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A Note on Ad Valorem and Per Unit Taxation in an Oligopoly Model

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This note shows that per unit taxation welfare dominates ad valorem taxation in an oligopoly model, when the number of consumers is sufficiently high compared to the number of oligopolists. It aims to provide an alternative perspective to existing literature arguing instead the dominance of ad valorem over per unit taxation in oligopoly frameworks. Our result is obtained in a simple example which uses a strategic market game formulation to study strategic behavior at a general equilibrium level.

Keywords: imperfect competition, strategic market game, commodity taxation.

JEL Classification: H22, L13, C72, D51.

1 Introduction

The comparison between ad valorem and per unit taxes is an old issue in the public finance literature. Under perfect competition, Suits and Musgrave (1953), and Bishop (1968) show that they are equivalent. On the contrary, under monopoly, an ad valorem tax is unambiguously welfare-superior to a per unit tax that raises the same yield (Suits and Musgrave, 1953; Skeath and Trandel, 1994). More recently, the same kind of result has been extended to the oligopoly set-up.[1] The dominance of ad valorem over per unit taxation is established by Delipalla

1 See Keen (1998) for a comprehensive survey.

and Keen (1992) for the case of a symmetric Cournot oligopoly, with both a fixed number of firms and entry, and by Demicolò and Matteuzzi (2000) for the asymmetric case. Anderson et al. (2001) show instead that per unit taxation may be welfare-superior under Bertrand competition with product differentiation.[2]

To tackle this problem, all these papers use partial equilibrium models.[3] One interesting question is thus whether the dominance of ad valorem over per unit taxes still holds when oligopoly is modelled at a general equilibrium level. In fact, a general equilibrium set-up allows us to grasp several implications of both distortions due to market power and commodity taxation. First, it permits us to take into account how the choices of agents who are not price-takers may influence the decisions of other agents participating in the economy, and thus the price formation mechanism. Second, it allows us to analyse how a tax on a good may affect not only the market for this good, but also the market for another good via the market price mechanism. Recently, Schrödér (2004) has shown that the result obtained in a partial equilibrium set-up extends to a Dixit-Stiglitz framework with differentiated products, increasing returns to scale, entry/exit and love for variety.[4] However, when imperfect competition is analyzed at a general equilibrium level, two main difficulties should be stressed: the profit maximization criterion may not be optimal from the shareholders' viewpoint, and the oligopoly equilibrium is not invariant with respect

2 These authors suggest that it is the mode of competition that is responsible for this result and not product differentiation. See also Colangelo and Galmarini (1997).

3 See, however, Myles (1996) for a general equilibrium model which analyses the optimal combination of ad valorem and per unit taxes to eliminate the welfare loss due to imperfect competition.

4 The Dixit-Stiglitz model of monopolistic competition is formulated in a general equilibrium context, in which competiton is nonstrategic and is due to consumers' preference for variety. In fact, each firm does not take into account its impact on other firms, and thus their reaction. It has, however, sufficient market power to set price above marginal cost, independently of the total number of firms (Gabszewicz and Thisse, 1999).

to the normalization rule used to normalize prices (Gabszewicz and Vial, 1972).[5] Such difficulties are so severe that, at this stage of research, the spectrum of general equilibrium models allowing for the presence of imperfect competition is rather narrow. Recently, the importance of these delicate problems has also been stressed by Myles (1995) and Salanié (2000). For this reason, it seems interesting to analyse the welfare comparison between ad valorem and per unit taxes within a general equilibrium set-up, being aware of both difficulties cited above. To this end, we use a recent approach by Gabszewicz and Michel (1997) who propose a class of examples which overcome both difficulties by using a strategic market game formulation (Shapley, 1976; Shapley and Shubik, 1977).[6] More precisely, such an approach allows us to solve the first difficulty because the profit criterion can be substituted by the owners' utility criterion when firms are supposed to be owned by single individuals. Further more, such an assumption also allows us to overcome the second difficulty: by using utilities as payoffs, the resulting definition of noncooperative equilibrium is invariant with respect to the normalization rule used to define absolute prices.

In this note, we present an example of a Cournot oligopoly with two types of agents and two goods. The first type of agents behaves competitively on the exchange market, and initially owns only the first good. The second good does not exist initially in the economy, but can be produced by the second type of agents through a linear technology which uses the first good as input. In contrast to the first type, the second type of agents behaves strategically on the exchange market. Accordingly, the

5 Both difficulties do not appear in a competitive economy. First, when the firm and its shareholders take prices as given, profit maximization leads to shareholders' utility maximization. Thus, all shareholders want the firm to maximize its profits. Second, in a competitive equilibrium, only relative prices are determined. Thus, the equilibrium is invariant with respect to the normalization rule used for defining absolute prices. For an overview of these problems and related attempts proposed in the literature to solve them, see d'Aspremont et al. (1999), and Gabszewicz and Thisse (1999).

6 When firms use prices as strategic variables, Dierker and Grodal (1999) avoid the normalization problem by assuming that firms maximize *shareholders' real wealth*, which takes shareholders' demand explicitly into account. Recently, Neary (2003) circumvents such difficulty, by assuming that firms are "large" in their own sector, but "small" in the economy as a whole, so that their production plans do not affect factor prices.

economy consists of some competitive agents who are only consumers, and other strategic agents who are simultaneously consumers and producers. In line with previous literature (Delipalla and Keen, 1992), we show that a conflict of interests arises between these two groups of agents: the competitive side of the market prefers ad valorem taxation, while the strategic side of the market prefers per unit taxation. To establish whether ad valorem or per unit taxes are preferred by an aggregate welfare point of view, in this kind of model it seems natural to let aggregate welfare depend on utility levels reached by both groups of agents under the two alternative tax regimes. By using such a formulation, we show that the usual result may not arise: a per unit tax welfare dominates an ad valorem tax when the number of consumers is sufficiently high compared to the number of oligopolists.

The structure of the paper is as follows. Section 2 presents the model of homogeneous oligopoly, and Sect. 3 compares the welfare properties of ad valorem and per unit taxation. Section 4 consists of a short conclusion.

2 The Model

Consider a productive economy with two goods, 1 and 2, and including $n+m$ agents, falling into two types.[7] Agents $i, i = 1, \ldots, n$, – The *consumers* – behave competitively on the market and their initial endowment consists only of good 1. Agents j, $j = 1, \ldots, m$, – The *oligopolists* – do not initially own any good, but each owns a linear technology which produces good 2 using good 1 as input. More precisely, consider the following economy. All agents have the same utility function U defined by

$$U(x^1, x^2) = x^1 x^2,$$

where x^1 and x^2 denote individual consumption of goods 1 and 2, respectively; while initial endowments are defined by

$$a_i = \left(\frac{1}{n}, 0\right), \quad i = 1, \ldots, n \tag{1}$$

and

7 This model of homogeneous oligopoly has been proposed by Gabszewicz and Michel (1997).

$$a_j = (0, 0), \quad j = 1, \ldots, m. \tag{2}$$

Furthermore, agents of type 2 own a linear technology, defined as

$$y_j = \frac{1}{\alpha} z_j, \quad \alpha > 0, \tag{3}$$

where y_j denotes the amount of good 2, which can be produced out of an amount z_j of good 1.[8] More specifically, notice that agents of type 2 have to take two distinct decisions. Firstly, they have to decide how much of good 2 to produce, which also determines via (3) the amount z_j of good 1 to buy from agents of type 1. Secondly, they have to choose which share q_j of the amount y_j produced of good 2 to send to the market for trade (and the resulting amount $y_j - q_j$ to keep for private consumption). Clearly the equilibrium exchange rate between good 1 and good 2 depends on the amount q_j of good 2 sent by each oligopolist j to the market. This amount influences the total supply $\sum_{k=1}^{m} q_k$ of good 2, compared with the fixed total supply $\sum_{k=1}^{n} a_k^1$ of good 1.[9] Consequently, each oligopolist j can individually manipulate the exchange rate by choosing the share q_j. This gives rise to a game whose *players* are the

8 See Grezzini (2000) – the discussion paper version of the present one – for an analysis of the asymmetric case with $\alpha_j > 0, j = 1, \ldots, m$.

9 This reflects differences in behavior between a competitive and non-competitive agent. The idea behind this point is the following. A competitive agent of type 1 comes with his total endowment in good 1 to a central market-place, where the sum of these endowments is supplied for trade. A price is announced, and this determines the income of each agent as the product of this price by his initial endowment. Then, each competitive agent maximises his utility by buying back a bundle of the commodities, the value of which does not exceed his income. Thus, "a trader has a *competitive behaviour* in a particular market if he/she supplies the market-place with his/her *total* initial endowment of the corresponding commodity" (Gabszewicz and Michel, 1997, p. 219). Alternalively, it may be that a non-competitive agent of type 2 would like to supply only a restricted share of his initial endowment of good 2, keeping for his own consumption the remaining share. Since this remaining share does not transit through the market, it does not affect the market clearing mechanism. "If a trader uses this opportunity for a particular good, we say that he/she has a *non-competitive* behaviour on the corresponding market" (ibidem). The reason why an agent would like to restrict his supply of good 2 "comes from the fact that the resulting equilibrium prices can give him/her better overall market opportunities than the equilibrium prices which would obtain, should he/she supply the market with his/her total initial endowment" (ibidem).

oligopolists, with *strategies* for oligopolists j, $j = 1, \ldots, m$, defined by pairs (q_j, y_j) with $q_j \leq y_j$.

Now consider that a commodity tax is levied on good 2. This tax can take the form of an ad valorem tax at a (tax-inclusive) rate t, $0 < t < 1$, or a per unit tax τ, $0 < \tau < \frac{1}{\alpha}$.[10] In the case of an ad valorem tax, the producer price for good 2 obtains as $P^2 = p^2(1-t)$, where p^2 is the consumer price for good 2; consequently, the total tax product is given by $R_t = tp^2 \sum_{k=1}^{m} q_k$. If, in contrast, a per unit tax is imposed, the producer price is defined by $P^2 = p^2 - \tau$, and accordingly the total tax product obtains as $R_\tau = \tau \sum_{k=1}^{m} q_k$.

Given a price vector (p^1, p^2), a competitive agent i, $i = 1, \ldots, n$, solves the problem

$$\max_{x^1, x^2} \quad x^1 x^2 \qquad s.t. \qquad x^1 + px^2 \leqslant \frac{1}{n},$$

giving rise to individual demand

$$x_i(p) = \left(\frac{1}{2n}, \frac{1}{2np}\right), \qquad i = 1, \ldots, n, \tag{4}$$

where $p = \frac{p^2}{p^1}$, and the total demand for good 2 equals $\frac{1}{2p}$. Thus, the indirect utility function S of consumers becomes

$$S(p) = \left(\frac{1}{2n}\right)\left(\frac{1}{2np}\right). \tag{5}$$

Now we proceed to the definition of the *payoffs* of the game among the oligopolists. To this end, assume that producer j has selected the strategy (q_j, y_j), $j = 1, \ldots, m$. At a price vector p, the profit of oligopolist j becomes

$$\pi_j(q_j, y_j) = p(1-t)q_j - z_j, \tag{6}$$

in the case of ad valorem taxation, and

10 In this model, taxes are expressed in real terms. Since the maximum amount of good 2 which can be produced out of good 1 is $1/\alpha$ via the linear technology in (3), we assume that $\tau < 1/\alpha$.

$$\pi_j(q_j, y_j) = (p - \tau)q_j - z_j, \tag{7}$$

in the case of per unit taxation. With this profit, he can buy an amount of good 1 equal to $p(1-t)q_j - \alpha y_j$, in the case of an ad valorem tax and $(p-\tau)q_j - \alpha y_j$, in the case of a per unit tax, yielding resulting utility payoffs

$$\big(p(1-t)q_j - \alpha y_j\big)(y_j - q_j), \tag{8}$$

and

$$\big((p-\tau)q_j - \alpha y_j\big)(y_j - q_j), \tag{9}$$

respectively.[11]

Given the strategies (q_j, y_j), $j = 1, \ldots, m$, the value of p at which supply equals the demand on the market for good 2 is given by

$$\sum_{k=1}^{m} q_k = \frac{1}{2p},$$

or

$$p = \frac{1}{2\sum_{k=1}^{m} q_k}. \tag{10}$$

By substituting this equilibrium exchange rate in the utility payoffs (8) and (9), we finally obtain the payoffs of the game, namely

11 Notice that utility payoffs (8) and (9) are obtained under the constraint that $\pi_j(q_j, y_j) \geqslant 0$, or

$$q_j \geqslant \frac{\alpha y_j}{p(1-t)},$$

in the case of ad valorem taxation (see (6)), and

$$q_j \geqslant \frac{\alpha y_j}{p - \tau},$$

in the case of per unit taxation (see (7)), Gabszewicz and Michel, 1997, p. 235). This implies that oligopolist j has to sell a share $q_j > 0$ of good 2 to be able to buy the amount z_j of good 1 from agents of type 1, allowing him to produce an amount $y_j > 0$ of good 2 via (3). In other words, oligopolist j could not avoide commodity taxation by keeping for his own consumption the whole amount of good 2 which he produces, i.e., $q_j = 0$, since this would lead to negative profits, i.e., $\pi_j(q_j, y_j) < 0$ (see (6) and (7)).

$$V(q_j, y_j) = \left(\frac{1-t}{2\sum_{k=1}^{m} q_k} q_j - \alpha y_j\right)(y_j - q_j); \quad j = 1, \ldots, m, \qquad (11)$$

in the case of ad valorem taxation, and

$$V(q_j, y_j) = \left(\left(\frac{1}{2\sum_{k=1}^{m} q_k} - \tau\right) q_j - \alpha y_j\right)(y_j - q_j); \quad j = 1, \ldots, m, \qquad (12)$$

in the case of per unit taxation. At an *oligopoly equilibrium*, V must be maximal with respect to q_j and y_j, given the strategies (q_k, y_k) chosen by the oligopolists k, $k \neq j$, and this must be satisfied for all j, $j = 1, \ldots, m$. The optimality conditions with respect to q_j and y_j give

$$\frac{1}{y_j - q_j} = \frac{\frac{1-t}{2Q} - \tau - \frac{(1-t)q_j}{2Q^2}}{\left(\frac{1-t}{2Q} - \tau\right) q_j - \alpha y_j} = \frac{\alpha}{\left(\frac{1-t}{2Q} - \tau\right) q_j - \alpha y_j}, \qquad (13)$$

with $Q = \sum_{k=1}^{m} q_k$, and where $0 < t < 1$ and $\tau = 0$, in the case of wholly ad valorem taxation, while $0 < \tau < \frac{1}{\alpha}$ and $t = 0$, in the case of wholly per unit taxation. From the second equality of the above equation, we obtain that for all j, $j = 1, \ldots, m$, the equality

$$1 - \frac{q_j}{Q} = 2\frac{\alpha + \tau}{1 - t} Q, \qquad j = 1, \ldots, m \qquad (14)$$

must hold at equilibrium. Summing up Eq. (14), we get, at equilibrium,

$$Q_h^* = \frac{(m-1)(1-t)}{2m(\alpha + \tau)}, \qquad h = t, \tau, \qquad (15)$$

and

$$p_h^* = \frac{m(\alpha + \tau)}{(m-1)(1-t)}, \qquad h = t, \tau; \qquad (16)$$

where the subscript h, $h = t, \tau$, denotes hereafter a variable obtained under ad valorem or per unit taxation. Accordingly, total tax revenue is equal to

$$R_t^* = tp_t^* Q_t^* = \frac{1}{2}t, \tag{17}$$

under ad valorem taxation, and

$$R_\tau^* = \tau Q_\tau^* = \frac{(m-1)\tau}{2m(\alpha+\tau)}, \tag{18}$$

under per unit taxation.[12] Furthermore, using (14) and (15), we obtain

$$q_{jh}^* = \frac{(m-1)(1-t)}{2m^2(\alpha+\tau)}, \quad j = 1, \ldots, m; \quad h = t, \tau \tag{19}$$

and

$$y_{jh}^* = \frac{(2m\alpha+\tau-\alpha)(1-t)}{4m^2\alpha(\alpha+\tau)}, \quad j = 1, \ldots, m; \quad h = t, \tau. \tag{20}$$

Since $m > 1$ the oligopoly equilibrium has m "active" firms both in the case of ad valorem and per unit taxation.

Finally, notice that, from (16), it is easy to check that the per unit taxes are over-shifted: the consumer price rises more than the increase in tax, i.e., $\frac{\partial p_\tau^*}{\partial \tau} > 1$. On the contrary, the result on the incidence of ad valorem taxation shows that consumer price may rise less than the increase in tax (under-shifting). Furthermore, from (15), it is easily checked that an increase in either a per unit tax or an ad valorem tax leads to a reduction in the total quantity of good 2, Q_h^*, $h = t, \tau$, which oligopolists are willing to exchange on the market, thus reinforcing the distortion already generated by their strategic behavior, i.e., $\frac{\partial Q_\tau^*}{\partial \tau} < 0$ and $\frac{\partial Q_t^*}{\partial t} < 0$.[13]

3 A Comparison Between Ad Valorem and Per Unit Taxes

In this section, we compare the welfare effects of ad valorem and per unit taxation. To perform this analysis, as a basis of comparison, we use ad valorem and per unit taxes that are *revenue-neutral*, at oligopoly equi-

12 Notice that in (17) total tax revenue under ad valorem taxation does not depend on the number m of firms which can be active at equilibrium.

13 Notice that these results are specific to oligopolistic models and in line with previous literature (see, for example, Delipalla and Keen, 1992).

librium. More precisely, we consider a shift from an ad valorem tax t to a per unit tax τ which raises an equal amount of tax revenue.[14] Specifically, from equating (17) and (18), the value of τ which is used as a basis of comparison is

$$\tau = \frac{m\alpha t}{m(1-t)-1}, \tag{21}$$

with $0 < \tau < \frac{1}{\alpha}$, under the assumption that $m > \frac{1}{1-t}$ and $\alpha < \left(\frac{m(1-t)-1}{mt}\right)^{\frac{1}{2}}$. By substituting (21) into (16), it is easily checked that, under per unit taxation, the price at equilibrium is

$$p^*_\tau = \frac{m\alpha}{m(1-t)-1}, \tag{22}$$

which is strictly greater than the price under ad valorem taxation obtained in (16).

To compare the welfare properties of ad valorem and per unit taxes, we consider their effects on the *aggregate welfare*, namely the sum of the utility levels of consumers i, $i = 1, \ldots, n$, and oligopolists j, $j = 1, \ldots, m$ at equilibrium. For this case, we state the following proposition.

Proposition: A revenue-neutral shift from an ad valorem tax t to a per unit tax τ increases aggregate welfare if and only if $\frac{n}{m^2} > \frac{4}{2-t}$.

Proof: By substituting (19) and (20) into (11) for the case of ad valorem taxation, and into (12) with τ given in (21) for the case of per unit taxation, the difference in the utility level for each oligopolist j, $j = 1, \ldots, m$, under ad valorem and per unit taxes obtains as

$$V(q^*_{jt}, y^*_{jt}) - V(q^*_{j\tau}, y^*_{j\tau}) = -\frac{(2-t)t}{16m^4\alpha}. \tag{23}$$

Similarly, by substituting (16) into (5) for the case of ad valorem taxation, and (22) into (5) for the case of per unit taxation, the difference in the utility level for each consumer i, $i = 1, \ldots, n$, is given by

14 Different bases of comparison may be used. For example, Delipalla and Keen (1992) consider a small tax shift that leaves total tax payments unchanged at the initial equilibrium price, but which is not fully revenue-neutral. See Suits and Musgrave (1953) for a discussion on this point.

$$S(p_t^*) - S(p_\tau^*) = \frac{t}{4n^2 m\alpha}. \tag{24}$$

Finally, from (23) and (24), the difference in aggregate welfare with ad valorem and per unit taxes is

$$n \cdot \frac{t}{4n^2 m\alpha} + m \cdot \left(-\frac{(2-t)t}{16m^4\alpha} \right) = \frac{t}{16nm^3\alpha}(4m^2 - n(2-t)), \tag{25}$$

which is strictly negative if and only if $\frac{n}{m^2} > \frac{4}{2-t}$. □

When ad valorem and per unit taxes are compared with respect to aggregate welfare, our proposition shows that per unit taxation welfare dominates ad valorem taxation that raises an equal amount of tax revenue, if and only if the number of consumers is sufficiently high compared to the number of oligopolists. However, behind the dominance of per unit over ad valorem taxation, there is a conflict of interests between oligopolists who prefer a per unit tax (see (23)), and consumers who prefer an ad valorem tax (see (24)).[15] Accordingly, when the number of consumers is sufficiently high compared to the number of oligopolists, as required by the condition underlying our proposition, one would expect that, from an aggregate welfare point of view, ad valorem taxation should be preferred to per unit taxation.[16]

In order to provide some intuitions for the reasons why we get the opposite result, let us consider n and m so that the condition underlying our proposition, i.e., $\frac{n}{m^2} > \frac{4}{2-t}$, is satisfied. Then, let us consider separately the welfare effects of a high number of consumers, given the number of

15 Oligopolists prefer a per unit tax because it reinforces the distortion due to market power more than an ad valorem tax. To see this, we have already noticed above that the equilibrium price is lower under ad valorem than per unit taxation, i.e., p_τ^* in (22) is strictly greater than p_t^* in (16). Furthermore, by comparing q_{jt}^* in (19) with $q_{j\tau}^*$ in (19), and y_{jt}^* in (20) with $y_{j\tau}^*$ in (20), in both cases under τ given in (21), it is easy to check that both the amount of good 2 produced and that sent to the market for trade are lower under per unit than ad valorem taxes, i.e., $q_{j\tau}^* < q_{jt}^*$ and $y_{j\tau}^* < y_{jt}^*$, $j = 1, \ldots, m$. Of course, the same kind of argument explains why consumers instead prefer an ad valorem tax.

16 Thus, in our set-up, a pure efficiency criterion could not be used to evaluate a revenue-neutral shift from an ad valorem to a per unit tax: consumers always obtain a higher utility level with an ad valorem tax than a per unit tax since $S(p_t^*) > S(p_\tau^*)$, $\forall t$, $0 < t < 1$ (see (24)), while oligopolists always obtain a higher utility level with a per unit tax than an ad valorem tax since $V(q_{jt}^*, y_{jt}^*) < V(q_{j\tau}^*, y_{j\tau}^*)$, $\forall t$, $0 < t < 1$ (see (23)).

oligopolists. When the number of consumers increases, two contrasting effects arise. On the one hand, the "weight" of consumers in the aggregate welfare also increases (see (25)). But, on the other hand, since the initial endowment per consumer is assumed to shrink as their number increases, their individual loss due to a revenue-neutral shift from ad valorem to per unit taxation decreases (see (24)).[17] Since the second effect dominates the first one, for all consumers the loss due to a shift from ad valorem to per unit taxation also decreases, when their number increases. Let us now consider the welfare effects related to the number of oligopolists, given the number of consumers. When m decreases, three effects arise: two of them affect oligopolists' welfare in opposite directions, and one affects consumers' welfare. From oligopolists' viewpoint, when their number decreases, firstly, their "weight" in aggregate welfare also decreases (see (25)). But, secondly, a reduction in the number of oligopolists leads to a decrease in the total amount of good 2 sent to the market for trade, and so to an increase in the relative price of such a good.[18] In other words, when m decreases, oligopolists' market power augments. But, the previous effect is stronger under per unit than ad valorem taxes, and thus the individual gain due to a revenue-neutral shift from ad valorem to per unit taxation increases as m decreases (see (23)).[19] Since this second effect dominates the first one, for all oligopolists the gain due to a shift from ad valorem to per unit taxation also increases when their number decreases. Finally, as expected, a decrease in m, i.e., an increase in oligopolists' market power, also leads to an increase in consumers' loss due to such a shift (see (24)). To sum up, when the number of consumers is sufficiently high compared to the number of oligopolists, oligopolists' gain from a revenue-neutral shift from ad valorem to per unit taxation more than offsets consumers' loss due to such a shift, and thus aggregate welfare is higher under per unit than ad valorem taxation.

17 The assumption that the initial endowment per consumer decreases as their number increases allows us to describe the competitive side of the market in such a way that the influence of each consumer becomes more negligible as their number increases. Different descriptions of the competitive side of the market could lead to a different result.

18 By using (15), and (16) with τ given in (21), it is easy to check that $\frac{\partial Q_h^*}{\partial m} > 0$, and thus $\frac{\partial p_h^*}{\partial m} < 0$, $h = t, \tau$.

19 From (15), with τ given in (21) for the case of per unit taxation, it is easily checked that $\frac{\partial Q_\tau^*}{\partial m} > \frac{\partial Q_t^*}{\partial m}$.

Finally, two remarks are in order. First, notice that the assumption underlying our proposition is not very demanding since it is in accordance with the assumption that consumers behave as price-takers.[20] Second, our result showing a conflict of interests between consumers and oligopolists is in line with previous literature. For example, it parallels that by Delipalla and Keen (1992) who show that consumers prefer ad valorem taxation in their role as consumers of both the taxed good and public expenditure, but prefer per unit taxation in their role as producers (profits are assumed to accrue to the representative consumer). In their framework, where the public interest lies depends on the *relative* social weight attached to consumer surplus and to profits: When they are weighted equally, the advantage lies with ad valorem taxation.[21] In contrast, in our set-up, when the social weight attached to consumers' and oligopolists' welfare is measured by the number of agents in each group, the condition underlying our Proposition requires that not only the number of consumers has to be sufficiently high compared to the number of oligopolists in *relative* terms, but also in *absolute* terms. Further, in our example, the size of each group is not only a measure for the social weight attached to consumers' and oligopolists' welfare, but it also affects the gain or loss each agent receives following a revenue-neutral shift from ad valorem to per unit taxation. Specifically, while the number of agents on the strategic side of the market affects both their own welfare and the welfare of consumers (see also Skeath and Trandel, 1994), our example also shows the crucial role played by the number of agents on the competitive side of the market, which affects their own welfare.

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20 Under the assumption of a single representative consumer, who is the sole owner of all oligopolistic firms, Delipalla and Keen (1992) show the dominance of ad valorem over per unit taxation. It is easy to check that, under this assumption, i.e., $n = 1$, we would also obtain the superiority of an ad valorem tax. However, in our context, this event has not any economic meaning since consumers represent the competitive side of the market, and for this reason their number has to be sufficiently high compared with the number of oligopolists which describe the strategic side.

21 Notice also other differences with our model. First, the paper by Delipalla and Keen (1992) considers an economy defined in a much more general set-up than the one considered here which, as we have already mentioned, considers a particular characterisation of imperfect competition, described via a strategic market game. Second, we do not refer to a standard optimal commodity tax problem since we compare aggregate welfare, namely the sum of the utility levels of consumers and oligopolists, under ad valorem and per unit taxes.

4 Concluding Remarks

This note compares the different welfare properties of ad valorem and per unit taxation in a model of oligopolistic interaction. It aims to provide an alternative perspective upon this issue with respect to existing literature which has shown, in a number of oligopolistic frameworks usually analyzed at a partial equilibrium level, the dominance of ad valorem over per unit taxation. To this end, our analysis has been cast into a simple example of a general equilibrium model. However, a remark dealing with the robustness of our approach should be pointed out. Even if we have shown that per unit taxation may welfare dominate ad valorem taxation in an oligopoly context, we have worked with a rather specific example of imperfectly competitive market. On the competitive side of the market, we have already mentioned the role played by the assumption on how the initial endowment of consumers varies when their number changes (see Footnote 17). On the strategic side of the market, we stress the fact that we have considered a particular oligopoly model, in which oligopolists are simultaneously producers and consumers. This stylized description is suitable to represent a world in which each firm would be owned by a single individual, but not that of enterprises owned by shareholders who do not have the same preferences (Gabszewicz and Michel, 1997). As stated in our introduction, since with the latter formulation several difficulties arise in modelling imperfect competition into a general equilibrium framework, we think that the study of taxation when agents have market power is a sufficiently important topic to initiate it under the rather special assumptions suggested in our example. It is clear, however, that our approach should be extended to broader noncompetitive contexts. This is an open field for further research.

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