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Size-dependent damping of a nanobeam using nonlocal thermoelasticity: extension of Zener, Lifshitz, and Roukes' damping model

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Abstract Thermoelastic damping is an important issue for micro-/nanoscale mechanical resonators. In this work, an Euler–Bernoulli beam with different types of mechanical boundary conditions at the two ends is adopted. Once it bends, the half compressed (stretched) will be heated (cooled), then a transverse heat conduction appears, and energy dissipation occurs. First, size-dependent thermoelasticity based on generalized thermodynamics is introduced by combining the nonlocal elasticity and hydrodynamic heat-conductive model heat conduction, and the governing equations of the nonlocal thermal Euler–Bernoulli beam are sequently formulated. Second, an analytical solution to the inverse quality factor is obtained by using the complex-frequency approach, and it is observed that the solution is related to the nonlocal parameter of both elastic and thermal fields, as well as material constants. Meanwhile, another numerical method to get the inverse quality factor is proposed. Third, the effects of nonlocal parameters of both thermal and elastic fields, the height of the beam, and the material constants on the quality factor are evaluated. Finally, conclusive remarks are summarized. The predicted results are expected to be beneficial to micro-/nanomechanical resonators design.

1 Introduction

Progressive advances of nanotechnology have been leading to great development of nanoelectromechanical systems (NEMS). An nanomechanical resonator, which is one of the important types of NEMS device, has recently attained a lot of interest due to a variety of promising applications, such as ultrasensitive mass detection, mechanical signal processing, probe microscopes scanning [1], and the charge probing of a quantum dot [2]. Beyond that, functionalization and hybrid structures make this emerging technology suitable for biological applications as well, for example, nanomechanical resonators' interaction with biomolecular [3] and label-free detection of bio-/chemical molecules at single-molecule (or atomic) resolution [4]. However, the performance of such devices is commonly limited by the deleterious effects of thermoelastic damping (TED) [5], and as a consequence, nanomechanical resonators with high quality factors (Q-factor) are required, and accurate prediction of the Q-factor is essential for designing high-performance mechanical resonators. Although fabrication methods for metallic beams of nanoscale size have been developed, very little is known about loss mechanisms in such beams. In this respect, it is significantly important to have an in-depth understanding on the energy dissipation mechanisms of NEMS resonators [6]. In technological applications, low dissipation is generally desirable, which makes a device more efficient, less susceptible to mechanical noise, and more sensitive. It is found that the piezoresistive effect may be adopted to compensate TED [7]. On the other hand, the dissipation is fundamentally necessary in some cases, for example, it may enable coupling to the environment and allow

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for the transduction of a signal from one system to another. If no dissipation process occurs, no actuation or detection would be possible [8]. Dissipation effects are commonly denoted by the Q-factor [9]:

- The Q-factor is defined as the ratio of stored energy versus dissipation energy for various damping mechanisms during one cycle of vibration. It represents the decay of vibrational energy over time.

Apparently, a higher Q-factor indicates low energy dissipation and corresponds to a longer decay time and vice versa. Generally speaking, the parameters affecting the Q-factor may be categorized as intrinsic and extrinsic. For the former, it is due to inherent flaws or defects in the structure of a resonator such as dislocations, grain boundaries, and crystalline impurities. The latter is owing to the interactions of a nanoresonator with its surrounding environment. Four major loss mechanisms for nanoresonators essentially contribute to the Q-factor, i.e., surface losses, clamping or support losses, gas damping losses, and TED losses. Surface and TED losses are intrinsic damping parameters, while clamping and gas damping losses are extrinsic damping factors. Among energy dissipation mechanisms, TED has been particularly identified as more crucial than other damping factors for a large range of micro-/nanomechanical resonators [10,11]. The process of TED for energy dissipation is a consequence of thermal currents generated by contraction/extension in elastic media:

- The bending of the media causes deformation of opposite signs on the upper and lower halves. One half is compressed and heated, and the other is stretched and cooled. Thus, heat conducting from high-temperature regimes to low-temperature regimes along the transverse direction inevitably occurs due to the produced temperature gradient, which further causes an increase in the entropy and it comes to energy dissipation.

The Q-factor of a resonator due to TED is directly related to its frequency response and also determines its performance. Accurate prediction of the Q-factor is crucial for designing high-performance microelectromechanical devices. Some approaches to predict the Q-factor caused by TED are introduced: The widely adopted one is the complex-frequency method [12], where the Q-factor is obtained as $Q^{-1} = 2\,|\mathrm{Im}(\omega)/\mathrm{Re}(\omega)|$, where ω is the complex frequency, Im and Re denote imaginary and real part of a complex number. Others include: thermal-energy method [13], finite element method-based method [14], and molecular dynamics simulation [15,16].

With the aid of the above-mentioned approaches, numerous works have been conducted for predicting the Q-factor induced by TED in the context of classical thermoelasticity: The seminal work on this topic is completed by Zener [17]. Based on classical coupled thermoelasticity theory [18], Lifshitz and Roukes [12] formulated an analytical solution to the Q-factor for micro-Euler–Bernoulli beam resonators, which arouses great research interests on TED, and leads to the blooming trend of such topic. Heat conduction along axial direction is neglected in this work [12,17], which is performd by conducting a systematical analysis with two-dimensional heat conduction in micromechanical resonators [19]. By using the spectral element method, the Q-factor of resonators based on Timoshenko beam theory is investigated [20]. A single-degree-of-freedom model for TED is proposed [21], from which the accuracy of Zener's results is proven. An analytical model for TED based on entropy generation is formulated [22]. The Q-factor of a rotating disk is considered [23], and it is observed that the maximum TED may be obtained by optimizing disk thickness, inner radius, and radial width. The vibration and Q-factors of a nanomechanical circular tube are studied, and the influences of the dimensions of the shell, the mode numbers, and initial stress are discussed [10]. The fact that TED depends significantly on the resonance mode shape was pointed out [24] and later formalized by Norris [25]. The effect of surface stress and geometrical shape of the cross section on thermoelastic dissipation of nanowires is considered [26,27]. And the role of tensile stress on damping of nanomechanical resonators is also clarified [28].

It seems that the effect of various factors on TED has been systematically investigated. However, the above-mentioned works are all built upon classical thermoelasticity, and as a result, they are not applicable to micro-/nanoscale issues, for which size effect is predominant. From heat conductive perspective, lots of non-Fourier models have been proposed, i.e., thermal wave model [29], dual-phase-lag model [30], thermomass model [31], inertial entropy theory [32], and nonlocal model [33]. For elasticity, the celebrated nonclassical models combined with size effect are: nonlocal model [34], strain gradient model [35], and couple stress model [36], etc. Combing elasticity with thermal wave model [29] comes to the generalized thermoelasticity (also denoted as LS model) [37]. Similarly, other nonclassical thermoelasticity results may be obtained by incorporating a non-Fourier law. Another widely adopted model is the one proposed by Green and Lindsay [38]. In the 1990s, Green and Naghdi [39,40] established a quite different thermoelasticity, also known as GN model.

It is found that nonclassical thermoelasticity has also been applied to predict TED of nanoresonators. For example, LS model is adopted for thermoelastic damping of a beam resonator [41], TED and frequency shift in micro-/nanoscale anisotropic beams [42], and microscale circular plate resonators [43]. Thermoelasticity based on the dual-phase-lag model is also adopted for TED analysis [44] and concludes that the result can partly explain the fact that the experimental results of the Q-factor tend to decrease monotonously when the size of the microresonators goes down to the nanometer scale. Beyond that, the size effect of elasticity is also considered, i.e., analytical expressions for the Q-factor of TED applying couple stress theory are presented [45,46]. Nonlocal elasticity is also applied in prediction of the Q-factor: TED in studies using LS thermoelastic model with nonlocal elasticity [47]. In 2014, TED of a double-walled carbon nanotube was investigated using the nonlocal shell theory [48]. Two works are reported in 2015, i.e., Rezazadeh et al. [49], Nazemizadeh and Bakhtiari-Nejad [50]: The former is devoted to transient responses, while the latter is designed for a Q-factor attributed to airflow damping and support losses.

An analysis for the Q-factor of mechanical resonators has been systematically reviewed, from which it is observed that:

- The phenomena observed from experiments, for example, the measured inverse Q-factor, is significantly larger than the thermoelastic limit [11], and the trend of the Q-factor tends to decrease monotonously with size miniaturizing and may not be explained;
- No prior works have considered the size effect for TED analysis in both heat conduction and elastic sense. At most, only the size effect of deformation is incorporated [45–50].

Sobolev [51] and Tzou [52] suggested that: Heat conduction at micro-/nanoscale is essentially nonlocal, and the classical heat conduction law should be further extended by introducing the material's characteristic length. The size effect of heat conduction is emphasized [53], where an abnormal result within the thermal wave model is eliminated by introducing a spatial size effect. The hydrodynamic heat-conductive model proposed by Guyer and Krumhansl [33] is essentially size dependent. The phonon scatterings are usually grouped into two categories: normal (N) process and resistive (R) process. The hydrodynamic model is merely a special case when the N process is the dominating phonon scattering, while Fourier's law is another special case, which is indeed a macroscopic heat transport equation when U-type R process is dominant.

In this work, the analysis of TED will be done with the aid complete nonlocal coupling of two fields, i.e., displacement and temperature by combining the hydrodynamic heat-conductive model and nonlocal elasticity. Based on the theoretical basis, an analytical solution to the Q-factor will be formulated. And it will be observed that the experimental phenomena [11] may be illustrated by the size effect of heat conduction. The present work is expected to be a direct continuation and a further perfection of Lifshitz and Roukes [12] by extending original results into nanodevices, essentially.

2 The concept of the Q-factor and two pioneering works

Although the Q-factor is directly related to the stored energy and dissipation energy, it is commonly calculated using the concept of complex Young's modulus or complex frequency with the aid of thermoelasticity theory. The formulation is complicated, and will be neglected here for brevity, in this work the Q-factor of the well-known one-degree-of-freedom mass–spring system will be presented, whose governing equation has the form as:

$$\ddot{x} + 2\varsigma\omega_0\dot{x} + \omega_0^2 x = 0 \;\text{ with }\; \varsigma = c/2\sqrt{km}, \omega_0 = \sqrt{k/m} \tag{1}$$

where x is the displacement of the mass, $\frac{1}{2\varsigma}$ is the Q-factor of the system [54], ω_0 denotes the frequency, c is the damping coefficient, and k and m are spring coefficient and mass, respectively. By assuming $x = f(x)\,\mathrm{e}^{i\omega t}$, and substituting it into Eq. (1), one obtains:

$$\omega^2 - 2\varsigma\omega_0\omega i - \omega_0^2 x = 0 \tag{2}$$

from which one gets: $\omega = \left(\sqrt{1-\varsigma^2} + \varsigma i\right)\omega_0$. As a result, the Q-factor may be related to the complex frequency as:

$$Q = \frac{1}{2\varsigma} \cong \frac{\sqrt{1-\varsigma^2}}{2\varsigma} = \frac{\mathrm{Re}\,(\omega)}{2\,\mathrm{Im}\,(\omega)}. \tag{3}$$

And correspondingly, the dissipation may be denoted as:

$$Q^{-1} = 2\left|\frac{\mathrm{Im}\,(\omega)}{\mathrm{Re}\,(\omega)}\right| \tag{4}$$

which will be adopted in this manuscript to investigate the TED of nanoscale devices. Before coming to this task, we would like to introduce two pioneering works of TED, firstly. Around the 1930s, to consider heating and finite thermalization time for real materials, Hooke's law was extended to include relaxation times of the strain and stress fields and comes to theory of anelasticity. Zener [17] adopted the theory to analyze thermoelastic dissipation by calculating the time averages of the stress, strain, and temperature, and obtained:

$$Q_{\text{Zener}}^{-1} = \frac{E\alpha_\theta^2 T_0}{\rho c_E} \frac{\omega\tau_{th}}{1+(\omega\tau_{th})^2} \quad \text{with} \quad \tau_{th} = \frac{h^2\,\rho c_E}{\pi^2 k} \tag{5}$$

where E is the elastic modulus, α_θ the thermal expansion coefficient, T_0 the reference temperature, ρ the mass density, c_E the specific heat, ω the resonant frequency, E the elastic coefficient, and h and k height of the beam and thermal conductivity, respectively. It is concluded that at low frequency, i.e., $\omega\tau_{th} << 1$, the isothermal regime holds, and the damping is low. On the other hand, for high frequency $\omega\tau_{th} >> 1$, the temperature gradient change is so quick that less heat conduction occurs. And the maximum dissipation may occur under the condition as $\omega = \tau_{th}^{-1}$.

Later, Lifshitz and Roukes [12] systematically investigated TED of Euler–Bernoulli beam resonators in the context of thermoelasticity and obtained an analytical solution as:

$$Q_{R-L}^{-1} = \frac{E\alpha_\theta^2 T_0}{\rho c_E}\left(\frac{6}{\psi^2} - \frac{6}{\psi^3}\frac{\sinh\psi + \sin\psi}{\cosh\psi + \cos\psi}\right) \text{ with } \psi = h\sqrt{\frac{\rho c_E \omega_0}{2k}} \tag{6}$$

which may be degenerated into

$$Q_{R-L}^{-1} = \frac{Eh^2\,\alpha_\theta^2 T_0}{10k} \frac{\omega}{1+\omega^2 h^4\, c_E^2 \big/ (96k^2)} \tag{7}$$

for thin rectangular beams [26]. It is noted that TED with surface stress was studied by Ru [26], who showed that for rectangular cross sections without surface effect the solution may be degenerated into

$$Q_{R-L}^{-1} = \frac{Eh^2\,\alpha_\theta^2 T_0}{10k} \frac{\omega}{1+\omega^2 h^4\, c_E^2 \big/ (100k^2)}. \tag{8}$$

It is observed that the results (7) and (8) are very close to each other. The present work will be focused on the size effect of heat conduction and elasticity at the micro-/nanoscales.

3 TED analysis of a nonlocal thermal Euler–Bernoulli beam

3.1 Theoretical basis: nonlocal thermoelasticity

Chan et al. [55] demonstrated that the nonlocal effects due to large temperature gradient, which exists over a distance that is small compared to the electron mean free path, can describe the heat confinement near the interface observed in their experiment. Apparently, the size effect may become significant when the characteristic length scale of the process is comparable with the mean free path of heat carriers. Guyer and Krumhansl [33] proposed a nonclassical heat-conductive law by solving the linearized phonon Boltzmann equation employing eigenvectors of the normal-process collision operator (also known as hydrodynamic heat-conductive model), as

$$q_i + \tau\frac{\partial q_i}{\partial t} = -k\nabla\theta + \zeta^2\left[\nabla^2 q_i + 2\nabla(\nabla\cdot q_i)\right] \tag{9}$$

where q_i denotes the heat flux, θ the temperature change, τ the relaxation time, k the heat conductivity, ζ denotes the nonlocal parameter of heat conduction, and the symbol ∇ represents the spatial gradient operator, i.e., $\nabla\theta = \{\partial_i\theta\}$, and ∇^2 denotes the Laplace operator. Recently, a similar expression as Eq. (9) is obtained within thermomass theory [56], which may be viewed as its microscopic interpretation.

On a separate front, Eringen [34] suggested that classical elasticity may fail as the external characteristic length (or time) approaches to the internal characteristic length (or time), and assumed that the stress field at a point in an elastic continuum not only depends on the strain field at the point but also on strains at all other points of the body. The model proposed by Eringen is commonly denoted as nonlocal elasticity, which has the differential form:

$$\left[1-\xi^2\nabla^2\right]\sigma_{ij}=C_{ijkl}\varepsilon_{kl} \tag{10}$$

where σ_{ij} is the strain tensor, ε_{ij} denotes the strain tensor, ξ is the nonlocal parameter of elasticity, C_{ijkl} are the elastic constants. Obviously, for macro-scale problems the characteristic length is relatively small, i.e., $\xi=0$, and Eq. (10) will be degenerated into the classical stress constitutive equation. And recently, the nonclassical stress constitutive equation (10) was reformulated by Polizzotto [57] using the complementary principle of virtual power (PVP). Here, to evaluate TED at the micro-/nanoscale, the above-mentioned nonclassical Fourier's law and stress constitutive equation will be combined to establish the size-dependent thermoelastic model. A theoretical formulation of nonlocal thermoelasticity is given by the present authors using generalized thermodynamics [58], and the coupled theory with size effect considered has been adopted for transient responses [58] and thermoelastic buckling of a nanobeam [59]. As summarized in the work [58], the theory may consist of the following equations:

- Equations of motion and energy conversation:

$$\sigma_{ji,j}=\rho\ddot{u}_i, \tag{11}$$

$$q_{i,i}=-\rho T_0\dot{\eta}; \tag{12}$$

- Nonlocal constitutive equation of stress and entropy:

$$\left(1-\xi^2\nabla^2\right)\sigma_{ij}=2\mu\varepsilon_{ij}+\lambda\varepsilon_{kk}\delta_{ij}-(3\lambda+2\mu)\alpha_\theta\theta\delta_{ij}, \tag{13}$$

$$\rho\eta=(3\lambda+2\mu)\alpha_\theta\varepsilon_{kk}+\frac{\rho c_E}{T_0}\theta; \tag{14}$$

- Geometric relations and generalized Fourier's law:

$$\varepsilon_{ij}=\frac{1}{2}\left(u_{i,j}+u_{j,i}\right), \tag{15}$$

$$\left(1-\zeta^2\nabla^2\right)q_i=-k\theta_{,i} \tag{16}$$

where u_i is the displacement and may be also denoted as $u,\ v,\ w$, respectively. And η denotes entropy. λ and μ are Lamé coefficients, α_θ denotes the linear thermal expansion coefficient. T_0 is the reference temperature. It is well known that:

$$\lambda=\frac{E\upsilon}{(1+\upsilon)(1-2\upsilon)},\quad \mu=\frac{E}{2(1+\upsilon)} \tag{17}$$

in which υ denotes Poisson's ratio. Combining (13) and (17), the constitutive equation in terms of E and υ may be:

$$\begin{aligned}\left(1-\xi^2\nabla^2\right)\sigma_{ij}&=\frac{E}{1+\upsilon}\varepsilon_{ij}+\frac{E\upsilon}{(1+\upsilon)(1-2\upsilon)}\varepsilon_{kk}\delta_{ij}-\frac{E\alpha_\theta}{1-2\upsilon}\theta\delta_{ij},\\ \rho\eta&=\frac{E\alpha_\theta}{1-2\upsilon}\varepsilon_{kk}+\frac{\rho c_E}{T_0}\theta.\end{aligned} \tag{18}$$

Thus far, the nonlocal thermoelastic framework is introduced. If nonlocal parameters of both heat conduction and elasticity are omitted, classical thermoelasticity will be obtained. Meanwhile, if deformation is not considered, it will degenerate into nonlocal heat conduction. Accordingly, nonlocal elasticity may be obtained once the thermal field is excluded.

3.2 Governing equations of a nonlocal thermal Euler–Bernoulli beam

As seen in Fig. 1, a micro-/nanoscale Euler–Bernoulli beam will be considered. Deformation of the mid-plane along the x and y-axis will be denoted by u and v, and the transverse deformation is given by w:

$$u(x,z,t)=-z\frac{\partial w}{\partial x},\quad v=0,\quad w(x,z,t)=w(x,t). \tag{19}$$

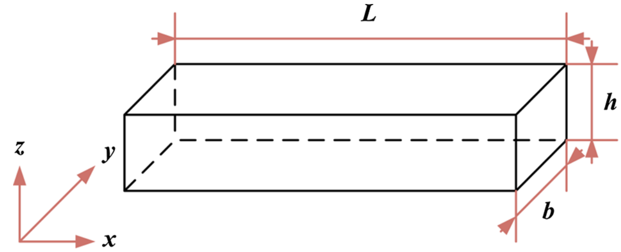

Fig. 1 (Color online) Configuration of the Euler–Bernoulli beam

Referring to (15), the strain may be:

$$\varepsilon_x = -z\frac{\partial^2 w}{\partial x^2}; \quad \varepsilon_y = 0; \quad \varepsilon_z = 0. \tag{20}$$

Substituting (20) into (18) and neglecting the Poisson effect yields:

$$\left(1-\xi^2\frac{\partial^2}{\partial x^2}\right)\sigma_x = E\varepsilon_x - E\alpha_\theta\theta = -Ez\frac{\partial^2 w}{\partial x^2} - E\alpha_\theta\theta, \tag{21.1}$$

$$\rho\eta = -E\alpha_\theta z\frac{\partial^2 w}{\partial x^2} + \frac{\rho c_E}{T_0}\theta, \tag{21.2}$$

which are the constitutive equations for an Euler–Bernoulli beam in thermal environment.

3.2.1 Governing equation of the elastic field

Firstly, the focus is placed on the formulation of the governing equation of deformation for an Euler–Bernoulli beam at the micro-/nanoscale. Similar to classical beam theory, the bending moment may be defined as $M = \int_{-h/2}^{h/2}\sigma_x bz\mathrm{d}z$, and one may get by further combining with (21.1):

$$M = \xi^2\frac{\partial^2 M}{\partial x^2} - EI\frac{\partial^2 w}{\partial x^2} - E\alpha_\theta M_\theta. \tag{22}$$

In Eq. (22), M_θ is defined as $M_\theta = \int_{-h/2}^{h/2}\theta bz\mathrm{d}z$, and $I = \frac{1}{12}bh^3$. Then, the shear force S may be obtained with the equation

$$S = \frac{\partial M}{\partial x}. \tag{23}$$

As is well known, the equation of a transversely vibrating beam is:

$$\frac{\partial^2 M}{\partial x^2} = \rho A\frac{\partial^2 w}{\partial t^2}. \tag{24}$$

Introducing (24) into (22), one has:

$$M = \xi^2\rho A\frac{\partial^2 w}{\partial t^2} - EI\frac{\partial^2 w}{\partial x^2} - E\alpha_\theta M_\theta. \tag{25}$$

Thus, the bending moment is obtained for a nonlocal thermal Euler–Bernoulli beam. From (24) and (25), the governing equation of transverse deformation w is obtained as:

$$EI\frac{\partial^4 w}{\partial x^4} + E\alpha_\theta\frac{\partial^2 M_\theta}{\partial x^2} + \left(1-\xi^2\frac{\partial^2}{\partial x^2}\right)\rho A\frac{\partial^2 w}{\partial t^2} = 0. \tag{26}$$

3.2.2 Governing equation of the thermal field

Similar to the work of Lifshitz and Roukes [12], heat conduction along the axial direction will be neglected. As a consequence, Eqs. (11), (15) will be reduced into:

$$\frac{\partial q_z}{\partial z} = -\rho T_0 \dot{\eta}, \tag{27}$$

$$\left(1 - \zeta^2 \frac{\partial^2}{\partial z^2}\right) q_z = -k \frac{\partial \theta}{\partial z} \tag{28}$$

from which one has:

$$k \frac{\partial^2 \theta}{\partial z^2} = \left(1 - \zeta^2 \frac{\partial^2}{\partial z^2}\right) \rho T_0.\dot{\eta}. \tag{29}$$

The governing equation of temperature may be obtained from Eqs. (29) and (21.2) as:

$$\chi \frac{\partial^2 \theta}{\partial z^2} = \left(1 - \zeta^2 \frac{\partial^2}{\partial z^2}\right) \dot{\theta} - \frac{\Delta E}{\alpha_\theta} z \frac{\partial^2 \dot{w}}{\partial x^2} \tag{30}$$

in which χ is the thermal diffusivity, as: $\chi = k \big/ \rho c_E$, and $\Delta E = \frac{T_0 E \alpha_\theta^2}{\rho c_E}$.

3.3 TED analysis

To predict TED of micro-/nanoresonators, we aim to solve the coupled governing Eqs. (26) and (30) for harmonic vibration. Firstly, let:

$$\begin{aligned} w(x,t) &= w_0(x)\, \mathrm{e}^{i\omega t}, \\ \theta(x,z,t) &= \theta_0(x,z)\, \mathrm{e}^{i\omega t}, \\ q(z,t) &= q_0(z)\, \mathrm{e}^{i\omega t}. \end{aligned} \tag{31}$$

Introducing (31) into the governing equation of temperature (30) yields:

$$\frac{\partial^2 \theta_0}{\partial z^2} = \frac{i\omega}{\chi + i\omega\varsigma^2} \left[\theta_0 - \frac{\Delta E}{\alpha_T} z \frac{\partial^2 w_0}{\partial x^2}\right] \tag{32}$$

whose solution may be assumed as

$$\theta_0 = A \sin mz + B \cos mz \tag{33}$$

where $m = \sqrt{\frac{i\omega}{\chi + i\omega\varsigma^2}}$. Introducing (33) into (28), it is obtained that $q_0 = \frac{-k}{1+\zeta^2 m^2} \frac{\partial \theta_0}{\partial z}$. Because the top and bottom surfaces are assumed to be adiabatic, as $q_0(\pm h/2) = 0$, the solution to temperature may be:

$$\theta = \frac{\Delta E}{\alpha_T} \frac{\partial^2 w_0}{\partial x^2} \left[z - \frac{\sin(mz)}{m \cos\left(\frac{mh}{2}\right)}\right] \mathrm{e}^{i\omega t}. \tag{34}$$

And then according to the definition $M_\theta = \int_{-h/2}^{h/2} \theta b z \mathrm{d}z$, one gets:

$$M_\theta = \frac{EI\Delta E}{E\alpha} \left[1 + \frac{12}{h^2 m^2} - \frac{24}{h^3 m^3} \tan\left(\frac{hm}{2}\right)\right] \frac{\partial^2 w_0}{\partial x^2} \mathrm{e}^{i\omega t}. \tag{35}$$

Inserting (35) into the governing equation of deformation (26) yields:

$$\frac{EI}{\rho A} \left[1 + f(\omega)\Delta E\right] \frac{\partial^4 w_0}{\partial x^4} - \left(1 - \xi^2 \frac{\partial^2}{\partial x^2}\right) \omega^2 w_0 = 0 \tag{36}$$

where $f(\omega) = 1 + \frac{12}{m^2h^2} - \frac{24}{m^3h^3}\tan\left(\frac{hm}{2}\right)$. Equation (36) is further rewritten as:

$$\frac{\partial^4 w_0}{\partial x^4} + \xi^2\beta^4\frac{\partial^2 w_0}{\partial x^2} - \beta^4 w_0 = 0 \quad \text{with} \quad \beta^4 = \frac{\omega^2}{a_0^2} \tag{37}$$

in which $a_0^2 = \frac{EI}{\rho A}[1 + f(\omega)\Delta E]$. If the term $f(\omega)\Delta E$ is omitted, then the flexural vibration of the nonlocal beam will be obtained:

$$\frac{\partial^4 w_0}{\partial x^4} + \xi^2\beta^4\frac{\partial^2 w_0}{\partial x^2} - \beta^4 w_0 = 0 \quad \text{with} \quad \beta^4 = \frac{\omega_{\text{NB}}^2}{a_{\text{NB}}^2} \quad \text{or} \quad \omega_{\text{NB}} = \beta^2 a_{\text{NB}} \tag{38}$$

where $a_{\text{NB}}^2 = \frac{EI}{\rho A}$, and the resonant frequency of the nonlocal beam will be denoted by ω_{NB} to distinguish it from the one of thermoelastic coupling. The solution of (38) is systematically discussed in the work [60]; however, the boundary condition C–S (Clamped–Simply supported) is not considered therein. In this work, the solution to the boundary condition C–S will be formulated. The characteristic equation of (37) has the form:

$$n^4 + \xi^2\beta^4 n^2 - \beta^4 = 0 \tag{39}$$

from which the roots are obtained as $n = \pm n_1;\ n = \pm i n_2$ with

$$n_1 = \beta\sqrt{\frac{\sqrt{\beta^4\xi^4 + 4} - \beta^2\xi^2}{2}}, \quad n_2 = \beta\sqrt{\frac{\sqrt{\beta^4\xi^4 + 4} + \beta^2\xi^2}{2}}. \tag{40}$$

Thus, the solution may be assumed as:

$$w_0 = A\sinh(n_1 z) + B\cosh(n_1 z) + C\sin(n_2 z) + D\cos(n_2 z) \tag{41}$$

in which A, B, C, and D rely on the boundary conditions for two ends of the resonator. Generally, three kinds of boundary conditions may be considered, i.e., clamped end, simply supported end, and free end:

$$\begin{aligned} w_0 &= 0; \quad \frac{\partial w_0}{\partial x} = 0 \text{ (Clamped)}, \\ w_0 &= 0; \quad M = 0 \text{ (Simply supported)}, \\ M &= 0; \quad S = 0 \text{ (Free)}. \end{aligned} \tag{42}$$

In Sect. 3.2.1, the bending moment has been derived. Neglecting the thermal effect and considering Eqs. (31) and (35), one has:

$$M = -\left(\xi^2\rho A\omega^2 w_0 + EI\frac{\partial^2 w_0}{\partial x^2}\right)\mathrm{e}^{i\omega t}, \tag{43}$$

and then the shear force may be obtained as:

$$S = \frac{\partial M}{\partial x} = -\left(\xi^2\rho A\omega^2\frac{\partial w_0}{\partial x} + EI\frac{\partial^3 w_0}{\partial x^3}\right)\mathrm{e}^{i\omega t}. \tag{44}$$

Thus, the boundary conditions (42) may be written in form of transverse displacement, as:

$$\begin{aligned} & w_0 = 0; \quad \frac{\partial w_0}{\partial x} = 0 \text{ (Clamped)}, \\ & w_0 = 0; \quad \xi^2\beta^4 w_0 + \frac{\partial^2 w_0}{\partial x^2} = 0 \text{ (Simply supported)}, \\ & \xi^2\beta^4 w_0 + \frac{\partial^2 w_0}{\partial x^2} = 0; \quad \xi^2\beta^4\frac{\partial w_0}{\partial x} + \frac{\partial^3 w_0}{\partial x^3} = 0 \text{ (Free)}. \end{aligned} \tag{45}$$

In the formulation, the C–S boundary condition will be adopted. Seeing (45), one has:

$$w_0(0) = \frac{\partial w_0}{\partial x}(0) = 0, \quad w_0(L) = \xi^2\beta^4 w_0(L) + \frac{\partial^2 w_0(L)}{\partial x^2} = 0.$$

With the aid of solution (41), it is obtained that:

$$n_2 \sinh(n_1 L)\cos(n_2 L) - n_1 \cosh(n_1 L)\sin(n_2 L) = 0. \tag{46}$$

It is noted that when the nonlocal parameter of elasticity is neglected, the characteristic equation (46) will be degenerated into its classical counterpart, $\sinh(\beta L)\cos(\beta L) - \cosh(\beta L)\sin(\beta L) = 0$. It is obtained from (46) and (40) that the the eigenvalue is dependent on the nonlocal parameter ξ. The value of the eigenvalue under given nonlocal parameter ξ may be also numerically solved from (46) and (40). And the results are shown in Table 1, where the nonlocal parameter and eigenvalue are normalized as ξ/L and βL, respectively. In addition, the results from Lu et al. [58] for the boundary conditions, i.e., S–S, C–C, and C–F, are also listed. From these results, it can be concluded that:

- The nonlocal parameter of elasticity decreases the eigenvalue for all boundary conditions, except for the first-order mode of C–F case. And the effect becomes larger for higher-order mode. As a result, the resonant frequency of a nonlocal beam $\omega_{\mathrm{NB}} = \beta^2 a_{\mathrm{NB}}$ is smaller than the classical one.

Thus far, the eigenvalue βL of the nonlocal beam is obtained, and how the nonlocal parameter ξ/L affects the eigenvalue is clear. The frequency of an Euler–Bernoulli beam using nonlocal elasticity may be obtained from (38) as: $\omega_{\mathrm{NB}} = \beta^2\sqrt{\frac{EI}{\rho A}} = \beta^2 h\sqrt{\frac{E}{12\rho}}$. Similarly, from Eq. (37), the frequency of a nonlocal thermal Euler–Bernoulli beam is:

$$\omega = \beta^2\sqrt{\frac{EI}{\rho A}\left[1 + f(\omega)\,\Delta E\right]} = \sqrt{1 + f(\omega)\,\Delta E}\,\omega_{\mathrm{NB}} \tag{47}$$

where $f(\omega) = 1 + \frac{12}{m^2h^2} - \frac{24}{m^3h^3}\tan\left(\frac{hm}{2}\right)$ with $m = \sqrt{\frac{i\omega}{\chi + i\omega\varsigma^2}}$, and $\Delta E = \frac{T_0 E \alpha_\theta^2}{\rho c_E}$. The dissipation has been presented in Sect. 2, as: $Q^{-1} = 2\left|\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)}\right|$. The algebraic equation of the complex frequency has been formulated, as shown in Eq. (47). It is observed that the nonlocal parameters of both heat conduction and elasticity, material constants, and the height of the beam have an effect on the dissipation. In principle, the complex frequency can be solved directly from (47), and then the energy dissipation will be obtained. In Eq. (47), the variable ΔE is relatively small, so (47) may be further expressed as:

$$\omega = \left[1 + \frac{\Delta E}{2} f(\omega)\right]\omega_{\mathrm{NB}}. \tag{48}$$

Later, two ways are applied to predict the dissipation of the micro-/nanoresonator.

Table 1 Eigenvalue βL versus nonlocal parameter ξ/L for four boundary conditions: S–S, C–C, C–F, and C–S

ξ/L	0	0.1	0.2	0.3	0.4
S–S ($\sin(n_2L) = 0$)					
$\beta_1 L$	3.14160	3.0685	2.8908	2.6800	2.4790
$\beta_2 L$	6.28320	5.7817	4.9581	4.3013	3.8204
C–C ($\xi^2 n_1 n_2 \sinh(n_1L)\sin(n_2L) + 2\cosh(n_1L)\cos(n_2L) - 2 = 0$)					
$\beta_1 L$	4.73000	4.59450	4.2766	3.9184	3.5923
$\beta_2 L$	7.85320	7.14020	6.0352	5.1963	4.5978
C–F ($2 + \xi^4 n_1^2 n_2^2 + \xi^2 n_1 n_2 \sinh(n_1L)\sin(n_2L) + 2\cosh(n_1L)\cos(n_2L) = 0$)					
$\beta_1 L$	1.87510	1.8792	1.8919	1.9154	1.9543
$\beta_2 L$	4.69410	4.5475	4.1924	3.7665	3.3456
C–S ($n_2\sinh(n_1L)\cos(n_2L) - n_1\cosh(n_1L)\sin(n_2L) = 0$)					
$\beta_1 L$	3.92660	3.82090	3.5701	3.2828	3.0176
$\beta_2 L$	7.06860	6.46490	5.5079	4.7688	4.2312
$\beta_3 L$	10.2102	8.65170	6.9204	5.8371	5.1213
$\beta_4 L$	13.3518	10.4688	8.0573	6.7143	5.8612

3.3.1 First way: analytical solution to the Q-factor

Because TED is very weak (Im $(\omega) << 1$), we can replace $f(\omega)$ in (48) with $f(\omega_{\mathrm{NB}})$, and then the frequency relation has the new form as:

$$\omega = \left[1 + \frac{\Delta E}{2} f(\omega_{\mathrm{NB}})\right]\omega_{\mathrm{NB}}. \tag{49}$$

So, the dissipation may be obtained as:

$$Q^{-1} = 2\left|\frac{\mathrm{Im}(\omega)}{\mathrm{Re}(\omega)}\right| = \frac{\Delta E\,|\mathrm{Im}[f(\omega_{\mathrm{NB}})]|}{1 + \frac{\Delta E}{2}|\mathrm{Re}[f(\omega_{\mathrm{NB}})]|} \cong \Delta E\,|\mathrm{Im}[f(\omega_{\mathrm{NB}})]|. \tag{50}$$

Now, the aim is to solve the imaginary part of the function

$$f(\omega_{\mathrm{NB}}) = 1 + \frac{24}{m_{\mathrm{NB}}^3 h^3}\left(\frac{hm_{\mathrm{NB}}}{2} - \tan\left(\frac{hm_{\mathrm{NB}}}{2}\right)\right) \tag{51}$$

in which $m_{\mathrm{NB}}h = \sqrt{\frac{i\omega_{\mathrm{NB}}}{\chi + i\omega_{\mathrm{NB}}\zeta^2}}h$ with $\omega_{\mathrm{NB}} = \beta^2 h\sqrt{\frac{E}{12\rho}}$. For convenience, it is written as:

$$m_{\mathrm{NB}}h = \sqrt{\frac{ip}{\chi/L + ip(\zeta/L)^2}}\frac{h}{L} \text{ with } p = (\beta L)^2\frac{h}{L}\sqrt{E/(12\rho)}. \tag{52}$$

Then, the real and imaginary part of $m_{\mathrm{NB}}h$ may be obtained as:

$$R_1 = \mathrm{Re}(m_{\mathrm{NB}}h) = \frac{1}{\sqrt{\phi^2 + (\zeta/L)^4}}\frac{\phi}{2\sqrt{\frac{-(\zeta/L)^2 + \sqrt{(\zeta/L)^4 + \phi^2}}{2}}}\frac{h}{L},$$

$$I_1 = \mathrm{Im}(m_{\mathrm{NB}}h) = \frac{1}{\sqrt{\phi^2 + (\zeta/L)^4}}\sqrt{\frac{-(\zeta/L)^2 + \sqrt{(\zeta/L)^4 + \phi^2}}{2}}\frac{h}{L} \tag{53}$$

where $\phi = \varphi\frac{1}{(\beta L)^2 h/L}$ with $\varphi = \frac{\sqrt{12}k}{\sqrt{\rho E c_E}L}$. Accordingly, $m_{\mathrm{NB}}h$ can be expressed as:

$$m_{\mathrm{NB}}h = R_1 + iI_1. \tag{54}$$

Introducing (54) into (51), one has the imaginary part of $f(\omega_{\mathrm{NB}})$:

$$\mathrm{Im}[f(\omega_{NE})] = \frac{-24R_1I_1}{\left(R_1^2 - I_1^2\right)^2 + 4R_1^2I_1^2} + \frac{24I_1\left(3R_1^2 - I_1^2\right)\sin(R_1) - 24R_1\left(R_1^2 - 3I_1^2\right)\sinh(I_1)}{[\cos(R_1) + \cosh(I_1)]\left[R_1^2\left(R_1^2 - 3I_1^2\right)^2 + I_1^2\left(3R_1^2 - I_1^2\right)^2\right]}. \tag{55}$$

And then, according to (50), it yields:

$$Q^{-1} = 24\Delta E\left|\frac{I_1\left(3R_1^2 - I_1^2\right)\sin(R_1) - 24R_1\left(R_1^2 - 3I_1^2\right)\sinh(I_1)}{[\cos(R_1) + \cosh(I_1)]\left[R_1^2\left(R_1^2 - 3I_1^2\right)^2 + I_1^2\left(3R_1^2 - I_1^2\right)^2\right]} - \frac{R_1I_1}{\left(R_1^2 - I_1^2\right)^2 + 4R_1^2I_1^2}\right|. \tag{56}$$

In Eq. (56), R_1 and I_1 are referred to the (53), which are related to the normalized height h/L, eigenvalue βL, nonlocal parameter of heat ζ/L, and material constants ρ, E, k, and c_E. It is noted that all these material constants are summarized as a variable, i.e., φ, and that the effect of the nonlocal parameter of elasticity is represented by the eigenvalue βL. Thus far, the analytical solution to the dissipation is obtained. In the next Section, the effect of all these factors on the energy dissipation of a micro-/nanoresonator will be evaluated. And now, another way to solve the Q-factor will be introduced.

3.3.2 Second way: numerical solution to the Q-factor

In this Subsection, we aim at directly solving the frequency relation (48). To this end, simplification will be made to the function $f(\omega) = 1 + \frac{12}{m^2h^2} - \frac{24}{m^3h^3}\tan\left(\frac{hm}{2}\right)$, where $\tan\left(\frac{hm}{2}\right)$ will be expanded using a Taylor series. As a consequence, it is obtained that:

$$f(\omega) = 1 + \frac{12}{m^2h^2} + \frac{24}{m^3h^3}\left[\frac{hm}{2} - \frac{1}{3}\left(\frac{1}{2}\right)^3(hm)^3 - \frac{2}{15}\left(\frac{1}{2}\right)^5(hm)^5 - \frac{17}{315}\left(\frac{1}{2}\right)^7(hm)^7 - \cdots\right]. \quad (57)$$

In this work, a micro-/nanoscale beam is considered. So, h is relatively small. By taking the first to third term of the Taylor expansion, and substituting into (48), one has:

$$\omega = \left[1 - \frac{\Delta E}{2}\frac{1}{10}(hm)^2\right]\beta^2 h\sqrt{\frac{E}{12\rho}}. \quad (58)$$

Table 2 Material constants of silicon

E (Gpa)	α_t (1/K)	ρ (kg/m^3)	c_E [J/(kg K)]	T_0 (K)
170	2.6e−6	2330	700	300

Table 3 The obtained Q^{-1} by two ways

βL	First way	Second way
3	1.2752e−007	1.2752e−007
6	5.1006e−007	5.1007e−007
9	1.1476e−006	1.1476e−006

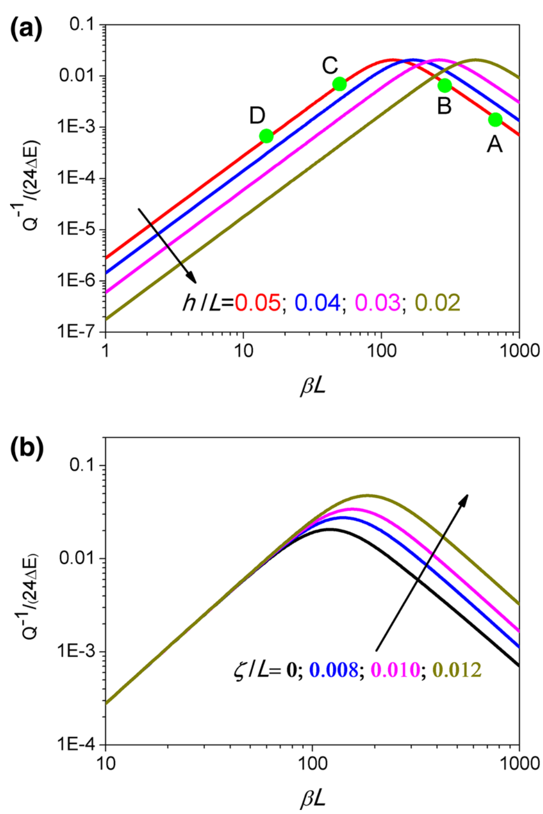

Fig. 2 (Color online) The dissipation $Q^{-1}/24\Delta E$ versus eigenvalue βL with **a** different height h/L, **b** nonlocal parameter of heat conduction ζ/L

Considering that $m = \sqrt{\frac{i\omega}{\chi + i\omega\zeta^2}}$, the frequency relation may be written in normalized form as:

$$\omega = \left[1 - \frac{\Delta E}{2}\frac{1}{10}\frac{i\omega}{\chi/L^2 + i\omega\left(\zeta/L\right)^2}\left(\frac{h}{L}\right)^2\right](\beta L)^2\,\frac{h}{L}\frac{1}{L}\sqrt{\frac{E}{12\rho}}. \tag{59}$$

From Eq. (59), the real and imaginary part of the complex frequency can be found. And then, the dissipation may be calculated using the relation $Q^{-1} = 2\left|\mathrm{Im}\,(\omega)/\mathrm{Re}\,(\omega)\right|$.

4 Results and discussions

In this Section, the effect of each factor, i.e., nonlocal parameter of heat conduction or elasticity, or material constants, on the dissipation of a micro-/nanobeam will be systematically investigated. The material of the resonator is selected as silicon, whose material constants are listed in Table 2 [20]. The length of the beam is adopted as $L = 2\times10^{-7}$ m. Firstly, the accuracy of the solving procedure shown in the last Section should be proven. Using the first way and the second one, the dissipation (Q^{-1}) for different eigenvalues under the conditions: $h/L = 0.05$, $\zeta/L = 0$ is obtained and presented in Table 3. It is observed that the results agree well between the two ways, so the accuracy of the numerical method is proven. In the rest of this part, the effect of normalized height h/L, eigenvalue βL, nonlocal parameter of heat ζ/L, and material constants on the dissipation will be clarified.

Presented in Fig. 2 is the distribution of the inverse Q-factor versus eigenvalue βL. It is observed that: as the eigenvalue decreases, the inverse Q-factor may increase (from point A to B in Fig. 2a), or decrease (from point C to D of Fig. 2a). There exists a maximum for the inverse Q-factor as shown in Fig. 2a. From Table 1 it is observed that: the nonlocal parameter of elasticity makes the eigenvalue smaller, and the effect is stronger for higher-order mode. Seeing Fig. 2a, the effect of the elastically nonlocal parameter may be divided into three classes: First, for lower eigenvalue, the higher nonlocal parameter makes the inverse Q-factor decrease (i.e.,

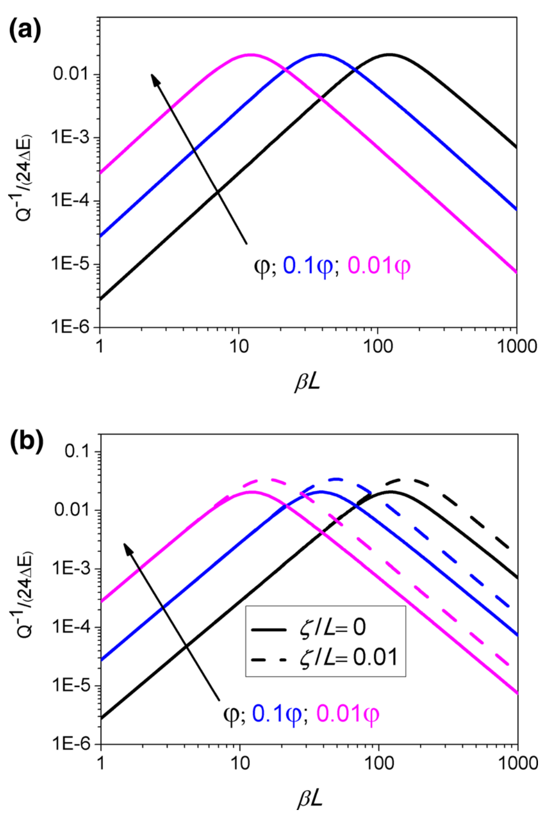

Fig. 3 (Color online) The dissipation $Q^{-1}/24\Delta E$ versus the eigenvalue βL with **a** different φ, **b** φ and also nonlocal parameter of heat conduction ζ/L

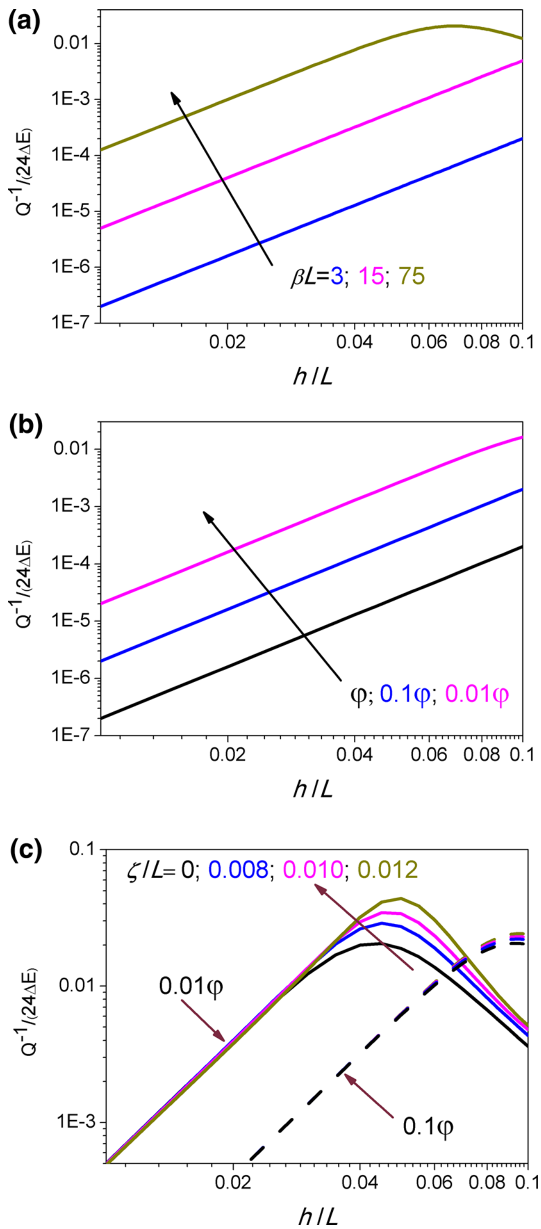

Fig. 4 (Color online) The dissipation $Q^{-1}/24\Delta E$ versus the normalized height h/L with **a** different eigenvalue βL, **b** φ and **c** φ and also nonlocal parameter of heat conduction ζ/L

from point C to point D in Fig. 2a). Second, for moderate eigenvalue, the inverse Q-factor may first increase, and then decrease (i.e., from point B to point C in Fig. 2a). And if the eigenvalue is comparatively large, Q^{-1} may increase when the nonlocal parameter of elasticity is considered (i.e., from point A to point B in Fig. 2a).

Shown in Fig. 2a, when the height of the beam becomes small, the distribution of the inverse Q-factor moves to the right side with the maximum unchanged, as a consequence, Q^{-1} decreases (increases) for those before (after) the maximum. From Fig. 2b, one may conclude that: when the size effect of heat conduction is considered, the maximum of the inverse Q-factor may be improved, which agrees well with the observation that the measured inverse Q-factor is significantly larger than the thermoelastic limit.

In the first way to calculate the inverse Q-factor, the effect of material constants has been summarized into one variable, as $\varphi = \frac{\sqrt{12}k}{\sqrt{\rho \bar{E}} c_E L}$. It is obtained from Fig. 3a that when the summarized variable φ decreases, the distribution of the inverse Q-factor moves to the left, as a result, Q^{-1} increases (decreases) for smaller

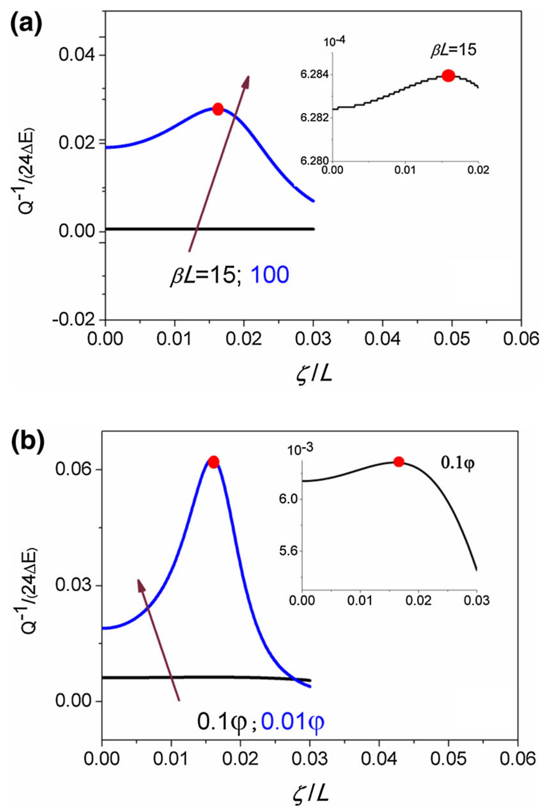

Fig. 5 (Color online) The dissipation $Q^{-1}/24\Delta E$ versus the normalized nonlocal parameter of heat conduction ζ/L with **a** different eigenvalue βL, **b** φ

(larger) eigenvalues. Also, once the nonlocal parameter of heat conduction is incorporated, the maximum will be increased (Fig. 3b).

Depicted in Fig. 4 is the inverse Q-factor versus height. It is observed that the inverse Q-factor decreases as the beam becomes thinner. In Fig. 4a, Q^{-1} is improved when the eigenvalue increases, which agrees well with the conclusion made from Fig. 2. From Fig. 4b it is obtained that the inverse Q-factor becomes larger as the summarized variable of material constants φ decreases, which is also consistent with the observation of Fig. 3. Beyond that, Fig. 4c indicates that when the summarized variable φ is smaller, the effect of the nonlocal parameter of heat conduction is more remarkable, that is, the maximum of Q^{-1} under small φ is higher than that for large φ with the same nonlocal parameter of temperature field.

Figure 5 is designed to show the inverse Q-factor versus the normalized nonlocal parameter of heat conduction, and it is observed that Q^{-1} firstly becomes larger as the nonlocal parameter ζ/L increases, and then the maximum will be obtained, after that, the inverse Q-factor may decrease. One may conclude that the thermoelastic limit of damping with size effect of the thermal field incorporated (red circles in Fig. 5) is larger than that of classical thermoelasticity. From Fig. 5a, it is also observed that the effect of the thermally nonlocal parameter is more efficient for larger eigenvalue. When the parameter φ is smaller, the effect of the nonlocal parameter of heat conduction is stronger, which can be concluded from Fig. 5b. In addition, it should be noted that: the thermally nonlocal parameter has a similar effect even for lower eigenvalue as shown in the inset of Fig. 5a, i.e., $\beta L = 15$, which may not be observed from Fig. 2b.

5 Concluding remarks

In this work, TED of a nanobeam is investigated. Although the issue has been considered in literature, both size effect of heat conduction and elasticity are not incorporated, similarly. From a theoretical aspect, the nonlocal thermoelasticity established upon generalized thermodynamics is adopted in this work, and thus both size effects are considered. In the present theoretical framework, it is noted that the spatial extensions of heat conduction and elasticity are similar. Specifically, the nonlocal thermal Euler–Bernoulli beam theory is established. To solve the inverse Q-factor, two ways are introduced: The first one is an analytical approach using the complex-frequency method, and the second one is a numerical way. Both ways are proven by comparison with each other. The inverse Q-factor is obtained and graphically depicted, from which it may be concluded:

- The nonlocal parameter of elasticity makes the inverse Q-factor increase (decrease) at higher (lower) frequency;
- Considering the size effect of heat conduction, the inverse Q-factor may be larger than that from classical thermoelasticity;
- The effect of the nonlocal parameter of both heat conduction and elasticity will be more significant for higher-order mode;
- The influence of the material constants on the inverse Q-factor can be summarized into one variable, as: $\varphi = \varphi(\rho^{-0.5}, E^{-0.5}, k, c_E^{-1})$. The inverse Q-factor will be larger, and the effect of the nonlocal parameter of heat conduction will be stronger for smaller variable φ.

The present work is a direct continuation and a further perfection of Lifshitz and Roukes [12], aiming at essentially extending the origin work into nanoscale. It is expected to be helpful in the further investigation on TED, numerically, theoretically, and also experimentally.

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