Acta Mech 229, 3537–3556 (2018)
https://doi.org/10.1007/s00707-018-2132-4

ORIGINAL PAPER

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A new macro-mechanical approach for investigation of damage zone effects on mixed mode I/II fracture of orthotropic materials

Received: 25 October 2017 / Revised: 2 February 2018 / Published online: 9 June 2018

Abstract In this paper a new criterion for fracture investigation of orthotropic materials with cracks under mixed mode I/II loading is presented. In this fracture criterion, orthotropic material will be considered as a reinforced isotropic material. It is supposed that the crack will grow in the matrix of the orthotropic material. A new definition named here as "isotropic–orthotropic stress reduction factor" (IO-SRF) is utilized to consider the effects of the fracture process zone by a macro-mechanics approach. Also, the stress reduction factors will present a valuable relationship between the orthotropic and isotropic fracture toughness. Experimental and finite element methods will be introduced for computing the stress reduction factors. The SRFs are calculated for samples of glass–epoxy as an orthotropic material and samples of epoxy as a related isotropic one. Experimental tests under mixed mode I/II are performed on glass–epoxy composite samples to evaluate the validity of the presented mixed mode fracture criterion. The results of experimental tests on composite samples show a good agreement with the results of the presented criterion. Thus, the proposed criterion could be utilized as an efficient criterion for investigating the fracture of orthotropic materials under mixed mode I/II loading.

1 Introduction

The application of composite materials is increasing in various industries. Most designers consider these materials because of their exclusive properties such as low stiffness to weight ratio. Structures designed with composite materials are under complicated loadings during their working life [1]. Existence of cracks and defects in these materials is more prevalent than in isotropic materials, and failure investigation could be effectively prepared by fracture mechanics theory [2] and lack of proper connection of the fibers to the matrix as well [3]. So, investigation of the fracture of composite materials finds a significant place in recent studies of fracture mechanics [4]. Nowadays, there are many comprehensive and reliable criteria for investigating the fracture of isotropic materials such as minimum strain energy density [5], maximum energy release rate [6] and maximum tangential stress [7]. These criteria are able to predict both the crack direction and crack initiation under mixed mode loading.

Also some criteria for investigating the facture of orthotropic materials under mixed mode I/II have been presented, which are divided into two major categories as experimental and analytical studies [8]. Most of the old presented fracture criteria are empirical and based on curve fitting on experimental data. Although the experimental approaches are suitable for investigation of the complicated fracture behavior of orthotropic materials and the ability of these methods for predicting the experimental results is acceptable, it is difficult to use these methods because they include at least two experimental constants [9]. Among the most common experimental criteria, Wu criterion could be noted that is presented for investigating the fracture of Balsa

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wood on 1967 [10]. In 1974, Leicester presented a linear interactive equation between stress intensities in mode I and mode II based on the experimental results for Pine (Pinus radiate) wood for investigating the wood fracture, which is a natural orthotropic material [11]. In 1976, Williams and Birch have investigated the fracture phenomenon for two wood species, Utile (Entandrophragma utile) and Scots pine (Pinus sylvestris), on specimens with straight and angled cracks. They concluded that the shear stress which creates slip has no effect on failure in mixed mode loading [12]. In 1979, Woo and Chow have investigated the fracture in mixed mode loading on two wood species, Kapur (Dryobalanops spp) and Gagil (Hopesegal), by the use of experimental specimens with central and edge cracks. Their experimental results showed that the fracture in mixed mode loading in wood depends on both stress intensity factors K_{I} and K_{II} [13]. Also, in 1982, Hunt and Crager presented an experimental equation for investigating the fracture of orthotropic materials under mixed mode which had two experimental constants [14]. In 1983, Mall et al. have done many mixed mode experiments on Red spruce (Picea rubens) wood specimens in TL crack system and definitely declared that there is an interaction between K_{I} and K_{II} at the moment of fracture initiation. They also introduced the interactive equation of Wu as the best criterion, based on the experimental results [15].

In most research in the field of analytical mixed mode fracture criteria for investigation of the orthotropic materials, the main approach was to extend the isotropic fracture criteria into orthotropic materials. In 2000, Jernkvist extended the most famous isotropic criteria such as minimum strain energy density, maximum energy release rate and maximum principal stress to fracture of wood specimens, which is a natural orthotropic material [16]. In another research in 2000, he performed some experimental tests on wood specimens and confirmed that the new extended energy criteria are too conservative [17]. He also declared that the common maximum tangential stress criterion does not have the necessary requirements for extension to orthotropic materials [16]. In 2008, Romanowicz et al. proposed a fracture criterion under mixed mode loading for orthotropic materials by a nonlocal stress approach and micro-mechanics analysis. They considered the effects of the process zone in their criterion by micro-point of view but unfortunately could not calculate the damage factor in their criterion [18]. Criticisms of this criterion have been presented in [19]. In 2006, Van Der Put proposed a criterion based on a new isotropic–orthotropic conversion by means of the Airy stress function. The result of this study was the extraction of real fracture energy; also an analytical relation between stress intensity factors and energy release rate was presented [20]. In 2010, Gohari and Fakoor [21] proposed a criterion for investigating the fracture of orthotropic materials like wood under mixed mode I/II loading. This method, named “reinforcement micro-crack” approach, has been presented for consideration of effects of the local damage area in the crack tip vicinity. In the same year [19], they proposed a general criterion for investigating the fracture of orthotropic materials under mixed mode I/II that is applicable for orthotropic materials with cracks at arbitrary angles with respect to the orthotropy axis. They also proposed a criterion for investigating the fracture of orthotropic materials under mixed mode I/II where the embedded damage factor was determined based on the strength of the orthotropic material along and perpendicular to the fibers [22]. In 2013, Fakoor and Rafiee [23] presented a criterion which predicts the crack initiation and crack growth direction in orthotropic specimens under mixed mode I/II loading. This criterion was based on the distribution of the maximum shear stress in the crack tip vicinity. Also, Zappalorto et al. have focused on inter-fiber failure in fiber-reinforced composites. In their study, material and geometrical parameters were highlighted. Fiber/matrix debonding and matrix failure were considered [24].

Wide research has been done for investigating the crack growth direction in orthotropic materials. Buczek et al. [25] proposed a criterion based on the ratio of tangential stress to anisotropic tensile strength in arbitrary plates in the crack tip vicinity. Gregory and Herakovich [26] investigated the effects of anisotropy and biaxial loading to understand the effective parameters determining crack growth direction. In this study, they used two models to investigate the stress field in the crack tip vicinity: the anisotropic elasticity method and the finite element method. Also three different criteria, the tangential stress ratio, polynomial tensor and strain energy density were investigated in the crack tip vicinity to predict the crack growth direction. Saouma et al. [27] proposed a theory based on a maximum tensile stress criterion that could be used for orthotropic materials. Buczek and Herakovich proposed a criterion for prediction of the crack growth direction in orthotropic materials like wood. In this criterion, the assumption is that the crack growth direction is predictable by the ratio of maximum normal stress to strength in the assumed plane. Therefore, the standard proposed model was based on the previous maximum normal stress that included the strength of the material [28].

By review of the mentioned references, it is clear that the main problem in the presentation of a suitable fracture criterion for investigating the mixed mode loading in orthotropic materials is presenting an appropriate approach which could consider the wasted energy in the crack tip fracture process zone (FPZ) due to toughening mechanisms. Various toughening mechanisms in the crack tip damage zone of an orthotropic material have been

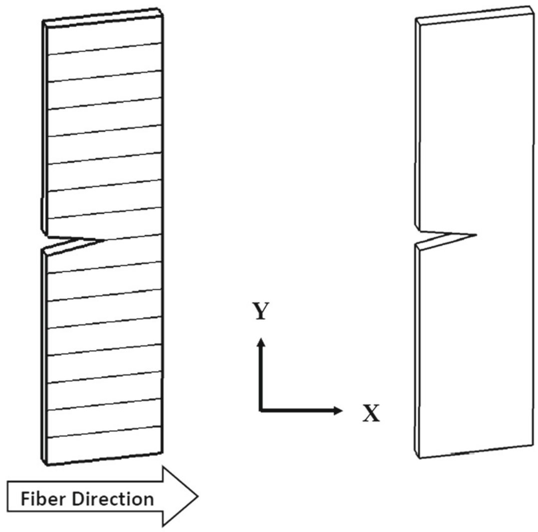

Fig. 1 Representative volume elements, **a** orthotropic material, **b** isotropic material

observed such as fiber bridging or micro-crack creation which can prevent the crack growth. So considering the effects of these mechanisms in a fracture criterion could obviously increase the accuracy of the proposed fracture criterion [29]. Since the FPZ has complicated nonlinear behavior, in recent proposed criteria the effects of the FPZ are not well considered, which leads to a conservative criterion, or the presented models for consideration of the effects of this zone are not so accurate and have not been validated by experimental methods [30].

The main purpose of this study is proposing a comprehensive mixed mode I/II fracture criterion which is able to consider the effects of the FPZ from a macro-mechanical point of view as well as the effects of fibers as reinforcements with a simple analytical relation. A new concept titled "isotropic–orthotropic stress reduction factor," IO-SRF, has been presented. In this concept, the fracture of orthotropic materials is investigated by a fiber-reinforced isotropic material model. This criterion is not dependent on unachievable material parameters which are difficult to extract from the standards, or for which there is not a defined standard test for extraction (such as mode II fracture toughness K_{IIc}), and most important the mentioned criterion could predict the fracture experimental data with a smaller error with respect to the other available criteria.

2 Problem statement

In this section, the proposed fracture model for extracting the fracture criterion for orthotropic materials under mixed mode I/II will be introduced. Also, general calculations and the formulation needed for extracting the fracture criterion based on a reinforced isotropic material will be presented. So, isotropic and orthotropic representative volume elements are necessary for modeling the presented criterion as shown in Fig. 1.

In this model, the orthotropic composite material is considered as an isotropic material and the fibers act as reinforcements in the isotropic matrix. The reason of presenting this model is that the fracture in orthotropic materials always appears in the isotropic base [31], even if the crack is not along the fibers (see Fig. 2).

In this model, we assume that the fibers decrease the normal and shear stress field in the crack tip vicinity by the coefficients η and ξ, respectively, like shown in Fig. 3.

As can be found from Fig. 3, the effect of adding fibers to the isotropic matrix (i.e., Fig. 3b) is considered as stress reduction factors. η is defined as the ratio of tensile stress in the orthotropic material to the isotropic material in the x direction (perpendicular to the loading direction), and ξ is the ratio of shear stress in the orthotropic material to the isotropic material as follows:

$$\eta = \frac{\sigma_x^{\text{ortho}}}{\sigma_x^{\text{iso}}}, \quad \xi = \frac{\sigma_{xy}^{\text{ortho}}}{\sigma_{xy}^{\text{iso}}}.$$

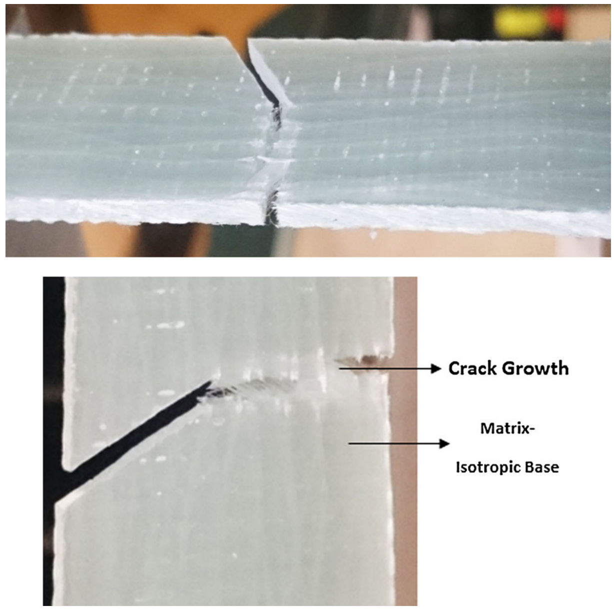

Fig. 2 Fracture of orthotropic material happening in the isotropic matrix along the fibers

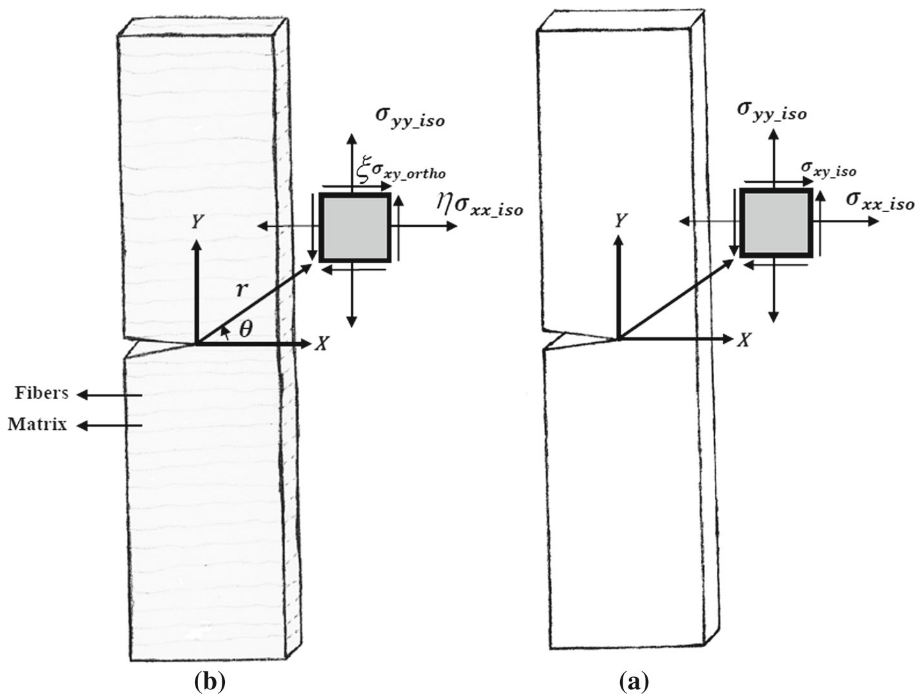

Fig. 3 Concept of stress reduction factors (η) and (ξ)

3 Theoretical background

In this section, extraction of mixed mode fracture equation for orthotropic materials, according to the models and assumptions in the previous section, will be performed. For extracting the fracture criterion, an isotropic–orthotropic conversion of the Airy stress function is utilized [20]. Fracture mechanics of most materials is based on the existence of a crack with nearly zero thickness and singularity in the crack tip. The existence of this crack makes it difficult to present an accurate criterion for investigating the material damage and describing the strength behavior of the material. As mentioned, the toughening process of crack tip in an orthotropic material depends on several phenomena and needs to investigate the crack tip damage from a microscopic point of view more carefully. But, generally the critical condition of crack initiation is investigated in most of the studies. Consequently, in this research, the failure condition in mixed mode also could be modeled by considering the critical condition when fracture occurs in the crack tip vicinity. In our presented approach, orthotropic materials are considered as an isotropic reinforced material and the Airy stress function is developed for orthotropic materials based on the reinforced matrix. The reinforcement is considered as a continuum material which satisfies equilibrium, compatibility and constitutive conditions. So, it is necessary that the Airy stress function be solved for available stresses in the isotropic matrix. For flat cracks along the principal axis of orthotropy, solutions of a 2D orthotropic solid are considered. The constitutive relations are as follows:

$$
\begin{aligned}
\varepsilon_x &= \frac{\sigma_x}{E_x} - \frac{\nu_{yx}\sigma_y}{E_y}, \\
\varepsilon_y &= \frac{\sigma_y}{E_y} - \frac{\nu_{yx}\sigma_x}{E_y}, \\
\gamma_{xy} &= \frac{\tau_{xy}}{G_{xy}}.
\end{aligned}
\tag{1}
$$

By defining φ as Airy stress function,

$$
\begin{aligned}
\sigma_x &= \frac{\partial^2\varphi}{\partial y^2}, \\
\sigma_y &= \frac{\partial^2\varphi}{\partial x^2}, \\
\tau_{xy} &= -\frac{\partial^2\varphi}{\partial x \partial y},
\end{aligned}
\tag{2}
$$

which satisfies the equilibrium equations. By substitution of Eqs. (2) into Eqs. (1),

$$
\begin{aligned}
\varepsilon_x &= C_{11}\frac{\partial^2\varphi}{\partial y^2} + C_{12}\frac{\partial^2\varphi}{\partial x^2}, \\
\varepsilon_y &= C_{12}\frac{\partial^2\varphi}{\partial y^2} + C_{22}\frac{\partial^2\varphi}{\partial x^2}, \\
\gamma_{xy} &= -C_{66}\frac{\partial^2\varphi}{\partial x \partial y},
\end{aligned}
\tag{3}
$$

in which the factors C_{ij} are components of the material elastic compliance tensor. In the compatibility condition we have the following equation:

$$
\frac{\partial^2\varepsilon_x}{\partial y^2} + \frac{\partial^2\varepsilon_y}{\partial x^2} = \frac{\partial^2\gamma_{xy}}{\partial x \partial y}. \tag{4}
$$

By substitution of Eqs. (3) into Eq. (4) we have:

$$
C_{22}\frac{\partial^4\varphi}{\partial x^4} + (C_{66} + 2C_{12})\frac{\partial^4\varphi}{\partial x^2 \partial y^2} + C_{11}\frac{\partial^4\varphi}{\partial y^4} = 0. \tag{5}
$$

For crack tip problems Eq. (5) could be selected as follows [20]:

$$
\left(\frac{\partial^2}{\partial x^2} + \alpha_1\frac{\partial^2}{\partial y^2}\right)\left(\frac{\partial^2}{\partial x^2} + \alpha_2\frac{\partial^2}{\partial y^2}\right)\varphi = 0, \tag{6}
$$

while we have:

$$
\begin{aligned}
\alpha_1\alpha_2 &= \frac{C_{11}}{C_{22}}, \\
\alpha_1 + \alpha_2 &= \frac{(C_{66} + 2C_{12})}{C_{22}}.
\end{aligned}
\tag{7}
$$

As discussed above, orthotropic failure is not determinative in cracked orthotropic materials. A composite orthotropic material is a reinforced material with fibers for which interaction of reinforcements in the isotropic matrix and also initial micro-cracks should be considered. So, any failure condition for orthotropic material based on matrix strength and Airy stress function should be solved for isotropic matrix stresses. Here, for an isotropic material with reinforcement, the following equation is considered to satisfy the equilibrium condition of matrix stresses:

$$\frac{\sigma_x}{\eta} = \frac{\partial^2 \varphi}{\partial y^2},$$
$$\sigma_y = \frac{\partial^2 \varphi}{\partial x^2},$$
$$\frac{\tau_{xy}}{\xi} = -\frac{\partial^2 \varphi}{\partial x \partial y}. \tag{8}$$

Here, instead of using the stress field of the orthotropic material, stresses in the isotropic matrix are considered that are reduced by factors η and ξ due to fiber effects and present a similar compatibility condition. By substitution of the total stresses into the compatibility equation (5) we have [20]:

$$C_{22}\frac{\partial^4 \varphi}{\partial x^4} + (\xi C_{66} + (1+\eta)C_{12})\frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + \eta C_{11}\frac{\partial^4 \varphi}{\partial y^4} = 0. \tag{9}$$

It is necessary for the stress function φ for the isotropic matrix to fulfill

$$\frac{\eta C_{11}}{C_{22}} = 1,$$
$$\frac{\xi C_{66} + (1+\eta)C_{12}}{C_{22}} = 2. \tag{10}$$

So we have:

$$\frac{\partial^4 \varphi}{\partial x^4} + 2\frac{\partial^4 \varphi}{\partial x^2 \partial y^2} + \frac{\partial^4 \varphi}{\partial y^4} = \nabla^4 \varphi = 0, \tag{11}$$

and consequently [20]:

$$\eta = \frac{C_{22}}{C_{11}} = \frac{E_1}{E_2},$$
$$\xi = \left(2 - \frac{C_{12}}{C_{22}} - \frac{C_{12}}{C_{11}}\right)\frac{C_{22}}{C_{66}} = (2 + \nu_{21} + \nu_{12})\frac{G_{12}}{E_2}. \tag{12}$$

Finally, the normal and shear stress reduction factors, i.e., η and ξ, will be calculated from Eq. (12).

4 Extraction of the fracture criterion

Proposed fracture criteria under mixed mode I/II for orthotropic materials based on experimental data are listed in Table 1.

Also analytical fracture criteria under mixed mode I/II for orthotropic materials are listed in Table 2.

As is clear from Tables 1 and 2, the general form of the fracture criteria under mixed mode I/II loading for orthotropic materials could be assumed as the following equation:

$$K_{\mathrm{I}}^2 + \rho K_{\mathrm{II}}^2 = K_{\mathrm{Ic}}^2. \tag{13}$$

In this equation, ρ is damage factor and contains the effects of the fracture process zone in the fracture criterion. Equation (13) is simple and has only two material constants, i.e., ρ and K_{Ic}. K_{Ic} is mode I fracture toughness and can be calculated simply by experiment or standard references. It is clear from Tables 1 and 2 that many researchers have recommended calculation of ρ. On the other hand, one of the important aspects of studies in investigating the fracture of orthotropic materials is proposing an appropriate method for calculation

Table 1 Proposed empirical fracture criteria under mixed mode I/II for orthotropic materials

Authors	Fracture criterion	Damage parameter
Wu [10]	$K_{\mathrm{I}} + \left(\frac{K_{\mathrm{I}c}}{K_{\mathrm{II}c}^2}\right) K_{\mathrm{II}}^2 - K_{\mathrm{I}c} = 0$	$\rho = \left(\frac{K_{\mathrm{I}c}}{K_{\mathrm{II}c}^2}\right), m = 1, n = 2$
Leicester [11]	$K_{\mathrm{I}} + \left(\frac{K_{\mathrm{I}c}}{K_{\mathrm{II}c}}\right) K_{\mathrm{II}} - K_{\mathrm{I}c} = 0$	$\rho = \left(\frac{K_{\mathrm{I}c}}{K_{\mathrm{II}c}}\right), m = 1, n = 1$
Williams and Birch [12]	$\frac{K_{\mathrm{I}}}{K_{\mathrm{I}c}} = 1$	–
Hunt and Croager [14]	$K_{\mathrm{I}} + \left((1.005)\frac{K_{\mathrm{I}c}}{K_{\mathrm{II}c}^{3.4}}\right) K_{\mathrm{II}}^{3.4} - 1 = 0$	$\rho = \left((1.005)\frac{K_{\mathrm{I}c}}{K_{\mathrm{II}c}^{3.4}}\right), m = 1, n = 3.4$
Mall et al. [15]	$K_{\mathrm{I}} + \left(\frac{K_{\mathrm{I}c}}{K_{\mathrm{II}c}^2}\right) K_{\mathrm{II}}^2 - 1 = 0$	$\rho = \left(\frac{K_{\mathrm{I}c}}{K_{\mathrm{II}c}^2}\right), m = 1, n = 2$

Table 2 Proposed analytical fracture criteria under mixed mode I/II for orthotropic materials

Authors	Fracture criteria	Damage parameter
Jernkvist [16]	$K_{\mathrm{I}}^2 + \beta_1 K_{\mathrm{II}}^2 - K_{\mathrm{I}c}^2 = 0$	$\rho = \beta_1 = \left(\frac{K_{\mathrm{I}c}}{K_{\mathrm{II}c}}\right)^2 = \frac{E_{\mathrm{I}}'}{E_{\mathrm{II}}'} = \sqrt{\frac{C_{11}'}{C_{22}'}}$
Jernkvist [16]	$K_{\mathrm{I}}^2 + \beta_2 K_{\mathrm{II}}^2 - K_{\mathrm{I}c}^2 = 0$	$\rho = \beta_2 = \left(\frac{K_{\mathrm{I}c}}{K_{\mathrm{II}c}}\right)^2$
Jernkvist [16]	$\frac{1}{(\beta_3 + \sqrt{\beta_4})}[\beta_3 K_{\mathrm{I}} + \sqrt{\beta_4 K_{\mathrm{I}}^2 + K_{\mathrm{II}}^2}] - K_{\mathrm{I}c} = 0$	$\beta_3 + \sqrt{\beta_4} = \frac{K_{\mathrm{II}c}}{K_{\mathrm{I}c}}$
Romanowicz et al. [18]	$K_{\mathrm{I}}^2 + \frac{c_{RL}}{c_R} K_{\mathrm{II}}^2 = K_{\mathrm{I}c}^2$	$\rho = \frac{c_{RL}}{c_R} = \left(\frac{K_{\mathrm{I}C}}{K_{\mathrm{II}c}}\right)^2$
Gowhari-Anaraki and Fakoor [21]	$K_{\mathrm{I}}^2 + \rho K_{\mathrm{II}}^2 - K_{\mathrm{I}c}^2 = 0$	$\rho = \frac{(5-\nu)(\xi\sqrt{\lambda}+\nu_{LR}\lambda)^2}{(10-3\nu)(1+0.5\nu_{LR}(1+\lambda))^2}$
Gowhari-Anaraki and Fakoor [22]	$K_{\mathrm{I}}^2 + \rho K_{\mathrm{II}}^2 - K_{\mathrm{I}c}^2 = 0$	$\rho = 2\left[\left(\frac{T_m}{T_M}\right) + \left(\frac{T_m}{T_M}\right)^2\right]$
Fakoor and Rafiee [23]	$K_{\mathrm{I}}^2 + \rho K_{\mathrm{II}}^2 - K_{\mathrm{I}c}^2 = 0$	$\rho = \left(\frac{K_{\mathrm{I}C}}{K_{\mathrm{II}c}}\right)^2$
Fakoor and Mehri [8]	$K_{\mathrm{I}}^2 + \rho K_{\mathrm{II}}^2 - K_{\mathrm{I}c}^2 = 0$	$\rho = \frac{(5-\nu)(\xi\sqrt{\lambda}+\nu_{LR}\lambda)^2}{(10-3\nu)(1+0.5\nu_{LR}(1+\lambda))^2}$

of this damage factor. If ρ is considered as $\rho = (K_{\mathrm{I}C}/K_{\mathrm{II}c})^2$, we will reach into the common Wu equation for mixed mode fracture of orthotropic materials. Employing the Wu criterion needs mode II fracture toughness, and as we know the calculation of this material property is difficult, this criterion is not welcome in application fields. Micro- or macro-mechanic approaches for the calculation of the damage factor ρ could be assumed. Using a micro-point of view for estimation of ρ is very complicated and needs many accurate and statistical investigations [32].

In this paper, for calculation of the damage factor ρ, a macro-mechanics approach and the common equation of Wu for orthotropic materials are utilized. By means of stress reduction factors η and ξ, the equations presented in Sect. 3 are rewritten as follows for calculation of the damage factor. For a crack with length of $2c$ in an infinite plane with a thickness of t, and in the main direction of orthotropy, the required energy for opening of crack sides with normal stress σ on crack plane is equal to

$$G_c^{\mathrm{ortho}} = \frac{\sigma^2 \pi c}{E_y} = \frac{(K_{\mathrm{I}c}^{\mathrm{ortho}})^2}{E_y}, \tag{14}$$

in which G_c^{ortho} is the critical fracture energy in the orthotropic material. As the loading condition in this paper is considered mixed mode I/II, Eq. (14) could be rewritten by utilizing superposition as follows:

$$G_c^{\mathrm{ortho}} = \frac{\left(K_{\mathrm{I}c}^{\mathrm{ortho}}\right)^2}{E_{\mathrm{I}}} + \frac{\left(K_{\mathrm{II}c}^{\mathrm{ortho}}\right)^2}{E_{\mathrm{II}}}. \tag{15}$$

So, for orthotropic material we have:

$$G_c^{\mathrm{ortho}} = \pi c \left(\frac{\sigma_x^2}{E_{\mathrm{I}}} + \frac{\tau_{xy}^2}{E_{\mathrm{II}}}\right)^{\mathrm{ortho}}. \tag{16}$$

By employing the concept of reinforced isotropic material we have:

$$G_c^{\text{ortho}} = \pi c \left(\frac{\left(\sigma_x^{\text{iso}}\right)^2 \eta^2}{E_{\text{I}}} + \frac{\left(\tau_{xy}^{\text{iso}}\right)^2 \xi^2}{E_{\text{II}}} \right). \tag{17}$$

In this equation σ_x^{iso} is the stress in the matrix of the orthotropic material along the x axis. τ_{xy}^{iso} is the shear stress in the isotropic matrix. By substitution of Eq. (12) into Eq. (17) we have:

$$G_c^{\text{ortho}} = \pi c \left(\frac{\eta (\sigma_x^{\text{iso}})^2 + \xi^2 (\tau_{xy}^{\text{iso}})^2}{E_{\text{II}}} \right). \tag{18}$$

By employing Eq. (16), the above equation takes the following form:

$$G_c^{\text{ortho}} = \frac{\eta G_{\text{I}}^{\text{iso}} E_{\text{I}}^{\text{iso}} + \xi^2 G_{\text{II}}^{\text{iso}} E_{\text{II}}^{\text{iso}}}{E_{\text{II}}^{\text{ortho}}}. \tag{19}$$

The left-hand side of Eq. (19) can be rewritten by using superposition of mode I and mode II energy:

$$G_{\text{I}}^{\text{ortho}} + G_{\text{II}}^{\text{ortho}} = \frac{\eta G_{\text{I}}^{\text{iso}} E_{\text{I}}^{\text{iso}} + \xi^2 G_{\text{II}}^{\text{iso}} E_{\text{II}}^{\text{iso}}}{E_{\text{II}}^{\text{ortho}}}. \tag{20}$$

Consequently, we have:

$$G_{\text{I}}^{\text{ortho}} E_{\text{II}}^{\text{ortho}} + \left(K_{\text{II}}^{\text{ortho}}\right)^2 = \eta \left(K_{\text{I}}^{\text{iso}}\right)^2 + \xi^2 \left(K_{\text{II}}^{\text{iso}}\right)^2, \tag{21}$$

and considering Eq. (14):

$$\frac{\left(K_{\text{I}}^{\text{ortho}}\right)^2}{E_{\text{I}}^{\text{ortho}}} E_{\text{II}}^{\text{ortho}} + \left(K_{\text{II}}^{\text{ortho}}\right)^2 = \eta \left(K_{\text{I}}^{\text{iso}}\right)^2 + \xi^2 \left(K_{\text{II}}^{\text{iso}}\right)^2. \tag{22}$$

By consideration of Eq. (12) we have:

$$\frac{\left(K_{\text{I}}^{\text{ortho}}\right)^2}{\eta} + \left(K_{\text{II}}^{\text{ortho}}\right)^2 = \eta \left(K_{\text{I}}^{\text{iso}}\right)^2 + \xi^2 \left(K_{\text{II}}^{\text{iso}}\right)^2. \tag{23}$$

The final equation is:

$$\left(K_{\text{I}}^{\text{ortho}}\right)^2 + \eta \left(K_{\text{II}}^{\text{ortho}}\right)^2 = \eta^2 \left(K_{\text{I}}^{\text{iso}}\right)^2 + \eta \xi^2 \left(K_{\text{II}}^{\text{iso}}\right)^2. \tag{24}$$

Equation (24) establishes a new relation between isotropic and orthotropic fracture toughness utilizing the stress reduction factors η and ξ.

As mentioned previously, for proposing a fracture criterion, the main purpose is to calculate the damage factor ρ in Eq. (13). The best estimation for the damage factor is $\rho = (K_{\text{IC}}/K_{\text{II}c})^2$ due to crossing the boundaries of the fracture limit curve (i.e., pure mode I and mode II fracture toughness). In pure mode I condition (i.e., $K_{\text{II}} = 0$) Eq. (24) will change as follows:

$$\left(K_{\text{I}c}^{\text{ortho}}\right)^2 = \eta^2 \left(K_{\text{I}c}^{\text{iso}}\right)^2. \tag{25}$$

Also, in pure mode II condition (i.e., $K_{\text{I}} = 0$) we have:

$$\left(K_{\text{II}c}^{\text{ortho}}\right)^2 = \xi^2 \left(K_{\text{II}c}^{\text{iso}}\right)^2. \tag{26}$$

Now the damage factor ρ is defined based on the Wu criterion as follows:

$$\rho = \left(\frac{K_{\text{I}c}^{\text{ortho}}}{K_{\text{II}c}^{\text{ortho}}}\right)^2 = \left(\frac{\eta}{\xi}\right)^2 \left(\frac{K_{\text{I}c}^{\text{iso}}}{K_{\text{II}c}^{\text{iso}}}\right)^2. \tag{27}$$

Equation (27) establishes a new relation between the fracture toughness of the orthotropic material and its isotropic matrix. Considering this equation, the ratio of fracture toughness of a reinforced material with fibers (orthotropic material) could be easily calculated with the related matrix properties (isotropic material). Thus, the damage factor ρ could be easily calculated utilizing the fracture toughness of mode I and mode II of the matrix and the stress reduction coefficients η and ξ.

5 Experimental tests

In this section necessary experimental tests for the determination of the fracture limit curve and the safe design area are defined. As is obviously clear, the proposed criterion is not dependent on unachievable fracture parameters such as mode I or II orthotropic fracture toughness. For implementation of this new fracture criterion, only fracture toughness of mode I and II of the isotropic matrix is necessary as well as the newly defined stress reduction factors η and ξ.

In this section, the critical loads are determined by experimental tests and the fracture toughness is calculated by utilizing the finite element method. Stress reduction factors are calculated by experimental tests and verified by the finite element method. Also, fracture tests under mixed mode I/II loading are done to verify the fracture curves resulting from the proposed criterion.

According to the proposed criterion, manufacturing of the orthotropic material as well as the isotropic matrix of the mentioned orthotropic material is required. In this section, glass–epoxy composite is considered as an orthotropic material and the matrix of this composite (i.e., epoxy) is considered as related isotropic material.

The resin transfer molding (RTM) method utilizing a vacuum pump is employed for preparing the glass–epoxy composites as an orthotropic material. In this method, glass fibers are put into a resin mold and then the resin is injected. After complete resin transmission, two sides of the resin and fiber set which is connected to the vacuum pump are highly compressed due to the created vacuum. Therefore, extra resin will transfer into the Blader textile and so air bubbles and unbalanced resin distribution do not occur. This method is more accurate and creates smooth surfaces in comparison with the manual method. Also, the possibility of air bubbles among the resin and fiber set that could increase the possibility of cracks is reduced.

Also, each layer of fibers can be impregnated with resin manually, and finally by connecting the fibers and resin set into the vacuum pump, the fibers will have completely dipped in the available resin in the set and the extra resin transfers to the Blader textile. After the set has dried, the glass–epoxy composite is ready. In Fig. 4 the manufacturing process of orthotropic material is illustrated.

5.1 Calculation of mechanical properties of glass–epoxy composite

In this section, elastic properties of glass–epoxy are calculated using standard tension tests. Ten specimens are prepared based on the ASTM D3039 standard to extract glass–epoxy mechanical properties along and perpendicular to the fibers (five specimens for each direction). The sample sizes are listed in Table 3.

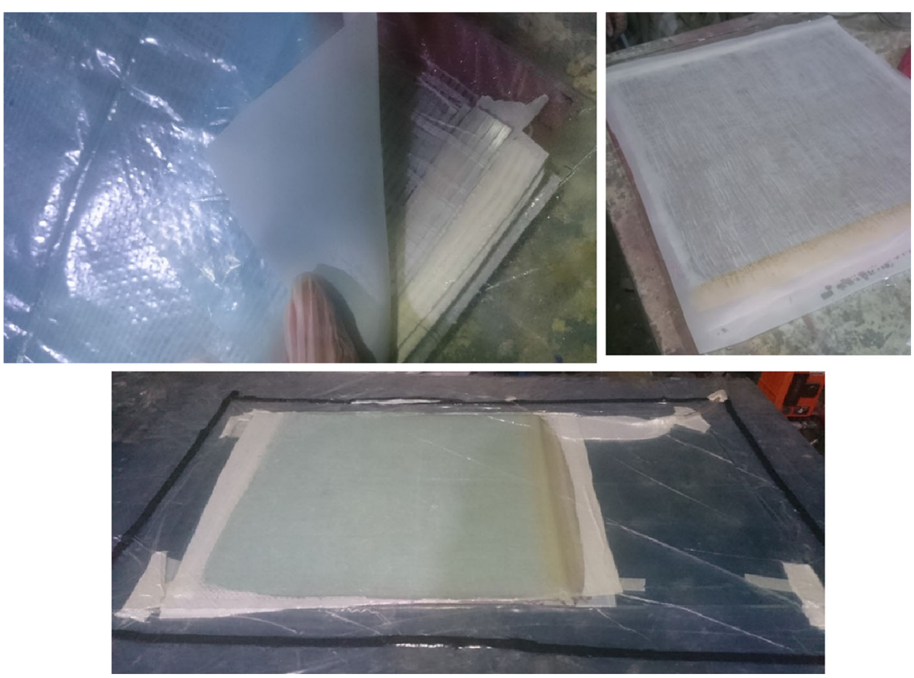

Fig. 4 Placing the Dacron textile and perforated film on the fibers and resin set

Table 3 Composite sample sizes prepared based according to the ASTM D3039 standard

Fiber orientation	Width (mm)	Overall length (mm)	Thickness (mm)
0°	15	250	3.2
90°	25	175	3.2

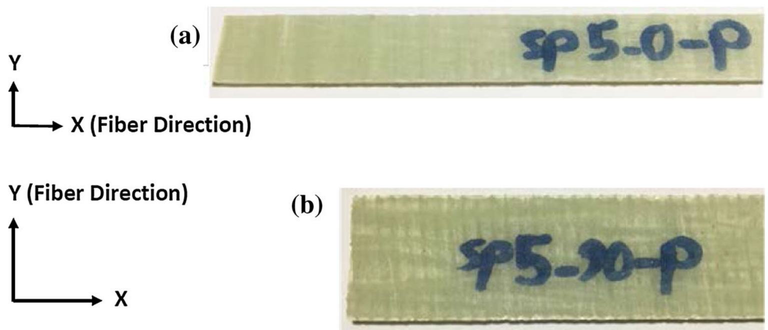

Fig. 5 Composite samples for calculation of mechanical properties **a** along the fibers, **b** perpendicular to the fibers

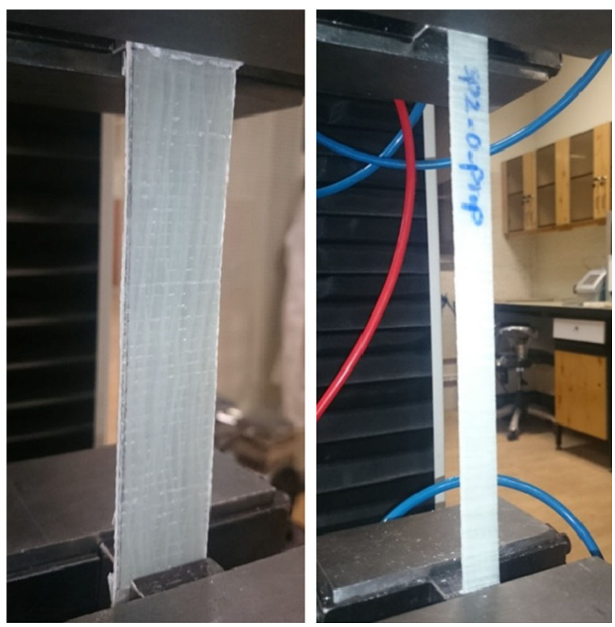

Fig. 6 Specimens under tensile test for calculation of the properties of orthotropic material along and perpendicular to the fibers according to the ASTM D3039 standard

Table 4 Mechanical properties of glass–epoxy based on ASTM D3039 standard

Fiber orientation	Area (mm^2)	F_{max} (N)	$x@F_{max}$ (mm)	S_{max} (MPa)	Elastic moduli (GPa)
0	15*3.2	17, 000	9.5	390.3	10.2
90	25*3.2	1250	1.3	16.5	2.5

The prepared composite samples are shown in Fig. 5.

The samples are undergoing tensile tests according to the ASTM D3039 standard. A sample under tensile test is illustrated in Fig. 6.

According to the standard, a linear slope section of the real stress–strain curve is equal to the elastic modulus. The final extracted properties by averaging from five sample results are listed in Table 4.

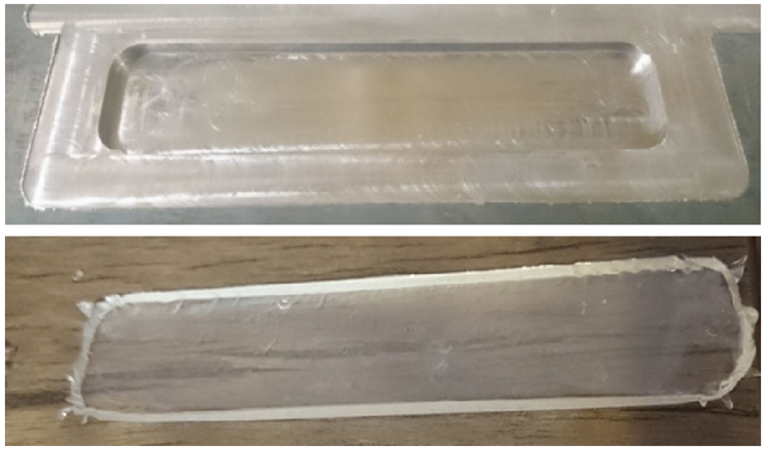

Fig. 7 Preparing isotropic samples from matrix of composite material

Table 5 Material properties of the matrix

Area (mm^2)	F_{max} (N)	$x@F_{max}$ (mm)	S_{max} (MPa)	Elastic moduli (GPa)
30*4.5	3829.6	5.8296	33.7737	0.754

As shown in Table 4, glass–epoxy composite elastic module along the fibers is 10.2 GPa and perpendicular to the fiber is 2.5 GPa. These values are acceptable in comparison with those reported in the literature [33].

5.2 Calculation of mechanical properties of epoxy

For investigating the matrix of orthotropic material, epoxy resin is considered as an isotropic material. Mixture of resin and hardener is performed in a steel mold for preparation of the isotropic samples. After the resin completely becomes dry, the samples detach from the mold and edges of the samples will be smooth by a rasp. The prepared samples are shown in Fig. 7.

Three samples are going under tension tests to calculate the properties of isotropic material. The results are gathered in Table 5.

The amount of elastic modules for epoxy resin is calculated as 754 MPa. This value is acceptable in comparison with the reported in the literature [33].

5.3 Calculation of stress reduction factors from experimental test results

5.3.1 Tension test of cracked orthotropic material

Since the properties of isotropic and orthotropic materials are calculated in previous sections from experimental tests, now orthotropic specimens with cracks along the fibers are tested under pure mode I loading; then, the resultant stress field will be compared with the results of isotropic material to calculate the stress reduction factors. For calculation of stress distribution on cracked orthotropic material under tension test, ASTM E1922 standard is utilized. The dimensions of the samples are as shown in Fig. 8.

Consequently, composite sample dimensions in this paper are listed in Table 6.

Loading conditions on prepared composite samples are as shown in Fig. 9.

The results of this tension tests are listed in Table 7.

Force–displacement diagram raised from the tension test is shown in Fig. 10.

The critical load on orthotropic specimen under pure mode I is estimated to be $F_c = 1250$ (N).

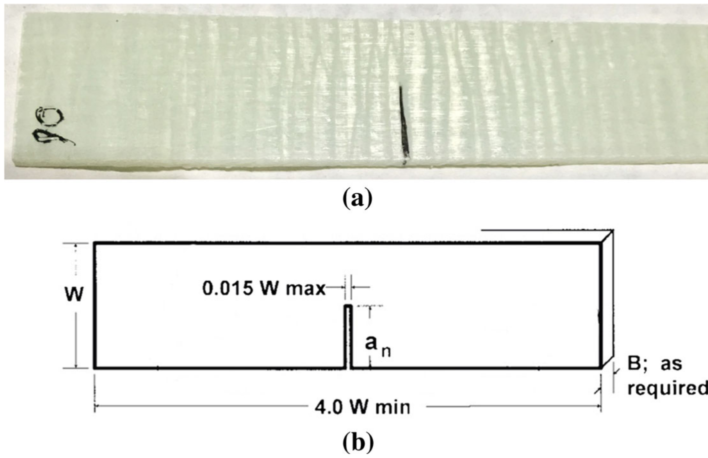

Fig. 8 **a** Composite sample prepared based on the ASTM E1922 standard, **b** dimensions of cracked samples under fracture test based on the ASTM E1922 standard

Table 6 Dimensions of prepared composite samples based on the ASTM E1922 standard

Fiber orientation	Width	Overall length	Thickness	Crack length
0°	30 mm	120 mm	3.2 mm	15 mm

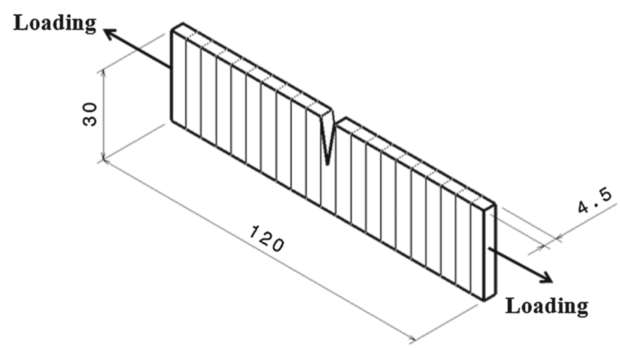

Fig. 9 Cracked composite sample under pure mode I loading

Table 7 Results from orthotropic material tension test under pure mode I loading

Fiber orientation	Area (mm^2)	F_{max} (N)	$x@F_{max}$ (mm)
0	3*3.2	1250	0.935

5.3.2 Tension test of cracked isotropic material

All mentioned procedures for calculation of the critical load and stress distributions in orthotropic material are kept in this section for isotropic material. Cracked isotropic specimens are going under pure mode I loading, and specimen dimensions are exactly the same as for the orthotropic material. Results of these tension tests are listed in Table 8.

The force–displacement diagram from this tension test is illustrated in Fig. 11.

The critical load of the isotropic specimen under pure mode I is estimated the to be $F_c = 1210\,(\text{N})$.

Since the values of the critical fracture load in the isotropic (epoxy) and the orthotropic (glass–epoxy composite) materials are almost the same, the principal assumption that the crack grows happens along the fibers in the isotropic base and low impact of the fibers in preventing the crack growth is proven again.

By employing the definition of the coefficient η as the ratio of stress along the loading direction in the orthotropic material relative to the isotropic the matrix we have: $\eta = \sigma_{yy}^{\text{ortho}}/\sigma_{yy}^{\text{iso}} = 1.77$.

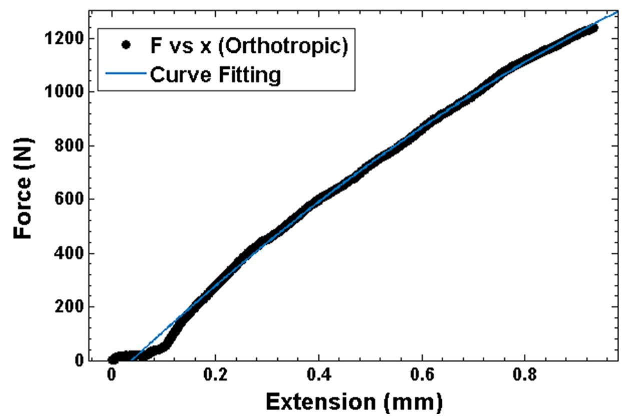

Fig. 10 Force–displacement curve of cracked composite sample (crack along the fibers) under pure mode I loading

Table 8 Results of cracked isotropic material tension test under pure mode I loading

Area (mm^2)	$F_{\max}$ (N)	$x@F_{\max}$ (mm)
3*3.2	1210.3	1.685

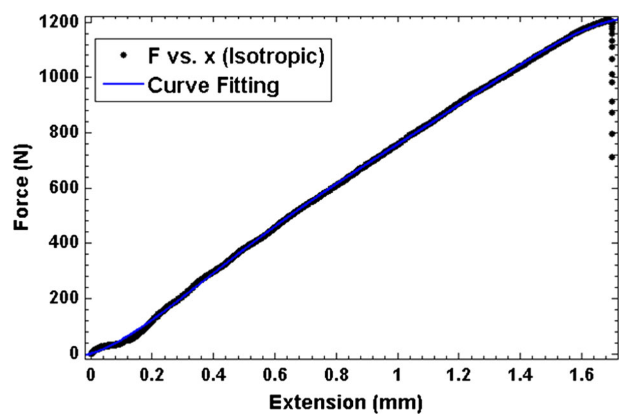

Fig. 11 Force–displacement curve of cracked isotropic sample under pure mode I loading

As a shear test was not performed on the specimens, only the finite element method is utilized to calculate the ξ coefficient.

6 Calculation of fracture toughness and stress reduction factors using FEM

In this section by employing finite element method, stress values for isotropic and orthotropic material are calculated under critical forces extracted from tension tests. A commercial finite element software and shell 8-node elements are utilized in this approach. As shown in Fig. 12, the orthotropic material is a 12-layer composite with 0° glass fibers (the same as experimental samples). Also, the mesh type in the composite material is S8R5 and in the epoxy material is S8R.

Stress distributions in the isotropic and in the orthotropic material are shown in Fig. 13.

Based on the definition of η we have $\eta = \sigma_{yy}^{\text{ortho}}/\sigma_{yy}^{\text{iso}} = 264\,\text{MPa}/145.4\,\text{MPa} = 1.8$ which is very close to the test results. Also, ξ is defined as the ratio of shear stress in the orthotropic material to the isotropic material and can be calculated as $\xi = \sigma_{xy}^{\text{ortho}}/\sigma_{xy}^{\text{iso}} = 5734\,\text{MPa}/3796\,\text{MPa} = 1.51$.

By the mentioned definition in this study, η and ξ show the normal and shear stress reduction amount, respectively, due to adding reinforcement (fibers) in the isotropic matrix. Now the value of the damage factor ρ will be calculated as follows:

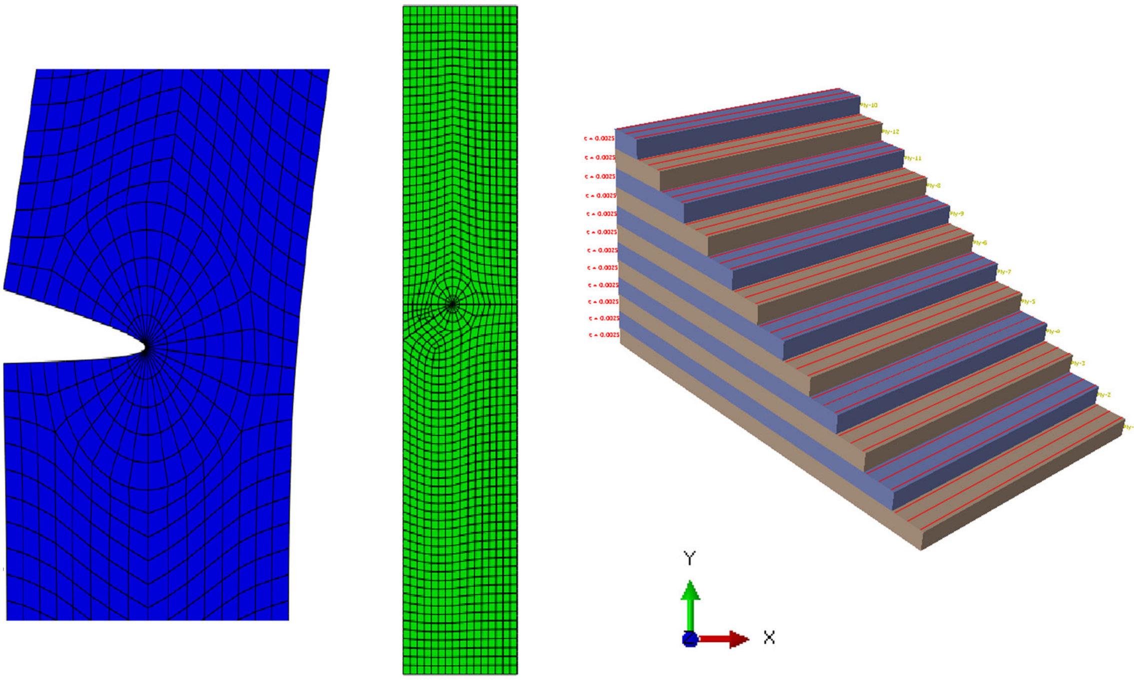

Fig. 12 Schematic of meshing and defined layers in composite material

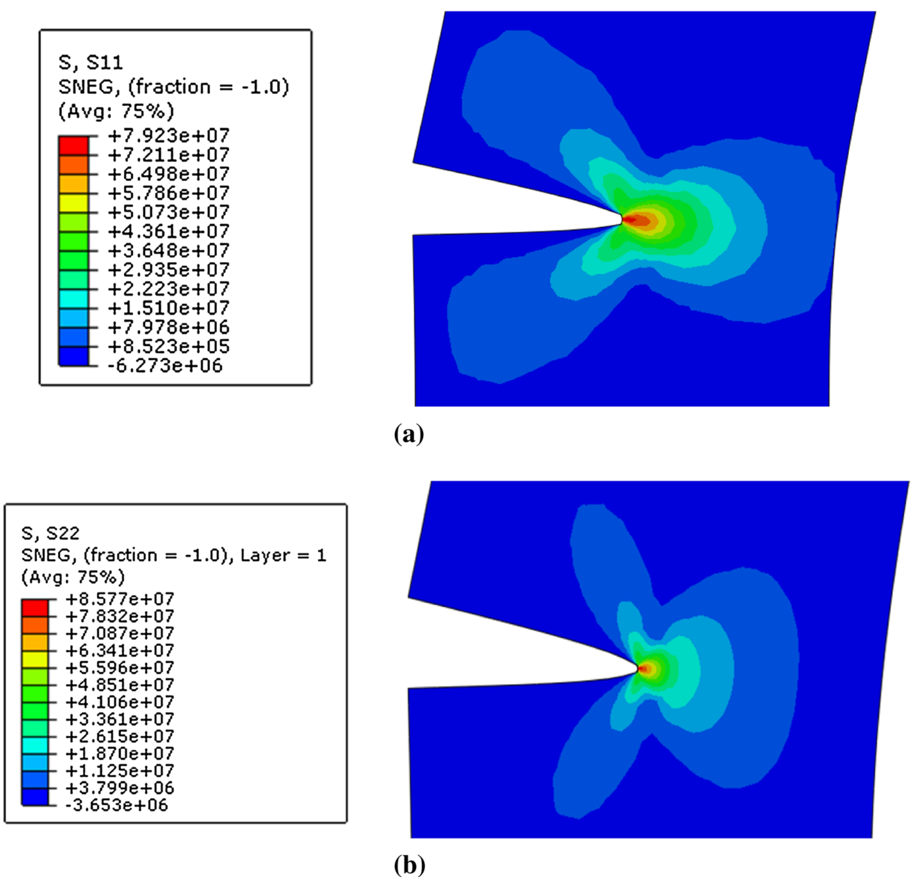

Fig. 13 Stress field around the crack, **a** isotropic material, **b** orthotropic material

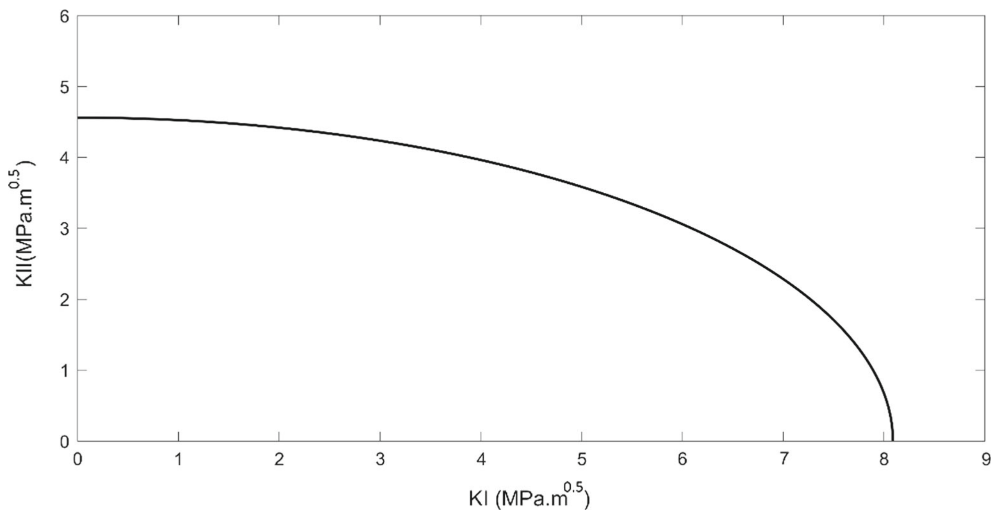

Fig. 14 Fracture limit curve related to proposed criterion

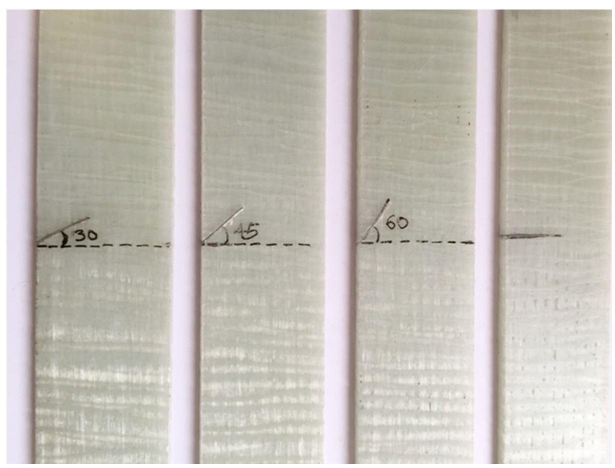

Fig. 15 Cracked specimens by various crack–fiber angle

$$\rho = \left(\frac{K_{\mathrm{I}c}^{\mathrm{ortho}}}{K_{\mathrm{II}c}^{\mathrm{ortho}}}\right)^2 = \left(\frac{\eta}{\xi}\right)^2 \left(\frac{K_{\mathrm{I}c}^{\mathrm{iso}}}{K_{\mathrm{II}c}^{\mathrm{iso}}}\right)^2 = \left(\frac{1.8}{1.51}\right)^2 \left(\frac{K_{\mathrm{I}c}^{\mathrm{iso}}}{K_{\mathrm{II}c}^{\mathrm{iso}}}\right)^2 = 3.15.$$

So, the fracture limit curve found from the proposed criterion can be plotted as illustrated in Fig. 14 for glass–epoxy.

For investigation of compatibility of this criterion with the nature of fracture of composite materials, experimental tests will be performed on some glass–epoxy specimens with arbitrary fiber–crack angles which creates various mixed mode loading situations.

7 Fracture experimental tests under mixed mode I/II

In this section, experimental fracture data under mixed mode I/II loading conditions are extracted. For preparing various mixed mode I/II loading conditions, a variation of the crack–fiber angle is considered. Therefore, cracked specimens with crack–fiber angles of 30°, 45° and 60° are utilized. Five specimens for each crack–fiber angle are prepared to take into account the scattering in the test results (see Fig. 15).

The critical fracture load is recorded for each specimen. Force–displacement diagrams extracted from the fracture tests are illustrated in Fig. 16.

The experimental results are gathered in Table 9.

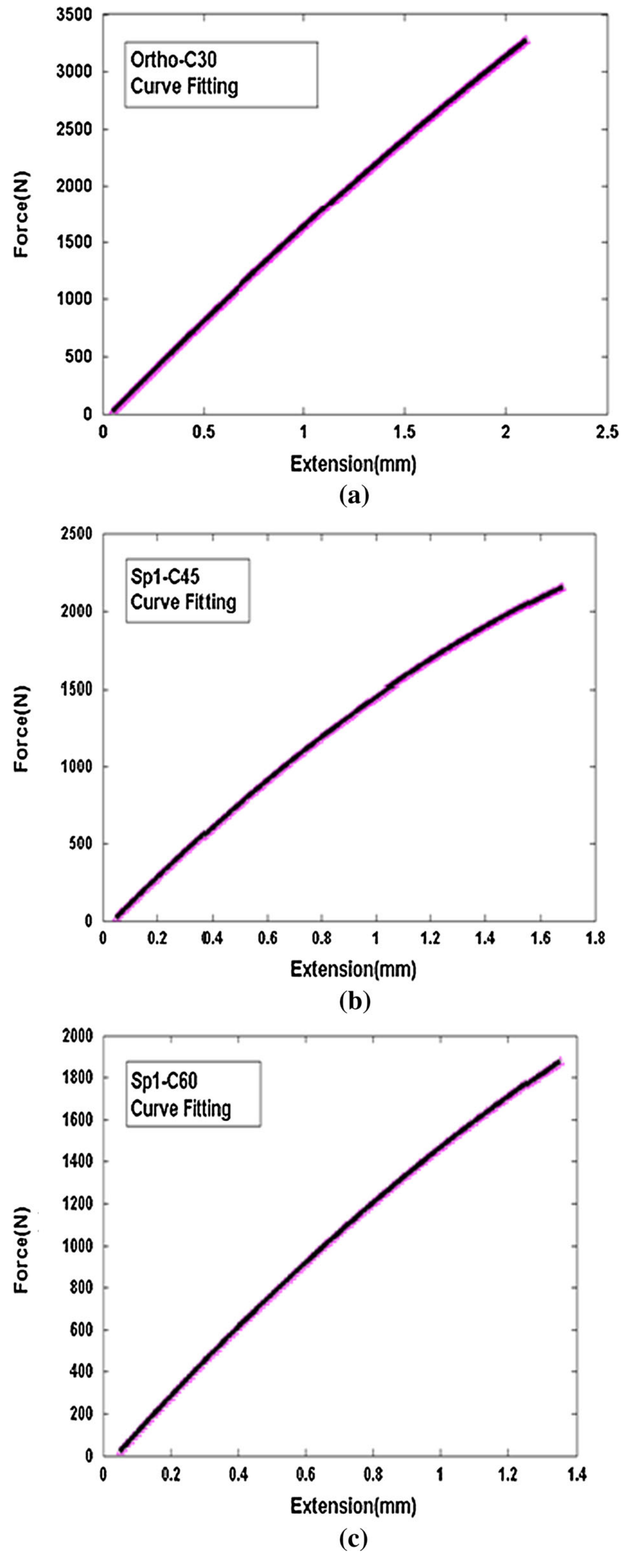

Fig. 16 Diagrams of force–displacement for crack–fiber angle of **a** 30°, **b** 45°, **c** 60°

Table 9 Results of experimental tests of the cracked specimens

No.	Crack–fiber angle (∘)	Displacement at fracture load (mm)	Critical fracture load (N)
1	0	0.93	1250
2	30	2.1	3275
3	45	1.65	2130
4	60	1.35	1873

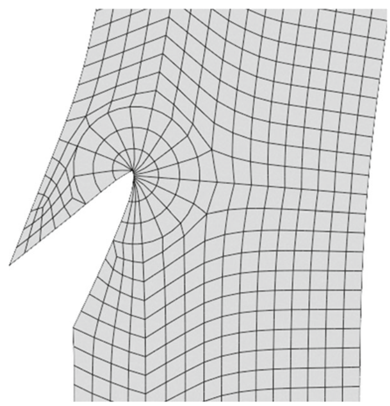

Fig. 17 A schematic of mesh configuration and size

The mentioned cracked specimens are simulated in finite element software by utilizing critical fracture loads extracted from the tests to achieve in mode I and mode II fracture toughness, i.e., $K_{\mathrm{I}c}^{\mathrm{iso}}$ and $K_{\mathrm{II}c}^{\mathrm{iso}}$.

8 Calculation of fracture toughness of specimens by the Finite Element Method

For calculation of $K_{\mathrm{I}c}^{\mathrm{iso}}$ and $K_{\mathrm{II}c}^{\mathrm{iso}}$ the critical fracture loads extracted from experimental tests are applied to finite element models of specimens. Elements and mesh type are similar to pure mode I models mentioned in Sect. 6. A schematic of mesh configuration and size is illustrated in Fig. 17.

Stress distribution around the crack tip extracted from finite element analysis is shown in Fig. 18.

The FE values of critical stress intensity factors are listed in Table 10.

The fracture limit curve is plotted in Fig. 19 in comparison with experimental data.

As is shown in Fig. 19, the proposed criterion has a good agreement with experimental data and this result shows the accuracy of the proposed fracture criterion as well as coincidence of this criterion with the nature of fracture of orthotropic materials.

9 Conclusion

In this paper, a new criterion for investigation of the fracture of orthotropic composite materials under mixed mode I/II is presented. A macro-mechanical approach is utilized for investigation and analysis of the complicated behavior of the crack tip damage zone. It is assumed that the orthotropic material is considered as a isotropic material reinforced with fibers. This assumption is due to experimental observations that the crack in orthotropic materials grows in the isotropic base. Based on this assumption, stress reduction factors η and ξ which are the ratio of stress in orthotropic material to isotropic material were defined. Although the proposed criterion has a simple form and considers the fracture process zone effects, implementation of this criterion is not dependent to unachievable fracture or damage zone properties. The damage factor included in this criterion was calculated by using defined stress reduction factors and fracture toughness of the isotropic matrix. Required experimental tests for calculation of the critical load and stress reduction factors were defined and performed. Also, by employing the finite element method, fracture toughness, stress reduction factors and finally the damage factor of the fracture limit curve were calculated. Finally, for investigating the accuracy of the proposed criterion, mixed mode loading tests were performed on orthotropic material. Results of mixed mode fracture tests show the correspondence of the proposed criterion with the fracture behavior of orthotropic materials.

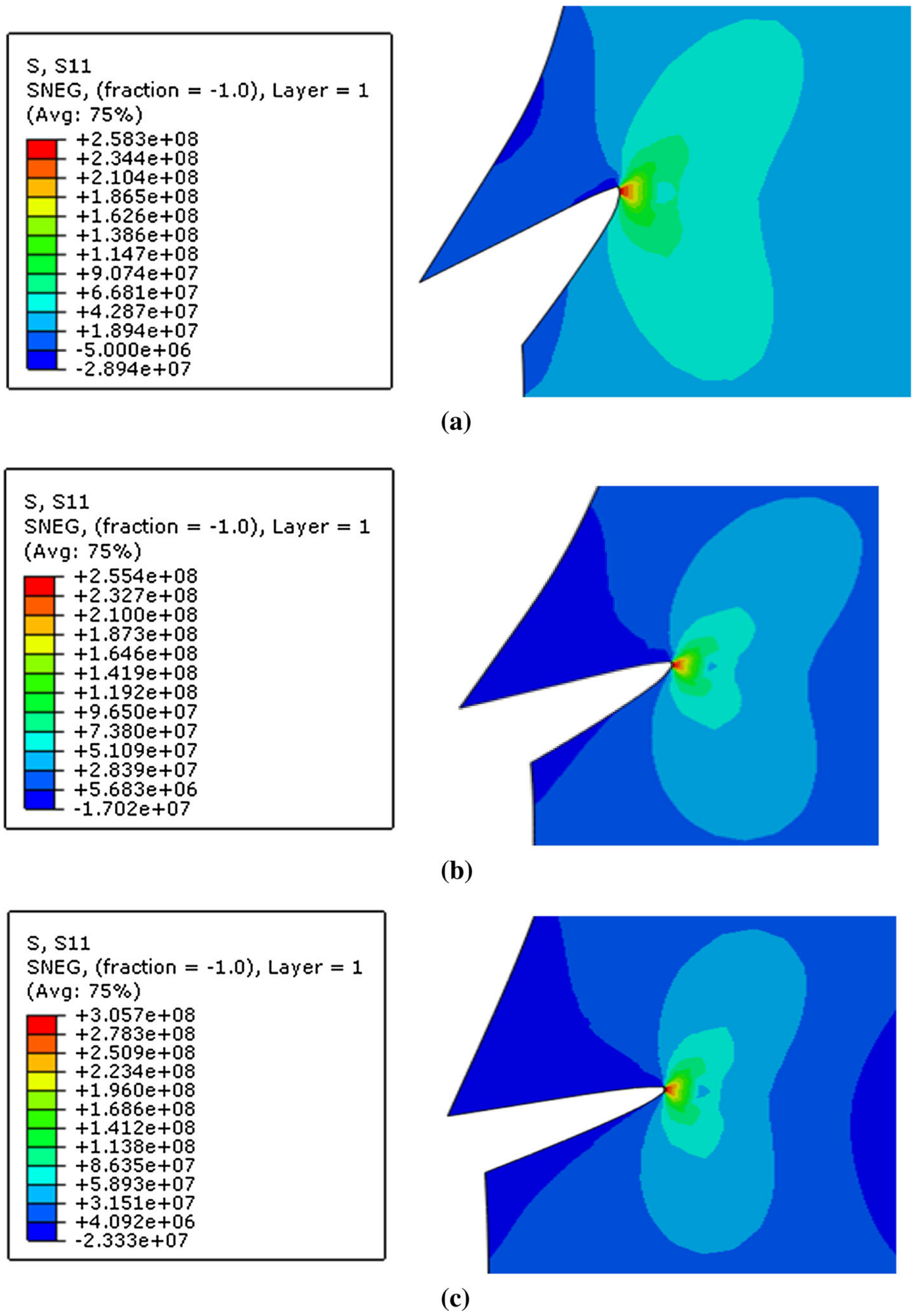

Fig. 18 Stress distribution around the crack tip extracted from finite element analysis with crack–fiber angle of **a** 30°, **b** 45°, **c** 60°

Table 10 Values of critical stress intensity factors extracted from FE analysis

Specimen no.	Crack–fiber angle (°)	$K_{Ic}^{iso}(\mathrm{MPa}\sqrt{m})$	$K_{IIc}^{iso}(\mathrm{MPa}\sqrt{m})$
1	0	8.09	0
2	30	3.94	3.84
3	45	5.3	3.58
4	60	6.91	2.5

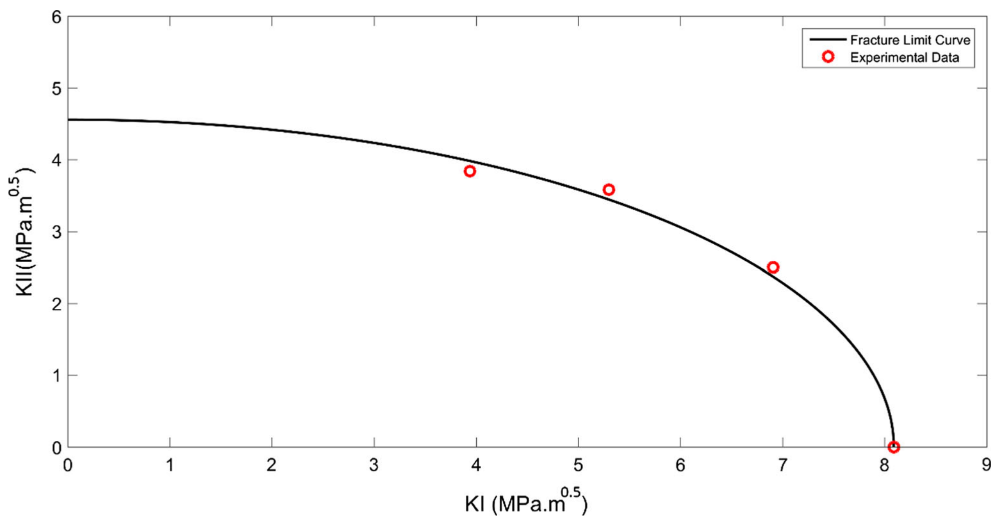

Fig. 19 Fracture limit curve in comparison with experimental data

References

1. Talreja, R.: Assessment of the fundamentals of failure theories for composite materials. Compos. Sci. Technol. **105**, 190–201 (2014)
2. Benveniste, Y., Aboudi, J.: Crack propagation in a laminated composite material modeled by a two-dimensional mixture theory. Acta Mech. **29**(1), 213–227 (1978)
3. Li, Y.D., Xiong, T., Zhao, H.: Interfacial fracture analysis of a piezoelectric-polythene composite cylindrical shell patch under axial shear. Acta Mech. **225**(2), 543 (2014)
4. Li, Y.D., Xiong, T., Cai, Q.G.: Coupled interfacial imperfections and their effects on the fracture behavior of a layered multiferroic cylinder. Acta Mech. **226**(4), 1183–1199 (2015)
5. Sih, G.C.: Strain-energy-density factor applied to mixed mode crack problems. Int. J. Fract. **10**(3), 305–321 (1974)
6. Hussain, M.A., Pu, S.L., Underwood, J.: Strain energy release rate for a crack under combined mode I and mode II. In: Fracture Analysis: Proceedings of the 1973 National Symposium on Fracture Mechanics, Part II. ASTM International (1974)
7. Erdogan, F., Sih, G.C.: On the crack extension in plates under plane loading and transverse shear. J. Basic Eng. **85**(4), 519–525 (1963)
8. Fakoor, M., Khansari, N.M.: Mixed mode I/II fracture criterion for orthotropic materials based on damage zone properties. Eng. Fract. Mech. **153**, 407–420 (2016)
9. Lin, W.H., Tsai, Y.M.: Fracture of hybrid laminates containing a pair of collinear cracks in the central layer. Acta Mech. **82**(3), 159–173 (1990)
10. Wu, E.M.: Application of fracture mechanics to anisotropic plates. J. Appl. Mech. **34**(4), 967–74 (1967)
11. Leicester, R.H.: Applications of linear fracture mechanics in the design of timber structures. In: Conference of the Australian Fracture Group, Melbourne, pp. 156–164 (1974)
12. Williams, J.G., Birch, M.W.: Mixed mode fracture in anisotropic media. ASTM STP **601**, 125–37 (1976)
13. Woo, C.W., Chow, C.L.: Mixed mode fracture in orthotropic media. In: Sih, G.C., Valluri, S.R. (eds.) Fracture Mechanics in Engineering Application, pp. 387–96. Sijthoff and Noordhoff, Rockville (1979)
14. Hunt, D.G., Croager, W.P.: Mode II fracture toughness of wood measured by a mixed-mode test method. J. Mater. Sci. Lett. **1**(2), 77–9 (1982)
15. Mall, S., Murphy, J.F., Shottafer, J.E.: Criterion for mixed mode fracture in wood. J. Eng. Mech. **109**(3), 680–690 (1983)
16. Jernkvist, L.O.: Fracture of wood under mixed mode loading: I. Derivation of fracture criteria. Eng. Fract. Mech. **68**(5), 549–563 (2001)
17. Jernkvist, L.O.: Fracture of wood under mixed mode loading: II. Experimental investigation of Picea abies. Eng. Fract. Mech. **68**(5), 565–576 (2001)
18. Romanowicz, M., Seweryn, A.: Verification of a non-local stress criterion for mixed mode fracture in wood. Eng. Fract. Mech. **75**(10), 3141–60 (2008)
19. Gowhari Anaraki, A.R., Fakoor, M.: General mixed mode I/II fracture criterion for wood considering T-stress effects. Mater. Des. **31**, 4461–4469 (2010)
20. Van der Put, T.A.C.M.: A new fracture mechanics theory for orthotropic materials like wood. Eng. Fract. Mech. **74**, 771–781 (2007)
21. Gowhari Anaraki, A.R., Fakoor, M.: Mixed mode fracture criterion for wood based on a reinforcement microcrack damage model. Mater. Sci. Eng. A **527**, 7184–7191 (2010)
22. Gowhari-Anaraki, A.R., Fakoor, M.: A new mixed-mode fracture criterion for orthotropic materials, based on strength properties. J. Strain Anal. IMechE (2010). https://doi.org/10.1243/03093247JSA667
23. Fakoor, M., Rafiee, R.,: Fracture investigation of wood under mixed mode I/II loading based on the maximum shear stress criterion. Strength Mater. **45**(3), 378–385 (2013)

24. Carraro, P.A., Zappalorto, M., Quaresimin, M.: A comprehensive description of interfibre failure in fibre reinforced composites. Theor. Appl. Fract. Mech. **79**, 91–97 (2015)
25. Buczek, M.B., Herakovich, C.T.: A normal stress criterion for crack extension direction in orthotropic composite materials. J. Compos. Mater. **19**(6), 544–553 (1985)
26. Gregory, M.A., Herakovich, C.T.: Predicting crack growth direction in unidirectional composites. J. Compos. Mater. **20**(1), 67–85 (1986)
27. Saouma, V.E., Ayari, M.L., Leavell, D.A.: Mixed mode crack propagation in homogeneous anisotropic solids. Eng. Fract. Mech. **27**(2), 171–184 (1987)
28. Buczek, M.B., Herakovich, C.T.: A normal stress criterion for crack extension direction in orthotropic composite materials. J. Compos. Mater. **19**(6), 544–53 (1985)
29. Wei-yang, Y., Shao-qin, Z., Yuan-duo, J.: On J-integrals in the plane fracture of composite materials. Appl. Math. Mech. **13**(3), 281–287 (1992)
30. Saucedo, L., Rena, C.Y., Ruiz, G.: Fully-developed FPZ length in quasi-brittle materials. Int. J. Fract. **178**(1–2), 97–112 (2012)
31. Fakoor, M., Rafiee, R., Sheikhansari, M.: The influence of fiber-crack angle on the crack tip parameters in orthotropic materials. In: Proceedings of the Institution of Mechanical Engineers, Part C: Journal of Mechanical Engineering Science, 0954406215617195 (2015)
32. He, Q.L., Wu, L., Li, M., Yu, H.: Prediction of mode I crack growth resistance based on a comparative investigation of J-integral and energy dissipation rate concept. Acta Mech. **215**(1–4), 175–191 (2010)
33. Agarwal, B.D., Broutman, L.J., Chandrashekhara, K.: Analysis and Performance of Fiber Composites. Wiley, Hoboken (2006)