Acta Mech 231, 4129–4144 (2020)
https://doi.org/10.1007/s00707-020-02751-2

ORIGINAL PAPER

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Rayleigh waves in a nonlocal thermoelastic layer lying over a nonlocal thermoelastic half-space

Received: 17 April 2020 / Revised: 29 May 2020 / Published online: 15 July 2020

Abstract The paper deals with Rayleigh wave propagation in a nonlocal thermoelastic layer, and the layer is lying over a nonlocal thermoelastic half-space. The problem is treated in the context of Eringen's nonlocal thermoelasticity and Green–Naghdi model type III of hyperbolic thermoelasticity. The frequency equation of Rayleigh waves is derived, and different cases are also discussed. The effect of the nonlocal parameter on phase velocity, attenuation coefficient, specific loss, and penetration depth is presented graphically.

1 Introduction

The theory of nonlocal elasticity has attracted the attention of many authors because of its early success in solving an old problem in fracture mechanics. The nonlocal elasticity solution of Eringen [1,2] showed that the stress at the tip of a crack is finite; it rises to a maximum and then diminishes with the distance from the crack tip. Eringen [3,4] found the nonlocal solution of the discrete dislocation problem. Nonlocal field theories contain very interesting physics, in fact, all physics, excluding quantum effects and elementary particle physics. This can be extended further to include the nonlocal mixture theory, diffusion, and other allied phenomena.

Some nonclassical thermoelasticity theories have been developed depending on the strategies to incorporate additional atomistic features based on Eringen's nonlocal elasticity theory [5] which is now well established. In the local elasticity model, Eringen [5] assumed that the stress field at a particular point in an elastic continuum not only depends on the strain field but also on strains at all other points of the body. Altan [6] studied the uniqueness in the linear theory of nonlocal elasticity. The nonlocal elasticity models characterized by the presence of nonlocality residuals of fields have been proposed by Eringen and Edelen [7]. Eringen extended the concept of nonlocality to various other fields in his works cited in [8–10].

Nonlocal elasticity theories are now well established and are being applied to the problems of wave propagation in elastic and thermoelastic solids. Pramanik and Biswas [11] investigated the propagation of Rayleigh surface waves in nonlocal thermoelastic solids. Biswas [12] considered the propagation of Rayleigh surface waves in a porous nonlocal thermoelastic orthotropic medium. Khurana and Tomar [13] studied wave propagation in a nonlocal microstretch solid. Jun et al. [14] discussed nonlocal thermoelasticity based on nonlocal heat conduction and nonlocal elasticity. Khurana and Tomar [15] investigated Rayleigh-type waves in a nonlocal micropolar solid half-space.

The generalized thermoelasticity theories have been developed with the aim of removing the paradox of infinite speed of heat propagation inherent in the classical coupled dynamical thermoelasticity theory (Biot [16]). Many new theories have been proposed to take care of this physical absurdity. Lord and Shulman [17] first modified Fourier's law by introducing the term representing the thermal relaxation time. The heat equation associated with this theory is a hyperbolic type and hence eliminates the paradox of infinite speed of

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thermal propagation. Then, Green and Lindsay [18] developed a more general theory of thermoelasticity in which Fourier's law of heat conduction is unchanged, whereas the classical energy equation and Duhamel–Neumann's relations are modified by introducing two constitutive constants having the dimensions of time. Later, Green and Naghdi [19–21] developed three models for generalized thermoelasticity of homogeneous isotropic materials, which are labelled as models I, II, and III. Green–Naghdi model type II is known as thermoelasticity without energy dissipation, and Green–Naghdi type III model is known as thermoelasticity with energy dissipation. Detailed information regarding these theories can be found in [22,23].

Dwan and Chakraborty [24] proposed Rayleigh waves in the context of Green-Lindsay's model of generalized thermoelasticity theory, and Rossikin and Shitikova [25] discussed nonstationary Rayleigh waves in the thermally insulated surfaces of some thermoelastic bodies of revolution. Singh et al. [26] considered propagation of the Rayleigh wave in an initially stressed transversely isotropic magneto-thermoelastic half-space with dual-phase-lag model. Biswas et al. [27] investigated Rayleigh surface wave propagation in orthotropic thermoelastic solids under three-phase-lag model. Biswas and Abo-Dahab [28] considered the effect of phase lags on Rayleigh waves in an initially stressed magneto-thermoelastic orthotropic medium. Abd-Alla and Al-Dawy [29] considered the effect of thermal relaxation times on Rayleigh waves in a generalized thermoelastic medium. Wojnar [30] examined Rayleigh waves in a thermoelastic medium with relaxation times. Biswas [31] reported Stroh analysis of Rayleigh waves in an anisotropic thermoelastic medium with three-phase-lag model. Biswas and Mukhopadhyay [32] employed the eigenfunction expansion method to characterize Rayleigh wave propagation in an orthotropic medium with three-phase-lag model.

In this article, Rayleigh wave propagation in an isotropic thermoelastic layer lying over an isotropic thermoelastic half-space is investigated. The problem is treated in the context of Green–Naghdi model type III based on Eringen's nonlocal thermoelasticity. Different frequency equations are derived as special cases which agree with the existing literature. In order to illustrate the theoretical developments, the computer simulated results with respect to phase velocity, attenuation coefficient, specific loss, and penetration depth are presented graphically.

2 Derivation of the model

We shall first establish the constitutive relations and field equations for a nonlocal thermoelastic medium with Green–Naghdi model type III of generalized thermoelasticity.

Consider a thermoelastic body having volume V, bounded by the surface S and occupying region B in R^3 at time t. Let the position of a typical point of B in the unbounded state be X_i and the position of the corresponding point in the deformed state be x_i. The displacement components u_i of the particle are given by $u_i = x_i - X_i$.

Let us denote the strain tensor by e_{ij}. In the linear theory, the Lagrangian strain tensor reduces to

$$e_{ij} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right).$$

Suppose $\theta = T - T_0$, where T_0 is the temperature of the material in its natural state assumed to be such that $\left|\frac{\theta}{T_0}\right| << 1$ and T is the absolute temperature of the material.

Within the context of linear theory and assuming that the initial body is free from stresses, we take the set of basic variables at two neighbouring points $\mathbf{x}$ and $\mathbf{x}'$, respectively, as

$$\begin{aligned} &\Pi = \left\{e_{ij}\left(\mathbf{x}\right), \theta\left(\mathbf{x}\right)\right\} \\ &\text{and } \Pi' = \left\{e_{ij}\left(\mathbf{x}'\right), \theta\left(\mathbf{x}'\right)\right\}. \end{aligned} \tag{1}$$

The strain energy function W for nonlocal thermoelastic materials can be written as

$$2W = C_{ijkl} e_{ij}\left(\mathbf{x}\right) e_{kl}\left(\mathbf{x}'\right) - \beta_{ij}\left[e_{ij}\left(\mathbf{x}\right)\theta\left(\mathbf{x}'\right) + e_{ij}\left(\mathbf{x}'\right)\theta\left(\mathbf{x}\right)\right] - a\theta\left(\mathbf{x}\right)\theta\left(\mathbf{x}'\right) \tag{2}$$

where the constitutive coefficients C_{ijkl}, a, β_{ij} are prescribed functions of $\mathbf{x}$ and $\mathbf{x}'$. C_{ijkl} are the elastic constants, β_{ij} are the thermal moduli, e_{ij} are the strain components, and $a = \frac{\rho C_v}{T_0}$.

We adopt the following symmetries in the constitutive coefficients as

$$C_{ijkl}\left(\mathbf{x}, \mathbf{x}'\right) = C_{klij}\left(\mathbf{x}, \mathbf{x}'\right) = C_{jikl}\left(\mathbf{x}, \mathbf{x}'\right), \beta_{ij}\left(\mathbf{x}, \mathbf{x}'\right) = \beta_{ji}\left(\mathbf{x}, \mathbf{x}'\right).$$

Following Eringen [1], the constitutive relations are obtained from

$$\Gamma = \int_V \left[\frac{\partial W}{\partial \Pi} + \left(\frac{\partial W}{\partial \Pi'} \right)^s \right] \mathrm{d}V\left(\mathbf{x}'\right) \tag{3}$$

where the superscript 's' represents the symmetry of that quantity with respect to interchange of $\mathbf{x}$ and $\mathbf{x}'$. The set $\Gamma = \left\{\tau_{ij}, -\eta\right\}$ is an ordered set with the set Π.

Thus, the force stress tensor τ_{ij} and the specific entropy η are obtained from relations (2) and (3) as

$$\tau_{ij} = \int_V \left[C_{ijkl}\left(\mathbf{x}, \mathbf{x}'\right) e_{kl}\left(\mathbf{x}'\right) - \beta_{ij}\left(\mathbf{x}, \mathbf{x}'\right) \theta\left(\mathbf{x}'\right)\right] \mathrm{d}V\left(\mathbf{x}'\right), \tag{4}$$

$$\rho\eta = \int_V \left[\beta_{ij}\left(\mathbf{x}, \mathbf{x}'\right) e_{ij}\left(\mathbf{x}'\right) + a\theta\left(\mathbf{x}'\right)\right] \mathrm{d}V\left(\mathbf{x}'\right). \tag{5}$$

For a centro-symmetric isotropic material, the constitutive coefficients reduce to

$$C_{ijkl} = \lambda\left(\mathbf{x}, \mathbf{x}'\right) \delta_{ij}\delta_{kl} + 2\mu\left(\mathbf{x}, \mathbf{x}'\right) \delta_{ik}\delta_{jl}, \beta_{ij} = \beta\left(\mathbf{x}, \mathbf{x}'\right)$$

where the material coefficients λ, μ, β are functions of $\left|\mathbf{x} - \mathbf{x}'\right|$.

Hence, the constitutive relations (4)–(5) become

$$\tau_{ij} = \int_V \left[\lambda\left(\left|\mathbf{x} - \mathbf{x}'\right|\right) \delta_{ij} e_{kk}\left(\mathbf{x}'\right) + 2\mu\left(\left|\mathbf{x} - \mathbf{x}'\right|\right) e_{ij}\left(\mathbf{x}'\right) - \beta\left(\left|\mathbf{x} - \mathbf{x}'\right|\right) \theta\left(\mathbf{x}'\right)\right] \mathrm{d}V\left(\mathbf{x}'\right), \tag{6}$$

$$\rho\eta = \int_V \left[\beta\left(\left|\mathbf{x} - \mathbf{x}'\right|\right) e_{ij}\left(\mathbf{x}'\right) + a\left(\left|\mathbf{x} - \mathbf{x}'\right|\right) \theta\left(\mathbf{x}'\right)\right] \mathrm{d}V\left(\mathbf{x}'\right). \tag{7}$$

For most of the materials, the cohesive zone is very small, and within that zone the intermolecular forces decrease rapidly with distance from the reference point. Hence, we consider that all constitutive coefficients attenuate with distance, e.g.

$$\lim_{(|\mathbf{x}-\mathbf{x}'|)\to\infty} \lambda\left(\left|\mathbf{x} - \mathbf{x}'\right|\right) \to 0.$$

We also consider that all the constitutive coefficients attenuate the same degree and they attain their maxima at $\mathbf{x} = \mathbf{x}'$.

Therefore, we can take the following relations between nonlocal and local coefficients:

$$\frac{\lambda\left(\left|\mathbf{x} - \mathbf{x}'\right|\right)}{\lambda_0} = \frac{\mu\left(\left|\mathbf{x} - \mathbf{x}'\right|\right)}{\mu_0} = \frac{\beta\left(\left|\mathbf{x} - \mathbf{x}'\right|\right)}{\beta_0} = \frac{a\left(\left|\mathbf{x} - \mathbf{x}'\right|\right)}{a_0} = G\left(\left|\mathbf{x} - \mathbf{x}'\right|\right); \tag{8}$$

here, the quantities in the denominator are Lamé constants coefficients. λ_0, μ_0 are well-known Lame's constants, $\beta_0 = (3\lambda_0 + 2\mu_0)\,\alpha_t$, α_t is the coefficient of linear thermal expansion, a is thermal constant, δ_{ij} is the Kronecker delta function, and the function $G\left(\left|\mathbf{x} - \mathbf{x}'\right|\right)$ is a nonlocal kernel representing the effect of distant interactions of material points between $\mathbf{x}$ and $\mathbf{x}'$.

Also, the integral of the nonlocal kernel $G\left(\left|\mathbf{x} - \mathbf{x}'\right|\right)$ over the domain of integration is unity, i.e.

$$\int_V G\left(\left|\mathbf{x} - \mathbf{x}'\right|\right) \mathrm{d}V = 1.$$

Hence, the kernel function G behaves as a Dirac delta function over the domain of influence. The function G attains its peak at $\left|\mathbf{x} - \mathbf{x}'\right| = 0$ and generally decays with increasing $\left|\mathbf{x} - \mathbf{x}'\right|$.

Eringen [5] has already shown that the function G satisfies the relation

$$\left(1 - \varepsilon^2\nabla^2\right) G\left(\left|\mathbf{x} - \mathbf{x}'\right|\right) = \delta\left(\left|\mathbf{x} - \mathbf{x}'\right|\right) \tag{9}$$

where $\varepsilon = e_0 a_{cl}$ is the elastic nonlocal parameter [1,5], a_{cl} being the internal characteristic length, and e_0 is a material constant. The internal characteristic length a_{cl} is the interatomic distance, e.g. length of C–C bond (0.142 nm in Carbon nanotube).

Applying the operator $\left(1 - \varepsilon^2 \nabla^2\right)$ on the constitutive relations (6), (7), owing to the relation (8) and the property (9), we obtain (after suppressing the subscript '0' from the constitutive coefficients)

$$\left(1 - \varepsilon^2 \nabla^2\right) \tau_{ij} = \tau_{ij}^L = \left\{2\mu e_{ij}(\mathbf{x}) + \left[\lambda e_{kk}(\mathbf{x}) - \beta\theta(\mathbf{x})\right]\delta_{ij}\right\}, \tag{10}$$

$$\left(1 - \varepsilon^2 \nabla^2\right) \rho\eta = (\rho\eta)^L = \left[\beta e_{kk}(\mathbf{x}) + a\theta(\mathbf{x})\right], \tag{11}$$

wherein the formula

$$\int f(x)\delta(x - a)\,\mathrm{d}x = f(a) \tag{12}$$

has been employed. The quantities τ_{ij}^L, and $(\rho\eta)^L$ correspond to the local thermoelastic solid.

The fourier law for the GN-III model in nonlocal thermoelasticity becomes:

$$\left(1 - \varepsilon^2 \nabla^2\right) \dot{q}_i = \dot{q}_i^L = -\left(K\dot{\theta}_{,i} + K^*\theta_{,i}\right) \tag{13}$$

where q_i are the components of the heat flux vector, K is the thermal conductivity, and K^* is the material constant characteristic of the theory.

3 Basic equations

The constitutive equations for isotropic thermoelastic material become:

(a) The energy equation for the linear theory of a thermoelastic material:

$$-\rho T_0 \dot{\eta} = q_{i,i}; \tag{14}$$

(b) The equations of motion (in the absence of body force):

$$\tau_{ij,j} = \rho\ddot{u}_i \tag{15}$$

where ρ is the mass density.

From the constitutive relations (10) and (11), Fourier's law (13), the energy equation (14), and the equation of motion (15), we obtain the field equations in terms of the displacement and temperature for a homogeneous isotropic nonlocal thermoelastic material in the absence of body forces as

$$(\lambda + \mu) u_{j,ij} + \mu u_{i,jj} - \beta\theta_{,i} = \left(1 - \varepsilon^2 \nabla^2\right) \rho\ddot{u}_i, \tag{16}$$

$$K\nabla^2\dot{\theta} + K^*\nabla^2\theta = \rho C_v \ddot{T} + \beta T_0 \ddot{e} \tag{17}$$

where $aT_0 = \rho C_v$, C_v is the specific heat at constant strain.

From Eq. (10), we get the stress components as

$$\left(1 - \varepsilon^2 \nabla^2\right) \tau_{ij} = \tau_{ij}^L = \lambda e_{kk}\delta_{ij} + 2\mu e_{ij} - \beta\theta\delta_{ij}. \tag{18}$$

4 Formulation of the problem in a nonlocal thermoelastic layer

Let us consider a medium which consists of a nonlocal, homogeneous and isotropic layer of a constant thickness H lying over a nonlocal, homogeneous, and isotropic thermoelastic half space (Fig. 1).

Let us consider a plane harmonic surface wave which propagates along the x-axis and which is polarised in the (x, z) plane.

We take $\vec{u}_1 = (u_1, 0, w_1)$ as the displacement vector in the layer, λ_1, μ_1 are Lamé constants in the layer, β_1 is the thermal modulus in the layer, ρ_1 is the mass density of the thermoelastic layer, θ_1 is the temperature above reference temperature of the layer, K_1 is the thermal conductivity of thermoelastic layer, K_1^* is the material constant characteristic of the theory for the layer, C_v is the specific heat at constant strain, and $\left(\tau_{ij}\right)_1$ are the stress components in the layer.

The basic governing equations of a nonlocal thermoelastic layer with Green–Naghdi type III model are obtained as

$$(\lambda_1+2\mu_1)\frac{\partial^2 u_1}{\partial x^2}+\mu_1\frac{\partial^2 u_1}{\partial z^2}+(\lambda_1+\mu_1)\frac{\partial^2 w_1}{\partial x\partial z}-\beta_1\frac{\partial\theta_1}{\partial x}=\left(1-\varepsilon^2\nabla^2\right)\rho_1\ddot{u}_1, \tag{19}$$

$$(\lambda_1+2\mu_1)\frac{\partial^2 w_1}{\partial x^2}+\mu_1\frac{\partial^2 w_1}{\partial z^2}+(\lambda_1+\mu_1)\frac{\partial^2 u_1}{\partial x\partial z}-\beta_1\frac{\partial\theta_1}{\partial z}=\left(1-\varepsilon^2\nabla^2\right)\rho_1\ddot{w}_1, \tag{20}$$

$$K_1^*\nabla^2\theta_1+K_1\nabla^2\dot{\theta}_1=\rho_1 C_v\ddot{\theta}_1+\beta_1 T_0\ddot{e}. \tag{21}$$

We define the dimensionless quantities as follows:

$$\left(x',z',\varepsilon'\right)=\frac{\omega_1^*}{c_1}\left(x,z,\varepsilon\right),\ \left(u_1',w_1'\right)=\frac{\omega_1^*}{c_1}\left(u_1,w_1\right),\ \theta_1'=\frac{\beta_1\theta_1}{\rho_1 c_1^2},\ t'=\omega_1^* t,\ \left(\tau_{ij}\right)_1'=\frac{\left(\tau_{ij}\right)_1}{\beta_1 T_0},$$

$$m_1=\frac{\mu_1}{\lambda_1+2\mu_1},\ m_2=\frac{\lambda_1+\mu_1}{\lambda_1+2\mu_1},\ m_3=\frac{\beta_1^2 T_0\omega_1^*}{\rho_1 K_1^*},\ m_4=\frac{K_1\omega_1^*}{K_1^*},\ m_5=\frac{\lambda_1}{\lambda_1+2\mu_1}$$

where $\omega_1^*=\frac{\rho_1 C_v c_1^2}{K_1^*}$ and $c_1=\sqrt{\frac{\lambda_1+2\mu_1}{\rho_1}}$ are the characteristic frequency and longitudinal wave velocity in the layer, respectively.

Using nondimensional quantities in Eqs. (19), (20), and (21) and dropping primes, we get

$$\frac{\partial^2 u_1}{\partial x^2}+m_1\frac{\partial^2 u_1}{\partial z^2}+m_2\frac{\partial^2 w_1}{\partial x\partial z}-\frac{\partial\theta_1}{\partial x}=\left(1-\varepsilon^2\nabla^2\right)\ddot{u}_1, \tag{22}$$

$$\frac{\partial^2 w_1}{\partial x^2}+m_1\frac{\partial^2 w_1}{\partial z^2}+m_2\frac{\partial^2 u_1}{\partial x\partial z}-\frac{\partial\theta_1}{\partial z}=\left(1-\varepsilon^2\nabla^2\right)\ddot{w}_1, \tag{23}$$

$$\nabla^2\theta_1+m_4\nabla^2\dot{\theta}_1=\ddot{\theta}_1+m_3\ddot{e}. \tag{24}$$

5 Formulation of the problem in a nonlocal thermoelastic half-space

We take $\vec{u}_2=(u_2,0,w_2)$ as the displacement vector in the half-space, λ_2, μ_2 are Lamé constants in the half-space , β_2 is the thermal modulus for the thermoelastic half-space, ρ_2 is the mass density of the thermoelastic half-space, θ_2 is the temperature above reference temperature of the half-space, K_2 is the thermal conductivity of the thermoelastic half-space, K_2^* is the material constant characteristic of the theory for the half-space, C_v' is the specific heat at constant strain, and $\left(\tau_{ij}\right)_2$ are the stress components in the half-space.

The basic governing equations of a nonlocal thermoelastic half-space with Green–Naghdi type III model are obtained as

$$(\lambda_2+2\mu_2)\frac{\partial^2 u_2}{\partial x^2}+\mu_2\frac{\partial^2 u_2}{\partial z^2}+(\lambda_2+\mu_2)\frac{\partial^2 w_2}{\partial x\partial z}-\beta_2\frac{\partial\theta_2}{\partial x}=\left(1-\varepsilon^2\nabla^2\right)\rho_2\ddot{u}_2, \tag{25}$$

$$(\lambda_2+2\mu_2)\frac{\partial^2 w_2}{\partial x^2}+\mu_2\frac{\partial^2 w_2}{\partial z^2}+(\lambda_2+\mu_2)\frac{\partial^2 u_2}{\partial x\partial z}-\beta_2\frac{\partial\theta_2}{\partial z}=\left(1-\varepsilon^2\nabla^2\right)\rho_2\ddot{w}_2, \tag{26}$$

$$K_2^*\nabla^2\theta_2+K_2\nabla^2\dot{\theta}_2=\rho_2 C_v'\ddot{\theta}_2+\beta_2 T_0\ddot{e}. \tag{27}$$

We define the dimensionless quantities as follows:

$$\left(x',z',\varepsilon'\right)=\frac{\omega^*}{c_2}\left(x,z,\varepsilon\right),\ \left(u_2',w_2'\right)=\frac{\omega^*}{c_2}\left(u_2,w_2\right),\ \theta_2'=\frac{\beta_2\theta_2}{\rho_2 c_2^2},\ t'=\omega^* t,\ \left(\tau_{ij}\right)_2'=\frac{\left(\tau_{ij}\right)_2}{\beta_2 T_0},$$

$$r_1=\frac{\mu_2}{\lambda_2+2\mu_2},\ r_2=\frac{\lambda_2+\mu_2}{\lambda_2+2\mu_2},\ r_3=\frac{\beta_2^2 T_0\omega^*}{\rho_2 K_2^*},\ r_4=\frac{K_2\omega^*}{K_2^*},\ r_5=\frac{\lambda_2}{\lambda_2+2\mu_2}$$

where $\omega^*=\frac{\rho_2 C_v' c_2^2}{K_2^*}$ and $c_2=\sqrt{\frac{\lambda_2+2\mu_2}{\rho_2}}$ are the characteristic frequency and longitudinal wave velocity in the half-space, respectively.

Using nondimensional quantities in Eqs. (25), (26), and (27), and dropping primes, we get

$$\frac{\partial^2 u_2}{\partial x^2} + r_1 \frac{\partial^2 u_2}{\partial z^2} + r_2 \frac{\partial^2 w_2}{\partial x \partial z} - \frac{\partial \theta_2}{\partial x} = \left(1 - \varepsilon^2 \nabla^2\right) \ddot{u}_2, \tag{28}$$

$$\frac{\partial^2 w_2}{\partial x^2} + r_1 \frac{\partial^2 w_2}{\partial z^2} + r_2 \frac{\partial^2 u_2}{\partial x \partial z} - \frac{\partial \theta_2}{\partial z} = \left(1 - \varepsilon^2 \nabla^2\right) \ddot{w}_2, \tag{29}$$

$$\nabla^2 \theta_2 + r_4 \nabla^2 \dot{\theta}_2 = \ddot{\theta}_2 + r_3 \ddot{e}. \tag{30}$$

6 Boundary conditions

Now we add boundary conditions which determine the properties of the wave field at the boundaries.

(a) Surface of the layer $z = 0$:
We assume the surface of the layer to be traction free and temperature free:
(i) $(\tau_{xz})_1 = 0$

$$\text{which gives } (\tau_{xz})_1^L = 0, \tag{31}$$

(ii) $(\tau_{zz})_1 = 0$,

$$\text{which gives } (\tau_{zz})_1^L = 0, \tag{32}$$

(iii)

$$\theta_1 = 0. \tag{33}$$

(b) Interface between the layer and the half-space $z = H$:
We require all displacement and stress components to be continuous across this interface:
(iv)

$$u_1 = u_2, \tag{34}$$

(v)

$$w_1 = w_2, \tag{35}$$

(vi)

$$(\tau_{xz})_1 = (\tau_{xz})_2\,,$$
$$\left(1 - \varepsilon^2 \nabla^2\right) (\tau_{xz})_1 = \left(1 - \varepsilon^2 \nabla^2\right) (\tau_{xz})_2\,.$$

We have

$$(\tau_{xz})_1^L = (\tau_{xz})_2^L\,, \tag{36}$$

(vii)

$$(\tau_{zz})_1 = (\tau_{zz})_2\,,$$
$$\left(1 - \varepsilon^2 \nabla^2\right) (\tau_{zz})_1 = \left(1 - \varepsilon^2 \nabla^2\right) (\tau_{zz})_2\,,$$

We have

$$(\tau_{zz})_1^L = (\tau_{zz})_2^L\,, \tag{37}$$

(viii)

$$\theta_1 = \theta_2. \tag{38}$$

(c) Infinite depth $z \to \infty$:
We require the displacements and temperature to diminish to zero at large depths,

$$u_2 \to 0,\ w_2 \to 0,\ \theta_2 \to 0.$$

This condition guarantees that the wave under consideration will have the character of a surface wave.

7 Solution of the problem in a nonlocal thermoelastic layer

For Rayleigh wave propagation in the layer along the $x-$direction, we take

$$(u_1, w_1, \theta_1)(x, z, t) = (f_1, g_1, h_1)(z) \exp[i(kx - \omega t)] \tag{39}$$

where $\omega = kc$ is the angular frequency of Rayleigh waves, c is the phase velocity, and k is the wave number.
Using Eq. (39) in Eqs. (22), (23), and (24), we get

$$\left(m_1 - \varepsilon^2\omega^2\right) D^2 f_1 + \left[\left(1 + \varepsilon^2 k^2\right)\omega^2 - k^2\right] f_1 + ikm_2 Dg_1 - ikh_1 = 0, \tag{40}$$

$$\left(1 - \varepsilon^2\omega^2\right) D^2 g_1 + \left[\left(1 + \varepsilon^2 k^2\right)\omega^2 - m_1 k^2\right] g_1 + ikm_2 Df_1 - Dh_1 = 0, \tag{41}$$

$$(1 - i\omega m_4) D^2 h_1 + \left[\omega^2 - (1 - i\omega m_4) k^2\right] h_1 + ikm_3\omega^2 f_1 + m_3\omega^2 Dg_1 = 0 \tag{42}$$

where $D \equiv \frac{d}{dz}$.
Eliminating f_1 and h_1 from Eqs. (40), (41), and (42), we get

$$\left(AD^6 + BD^4 + CD^2 + E\right) g_1(z) = 0.$$

In a similar way, we can write

$$\left(AD^6 + BD^4 + CD^2 + E\right)(f_1(z), g_1(z), h_1(z)) = 0 \tag{43}$$

in which

$$\begin{aligned} A &= n_5 n_8 (n_3 - n_4 n_5), \\ B &= (n_3 - n_4 n_5)(n_5 n_9 + n_6 n_8 + n_{11}) - n_4 n_5 n_6 n_8, \\ C &= n_6 n_9 (n_3 - n_4 n_5) - n_4 n_6 (n_5 n_9 + n_6 n_8 + n_{11}), \\ E &= -n_4 n_6^2 n_9, \end{aligned}$$

where $n_1 = \left(m_1 - \varepsilon^2\omega^2\right)$, $n_2 = \left[\left(1 + \varepsilon^2 k^2\right)\omega^2 - k^2\right]$, $n_3 = ikm_2$, $n_4 = ik$, $n_5 = \left(1 - \varepsilon^2\omega^2\right)$,
$n_6 = \left[\left(1 + \varepsilon^2 k^2\right)\omega^2 - m_1 k^2\right]$, $n_7 = ikm_2$, $n_8 = (1 - i\omega m_4)$, $n_9 = \left[\omega^2 - (1 - i\omega m_4) k^2\right]$,
$n_{10} = ikm_3$, $n_{11} = m_3\omega^2$.

From Eq. (43), we get

$$\left(D^2 + \eta_1^2\right)\left(D^2 + \eta_2^2\right)\left(D^2 + \eta_3^2\right)[f_1(z), g_1(z), h_1(z)] = 0. \tag{44}$$

Now we obtain

$$f_1(z) = \sum_{n=1}^{3} A_n \cos\eta_n z + \sum_{n=1}^{3} B_n \sin\eta_n z, \tag{45}$$

$$g_1(z) = \sum_{n=1}^{3} C_n \cos\eta_n z + \sum_{n=1}^{3} D_n \sin\eta_n z, \tag{46}$$

$$h_1(z) = \sum_{n=1}^{3} E_n \cos\eta_n z + \sum_{n=1}^{3} F_n \sin\eta_n z. \tag{47}$$

Now the displacements and temperature are obtained as

$$u_1(x, z, t) = \left[\sum_{n=1}^{3} A_n \cos\eta_n z + \sum_{n=1}^{3} B_n \sin\eta_n z\right] \exp[i(kx - \omega t)], \tag{48}$$

$$w_1(x, z, t) = \left[\sum_{n=1}^{3} C_n \cos\eta_n z + \sum_{n=1}^{3} D_n \sin\eta_n z\right] \exp[i(kx - \omega t)], \tag{49}$$

$$\theta_1(x, z, t) = \left[\sum_{n=1}^{3} E_n \cos\eta_n z + \sum_{n=1}^{3} F_n \sin\eta_n z\right] \exp[i(kx - \omega t)] \tag{50}$$

where we take $C_n = c_n A_n$, $D_n = d_n B_n$, $E_n = e_n A_n$, $F_n = f_n B_n$.

The stresses are obtained as follows:

$$
\begin{aligned}
(\tau_{xx})_1^L &= \frac{\partial u_1}{\partial x} + m_5 \frac{\partial w_1}{\partial z} - \theta_1 \\
&= \left[\sum_{n=1}^{3} (ikA_n + m_5 \eta_n d_n B_n - e_n A_n) \cos \eta_n z \right. \\
&\quad \left. + \sum_{n=1}^{3} (ikB_n - m_5 \eta_n c_n A_n - f_n B_n) \sin \eta_n z \right] \exp\left[i\left(kx - \omega t\right)\right], \\
(\tau_{xz})_1^L &= \frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x} \\
&= \left[\sum_{n=1}^{3} (\eta_n B_n + ikc_n A_n) \cos \eta_n z + \sum_{n=1}^{3} (-\eta_n A_n + ikd_n B_n) \sin \eta_n z \right] \exp\left[i\left(kx - \omega t\right)\right], \\
(\tau_{zz})_1^L &= \frac{\partial w_1}{\partial z} + m_5 \frac{\partial u_1}{\partial x} - \theta_1 \\
&= \left[\sum_{n=1}^{3} (im_5 A_n - e_n A_n + d_n \eta_n B_n) \cos \eta_n z + \sum_{n=1}^{3} (ikm_5 B_n - \eta_n c_n A_n - f_n B_n) \sin \eta_n z \right] \\
&\quad \times \exp\left[i\left(kx - \omega t\right)\right].
\end{aligned}
$$

8 Solution of the problem in the nonlocal thermoelastic half-space

For Rayleigh wave propagation in the half-space along x-direction, we take

$$
(u_2, w_2, \theta_2)(x, z, t) = (f_2, g_2, h_2)(z) \exp\left[i\left(kx - \omega t\right)\right]. \tag{51}
$$

Using Eq. (51) in Eqs. (28), (29), and (30), we get

$$
\left(r_1 - \varepsilon^2\omega^2\right) D^2 f_2 + \left[\left(1 + \varepsilon^2 k^2\right)\omega^2 - k^2\right] f_2 + ikr_2 Dg_2 - ikh_2 = 0, \tag{52}
$$

$$
\left(1 - \varepsilon^2\omega^2\right) D^2 g_2 + \left[\left(1 + \varepsilon^2 k^2\right)\omega^2 - r_1 k^2\right] g_2 + ikr_2 Df_2 - Dh_2 = 0, \tag{53}
$$

$$
(1 - i\omega r_4) D^2 h_2 + \left[\omega^2 - (1 - i\omega r_4) k^2\right] h_2 + ikr_3\omega^2 f_2 + r_3\omega^2 Dg_2 = 0 \tag{54}
$$

where $D \equiv \frac{d}{dz}$.

Eliminating f_2 and h_2 from Eqs. (52), (53), and (54), we get

$$
\left(A' D^6 + B' D^4 + C' D^2 + E'\right) g_2(z) = 0.
$$

In a similar way, we can write

$$
\left(A' D^6 + B' D^4 + C' D^2 + E'\right) \left(f_2(z), g_2(z), h_2(z)\right) = 0 \tag{55}
$$

in which

$$
\begin{aligned}
A' &= s_5 s_8 (s_3 - s_4 s_5), \\
B' &= (s_3 - s_4 s_5)(s_5 s_9 + s_6 s_8 + s_{11}) - s_4 s_5 s_6 s_8, \\
C' &= s_6 s_9 (s_3 - s_4 s_5) - s_4 s_6 (s_5 s_9 + s_6 s_8 + s_{11}), \\
E' &= -s_4 s_6^2 s_9,
\end{aligned}
$$

where

$$
\begin{aligned}
& s_1 = \left(r_1 - \varepsilon^2\omega^2\right), s_2 = \left[\left(1 + \varepsilon^2 k^2\right)\omega^2 - k^2\right], s_3 = ikr_2, s_4 = ik, s_5 = \left(1 - \varepsilon^2\omega^2\right), \\
& s_6 = \left[\left(1 + \varepsilon^2 k^2\right)\omega^2 - r_2 k^2\right], s_7 = ikr_2, s_8 = (1 - i\omega r_4), s_9 = \left[\omega^2 - (1 - i\omega r_4) k^2\right], \\
& s_{10} = ikr_3, s_{11} = r_3\omega^2.
\end{aligned}
$$

From Eq. (55), we get

$$
\begin{aligned}
&\left(D^2-k_1^2\right)\left(D^2-k_2^2\right)\left(D^2-k_3^2\right)\left[f_2(z), g_2(z), h_2(z)\right]=0, \\
&f_2(z)=\sum_{n=1}^{3} X_n \exp\left(-k_n z\right)+\sum_{n=1}^{3} L_n \exp\left(k_n z\right), \\
&g_2(z)=\sum_{n=1}^{3} Y_n \exp\left(-k_n z\right)+\sum_{n=1}^{3} M_n \exp\left(k_n z\right), \\
&h_2(z)=\sum_{n=1}^{3} Z_n \exp\left(-k_n z\right)+\sum_{n=1}^{3} N_n \exp\left(k_n z\right).
\end{aligned} \tag{56}
$$

For bounded solution at $z \to \infty$, we take $L_n = M_n = N_n = 0$.
So, we get

$$f_2(z)=\sum_{n=1}^{3} X_n \exp\left(-k_n z\right), \tag{57}$$

$$g_2(z)=\sum_{n=1}^{3} Y_n \exp\left(-k_n z\right), \tag{58}$$

$$h_2(z)=\sum_{n=1}^{3} Z_n \exp\left(-k_n z\right). \tag{59}$$

Using Eqs. (57)–(59) in Eqs. (52)–(54), we get

$$Y_n = y_n X_n, \quad Z_n = z_n X_n$$

where

$$
\begin{aligned}
y_n &= \frac{k_n\left[s_1 k_n^2+\left(s_2-s_4 s_7\right)\right]}{\left[\left(s_3-s_4 s_5\right) k_n^2-s_4 s_6\right]}, \\
z_n &= \frac{s_1 s_{11} k_n^2+\left(s_2 s_{11}-s_3 s_{10}\right)}{s_3 s_6 k_n^2+\left(s_3 s_9+s_4 s_{11}\right)}.
\end{aligned}
$$

Now replacing z by $(z-H)$, we get displacements, temperature, and stresses as follows:

$$u_2=\sum_{n=1}^{3} X_n \exp\left[-k_n(z-H)\right] \exp[i(kx-\omega t)], \tag{60}$$

$$w_2=\sum_{n=1}^{3} y_n X_n \exp\left[-k_n(z-H)\right] \exp[i(kx-\omega t)], \tag{61}$$

$$
\begin{aligned}
\theta_2 &= \sum_{n=1}^{3} z_n X_n \exp\left[-k_n(z-H)\right] \exp[i(kx-\omega t)], \\
\left(\tau_{xx}\right)_2^L &= \frac{\partial u_2}{\partial x}+r_5 \frac{\partial w_2}{\partial z}-\theta_2 \\
&= \sum_{n=1}^{3}\left(ik-r_5 y_n k_n-z_n\right) X_n \exp\left[-k_n(z-H)\right] \exp[i(kx-\omega t)], \\
\left(\tau_{xz}\right)_2^L &= \frac{\partial u_2}{\partial z}+\frac{\partial w_2}{\partial x}
\end{aligned}
$$

$$= \sum_{n=1}^{3} (iky_n - k_n) X_n \exp\left[-k_n (z - H)\right] \exp\left[i (kx - \omega t)\right],$$

$$(\tau_{zz})_2^L = \frac{\partial w_2}{\partial z} + r_5 \frac{\partial u_2}{\partial x} - \theta_2$$

$$= \sum_{n=1}^{3} (ikr_5 - k_n y_n - z_n) X_n \exp\left[-k_n (z - H)\right] \exp\left[i (kx - \omega t)\right]. \tag{62}$$

9 Derivation of the frequency equation

Using the boundary conditions (31), (32), and (33), we get

$$\sum_{n=1}^{3} (\eta_n B_n + ikc_n A_n) = 0, \tag{63}$$

$$\sum_{n=1}^{3} \left[(ikm_5 - e_n) A_n + d_n \eta_n B_n\right] = 0, \tag{64}$$

$$\sum_{n=1}^{3} e_n A_n = 0. \tag{65}$$

From Eq. (63), we get

$$B_n = \delta_n A_n \tag{66}$$

in which $\delta_n = -\frac{ikc_n}{\eta_n}$.

From Eq. (65), we get

$$A_3 = -\frac{e_1}{e_3} A_1 - \frac{e_2}{e_3} A_2. \tag{67}$$

Using Eq. (66) and Eqs. (45)–(47) in Eqs. (40)–(42), we get

$$c_n = \frac{n_6 + (n_7 - f_n)\, \eta_n \delta_n}{n_5 \eta_n^2}, \; d_n = \frac{(n_7 \eta_n - e_n \eta_n - n_6 \delta_n)}{n_5 \eta_n^2 \delta_n},$$

$$e_n = f_n = \frac{(n_2 n_{11} + n_3 n_{10}) - n_1 n_{11} \eta_n^2}{(n_4 n_{11} - n_3 n_9) + n_3 n_8 \eta_n^2},$$

Using the boundary conditions (34)–(38), we get

$$\alpha_1 A_1 + \alpha_2 A_2 - X_1 - X_2 - X_3 = 0, \tag{68}$$

$$\gamma_1 A_1 + \gamma_2 A_2 - y_1 X_1 - y_2 X_2 - y_3 X_3 = 0, \tag{69}$$

$$l_1 A_1 + l_2 A_2 - l_3 X_1 - l_4 X_2 - l_5 X_3 = 0, \tag{70}$$

$$p_1 A_1 + p_2 A_2 - p_3 X_1 - p_4 X_2 - p_5 X_3 = 0, \tag{71}$$

$$q_1 A_1 + q_2 A_2 - z_1 X_1 - z_2 X_2 - z_3 X_3 = 0. \tag{72}$$

We have five homogeneous equations in terms of five unknowns. The system of equations has a nontrivial solution if

$$\begin{vmatrix} \alpha_1 & \alpha_2 & -1 & -1 & -1 \\ \gamma_1 & \gamma_2 & -y_1 & -y_2 & -y_3 \\ l_1 & l_2 & -l_3 & -l_4 & -l_5 \\ p_1 & p_2 & -p_3 & -p_4 & -p_5 \\ q_1 & q_2 & -z_1 & -z_2 & -z_3 \end{vmatrix} = 0.$$

Expanding the determinant, we get

$$\alpha_1 P_1 - \alpha_2 P_2 - P_3 + P_4 - P_5 = 0 \tag{73}$$

in which P_n $(n = 1, 2, \ldots, 5)$ are mentioned in the Appendix.

Equation (73) is the frequency equation of Rayleigh waves in a nonlocal thermoelastic layer lying over a nonlocal thermoelastic half-space.

10 Discussion of the frequency equation

Considering various particular values of the parameters, we can obtain the following different results in an isotropic medium:

(a) The frequency equation reduces to the case of the theory of classical coupled thermoelasticity (C T) when we put $K_1^* = K_2^* = 0$.
(b) The frequency equation reduces to the case of GN model type II when we put $K_1 = K_2 = 0$.
(c) The frequency equation reduces to the case of local thermoelasticity if we put $\varepsilon = 0$.
(d) In the absence of a temperature field, the frequency equation of Rayleigh waves in a local elastic thermoelastic layer agrees with that of Singhal and Sahu [33].

11 Solution of the frequency equation

In general, wavenumber (k) and hence phase velocity (c) are complex quantities. If we take

$$c^{-1} = V^{-1} + i\omega^{-1} Q, \tag{74}$$

the wavenumber can be expressed as $k = R + iQ$ where $R = \frac{\omega}{V}$ in which V and Q are real. V is the propagation speed, and Q is the attenuation coefficient of Rayleigh waves.

12 Specific loss

The specific loss (SL) is the ratio of energy (ΔW) dissipated in taking specimen through cycle, to elastic energy (W) stored in a specimen when the strain is at maximum. The specific loss is the most direct way of defining internal friction for a material (Puri and Cowin [34]). For a sinusoidal surface wave of small amplitude, Kolsky [35] shows that the specific loss $\frac{\Delta W}{W}$ equals 4π times the absolute value of the ratio of imaginary part of k to the real part of k, that is ,

$$\mathrm{SL} = \frac{\Delta W}{W} = 4\pi \left| \frac{\mathrm{Im}(k)}{\mathrm{Re}(k)} \right| = 4\pi \left| \frac{VQ}{\omega} \right|.$$

13 Special cases

In the absence of a layer, i.e., if we take $H = 0$ then the frequency of Rayleigh waves reduces to the frequency equation of Rayleigh waves in case of a thermoelastic half-space. We discuss some special cases of the frequency equation in a local thermoelastic half-space as follows:

Case (1) The frequency equation of surface waves in an isotropic half-space with classical coupled thermoelasticity is obtained as follows:

$$\left[\left(2 - \frac{c^2}{c_3^2}\right)^2 (\gamma_1 + \gamma_2) - 4\gamma_3 \left(\gamma_1\gamma_2 + 1 - \frac{k^2c^2}{c_2^2}\right) \right] = 0 \tag{75}$$

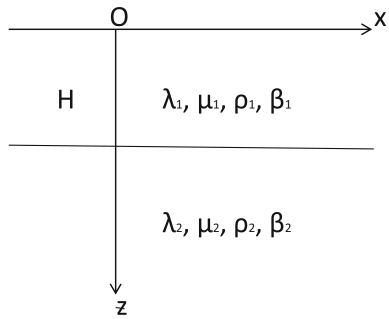

Fig. 1 Geometry of the problem

Equation (75) is similar to the result obtained in Nowinski [36], where $\gamma_1^2 = 1 - \frac{\xi_1^2}{k^2}, \gamma_2^2 = 1 - \frac{\xi_2^2}{k^2}, \gamma_3^2 = 1 - \frac{\zeta^2}{k^2}, \zeta^2 = \frac{k^2c^2}{c_3^2}$, and ξ_1^2 and ξ_2^2 are the roots of the biquadratic equation

$$\xi^4 - \left[\frac{k^2c^2}{c_3^2} + (1+\kappa)\frac{ikc\rho_2 C_v'}{K_2}\right]\xi^2 + \frac{ik^3c^3\rho_2 C_v'}{K_2 c_2^2} = 0 \tag{76}$$

in which $\kappa = \frac{T_0\beta_2^2}{\rho_2^2 c_2^2 C_v'}$ and $c_2^2 = \frac{\lambda_2+2\mu_2}{\rho_2}, c_3^2 = \frac{\mu_2}{\rho_2}$.

The results of the paper for an isotropic half-space with classical coupled thermoelasticity agree with Abd-Alla and Al-Dawy [29] and Wojnar [30].

Case (2) If we take $K_2^* = 0$ and if we add a thermal relaxation time, then the paper agrees with the results of Abd-Alla and Al-Dawy [29], Nayfeh and Nemat-Nasser [37] and Agrawal [38] in case of the Lord–Shulman model.

Case (3) If we take $K_2^* = 0$ and if we add two thermal relaxation times, then the paper agrees with the results of Abd-Alla and Al-Dawy [29], Wojnar [30], and Agarwal [38] in case of Green–Lindsay model.

Case (4) Neglecting thermal parameters, i.e. when there is no coupling between temperature and strain field, the frequency equation of Rayleigh waves in an isotropic elastic half-space is obtained as

$$\left(2 - \frac{c^2}{c_3^2}\right)^2 = 4\left(1 - \frac{c^2}{c_2^2}\right)^{\frac{1}{2}}\left(1 - \frac{c^2}{c_3^2}\right)^{\frac{1}{2}} \tag{77}$$

where $c_2^2 = \frac{\lambda_2+2\mu_2}{\rho_2}, c_3^2 = \frac{\mu_2}{\rho_2}$.

14 Numerical discussion

For numerical computation, we take the data values of copper material as follows (Biswas [39]):

$$\begin{aligned}
&\lambda_1 = \lambda_2 = 7.76 \times 10^{10}\,\mathrm{Kg\,m^{-1}\,s^{-2}}, \mu_1 = \mu_2 = 3.86 \times 10^{10}\mathrm{Kg\,m^{-1}\,s^{-2}},\\
&\beta_1 = \beta_2 = 1.78 \times 10^{-5}\,\mathrm{K^{-1}}, \quad \rho_1 = \rho_2 = 8954\,\mathrm{Kg\,m^{-3}}, K_1 = K_2 = 386\,\mathrm{W\,m^{-1}\,K^{-1}},\\
&K_1^* = K_2^* = 124\mathrm{W\,m^{-1}\,K^{-1}\,s^{-1}}, T_0 = 293\mathrm{K},\\
&C_v = C_v' = 383.1\mathrm{J\,Kg^{-1}\,K^{-1}}, H = 2\mathrm{m}, e_0 = 0.39, a_{cl} = 0.5 \times 10^{-9}\mathrm{m}.
\end{aligned}$$

In Fig. 2, the variation of phase velocity with respect to frequency is presented. It is observed that the phase velocity increases with the increase in frequency. The phase velocity for local thermoelastic medium is larger than the phase velocity for a nonlocal thermoelastic medium.

In Fig. 3, the variation of the attenuation coefficient with respect to frequency is presented. It is observed that the attenuation coefficient decreases with the increase in frequency. The attenuation coefficient for a local thermoelastic medium is larger than the attenuation coefficient for a nonlocal thermoelastic medium.

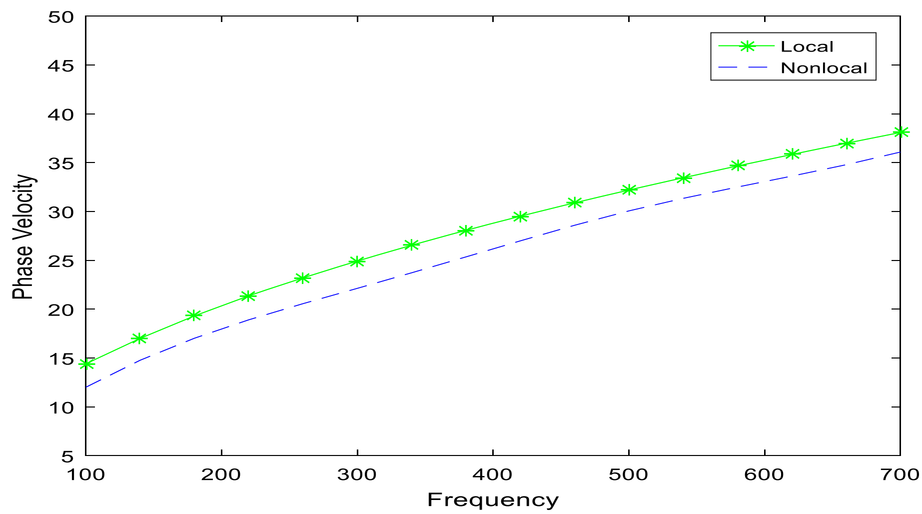

Fig. 2 Variation of phase velocity with respect to frequency

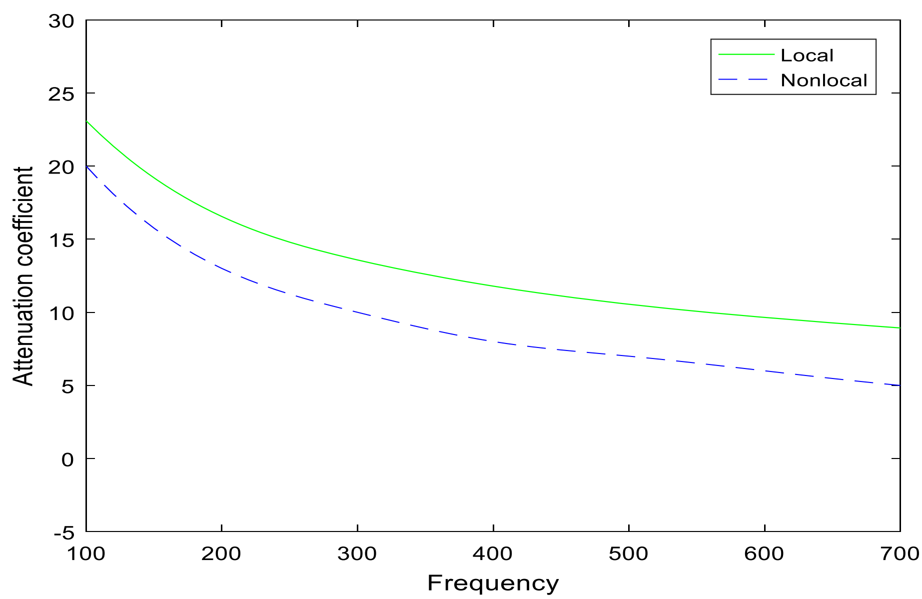

Fig. 3 Variation of attenuation coefficient with respect to frequency

In Fig. 4, the variation of penetration depth with respect to frequency is presented. It is observed that the penetration depth increases with the increase in frequency. The penetration depth for nonlocal thermoelastic medium is larger than the penetration depth for a local thermoelastic medium.

In Fig. 5, the variation of specific loss with respect to frequency is presented. It is observed that the specific loss decreases with the increase in frequency. The specific loss for a nonlocal thermoelastic medium is larger than the specific loss for a local thermoelastic medium.

15 Conclusions

In this article, Rayleigh wave propagation in a nonlocal thermoelastic layer lying over a nonlocal thermoelastic half-space is investigated with the Green–Naghdi model type III based on Eringen's nonlocal thermoelasticity theory. The frequency equation of a Rayleigh wave is derived, and different cases are discussed. Different characteristics of wave propagation are computed numerically and presented graphically.

From the theoretical and numerical discussion, we can conclude the following remarks:

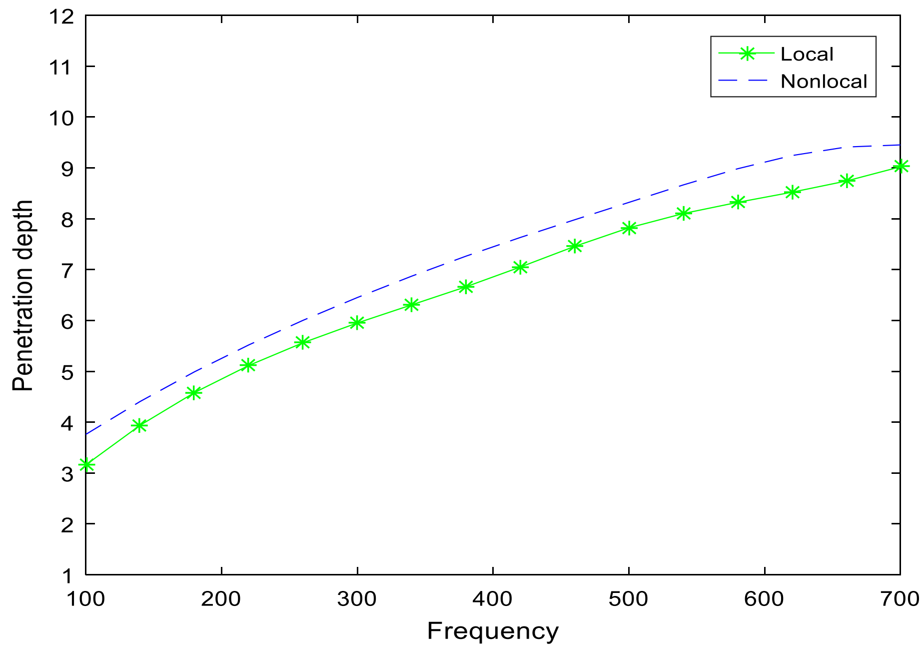

Fig. 4 Variation of penetration depth with respect to frequency

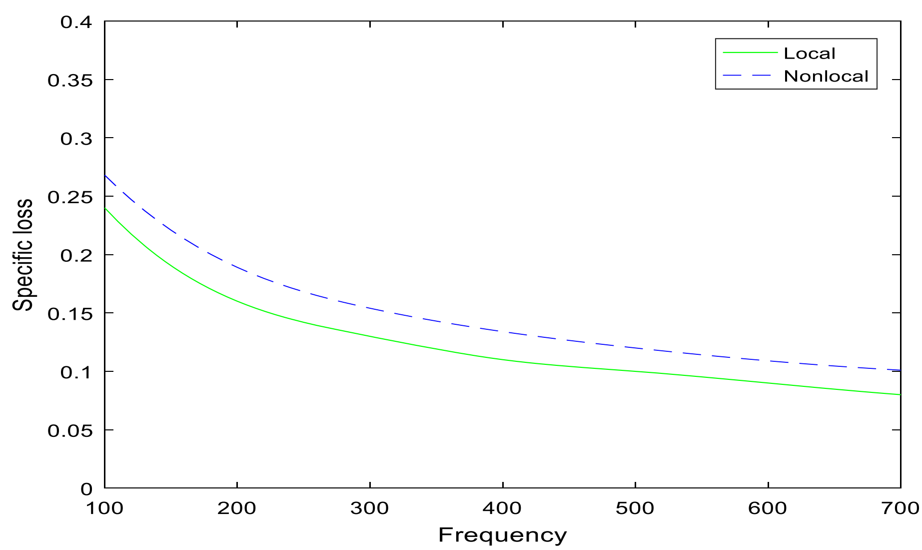

Fig. 5 Variation of specific loss with respect to frequency

(a) Phase velocity and penetration depth of Rayleigh waves increase with the increase in frequency.
(b) Attenuation coefficient and specific loss of Rayleigh waves decrease with the increase in frequency.
(c) Phase velocity and attenuation coefficient for a local thermoelastic medium are larger than phase velocity and attenuation coefficient for a nonlocal thermoelastic medium.
(d) Penetration depth and specific loss for a nonlocal thermoelastic medium are larger than penetration depth and specific loss for a local thermoelastic medium.

Compliance with ethical standards

Conflict of interest There is no conflict of interests.

Appendix

$$
\begin{aligned}
\alpha_1 &= \delta_1 \sin \eta_1 H + \cos \eta_1 H - \frac{e_1}{e_3}\left(\delta_3 \sin \eta_3 H + \cos \eta_3 H\right),\\
\alpha_2 &= \delta_2 \sin \eta_2 H + \cos \eta_2 H - \frac{e_2}{e_3}\left(\delta_3 \sin \eta_3 H + \cos \eta_3 H\right),\\
\gamma_1 &= c_1 \cos \eta_1 H + d_1\delta_1 \sin \eta_1 H - \frac{e_1}{e_3}\left(c_3 \cos \eta_3 H + d_3\delta_3 \sin \eta_3 H\right),\\
\gamma_2 &= c_2 \cos \eta_2 H + d_2\delta_2 \sin \eta_2 H - \frac{e_2}{e_3}\left(c_3 \cos \eta_3 H + d_3\delta_3 \sin \eta_3 H\right),\\
l_1 &= (ikc_1 + \delta_1\eta_1)\cos \eta_1 H + (ikd_1\delta_1 - \eta_1)\sin \eta_1 H\\
&\quad - \frac{e_1}{e_3}\left[(ikc_3 + \delta_3\eta_3)\cos \eta_3 H + (ikd_3\delta_3 - \eta_3)\sin \eta_3 H\right],\\
l_2 &= (ikc_2 + \delta_2\eta_2)\cos \eta_2 H + (ikd_2\delta_2 - \eta_2)\sin \eta_2 H\\
&\quad - \frac{e_2}{e_3}\left[(ikc_3 + \delta_3\eta_3)\cos \eta_3 H + (ikd_3\delta_3 - \eta_3)\sin \eta_3 H\right],\\
l_3 &= iky_1 - k_1,\\
l_4 &= iky_2 - k_2,\\
l_5 &= iky_3 - k_3,\\
p_1 &= (ikm_5 - e_1 + d_1\eta_1\delta_1)\cos \eta_1 H + (-c_1\eta_1 + ikm_5\delta_1 - f_1\delta_1)\sin \eta_1 H\\
&\quad - \frac{e_1}{e_3}\left[\begin{array}{l}(ikm_5 - e_3 - d_3\delta_3\eta_3)\cos \eta_3 H\\ +(-c_3\eta_3 + ikm_5\delta_3 - f_3\delta_3)\sin \eta_3 H\end{array}\right],\\
p_2 &= (ikm_5 - e_2 + d_2\eta_2\delta_2)\cos \eta_2 H + (-c_2\eta_2 + ikm_5\delta_2 - f_2\delta_2)\sin \eta_2 H\\
&\quad - \frac{e_2}{e_3}\left[\begin{array}{l}(ikm_5 - e_3 - d_3\delta_3\eta_3)\cos \eta_3 H\\ +(-c_3\eta_3 + ikm_5\delta_3 - f_3\delta_3)\sin \eta_3 H\end{array}\right],\\
p_3 &= ikr_5 - k_1y_1 - z_1,\\
p_4 &= ikr_5 - k_2y_2 - z_2,\\
p_5 &= ikr_5 - k_3y_3 - z_3,\\
q_1 &= e_1 \cos \eta_1 H + f_1\delta_1 \sin \eta_1 H - \frac{e_1}{e_3}\left(e_3 \cos \eta_3 H + f_3\delta_3 \sin \eta_3 H\right),\\
q_2 &= e_2 \cos \eta_2 H + f_2\delta_2 \sin \eta_2 H - \frac{e_2}{e_3}\left(e_3 \cos \eta_3 H + f_3\delta_3 \sin \eta_3 H\right),\\
P_1 &= \gamma_2\left\{-l_3(p_4z_3 - z_2p_5) + l_4(p_3z_3 - z_1p_5) + l_5(p_3z_2 - z_1p_4)\right\}\\
&\quad + y_1\left\{l_2(z_3p_4 - z_2p_5) + l_4(-p_2z_3 + q_2p_5) - l_5(-p_2z_2 + p_4q_2)\right\}\\
&\quad - y_2\left\{l_2(z_3p_3 - z_1p_5) + l_3(-p_2z_3 + p_5q_2) - l_5(-p_2z_1 + q_2p_3)\right\}\\
&\quad + y_3\left\{l_2(p_3z_2 - p_4z_1) + l_3(-p_2z_2 + p_4q_2) - l_4(-p_2z_1 + p_3q_2)\right\},\\
P_2 &= \gamma_1\left\{-l_3(p_4z_3 - z_2p_5) + l_4(p_3z_3 - z_1p_5) + l_5(p_3z_2 - z_1p_4)\right\}\\
&\quad + y_1\left\{l_1(z_3p_4 - z_2p_5) + l_4(-p_1z_3 + q_1p_5) - l_5(-p_1z_2 + p_4q_1)\right\}\\
&\quad - y_2\left\{l_1(z_3p_3 - z_1p_5) + l_3(-p_1z_3 + p_5q_1) - l_5(-p_1z_1 + q_1p_3)\right\}\\
&\quad + y_3\left\{l_1(p_3z_2 - p_4z_1) + l_3(-p_1z_2 + p_4q_1) - l_4(-p_1z_1 + p_3q_1)\right\},\\
P_3 &= \gamma_1\left\{l_2(p_4z_3 - z_2p_5) + l_4(-p_2z_3 + q_2p_5) - l_5(-p_2z_2 + q_2p_4)\right\}\\
&\quad - \gamma_2\left\{l_1(z_3p_4 - z_2p_5) + l_4(-p_1z_3 + q_1p_5) - l_5(-p_1z_2 + p_4q_1)\right\}\\
&\quad - y_2\left\{l_1(z_3p_2 + q_2p_5) - l_2(-p_1z_3 + p_5q_1) - l_5(p_1q_2 - q_1p_2)\right\}\\
&\quad + y_3\left\{l_1(-p_2z_2 + p_4q_2) - l_2(-p_1z_2 + p_4q_1) - l_4(p_1q_2 - p_2q_1)\right\},\\
P_4 &= \gamma_1\left\{l_2(p_3z_3 - z_1p_5) + l_3(-p_2z_3 + q_2p_5) - l_5(-p_2z_1 + q_2p_3)\right\}\\
&\quad - \gamma_2\left\{l_1(z_3p_3 - z_1p_5) + l_3(-p_1z_3 + q_1p_5) - l_5(-p_1z_1 + p_3q_1)\right\}\\
&\quad - y_1\left\{l_1(-z_3p_2 + q_2p_5) - l_2(-p_1z_3 + p_5q_1) - l_5(p_1q_2 - q_1p_2)\right\}\\
&\quad + y_3\left\{l_1(-p_2z_1 + p_3q_2) - l_2(-p_1z_1 + p_3q_1) - l_3(p_1q_2 - p_2q_1)\right\},
\end{aligned}
$$

$$
\begin{aligned}
P_5 = \gamma_1 \{ & l_2 (p_3 z - p_4 z_1) + l_3 (-p_2 z_2 + q_2 p_4) - l_4 (-p_2 z_1 + q_2 p_3)\} \\
& -\gamma_2 \{l_1 (z_3 p_2 - p_4 z_1) + l_3 (-p_1 z_2 + q_1 p_4) - l_4 (-p_1 z_1 + p_3 q_1)\} \\
& -y_1 \{l_1 (-z_2 p_2 + q_2 p_4) - l_2 (-p_1 z_2 + p_4 q_1) - l_4 (p_1 q_2 - q_1 p_2)\} \\
& +y_2 \{l_1 (-p_2 z_1 + p_3 q_2) - l_2 (-p_1 z_1 + p_3 q_1) - l_3 (p_1 q_2 - p_2 q_1)\}.
\end{aligned}
$$

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