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# On the efficiency of indirect taxes in differentiated oligopolies with asymmetric costs

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**Abstract** This paper shows that unit taxation can be welfare superior to ad valorem taxation in asymmetric and differentiated oligopolies if the goods are sufficiently differentiated, the cost variance is sufficiently large and the ad valorem tax rate is sufficiently high. Moreover, this result holds under either Cournot competition or Bertrand competition.

**Keywords** Unit tax · Ad valorem tax · Cournot competition · Bertrand competition

**JEL Classification** D43 · H21 · L13

## 1 Introduction

The recognition that unit (or specific) taxation and ad valorem taxation may lead to different outcomes under imperfect competition dates back to Cournot (1838) and Wicksell (1896). Suits and Musgrave (1953) were the first to show in a general monopoly setting that ad valorem taxation is welfare superior to unit taxation in that the former yields a larger total surplus than the latter with the same tax revenue. Based on product homogeneity and Cournot competition, a recurrent finding in the literature on taxation in an oligopolistic industry is that ad valorem taxation welfare dominates unit taxation. In particular, Delipalla and Keen (1992) and Anderson et al. (2001b) extend the classical result on the superiority of ad valorem taxation in the monopoly setting

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to Cournot oligopolies.[1] These results confirm the insight that oligopolistic price distortion is exacerbated less by an ad valorem tax than by a unit tax in imperfectly competitive markets.

By studying a differentiated goods oligopoly with asymmetric costs, we show in this paper that unit taxation can be welfare superior to ad valorem taxation if the goods are sufficiently differentiated, the cost variance is sufficiently large, and the ad valorem tax rate is sufficiently high. Moreover, this result holds under either Cournot competition or Bertrand competition. We first show how average output levels and output variances affect tax revenue, consumer surplus and total surplus in an asymmetric and heterogeneous oligopoly by obtaining closed-form expressions for these measures. We further show how these measures are affected differently under the two tax regimes by analyzing the different weights in their expressions. Finally, we show that either direction of welfare domination between the two tax regimes is possible.

The intuition for our new result is as follows. Both imperfect competition and indirect taxation create price distortions, but price distortion of imperfect competition is exacerbated less by ad valorem taxation than by unit taxation. Cost asymmetry can alleviate the welfare loss from these distortions since the more efficient firms will take up larger output shares and therefore lower the total cost of producing a given level of output. With homogeneous goods, cost asymmetry strengthens the position of ad valorem taxation due to higher efficiency in production allocation. However, when goods are heterogeneous, output variation can make a sufficiently high contribution to total surplus under unit taxation relative to ad valorem taxation. When the above mentioned sufficient conditions hold, this bolsters the position of unit taxation so much so that the afore-mentioned welfare superiority result can be reversed.

In the ensuing 50 odd years since Suits and Musgrave (1953), many developments have been advanced in regards to the two common forms of commodity taxation. Keen (1998) provides a comprehensive review of both theoretical and empirical results in the earlier literature. While the earlier theoretical literature mostly focused on monopoly and homogeneous Cournot oligopolies with identical cost functions, recent studies have both extended and broadened the market structure and mode of competition adopted in the earlier literature. Denicolo and Matteuzzi (2000) and Anderson et al. (2001b) show that the superiority of ad valorem taxation is still valid in a homogeneous Cournot market when firms have non-identical costs. Anderson et al. (2001b), Schröder (2004), and Liu and Saving (2005) all study the relative efficiency issue in the Bertrand price competition setting with differentiated goods. In particular, both Schröder (2004) and Liu and Saving (2005) work with identical costs and a symmetric demand system derived from a form of the Dixit–Stiglitz (1977) type utility function. They establish that in their symmetric markets the superiority of ad valorem taxation continues to hold under Bertrand competition.[2] Anderson et al. (2001b) obtain a similar conclusion

[1] Skeath and Trandel (1994) find that the stronger result that ad valorem taxation Pareto dominates (i.e., greater tax revenue, profit and consumer surplus) unit taxation always holds in the monopoly setting and holds true in a Cournot oligopoly when tax level is high.

[2] By advancing a comparative profitability analysis between ad valorem and unit taxes, Liu and Saving (2005) give a political economy explanation of tax policy in that the government may be following producers' interest in choosing a tax policy. In this view, it may be rational for a government that favors producers' interests to adopt unit taxation since it gives rise to higher firm profits in some circumstances.

working with a general symmetric demand system and symmetric costs. However, they illustrate by using the Hotelling linear city duopoly model that the superiority result can be reversed if firms have different costs.[3] Anderson et al. (2001a) also study the two common tax forms under Bertrand competition in a symmetric market. However, their focus is on tax incidence. They show that results under Bertrand competition with differentiated products largely corroborate Cournot markets with homogeneous good in that the particular type of excise tax can have different implications for tax incidence, overshifting of taxes can occur, and firm profits can rise under either taxes.

Recent studies have also examined the two tax regimes under monopsony (Hamilton 1999) and in a general equilibrium setting (Grazzini 2006; Blackorby and Murty 2007). Hamilton (1999) shows that the relative efficiency result under monopoly is reversed under monopsony. Grazzini (2006) studies a special two-sector general equilibrium model where one of the sectors is composed of a Cournot oligopolistic industry. She shows that in this model, unit taxation welfare dominates ad valorem taxation. Blackorby and Murty (2007) study a general equilibrium model with a monopoly sector and show that the set of unit-tax Pareto optima is the same as the set of ad valorem-tax Pareto optima.

Although unit taxation has been shown to be welfare superior to ad valorem taxation under certain circumstances in some settings (i.e., Bertrand, monopsony, or general equilibrium settings), our result that unit taxation can welfare dominate ad valorem taxation under Cournot competition is new. Our result on the possibility of reversal of comparative ranking of the two tax regimes under Bertrand competition corroborates existing findings in the literature (Anderson et al. 2001b). Both the empirical relevance of oligopolistic market structures and the overwhelming applications of oligopoly models in economic research attest to the importance of a better understanding of tax policies in oligopolistic markets. Based on their conclusion that ad valorem taxation is welfare superior to unit taxation in quantity competition and their example of welfare domination of unit taxation over ad valorem taxation under price competition, Anderson et al. (2001b) state in their conclusion that "This leaves open the question whether it is the mode of competition or the introduction of product differentiation that is primarily responsible for the difference in results" (p. 249). We show that it is not the mode of competition but rather product differentiation and cost asymmetry that are responsible for the difference in results.

The rest of the paper is organized as follows. Section 2 presents the basic model and the solutions under Cournot and Bertrand competition. Sections 3 and 4 investigate the relative efficiency of unit and ad valorem taxation under Cournot and Bertrand competition, respectively. Section 5 provides some concluding remarks. The appendix provides proofs.

[3] While the present paper focuses on the short run setting in which the number of firms is fixed, both Liu and Saving (2005) and Anderson et al. (2001b) consider short run as well as long run competition in which the number of firms is variable due to entry and exit. Liu and Saving's long run model is essentially a monopolistic competition model. They confirm their short-run finding of welfare dominance of ad valorem tax over unit tax in their long run model. Anderson et al. (2001b) use a discrete choice model and find that unit taxation welfare dominates ad valorem taxation in the long run. Kay and Keen (1983) also provide an example in which unit taxation welfare dominates ad valorem taxation under price competition in the long run.

## 2 The basic model

Consider an $n$-firm differentiated goods oligopoly model that is a direct generalization of the duopoly model developed by Dixit (1979). There is a representative consumer with the following quasi-linear utility function:

$$U = I + \alpha q^T e - \frac{q^T H q}{2}. \tag{1}$$

In (1), $I$ is a composite measure of the consumer's consumption of all other goods, $q$ is an $n \times 1$ column vector of outputs, $e$ is an $n \times 1$ column vector of ones, $H \equiv (1-\gamma)I_n + \gamma E_{nn}$ is the consumer's Hessian matrix for preferences ($I_n$ is the $n \times n$ identity matrix and $E_{nn}$ is an $n \times n$ matrix of ones), and $\gamma \in (-1/(n-1), 1]$ is the substitution rate with $\gamma > 0$ ($=0$ or $<0$) representing substitute (independent or complementary) goods.[4]

Let $p$ denote the $n \times 1$ column vector of prices, $M$ the consumer's income, and the composite good's price be normalized to 1. Maximizing $U$ subject to the budget constraint that $p^T q + I \leq M$ gives the following inverse demand equations:

$$p = \alpha e - Hq. \tag{2}$$

Let $c$ denote the $n \times 1$ column vector of constant unit costs of production. For convenience, we assume that $c_1 \leq c_2 \leq \cdots \leq c_n$, so firm 1 is the most efficient firm and firm $n$ is the least efficient firm. To keep the same approach as adopted in the literature and also for tractability, we assume that all firms are taxed at the same rate $t (t > 0)$ per unit of output under unit taxation and the same rate of $\tau (0 < \tau < 1)$ fraction of gross revenue under ad valorem taxation.[5]

### 2.1 Quantity competition

We first present the equilibrium under unit and ad valorem taxation when firms compete in quantities à la Cournot. With a unit tax at the rate of $t$ per unit of output, firm $i$'s profit function is given by $\pi_i = p_i q_i - c_i q_i - t q_i$, where $p_i$ is given by (2). Assuming

[4] The requirement that $\gamma > -1/(n-1)$ guarantees that the preferences are concave (i.e., the Hessain matrix H is negative semi-definite).

[5] This assumption applies to all of the papers cited in the introduction section that study taxes in a differentiated goods model. In reality, unit and ad valorem tax rates are usually the same for similar products. For example, different grade gasoline products are levied at the same unit (excise) rates; cigarettes of different brands and makes are taxed at the same unit rates; cars of all sizes are taxed at the same ad valorem rates in each locale that has a sales tax on automobiles; etc. Obviously, imposing different tax rates for similar products would face many difficulties in implementation because of likely ambiguity and manipulation in the classification and naming of products. Although not modeled in the present paper, another justification for the use of the same tax rates across different goods is the likely presence of informational asymmetry between the government and industry insiders (firms).

that all firms produce a positive quantity in the Cournot equilibrium (or Cournot–Nash equilibrium),[6] it is straightforward to find the equilibrium output and price levels as given by[7]

$$q^u = \bar{q}^u e + \frac{1}{2-\gamma}(\bar{c}e - c), \tag{3}$$

$$p^u = \bar{p}^u e + \frac{1}{2-\gamma}(c - \bar{c}e), \tag{4}$$

where $\bar{c} = c^T e/n$ denotes the industry's average unit cost,

$$\bar{q}^u = \frac{\alpha - \bar{c} - t}{2+(n-1)\gamma} \quad \text{and} \quad \bar{p}^u = \frac{\alpha + [1+(n-1)\gamma](\bar{c}+t)}{2+(n-1)\gamma} \tag{5}$$

are the industry's average output and average price, respectively. It is obvious from (3) and (4) that $q_1^u \geq \cdots \geq q_n^u$ and $p_1^u \leq \cdots \leq p_n^u$, namely firms' equilibrium outputs (prices) are ranked according to their efficiency levels with the most efficient firm 1 producing the most (charging the lowest price) and the least efficient firm n producing the least (charging the highest price). However, if $c_1 = c_2 = \cdots = c_n$ then all firms produce the same level of output and charge the same price.

With an ad valorem tax levied at the rate of $\tau$ fraction of gross revenue, firm $i$'s profit function is $\pi_i = (1-\tau)p_i q_i - c_i q_i$. Assuming that all firms produce a positive quantity in the (unique) Cournot equilibrium, the equilibrium output and price levels are given by

$$q^a = \bar{q}^a e + \frac{1}{(1-\tau)(2-\gamma)}(\bar{c}e - c), \tag{6}$$

$$p^a = \bar{p}^a e + \frac{1-\gamma}{(1-\tau)(2-\gamma)}(c - \bar{c}e), \tag{7}$$

where

$$\bar{q}^a = \frac{\alpha - \frac{\bar{c}}{1-\tau}}{2+(n-1)\gamma} \quad \text{and} \quad \bar{p}^a = \frac{\alpha + [1+(n-1)\gamma]\frac{\bar{c}}{1-\tau}}{2+(n-1)\gamma} \tag{8}$$

are the industry's average output and average price at the equilibrium. From (6) and (7), $q_1^a \geq \cdots \geq q_n^a$ and $p_1^a \leq \cdots \leq p_n^a$. If $c_1 = c_2 = \cdots = c_n$ then all firms produce the same output level and charge the same price.

Comparing (5) and (8), one sees that if $\tau = t/(\bar{c}+t)$ then $\bar{q}^u = \bar{q}^a$. Since much of our arguments and discussions below will revolve around equal average outputs (hence equal total outputs) under the two tax regimes, for convenience, we highlight

[6] It is well-known that such equilibrium is unique in the present linear model.

[7] Here and henceforth superscripts "u" and "a" denote equilibrium values under unit taxation and ad valorem taxation, respectively.

the above condition relating the unit tax rate $t$ and the ad valorem tax rate $\tau$ as an assumption given below:

**Assumption 1** $\tau = \frac{t}{\bar{c}+t}$.

Applying $\bar{q}^u = \bar{q}^a$, subtracting (3) from (6) and (4) from (7) yields

$$q^a - q^u = \frac{\tau}{(1-\tau)(2-\gamma)}(\bar{c}e - c), \quad \text{and}$$
$$p^a - p^u = \frac{\tau(1-\gamma)}{(1-\tau)(2-\gamma)}(c - \bar{c}e).$$

Hence, if total outputs are the same under the two tax regimes then all efficient firms (i.e., $c_i - \bar{c} < 0$) produce more and charge lower prices under ad valorem taxation than under unit taxation, and all inefficient firms produce less and charge higher prices under ad valorem taxation than under unit taxation.

## 2.2 Price competition

We next present the equilibrium under unit and ad valorem taxation when firms compete in prices à la Bertrand. Assuming that goods are less than perfect substitutes ($\gamma < 1$),[8] inverting the demand system (2) gives the following direct demand equations:[9]

$$q = \frac{\alpha}{1+(n-1)\gamma}e - \frac{3+(n-2)\gamma}{(1-\gamma)[1+(n-1)\gamma]}p + \frac{1}{(1-\gamma)[1+(n-1)\gamma]}Hp. \tag{9}$$

With a unit tax at the rate $t$, firm $i$'s profit function is $\pi_i = p_i q_i - c_i q_i - t q_i$. Assuming that all firms produce a positive quantity in the Bertrand equilibrium (or Bertrand–Nash equilibrium), it is straightforward to find the equilibrium price and output levels as given by[10]

$$p^{u^*} = \bar{p}^{u^*}e + \frac{1+(n-2)\gamma}{2+(2n-3)\gamma}(c - \bar{c}e), \tag{10}$$

$$q^{*} = \bar{q}^{u^*}e + \frac{1+(n-2)\gamma}{(1-\gamma)[2+(2n-3)\gamma]}(\bar{c}e - c), \tag{11}$$

[8] The case of perfect substitutes requires a separate analysis since only the most efficient firm(s) will produce in the Bertrand equilibrium. Our focus in this paper is on when all n firms produce a positive output in equilibrium.

[9] It is worthwhile to note that the demand equations in (9) are only sensible when the requirement $\gamma > -1/(n-1)$ is satisfied. This parameter requirement also ensures that the second-order condition for an interior solution under Bertrand competition holds.

[10] To distinguish from the Cournot solution, we add * to the superscript to denote the Bertrand solution.

where $\bar{p}^{u^*}$ and $\bar{q}^{u^*}$ are the average price and average output at equilibrium with a unit tax, as given by

$$\bar{p}^{u^*} = \frac{(1-\gamma)(\alpha - \bar{c} - t)}{2 + (n-3)\gamma} + \bar{c} + t \quad \text{and} \quad \bar{q}^{u^*} = \frac{[1+(n-2)\gamma](\alpha - \bar{c} - t)}{[1+(n-1)\gamma][2+(n-3)\gamma]}. \quad (12)$$

With an ad valorem tax levied at the rate $\tau$, firm $i$'s profit function is $\pi_i = (1-\tau) p_i q_i - c_i q_i$. Applying the demand system (9) and assuming that all $n$ firms produce a positive quantity in the Bertrand equilibrium, the equilibrium price and output levels are given by

$$p^{a^*} = \bar{p}^{a^*} e + \frac{1+(n-2)\gamma}{(1-\tau)[2+(2n-3)\gamma]}(c - \bar{c}e), \quad (13)$$

$$q^{a^*} = \bar{q}^{a^*} e + \frac{1+(n-2)\gamma}{(1-\tau)(1-\gamma)[2+(2n-3)\gamma]}(\bar{c}e - c), \quad (14)$$

where $\bar{p}^{a^*}$ and $\bar{q}^{a^*}$ are the average price and average output at equilibrium with an ad valorem tax, as given by

$$\bar{p}^{a^*} = \frac{(1-\gamma)\left(\alpha - \frac{\bar{c}}{1-\tau}\right)}{2+(n-3)\gamma} + \frac{\bar{c}}{1-\tau} \quad \text{and} \quad \bar{q}^{a^*} = \frac{[1+(n-2)\gamma](\alpha - \frac{\bar{c}}{1-\tau})}{[1+(n-1)\gamma][2+(n-3)\gamma]}. \quad (15)$$

By (10)–(11) and (13)–(14), firms' outputs (prices) in price competition under both tax regimes are ranked in the same (opposite) order of their unit costs, which is analogous to the ranking in quantity competition.

Comparing (12) and (15), it follows immediately that if Assumption 1 holds then $\bar{q}^{u^*} = \bar{q}^{a^*}$ and $\bar{p}^{u^*} = \bar{p}^{a^*}$. Applying $\bar{q}^{u^*} = \bar{q}^{a^*}$, subtracting (10) from (13) and (11) from (14) yields

$$p^{a^*} - p^{u^*} = \frac{\tau[1+(n-2)\gamma]}{(1-\tau)[2+(2n-3)\gamma]}(c - \bar{c}e), \quad \text{and}$$

$$q^{a^*} - q^{u^*} = \frac{\tau[1+(n-2)\gamma]}{(1-\tau)(1-\gamma)[2+(2n-3)\gamma]}(\bar{c}e - c).$$

Hence, if the total quantities are the same under the two tax regimes in price competition, all efficient firms (i.e., $c_i - \bar{c} < 0$) charge lower prices (and produce more) under ad valorem taxation than under unit taxation, and all inefficient firms charge higher prices (and produce less) under ad valorem taxation than under unit taxation.

The following basic properties of unit and ad valorem taxes and their effects on output and price variances are easily verified. (1) In both price and quantity competition, unit taxation always decreases each firm's output, and it has no effects on price and output variances. (2) In both price and quantity competition, ad valorem taxation always decreases each firm's output. Output variances under ad valorem taxation are always greater than under unit taxation, and comparison of price variances between the two tax regimes is ambiguous. (3) In quantity competition, output and price variances under unit taxation are identical, and output variance under ad valorem taxation is greater (less) than price variance if and only if goods are substitutes (complements). (4) In price competition, output variances under both tax regimes are greater (less) than price variances if and only if goods are substitutes (complements).

## 3 Efficiency comparison of unit and ad valorem taxes under Cournot competition

In this section we compare unit and ad valorem taxes under Cournot competition. For this purpose, we first derive expressions for tax revenue (TR), total profits (Π), consumer surplus (CS), and total surplus (TS) under the two tax regimes in terms of output levels at the Cournot equilibrium. As with the most common approach in the literature, total surplus is measured as the sum of consumer surplus, firm profits and tax revenue (i.e., TS = CS + Π + TR).

We start with the measurement of consumer surplus. Applying the maximizing conditions (2), consumer surplus is given by

$$\text{CS} = U - (I + p^T q) = \frac{q^T H q}{2}.$$

Substituting the identity $H \equiv (1-\gamma)I_n + \gamma E_{nn}$, one obtains

$$\text{CS} = \frac{1-\gamma}{2}(q - \bar{q}e)^T(q - \bar{q}e) + \frac{n[1+(n-1)\gamma]}{2}(\bar{q})^2. \tag{16}$$

Hence, by using (4) and (16), we have the following expressions for tax revenue, total profits, consumer surplus, and total surplus under unit taxation:

$$\text{TR}^u = nt\bar{q}^u, \tag{17}$$

$$\Pi^u = (q - \bar{q}e)^T(q - \bar{q}e) + n(\bar{q})^2, \tag{18}$$

$$\text{CS}^u = \frac{1-\gamma}{2}(q^u - \bar{q}^u e)^T(q^u - \bar{q}^u e) + \frac{n[1+(n-1)\gamma]}{2}(\bar{q}^u)^2, \tag{19}$$

$$\text{TS}^u = \frac{3-\gamma}{2}(q^u - \bar{q}^u e)^T(q^u - \bar{q}^u e) + \frac{n[3+(n-1)\gamma]}{2}(\bar{q}^u)^2 + nt\bar{q}^u, \tag{20}$$

where $q^u$ and $\bar{q}^u$ are given by (3) and (5), respectively.

Similarly, by using (7) and (16), we have the following expressions for the above measures under ad valorem taxation:

$$\text{TR}^a = \tau[n(\bar{q}^a)^2 - (1-\gamma)(q^a - \bar{q}^a e)^T(q^a - \bar{q}^a e)] + \frac{n\bar{c}\tau}{1-\tau}\bar{q}^a, \tag{21}$$

$$\Pi^a = (1-\tau)[(q^a - \bar{q}^a e)^T(q^a - \bar{q}^a e) + n(\bar{q}^a)^2], \tag{22}$$

$$\mathrm{CS}^a = \frac{1-\gamma}{2}(q^a - \bar{q}^a e)^T (q^a - \bar{q}^a e) + \frac{n[1+(n-1)\gamma]}{2}(\bar{q}^a)^2, \tag{23}$$

$$\mathrm{TS}^a = \frac{3-\gamma-2\tau(2-\gamma)}{2}(q^a - \bar{q}^a e)^T (q^a - \bar{q}^a e) + \frac{n[3+(n-1)\gamma]}{2}(\bar{q}^a)^2 + \frac{n\bar{c}\tau}{1-\tau}\bar{q}^a, \tag{24}$$

where $q^a$ and $\bar{q}^a$ are given by (6) and (8), respectively.

A direct observation of (17)–(24) reveals that all of the expressions are linear combinations of average output, average output squared and output variance. When average output is the same, comparison between the two tax regimes for any measurement rests on the difference in output variance between the two tax regimes and on the weights placed on the three terms.

We first point out a special case that produces a straightforward result. That is, under Assumption 1, if $c_1 = \cdots = c_n$ then $\mathrm{TR}^a > \mathrm{TR}^u$, $\mathrm{CS}^u = \mathrm{CS}^a$ and $\mathrm{TS}^u = \mathrm{TS}^a$. In this case, output variance is zero under either tax policy. Consequently, there are no changes in prices, outputs or consumer surplus, and there is simply a shift from profits to tax revenues. Applying the same argument as that pointed out by Anderson et al. (2001b) in their homogeneous good model, continuity implies that slightly lowering ad valorem tax rate from that determined by Assumption 1 will raise total surplus under ad valorem taxation while still keeping the tax revenue higher than that under unit taxation. Namely, if $c_1 = \cdots = c_n$ in our heterogeneous goods model then under Cournot competition ad valorem taxation is always welfare superior to unit taxation. For ease of presentation, we assume in the rest of this section that unit costs are not all equal. This assumption implies that the cost variance $\sigma_c^2 > 0$.

Lemma 1 compares each of the above four performance measures between the two tax regimes, namely tax revenue, consumer surplus and total surplus.

**Lemma 1** *Consider Cournot competition and assume Assumption 1. The following three properties of ad valorem taxation hold*:

- (i) *It provides a higher (lower) tax revenue than unit taxation does if and only if the ad valorem tax rate $\tau$ is below (above) a critical value, or precisely,* $\mathrm{TR}^a > \mathrm{TR}^u \Leftrightarrow \tau < \tau_{\mathrm{TR}}$, *where $\tau_{\mathrm{TR}}$ is given in* (34).
- (ii) *It provides a higher (equal) consumer surplus than unit taxation does for all substitution rate less than* 1 *(equal to* 1*), or precisely,* $\mathrm{CS}^a > \mathrm{CS}^u$ *if* $-1/(n-1) \leq \gamma < 1$, *and* $\mathrm{CS}^a = \mathrm{CS}^u$ *if* $\gamma = 1$.
- (iii) *It provides a higher (lower) total surplus than unit taxation does if and only if $\tau$ is below (above) a critical value, or precisely,* $\mathrm{TS}^a > \mathrm{TS}^u \Leftrightarrow \tau < \tau_{\mathrm{TS}} \equiv 2/(3-\gamma)$.

Recall that tax revenue under unit taxation depends only on average output (see (17)), and that tax revenue under ad valorem taxation depends negatively on output variance and positively on average output (see (21)). When average output is the same under the two tax regimes, which regime generates the larger tax revenue depends on the balance between the positive effect of average output and the negative effect of output variance under ad valorem taxation. This balance is completely characterized by an upper bound on the ad valorem tax rate (i.e., $\tau_{TR}$ in (34)), or alternatively, by an upper bound on the cost variance (i.e., $\sigma_{TR}$ in (33)).

An examination of (19) and (23) shows that consumer surplus is given by the same linear combination of output variance and average output squared. When average output is the same, output variance is greater under ad valorem taxation. It follows that ad valorem taxation will give rise to a larger consumer surplus except in the case the coefficient in front of the output variance term is zero. This happens only when the goods are perfect substitutes (i.e., $\gamma = 1$). Part (ii) of Lemma 1 also implies that in order for unit taxation to yield a larger consumer surplus it has to generate a larger output level than that under ad valorem taxation since output variance under ad valorem taxation is always greater than that under unit taxation.

The following proposition provides sufficient conditions such that ad valorem taxation is welfare superior to unit taxation in that the former yields greater total surplus than the latter while generating at least as much tax revenue.

**Proposition 1** *Consider Cournot competition and assume Assumption 1. Ad valorem taxation is welfare superior to unit taxation under the following two conditions:* (i) $\tau = t/(\bar{c} + t) \leq \tau_{TR}$; *and* (ii) $\tau = t/(\bar{c} + t) < \tau_{TS}$, *where* $\tau_{TR}$ *and* $\tau_{TS}$ *are the same as in Lemma 1.*

By Proposition 1, given a unit tax rate $t$, if the ad valorem tax rate $\tau$ given by $\tau = t/(\bar{c} + t)$ satisfies conditions (i) and (ii) in the proposition, then the ad valorem tax at rate $\tau$ will lead to greater total surplus than the unit tax at rate $t$ while generating at least as much tax revenue as that under the unit tax. Of course, for an arbitrary unit tax rate the ad valorem tax rate $\tau$ defined above may not always satisfy conditions (i) and (ii) in Proposition 1. This makes it possible that in some cases unit taxation can be welfare superior to ad valorem taxation, as shown in Proposition 2.

As an immediate corollary to Proposition 1, a result obtained first by Anderson et al. (2001b) for a more general homogeneous good setting is re-established for our linear model with heterogeneous goods.

**Corollary 1** *Under Cournot competition, if goods are perfect substitutes* ($\gamma = 1$) *then ad valorem taxation is always welfare superior to unit taxation.*

The next proposition provides sufficient conditions such that unit taxation is welfare superior to ad valorem taxation in that the former yields greater total surplus than the latter while generating at least as much tax revenue.

**Proposition 2** *Consider Cournot competition and assume Assumption 1. Unit taxation is welfare superior to ad valorem taxation under the following two conditions:* (i) $\tau \geq \tau_{TR}$, *and* (ii) $t > \tau_{TS}$, *where* $\tau_{TR}$ *and* $\tau_{TS}$ *are the same as in Lemma 1.*

The conditions in this proposition are satisfied if the substitution coefficient ($\gamma$) is sufficiently small (i.e., sufficiently less than 1), the cost variance ($\sigma_c^2$) is sufficiently large and the ad valorem tax rate ($\tau$) is sufficiently high. Moreover, condition (i) in Proposition 2 is more easily satisfied the larger the number of firms ($n$) is.

Obviously, the two sets of sufficient conditions in Propositions 1 and 2 are not exhaustive of all possibilities. For example, if only one of the two conditions in Proposition 2 is satisfied then we don't have definite conclusions. Similarly, if only one of the two conditions in Proposition 1 is satisfied we don't have a conclusion either. However, in terms of identifying the possibility of welfare superiority between unit taxation and ad valorem taxation, the two propositions together point out that under Cournot competition either direction of welfare domination is possible when goods are differentiated and unit costs are not identical.

## 4 Efficiency comparison of unit and ad valorem taxes under Bertrand competition

In this section we compare unit and ad valorem taxes under Bertrand competition. We have the following expressions for tax revenue, total profits, total profits plus tax revenue, consumer surplus, and total surplus under unit taxation in terms of output levels in the Bertrand equilibrium:

$$\mathrm{TR}^{u^*} = nt\bar{q}^{u^*}, \tag{25}$$

$$\Pi^{u^*} = \frac{(1-\gamma)[1+(n-1)\gamma]}{1+(n-2)\gamma}[(q^{u^*}-\bar{q}^{u^*}e)^T(q^{u^*}-\bar{q}^{u^*}e)+n(\bar{q}^{u^*})^2], \tag{26}$$

$$\mathrm{CS}^{u^*} = \frac{1-\gamma}{2}(q^{u^*}-\bar{q}^{u^*}e)^T(q^{u^*}-\bar{q}^{u^*}e)+\frac{n[1+(n-1)\gamma]}{2}(\bar{q}^{u^*})^2, \tag{27}$$

$$\begin{aligned}\mathrm{TS}^{u^*} =& \frac{(1-\gamma)[3+(3n-4)\gamma]}{2[1+(n-2)\gamma]}(q^{u^*}-\bar{q}^{u^*}e)^T(q^{u^*}-\bar{q}^{u^*}e)\\ &+\frac{n[3+(n-4)\gamma][1+(n-1)\gamma]}{2[1+(n-2)\gamma]}(\bar{q}^{u^*})^2+nt\bar{q}^{u^*},\end{aligned} \tag{28}$$

where $q^{u^*}$ and $\bar{q}^{u^*}$ are given by (11) and (12), respectively.

These measures under ad valorem taxation and Bertrand competition are given by:

$$\begin{aligned}\mathrm{TR}^{a^*} =& -\tau(1-\gamma)(q^{a^*}-\bar{q}^{a^*}e)^T(q^{a^*}-\bar{q}^{a^*}e)\\ &+\frac{n\tau(1-\gamma)[1+(n-1)\gamma]}{1+(n-2)\gamma}(\bar{q}^{a^*})^2+\frac{n\bar{c}\tau}{1-\tau}\bar{q}^{a^*},\end{aligned} \tag{29}$$

$$\Pi^{a^*} = \frac{(1-\tau)(1-\gamma)[1+(n-1)\gamma]}{1+(n-2)\gamma}[(q^{a^*}-\bar{q}^{a^*}e)^T(q^{a^*}-\bar{q}^{a^*}e)+n(\bar{q}^{a^*})^2], \tag{30}$$

$$\mathrm{CS}^{a^*} = \frac{1-\gamma}{2}(q^{a^*}-\bar{q}^{a^*}e)^T(q^{a^*}-\bar{q}^{a^*}e)+\frac{n[1+(n-1)\gamma]}{2}(\bar{q}^{a^*})^2, \tag{31}$$

$$\begin{aligned}\mathrm{TS}^{a^*} =& \frac{(1-\gamma)\{[3+(3n-4)\gamma]-2\tau[2+(2n-3)\gamma]\}}{2[1+(n-2)\gamma]}(q^{a^*}-\bar{q}^{a^*}e)^T(q^{a^*}-\bar{q}^{a^*}e)\\ &+\frac{n[3+(n-4)\gamma][1+(n-1)\gamma]}{2[1+(n-2)\gamma]}(\bar{q}^{a^*})^2+\frac{n\bar{c}\tau}{1-\tau}\bar{q}^{a^*},\end{aligned} \tag{32}$$

where $q^{a^*}$ and $\bar{q}^{a^*}$ are given by (14) and (15), respectively.

As in the case of Cournot competition, if $c_1 = \cdots = c_n$ then Assumption 1 implies that $\mathrm{TR}^{a^*} > \mathrm{TR}^{u^*}$, $\mathrm{CS}^{u^*} = \mathrm{CS}^{a^*}$ and $\mathrm{TS}^{u^*} = \mathrm{TS}^{a^*}$. Applying again the argument by Anderson et al. (2001b), continuity implies that slightly lowering ad valorem tax rate from that determined by Assumption 1 will raise total surplus under ad valorem taxation while still keeping the tax revenue higher than that under unit taxation. Namely, if $c_1 = \cdots = c_n$ then in our Bertrand competition model ad valorem taxation is always welfare superior to unit taxation. We assume in the rest of this section that unit costs are not all equal, which implies that $\sigma_c^2 > 0$.

Lemma 2 summarizes the effects of the two tax regimes on tax revenue, tax revenue plus profits, consumer surplus and total surplus, which are analogous to the properties in quantity competition given in Lemma 1.

**Lemma 2** *Consider Bertrand competition and assume Assumption 1. The following three properties of ad valorem taxation hold:*

(i) *It provides a higher (lower) tax revenue than unit taxation does if and only if the ad valorem tax rate $\tau$ is below (above) a critical value, or precisely,* $\mathrm{TR}^{a^*} > \mathrm{TR}^{u^*} \Leftrightarrow \tau \leq \tau^*_{\mathrm{TS}}$, *where $\tau^*_{\mathrm{TR}}$ is given in (36).*
(ii) *It provides a higher (equal) consumer surplus than unit taxation does for all substitution rate less than* 1 (*equal to* 1)*, or precisely,* $\mathrm{CS}^{a^*} > \mathrm{CS}^{u^*}$ *if* $-1/(n-1) \leq \gamma < 1$, *and* $\mathrm{CS}^{a^*} = \mathrm{CS}^{u^*}$ *if* $\gamma = 1$.
(iii) *It provides a higher (lower) total surplus than unit taxation does if and only if $\tau$ is below (above) a critical value, or precisely,* $\mathrm{TS}^{a^*} > \mathrm{TS}^{u^*} \Leftrightarrow \tau < \tau^*_{\mathrm{TS}} \equiv 2[1 + (n-1)\gamma]/[3 + (3n-4)\gamma]$.

The following proposition provides sufficient conditions such that under Bertrand competition ad valorem taxation is welfare superior to unit taxation in that the former yields greater total surplus than the latter while generating at least as much tax revenue.

**Proposition 3** *Consider Bertrand competition and assume Assumption 1. Ad valorem taxation is superior to unit taxation under the following two conditions:* (i) $\tau = t/(\bar{c} + t) \leq \tau^*_{\mathrm{TR}}$; *and* (ii) $\tau = t/(\bar{c} + t) < \tau^*_{\mathrm{TS}}$, *where $\tau^*_{\mathrm{TR}}$ and $\tau$ are the same as in Lemma 2.*

By Proposition 3, given a unit tax rate $t$, if the ad valorem tax rate $\tau$ given by $\tau = t/(\bar{c}+t)$ satisfies conditions (i) and (ii) in the proposition then the ad valorem tax at the rate $\tau$ will lead to greater total surplus than the unit tax at rate $t$ while generating at least as much tax revenue as that under the unit tax. Of course, for an arbitrary unit tax rate the ad valorem tax rate $\tau$ defined above may not always satisfy conditions (i) and (ii) in Proposition 3. This makes it possible that in some cases unit taxation can be welfare superior to ad valorem taxation, as shown in Proposition 4.

The next proposition provides sufficient conditions such that under Bertrand competition unit taxation is welfare superior to ad valorem taxation in that the former yields greater total surplus than the latter while generating at least as much tax revenue.

**Proposition 4** *Consider Bertrand competition and assume Assumption 1. Unit taxation is superior to ad valorem taxation under the following two conditions:* (i) $\tau = t/(\bar{c} + t) \geq \tau^*_{\mathrm{TR}}$*; and* (ii) $\tau = t/(\bar{c} + t) > \tau^*_{\mathrm{TS}}$, *where $\tau^*_{\mathrm{TR}}$ and $\tau^*_{\mathrm{TS}}$ are the same as in Lemma 2.*

The conditions in this proposition are satisfied if the substitution coefficient ($\gamma$) is sufficiently small (i.e., sufficiently less than 1), the cost variance ($\sigma_c^2$) is sufficiently large and the ad valorem tax rate ($\tau$) is sufficiently high. Moreover, condition i) in Proposition 4 is more easily satisfied the larger the number of firms ($n$) is.

Obviously, the two sets of sufficient conditions in Propositions 3 and 4 are not exhaustive of all possibilities. However, in terms of identifying the possibility of welfare superiority between unit taxation and ad valorem taxation, the two propositions together point out that under Bertrand competition either direction of welfare domination is possible when goods are differentiated and unit costs are not identical. That is, qualitatively the same conclusion as in the case of differentiated goods Cournot oligopoly studied in the previous section holds under Bertrand competition.

## 5 Concluding remarks

Based on product homogeneity and Cournot quantity competition, a recurrent finding in the literature is that ad valorem taxation is welfare superior to unit taxation in non-competitive markets. This paper has shown that in a heterogeneous goods oligopoly with asymmetric costs the above result may not hold. Moreover, this conclusion holds under either Cournot or Bertrand competition. More specifically, the paper has shown that, under either mode of competition, if the substitution coefficient is sufficiently small, the cost variance is sufficiently large and the ad valorem tax rate is sufficiently high then unit taxation can be welfare superior to ad valorem taxation. In particular, we have shown that in a heterogeneous goods Cournot oligopoly with asymmetric costs, either direction of welfare domination between unit taxation and ad valorem taxation is possible.

The above results are obtained using the Dixit-type quasi-linear utility function which leads to a symmetric linear demand system. Although the simple framework adopted here suffices for our very limited objective in illustrating that unit taxation can welfare dominate ad valorem taxation under either Cournot or Bertrand competition, such a demand system is obviously highly special and leaves much to be desired in generality.[11] By continuity, one does expect that similar results should obtain if small degrees of asymmetry are introduced into the demand system (e.g., the common demand parameter $\alpha$ is replaced by different values for different goods or the common substitution parameter $\gamma$ is replaced by different substitution parameters across goods).[12] We believe that the two kinds of asymmetries (cost asymmetry and product heterogeneity) we assume in this paper are of general relevance in obtaining a reverse welfare ranking between unit and ad valorem taxes in an oligopolistic setting.

[11] Our linear demand system does allow relatively simple expressions for the solutions under Cournot and Bertrand competitions when costs are asymmetric. However, it is unknown whether explicit solutions can be found in the symmetric nonlinear demand system adopted by Schröder (2004) and Liu and Saving (2005) if firms have asymmetric costs.

[12] Although one may use the inverse matrix formula presented in Zhao and Howe (2007) to derive explicit Cournot and Bertrand solutions for the case when the common demand parameter $\alpha$ and/or the common substitution parameter $\gamma$ is replaced by different values for different goods, the resulting solutions are too complex to obtain delineating conditions in the comparative study of unit and ad valorem taxes.

Of course, one is much less likely to obtain delineating conditions in a more general model or a model with different preference structures so that the above ranking result holds.

In addition to product differentiation, the other key assumption in obtaining the main result of this paper is (unit or marginal) cost asymmetry. Given that marginal costs cannot be directly observed, there is scant direct empirical evidence documenting cost differences among competing firms.[13] However, that has not prevented economists from making cost asymmetry assumptions in many important areas of research. For example, the whole premise of the literature on process licensing is the assumption that the licensor has a more advanced technology that corresponds to a lower marginal cost than the potential licensees (e.g., Kamien and Tauman 2003). These cost differences do not disappear post-licensing whenever any output based royalty is used in licensing. Cost asymmetry as well as cost synergy also plays an important role in the literature on horizontal mergers (e.g., Farrell and Shapiro 1990). In this story, it is assumed that the merging firms have different efficiency (cost) levels pre-merger and cost synergy enables the less efficient firm to become more efficient post-merger (Heubeck et al. 2006). Cost asymmetry has also been incorporated in numerous studies in the international trade literature. For example, the book by Lahiri and Ono (2004) presents a series of theoretical studies of important issues in international trade premised on the assumption that firms have asymmetric costs. All of these studies lend support to the assumption of cost asymmetry in the present paper.

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## Appendix

*Proof of Lemma 1* Part (i): By Assumption 1 and by (17) and (21),

$$\mathrm{TR}^a - \mathrm{TR}^u = \tau[n(\bar{q}^a)^2 - (1-\gamma)(q^a - \bar{q}^a e)^T (q^a - \bar{q}^a e)].$$

Applying (6) and (8) leads to

$$\mathrm{TR}^a - \mathrm{TR}^u = \frac{\tau n}{(1-\tau)^2[2+(n-1)\gamma]^2}\Bigg[(\alpha(1-\tau)-\bar{c})^2 - \frac{(1-\gamma)\sigma_c^2[2+(n-1)\gamma]^2}{(2-\gamma)^2}\Bigg].$$

Hence,

$$\mathrm{TR}^a > \mathrm{TR}^u \Leftrightarrow \sigma_c^2 < \sigma_{\mathrm{TR}} \equiv \frac{(2-\gamma)^2[\alpha(1-\tau)-\bar{c}]^2}{(1-\gamma)[2+(n-1)\gamma]^2}. \tag{33}$$

[13] Casual empiricism suggests that firms are different, sometimes even firms in the same industry are very different (e.g., GM vs. Toyota, Dell vs. Apple). Unfortunately, there is little empirical research documenting these differences.

Alternatively,

$$\mathrm{TR}^a > \mathrm{TR}^u \Leftrightarrow [\alpha(1-\tau)-\bar{c}]^2 - \frac{(1-\gamma)\sigma_c^2[2+(n-1)\gamma]^2}{(2-\gamma)^2} > 0$$
$$\Leftrightarrow \tau < \tau_{\mathrm{TR}} \equiv 1 - \frac{\bar{c}(2-\gamma)+\sigma_c[2+(n-1)\gamma]\sqrt{1-\gamma}}{\alpha(2-\gamma)}. \quad (34)$$

Part (ii): Substituting $\bar{q}^u = \bar{q}^a$ into (19) and (23) leads to

$$\mathrm{CS}^a - \mathrm{CS}^u = \frac{\tau(2-\tau)(1-\gamma)}{2(1-\tau)^2}(q^u - \bar{q}^u e)^T(q^u - \bar{q}^u e),$$

which leads to the conclusion.

Part (iii): Substituting Assumption 1 (i.e., $\bar{q}^u = \bar{q}^a$) into (20) and (24) leads to

$$\mathrm{TS}^a - \mathrm{TS}^u = \frac{\tau[2-\tau(3-\gamma)]}{2(1-\tau)^2}(q^u - \bar{q}^u e)^T(q^u - \bar{q}^u e),$$

which leads to

$$\mathrm{TS}^a > \mathrm{TS}^u \Leftrightarrow \tau < \tau_{\mathrm{TS}} \equiv \frac{2}{3-\gamma}.$$

□

*Proof of Proposition 1* By part (i) of Lemma 1, $\tau \le \tau_{\mathrm{TR}}$ implies that tax revenue under ad valorem taxation is at least as high as that under unit taxation. By part (iv) of Lemma 1, $\tau < \tau_{\mathrm{TR}}$ implies that total surplus is higher under ad valorem taxation than under unit taxation. Hence, the two conditions guarantee that ad valorem taxation leads to higher total surplus while generating at least as much in tax revenue as unit taxation. That is, ad valorem taxation is welfare superior to unit taxation. □

*Proof of Corollary 1* If $\gamma = 1$ then conditions (i) and (ii) in Proposition 1 hold obviously for any $\tau \in (0, 1)$ since $\tau_{\mathrm{TR}} = \tau_{\mathrm{TS}} = 1$. Hence, setting the ad valorem tax rate at $\tau = t/(\bar{c}+t)$ leads to the conclusion in Proposition 1. □

*Proof of Proposition 2* Note that the two conditions in this proposition do not involve the unit tax rate. For any ad valorem tax rate $\tau$ that satisfies conditions (i) and (ii) above, choose the unit tax rate $t$ at $t = \bar{c}\tau/(1-\tau)$. That is, set the unit tax rate such that Assumption 1 holds. Then, by Lemma 1, condition (i) above implies that tax revenue under unit taxation is at least as high as that under ad valorem taxation. By Lemma 4, condition (ii) above implies that total surplus is higher under unit taxation than under ad valorem taxation. Hence, under conditions (i) and (ii) above, the unit tax with rate $t = \bar{c}\tau/(1-\tau)$ leads to higher total surplus while generating at least as much in tax revenue as the ad valorem tax with rate $\tau$. That is, unit taxation is welfare superior to ad valorem taxation. □

*Proof of Lemma 2* Part (i): By (25) and (29),

$$\mathrm{TR}^{a^*} - \mathrm{TR}^{u^*} = \tau(1-\gamma)\left\{\frac{n[1+(n-1)\gamma]}{1+(n-2)\gamma}(\bar{q}^{a^*})^2 - (q^{a^*} - \bar{q}^{a^*}e)^T(q^{a^*} - \bar{q}^{a^*}e)\right\}.$$

Applying (15) in the above expression, we have

$$\mathrm{TR}^{a^*} - \mathrm{TR}^{a^*} = \frac{n\tau[1+(n-2)\gamma]^2}{(1-\gamma)(1-\tau)^2[2+(2n-3)\gamma]^2} \times \left\{\frac{(1-\gamma)^2[2+(2n-3)\gamma]^2[\alpha(1-\tau)-\bar{c}]^2}{[1+(n-1)\gamma][1+(n-2)\gamma][2+(n-3)\gamma]^2} - \sigma_c^2\right\}.$$

Hence,

$$\mathrm{TR}^{a^*} > \mathrm{TR}^{a^*} \Leftrightarrow \sigma_c^2 < \frac{(1-\gamma)^2[2+(2n-3)\gamma]^2[\alpha(1-\tau)-\bar{c}]^2}{[1+(n-1)\gamma][1+(n-2)\gamma][2+(n-3)\gamma]^2}, \quad (35)$$

which is equivalent to $\mathrm{TR}^{a^*} > \mathrm{TR}^{u^*} \Leftrightarrow$

$$\tau < \tau^*_{\mathrm{TR}} \equiv 1 - \frac{1}{\alpha}\left\{\bar{c} + \frac{\sigma_c[2+(n-3)\gamma]\sqrt{[1+(n-1)\gamma][1+(n-2)\gamma]}}{(1-\gamma)[2+(2n-3)\gamma]}\right\}. \quad (36)$$

Part (ii): The proof is the same as the proof of part (ii) of Lemma 1.
Part (iii): Applying $t = \frac{\tau\bar{c}}{1-\tau}$ and $\bar{q}^u = \bar{q}^a$ in (28) and (32), one has

$$\mathrm{TS}^{a^*} - \mathrm{TS}^{u^*} = \frac{n\tau[1+(n-2)\gamma]\sigma_c^2}{2(1-\gamma)(1-\tau)^2[2+(2n-3)\gamma]^2} \times \{2[1+(n-1)\gamma] - \tau[3+(3n-4)\gamma]\}.$$

Hence,

$$\mathrm{TS}^{a^*} > \mathrm{TS}^{u^*} \Leftrightarrow \tau < \tau^*_{\mathrm{TS}} \equiv \frac{2[1+(n-1)\gamma]}{3+(3n-4)\gamma}.$$

□

*Proof of Proposition 3* The proof of this proposition is similar to that of Proposition 1. □

*Proof of Proposition 4* The proof of this proposition is similar to that of Proposition 2. □

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