{\partial q_{\sim i}}{\partial q_i}$ is the conjecture that one firm makes about how its rival's output will change in response to changes in its own output.

Following Perry (1982), we define $\rho_0 = \frac{\partial Br_1}{\partial q_0}$ and $\rho_1 = \frac{\partial Br_0}{\partial q_1}$ as the actual output changes of the rival's output in response to a change in own output as taken from the rival's best response functions. The respective values are

$$\begin{aligned} \rho_1 &= -\frac{\gamma_0 + k\gamma_0 + 1}{\gamma_0 + k + 1} \\ \rho_0 &= -\frac{1}{\gamma_1 + k + 2} \end{aligned} \tag{3}$$

Consistent conjectures require that the conjectured responses equal the actual responses for each firm: $\gamma_i = \rho_i, i = 0, 1$. Solving two equations and two unknowns generates the consistent conjectures:

$$\begin{aligned} \gamma_0^* &= -\frac{2k + 2}{2 + 3k + k^2 + \Theta} \\ \gamma_1^* &= -\frac{2 + 3k + k^2 - \Theta}{2k + 2} \end{aligned} \tag{4}$$

[6] As frequently pointed out, if marginal costs are equal and constant, the welfare maximizing public firm simply produces the competitive quantity displacing all private output (DeFraja and Delbono 1989). More generally, if marginal costs are constant but higher for the public firm, the total output is that associated with the public firm's marginal cost. The private firm produces output equal to the difference in marginal costs and public firm produces the remainder.

Table 1 Comparing the equilibriums when $k = 1$

	CNE (public)	CCE (public)	CNE (part private)	CCE (part private)
λ	0	0	0.200	0.329
γ_0	0	-0.354	0	-0.365
γ_1	0	-0.177	0	-0.260
q_0	$0.400a$	$0.352a$	$0.357a$	$0.320a$
q_1	$0.200a$	$0.230a$	$0.214a$	$0.248a$
Q	$0.600a$	$0.582a$	$0.571a$	$0.568a$
P	$0.400a$	$0.418a$	$0.429a$	$0.432a$
$\pi_0 + \pi_1$	$0.140a^2$	$0.155a^2$	$0.158a^2$	$0.163a^2$
CS	$0.180a^2$	$0.169a^2$	$0.163a^2$	$0.161a^2$
W	$0.320a^2$	$0.324a^2$	$0.321a^2$	$0.325a^2$

CNE Cournot–Nash equilibrium, *CCE* consistent conjectures equilibrium
$W = CS + \pi_0 + \pi_1$

where $\Theta = \sqrt{8k + 13k^2 + 6k^3 + k^4}$. Substituting the $\gamma's$ from (4) back into (2) and solving simultaneously for $q's$ yields the equilibrium quantities:

$$q_0^* = \frac{\Theta + 1 + 5k + k^2}{(k+3)\Theta + \Theta^2/k} a$$
$$q_1^* = \frac{[(k+2)\Theta + 2 + 8k + 5k^2 + k^3](2 + 2k)}{[(k+3)\Theta + \Theta^2/k](\Theta + 2 + 3k + k^2)} a \tag{5}$$

The equilibrium in the CCE can be shown to have more nearly symmetric output between the two firms, lower total output, higher price and higher welfare compared to the CNE. Despite its reduced total output and the associated higher price, the CCE emerges with greater welfare. This, results because of the lower production costs generated by the smaller output asymmetry between the firms. To illustrate we present values for both the CCE and CNE when $k = 1$ in the first two columns of Table 1.

Using this general expression for the equilibrium we now consider the optimal degree of privatization.

3 Partial privatization

We follow Matsumura (1998) by considering a new first stage in which the government sets a socially optimal extent of privatization, λ. The previously public firm now maximizes

$$G = (1 - \lambda)W + \lambda\pi_0 \tag{6}$$

where $0 \leq \lambda \leq 1$. Matsumura and those that follow think of λ as the share of the public firm sold to private parties. The objective function thus reflects a weighted average of the underlying ownership structure.

To simplify the presentation, we let $k = 1$ but note in the appendix the more general results. The simultaneous quantity game generates best response functions:

$$\begin{aligned} Br_0 : q_0 &= \frac{a(\gamma_0 + 1 - \lambda\gamma_0) - (2\gamma_0 - 2\lambda\gamma_0 + 1)q_1}{\gamma_0 + \lambda + 2} \\ Br_1 : q_1 &= \frac{a - q_0}{\gamma_1 + 3} \end{aligned} \tag{7}$$

The actual behavioral responses along those functions are

$$\begin{aligned} \rho_1 &= -\frac{2\gamma_0 - 2\lambda\gamma_0 + 1}{\gamma_0 + \lambda + 2} \\ \rho_0 &= -\frac{1}{\gamma_1 + 3} \end{aligned} \tag{8}$$

The condition for the CCE requires that $\gamma_i = \rho_i$:

$$\begin{aligned} \gamma_0 &= -\frac{2\lambda + 4}{6 + 3\lambda + \Gamma} \\ \gamma_1 &= -\frac{6 + 3\lambda - \Gamma}{2\lambda + 4} \end{aligned} \tag{9}$$

where $\Gamma = \sqrt{28 + 16\lambda + \lambda^2}$ and $\gamma_i < 0, i = 0, 1.\ \forall\lambda \in [0, 1]$.

Substituting (9) into the best responses in (7) and solving for the quantities yields:

$$\begin{aligned} q_0 &= \frac{(1 + \lambda)\Gamma + \lambda + 12 - \lambda^2}{(4 + 3\lambda)\Gamma + \Gamma^2} a \\ q_1 &= \frac{\left[(3 + 2\lambda)\Gamma + 16 + 15\lambda + 2\lambda^2\right](4 + 2\lambda)}{\left[(4 + 3\lambda)\Gamma + \Gamma^2\right](6 + 3\lambda + \Gamma)} a \end{aligned} \tag{10}$$

Substituting these into the welfare function (1) and maximizing with respect to λ yields $\lambda = 0.329$ allowing us to state the initial theorem.

Proposition 1 *The optimal extent of privatization is larger in the CCE, $\lambda^* = 0.329$, than in the CNE, $\lambda^* = 0.200$.*

Proof The CCE result follows from the derivation above while that for the CNE come from constraining $\gamma_0 = \gamma_1 = 0$ in (7) and solving for the quantities, returning those to (1) and maximizing with respect to λ (Matsumura 1998).

Table 1 illustrates the difference between the two optimal partially privatized equilibriums in columns 3 and 4. The CCE has a greater extent of privatization, more nearly similar outputs between the two firms, greater profit, lower consumer surplus but greater total welfare.

The critical difference in the optimal privatization results because in the CNE privatization influences only the objective function (6) not the behavioral assumptions. In the CCE privatization also changes the behavioral assumptions by causing the public firm to recognize that private firm output is a greater strategic substitute (ρ_0 becomes more negative). A given increase in public firm output to increase welfare causes a larger decrease in private firm output and greater cost asymmetry. The government thus has an incentive for greater privatization to reduce this asymmetry. Moreover, in the CCE privatization also causes the private firm to recognize that public firm output is a smaller strategic substitute (ρ_1 becomes less negative). A given decrease in private output generates a smaller increase in public firm output encouraging larger private firm reductions in output further increasing asymmetry. Both of these influences require greater privatization in order to reduce the inefficient asymmetry and achieve optimal welfare.

Thus, behind our initial proposition is the intuition that as partial privatization increases, the consistent conjectures become more negative encouraging further privatization.

Proposition 2 *Consistent conjectures decrease (become more negative) as the degree of privatization increases,* $\frac{\partial \gamma_i}{\partial \lambda} < 0, i = 0, 1, \forall \lambda \in [0, 1]$.

Proof This follows directly from the derivatives of (9).

While our presentation assumes $k = 1$, this result holds for any k as shown in the appendix. The more negative conjectures resulting from increasing privatization cause the output of the public firm to decrease and the output of the private firm to increase as revealed by the derivates of best response functions in (7) with respect to the conjectures. Thus, partial privatization generates greater substitutability which results in a larger optimal degree of privatization.

We note again that Proposition 1 can be confirmed for any k but that there is not a closed form solution for λ in terms of k. We have, however, shown a larger optimal extent of privatization in the CCE for all values of k checked through a grid search simulation available from the authors upon request. The implied policy recommendation should be a substantially larger degree of privatization in those circumstances in which firms may be anticipated to dynamically adjust their perceptions of their rival's behavior. In the case of $k = 1$, the extent of privatization with the CCE is two-thirds larger than that with the CNE.

4 Sensitivity analysis

Our fundamental proposition that privatization should be more extensive in the CNE depends upon a number of assumptions and comparisons that we now explore. We start by recognizing the limited nature of our comparison between the CCE and the CNE. The Cournot equilibrium, $\gamma_0 = \gamma_1 = 0$, is only one of a infinite variety of shared conjectural variations for the two firms, $\gamma_0 = \gamma_1 = \gamma$ where $\gamma \in (-1, 0)$. Thus, one might wonder whether the difference we present truly flows from the behavioral difference associated with the CCE or simply from the different conjectures associated with the two equilibria we compare.

To explore this we reproduce the derivation through (7) under the assumption that $\gamma_0 = \gamma_1 = \gamma$. Solving the two best response functions yields the quantities of each firm as a function of the shared conjecture γ and the privatization share λ Returning these to (1) and maximizing yields a closed form solution for the optimal extent of privatization:

$$\lambda^* = \frac{\gamma^2 + 3\gamma + 1}{2\gamma^2 + 7\gamma + 5} \tag{11}$$

Note that when $\gamma = 0$, (11) yields the optimal privatization associated with the CNE, $\lambda^* = 0.200$.

The result in (11) allows us to broaden the comparison in our initial proposition.

Proposition 3 *The optimal extent of privatization is larger in the CCE, $\lambda^* = 0.329$, than in any shared conjecture $\gamma \in (-1, 0)$ including the CNE.*

Proof From (11) when $\gamma = 0$, $\lambda^* = 0.200$ and $\partial\lambda^*/\partial\gamma > 0$.

Thus, all shared conjectures between Bertrand and Cournot have an optimal privatization share smaller than that associated with Cournot which is already smaller than that associated with the CCE. This makes clear that it is not the size of the conjecture per se that is driving the additional privatization. It is the behavioral assumption of the CCE that causes conjectures and behavior to change with partial privatization that generates the larger share.

The next assumption we explore is that of the shared cost structure used so often in the literature. While common, it is by no means the only reasonable assumption. In some cases, researchers consider a public firm with higher costs (White 2001). This can be implemented in several ways but perhaps the most straightforward retains convex costs adds a linear term for the public firm. Thus, the private cost structure remains as before, $C(q_1) = \varphi + \frac{1}{2}kq_1^2$, while that of the public firm becomes $C(q_0) = \varphi + \delta q_O + \frac{1}{2}kq_0^2$. There exists a constant wedge of δ per unit driven between the cost of the public and private firm. It is clear that the introduction of this additional inefficiency increases the optimal degree of privatization in either the CNE or the CCE. Following precisely the steps outlined in the previous section, a slightly more complicated version of the outputs in (10) can be derived and then returned to the welfare function. The resulting optimization yields the privatization share in the CCE as a function of the demand intercept, a, and the linear cost parameter δ. Figure 1 graphs the optimal privatization share as a function of the proportion (δ/a) for the CCE together with the analogous relationship for the CNE. Note that when $\delta = 0$, the respective privatization shares of 0.329 and 0.200 are returned. As δ increases, the privatization ratio grows for both equilibria but grows more quickly for the CCE. Thus, the introduction of the cost wedge does nothing to change the initial proposition that the extent of privatization will be greater with consistent conjectures.

As a further display of robustness we retain the original shared convex costs but imagine alternative slopes for the marginal cost curve, k. We start by noting that as the slope of the marginal cost curve shrinks, the CCE converges toward Bertrand conjectures.

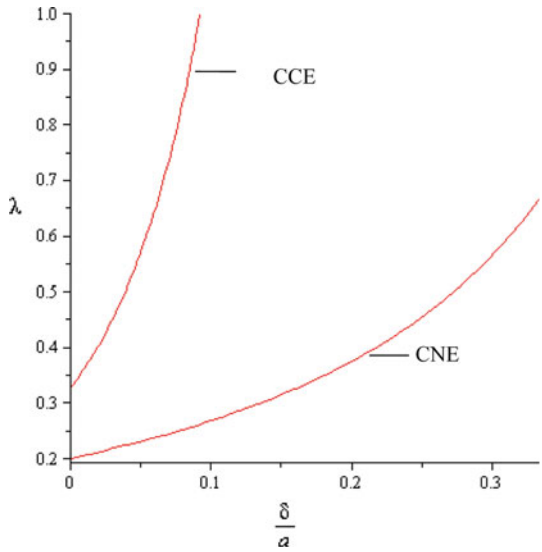


Fig. 1 Optimal privatization ratio (λ) for different values of the public firm's linear cost parameter (δ/a)

Proposition 4 (i) *The flatter is the marginal cost curve, the greater is the extent of strategic substitution,* $\frac{d\gamma_i^*}{dk} > 0, i = 0, 1$. (ii) *The consistent conjectures for* $0 < k < \infty$ *are* $-1 < \gamma < 0$ *as* $\lim_{k\to 0} \gamma_i = -1$ *and* $\lim_{k\to\infty} \gamma_i = 0$.

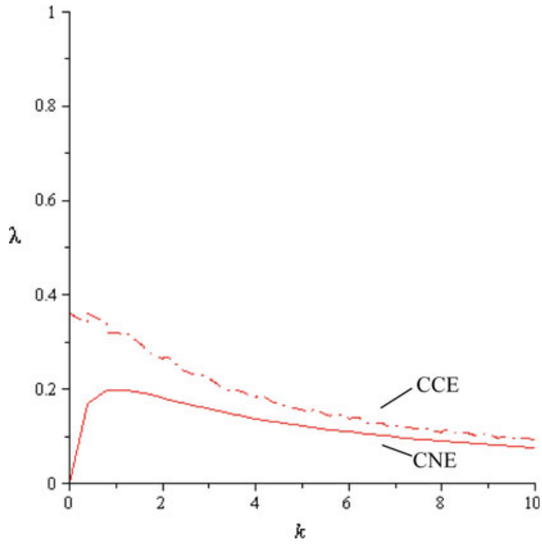
Proof Both contentions follow immediately from Eq. (4).

This represents the mixed oligopoly equivalent of Perry (1982) showing that for a private oligopoly the consistent conjectures will be between Cournot and Bertrand (competitive) depending on the marginal cost slope.

The greater strategic substitutability (resulting from the more negative conjectures) limits the willingness of the public firm to increase output. When compared to the CNE, increased output from the public firm brings larger decreased output from the private firm. This makes the asymmetry in cost greater for a given level of total market output reducing the extent to which the welfare maximizing public firm will increase output. Figure 2 graphs the optimal privatization share as a function of the slope of marginal (k) for the CCE together with the analogous relationship for the CNE. These follow from again the open form solution that generated Proposition 1 but instead of substituting $k = 1$, the values of λ are plotted against the values of k. To do this we assumed $a = 10$ although this is not critical. As k grows, the resulting reduction in production asymmetry in the CCE appears to reduce the optimal extent of privatization moving it closer to the CNE.

As a final sensitivity test, we briefly consider enlarging the number of private firms. We know that in the CNE as the number of private firms grows, the equilibrium increasingly approaches the competitive case diminishing the realm fin which the public firm can increase welfare (DeFraja and Delbono 1989). Reflecting this, in the CNE the

Fig. 2 Optimal privatization ratio, λ, given the marginal cost slope, k



optimal privatization share increases with the number of private firms, $n - 1$:

$$\lambda^* = \frac{(n-1)k}{1 + k + k^2 + nk} \tag{12}$$

For the CCE, our attempt to generalize has been far less successful. To at least give a flavor, we imagine a case with three firms, a public firm and two private firms. We define $\gamma_{ij} = \frac{\partial q_i}{\partial q_j}$ and $\rho_{ij} = \frac{\partial Br_i}{\partial q_j}$. The six derivatives of the best response functions are as follows

$$\begin{aligned} \rho_{0i} &= -\frac{(1-\lambda)(k\gamma_{i0} + \gamma_{i0} + \gamma_{j0}) + 1}{\gamma_{i0} + \gamma_{j0} + k + 1} \\ \rho_{ij} &= -\frac{1}{\gamma_{ji} + \gamma_{0i} + k + 2} \end{aligned} \tag{13}$$

where $i = 1, 2,\ j = 0, 1, 2$ and $i \neq j$. Therefore, six consistent conjecture conditions, $\gamma_{ij} = \rho_{ij}$, implicitly generate equilibrium and optimal quantities. Yet, the solution of the consistent conjectures does not generate a closed solution as it did in the two firm case (9). Thus, the consistent conjectures remain implicit and depend on both k and λ. To proceed we assumed $k = 1$ and began a grid search over λ generating conjectures, outputs and so welfare for each chosen value of λ. This process showed welfare maximized at complete privatization $\lambda^{CCE} = 1$, with all three firms sharing consistent conjectures of -0.5. This result hints that the realm for a partially public firm may be limited in multiple firm cases of the CCE. Moreover, the welfare exceeds that with optimal privatization in the CNE which has a privatization share of only 1/3. While supportive of the general finding of greater optimal privatization in the CNE, a more complete characterization of the case beyond duopoly must wait for further research and may not be easily tractable.

5 A mixed duopoly with a foreign firm

In this section, we imagine that the private firm which competes against the public firm is foreign. As a consequence, the profit of the private firm is assumed to be expatriated and does not form part of domestic welfare (Fjell and Pal 1996). The possibility of a public firm competing against foreign private firms has been extensively studied and has been used to study open door policies, cross-border mergers and strategic trade policy (see for example Pal and White 1998). More recently, Lin (2007) has derived the optimal extent of partial privatization assuming a Cournot–Nash Equilibrium and shows that replacing a domestic private firm with a foreign private firm reduces the optimal extent of privatization. Moreover, Kalashnikov et al. (2009) argue for the importance of considering the conjectural variations equilibrium for mixed oligopolies that include foreign firms. Yet, no one has considered what optimal partial privatization would look like in such a CCE and how it would compare with the otherwise similar CNE. Again, as the CCE can provide a shortcut to a fully dynamic model, such an undertaking is valuable.

Following convention, we assume the welfare objective of a fully public firm and of the domestic government to be the sum of consumer surplus and domestic profit. Foreign production can add to consumer surplus by lowering the price but foreign profit fails to enter the domestic welfare function as it is transferred out the country:

$$W = CS + \pi_0 \tag{14}$$

The foreign private firm still maximizes its profit and the public firm maximizes the objective function, G from (6) given (14):

$$G = (1 - \lambda)CS + \pi_0 \tag{15}$$

The first order conditions from the simultaneous quantity game generate best response functions in terms of the parameters and the conjectures:

$$\begin{aligned} Br_0 : q_0 &= \frac{a - (\lambda\gamma_0 + \lambda - \gamma_0)q_1}{\lambda\gamma_0 + \lambda + 2} \\ Br_1 : q_1 &= \frac{a - q_0}{\gamma_1 + 3} \end{aligned} \tag{16}$$

The actual behavioral response along those functions can again be determined:

$$\begin{aligned} \rho_1 &= -\frac{\lambda\gamma_0 + \lambda - \gamma_0}{\lambda\gamma_0 + \lambda + 2} \\ \rho_0 &= -\frac{1}{\gamma_1 + 3} \end{aligned} \tag{17}$$

The condition for the CCE requires that $\gamma_i = \rho_i$ which generates:

$$\gamma_0 = -\frac{2\lambda + 4}{6 + 3\lambda + \Gamma}$$
$$\gamma_1 = -\frac{6 + 3\lambda - \Gamma}{2\lambda + 4} \tag{18}$$

where $\Gamma = \sqrt{28 + 16\lambda + \lambda^2}$. Note that (18) is identical to (9) implying that Proposition 4 still holds and that the consistent conjectures continue to decrease (become more negative) as the degree of privatization increases for any k.

Substituting (18) into the best responses in (16) and solving for the quantities yields:

$$q_0 = \frac{(3 - \lambda)\Gamma + \lambda + 12 - \lambda^2}{(6 + \lambda)\Gamma + \Gamma^2} a$$
$$q_1 = \frac{\left[(3 + 2\lambda)\Gamma + 16 + 15\lambda + 2\lambda^2\right](4 + 2\lambda)}{\left[(6 + \lambda)\Gamma + \Gamma^2\right](6 + 3\lambda + \Gamma)} a \tag{19}$$

By setting $\lambda = 0$ in (19), the fully public CCE can be compared to an analogous CNE that also imposes that $\gamma_0 = \gamma_1 = 0$. This shows that CCE with a foreign firm has lower output, higher price and profits for both firms and greater welfare than the CNE (see Table 2 for the comparison).

Substituting the quantities from (19) into the welfare function (14) and maximizing with respect to λ generates the optimal degree of privatization: $\lambda^* = 0.038$. The smaller degree of privatization in the presence of a foreign firm conforms to the results from the CNE as shown by Lin (2007). Privatization continues to cause the public firm to produce less decreasing total output and consumer surplus. In a fully domestic oligopoly this loss in consumer surplus is offset by both an increase in the profit of the previously public firm and an increase in the profit of the private firm. With a foreign

Table 2 Comparing the equilibriums when $k = 1$ for case with a foreign firm

	CNE (public)	CCE (public)	CNE (part private)	CCE (part private)
λ	0	0	0.091	0.038
γ_0	0	−0.354	0	−0.356
γ_1	0	−0.177	0	−0.188
q_0	$0.500a$	$0.467a$	$0.471a$	$0.458a$
q_1	$0.167a$	$0.189a$	$0.176a$	$0.193a$
Q	$0.667a$	$0.656a$	$0.647a$	$0.651a$
P	$0.333a$	$0.344a$	$0.353a$	$0.349a$
π_0	$0.042a^2$	$0.052a^2$	$0.057a^2$	$0.56a^2$
π_1	$0.041a^2$	$0.047a^2$	$0.045a^2$	$0.048a^2$
CS	$0.222a^2$	$0.215a^2$	$0.209a^2$	$0.212a^2$
W	$0.264a^2$	$0.267a^2$	$0.265a^2$	$0.268a^2$

CNE Cournot–Nash equilibrium, *CCE* consistent conjectures equilibrium
$W = CS + \pi_0$

firm the only offset is the increase in the profit of the previously public firm. As a consequence, the optimal extent of privatization chosen by the government is smaller.

What is surprising is that the optimal extent of privatization in the presence of a foreign firm is actually far smaller in the CCE than in the CNE. This reverses the ordering we derived for the fully domestic oligopoly.

Proposition 5 *With a private foreign firm the optimal extent of privatization is smaller in the CCE,* $\lambda^* = 0.038$, *than in the CNE,* $\lambda^* = 0.091$.

Proof The CCE result follows from the derivation above while those for the CNE come from constraining $\gamma_0 = \gamma_1 = 0$ in (16) and solving for the quantities, returning those to (14) and maximizing with respect to λ.

The reversal in the ordering of the extent of privatization does not originate in the relationship between the conjectures as we emphasized that (18) and (9) are identical. Simply put, the response functions for the conjectures do not change as a result of replacing the domestic private firm with a foreign firm. Moreover, the best response function in quantities for the private firm is identical regardless of whether it is foreign or domestic (compare 10 and 19). The source of the reversal is in the best response function in quantities for the previously public firm. See Table 2 for a comparison of the partially privatized CNE and CCE.

The critical point is that the slope of the reaction function for the previously public firm under CCE is flatter (closer to 0) than under CNE when the private firm is domestic. Yet, it is steeper (closer to -1) than that under CNE when the private firm is foreign. This explains the reversal in the extent of privatization.

Proposition 6 *The strategic substitutability of private firm output in the CCE is lower than that in the CNE when that firm is domestic but higher than that in the CNE when that firm is foreign.*

Proof In the neighborhood of the optimal partial privatization, substituting (9) into (8) generates $\rho_1|_{\gamma=\gamma^{\text{CCE}}} - \rho_1|_{\gamma=\gamma^{\text{CNE}}} > 0$ for a domestic mixed oligopoly ($\rho_1 < 0$), but substituting (18) into (17) generates $\rho_1|_{\gamma=\gamma^{\text{CCE}}} - \rho_1|_{\gamma=\gamma^{\text{CNE}}} < 0$ for a mixed oligopoly with a foreign firm ($\rho_1 < 0$).

Understanding this proposition provides intuition for the reversal from Propositions 1 and 5. In the CCE privatization also causes the public firm to recognize that private firm output is a greater strategic substitute (ρ_0 becomes more negative). A given increase in public firm output to increase welfare causes a larger decrease in private firm output and greater cost asymmetry. First, as ρ_0 is now more negative under the CCE than under the CNE, privatization causes a larger private firm output response and more profit expatriated in the CCE than the CNE. Second, the fact that ρ_1 is more negative under the CCE than the CNE encourages the foreign firm to produce more with privatization (because increases of its output are associated with smaller reductions in public output). Both of these points imply that the government should adopt a smaller extent of privatization in the CCE. In summary, the different objective function and the opposite relative steepness of the public firm reaction function explains the reversal in the extent of privatization. Among the policy implications is that foreign acquisition of the domestic private firm should be responded to with a decreased degree of privatization.[7] The reduction would by approximately half (0.200 to 0.091) in the CNE but would be 90% (0.320 to 0.038) in the CCE.[8]

As a final technical note, we emphasize that while the process of privatization increases the strategic substitutability in all cases, $\frac{d\rho_1}{d\lambda} < 0$, the rate at which it does so varies across cases. The rate of increase for the CCE exceeds that for CNE when the private firm is domestic, i.e. $\frac{d(\rho_1|_{\gamma=\gamma^{\mathrm{CCE}}} - \rho_1|_{\gamma=\gamma^{\mathrm{CNE}}})}{d\lambda} < 0$. The rate of increase for the CNE exceeds that for CCE when the private firm is foreign, i.e. $\frac{d(\rho_1|_{\gamma=\gamma^{\mathrm{CCE}}} - \rho_1|_{\gamma=\gamma^{\mathrm{CNE}}})}{d\lambda} > 0$. As, the first difference is negative and second difference is positive, we note that the process of privatization decreases the differences in substitutability across the two equilibriums.

6 Conclusions

Several points of the paper deserve concluding emphasis. First, regardless of equilibrium concept, the extent of optimal privatization is substantially greater when the public firm faces a domestic private firm rather than a foreign private firm. Second, the extent of this difference depends on underlying equilibrium concept. In the CCE, the difference is a multiple of 10 while in CNE, it is about double. Third, the ordering of the optimal extent of privatization in the two major equilibrium concepts differs by the ownership of the private firm. When the private firm is domestic, the extent of privatization is greater in the CCE. When the private firm is foreign, the extent of privatization is greater in the CNE. Fourth, we show that in the CCE, the very process of privatization generates greater strategic substitutability and that this influences the optimal extent of privatization

Policy makers should thus account for two offsetting influences when considering the extent of privatization. If they believe long lived rivals have engaged in dynamic adaption (CCE) and the private rival is domestic, the extent of privatization should be greatest. It should decline if the rival is domestic but the dynamic adaption is unlikely (CNE). It is should decline still further if the rival is foreign and the dynamic adaption is unlikely (CNE). It should be smallest if rival is foreign and dynamic adoption is likely (CCE). This ordering stands as one contribution of this research.

Future work may move in several broad directions. First, further sensitivity tests may be warranted. Our effort to expand beyond duopoly is only suggestive and more work is warranted. If successful, this could allow consideration of endogenizing the number of firms by considering free entry and exit (Matsumura and Kanda 2005). Second, this work might be used in the host of applications in which the optimal degree of privatization has been derived using the Cournot assumption. These include the presence of both domestic and foreign firms, the presence of an R&D rivalry and many

[7] Obviously, if the foreign private firm is acquired by domestic concerns, this is simply reversed with share of privatization increasing.

[8] While these changes assume $k = 1$, alternative marginal slopes such as 0.5, 2 and 3 also show dramatically different implied reductions. These simulations are available from the authors upon request.

others. Moreover, generalizing the CCE beyond the mixed duopoly and examining the endogenous timing of moves (Pal 1998) in the CCE could also be examined.

Appendix: Proposition 2 for general values of *k*

The reaction functions in (7) and the consistent conjectures in (9) will now be in terms of k:

$$\begin{aligned} \gamma_0 &= -\frac{(2+2k+2\lambda)}{(2+3k+2\lambda+k^2+\lambda k+\Psi)} \\ \gamma_1 &= -\frac{2+3k+k^2+2\lambda+\lambda k-\Psi}{2+2\lambda+2k} \end{aligned} \tag{A1}$$

where (9) can be recovered by substituting $k = 1$. Critically, Proposition 3 still holds:

$$\begin{aligned} \partial\gamma_0/\partial\lambda &= -\frac{(8k+4k^2)(1+\lambda+k)}{\Psi(2+3k+2\lambda+k^2+\lambda k+\Psi)^2} < 0 \\ \partial\gamma_1/\partial\lambda &= -\frac{2k+k^2}{\Psi(1+\lambda+k)} < 0 \end{aligned} \tag{A2}$$

where $\Psi = \sqrt{k(8+5k+k\lambda+k^2)(1+\lambda+k)}$.

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