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A novel methodology for wing sizing of bio-inspired flapping wing micro air vehicles: theory and prototype

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Abstract To design efficient flapping wing micro air vehicles (FWMAVs), a comprehensive sizing method based on theoretical and statistical analyses is proposed and experimentally verified. This method is composed of five steps including defining and analyzing the MAV mission, determining the flying modes, defining the wing shape and aspect ratio of the wing, applying the constraint analysis based on the defined mission, and estimating the weights of the electrical and structural components of the bio-inspired flapping wing micro air vehicle. To define the vehicle mission and flight plan, path analysis is performed based on the defined mission, the speed of cruise and turning, the turning radius and climatic conditions in the flight area. Following the defined mission analysis, the appropriate modes of flying (i.e., flapping, gliding, hovering, bounding, and soaring) for the flapping wing bird are recognized. After that, the wing shape and the wing aspect ratio are determined based on the defined flight modes. To estimate the wing loading, a constraint analysis is exploited in which flight equitation is simulated based on the modes and missions of the flight. Along with the four listed steps, a statistical method is employed to estimate the FWMAV weight for a well-defined mission. Based on the offered method for wing sizing of flapping wings, a FWMAV named Thunder I has been designed, fabricated, and tested. This developed methodology is very beneficial by giving guidelines for the design of efficient bio-inspired FWMAVs.

1 Introduction

The popularity of drones as their broad spectrum of applications, such as military surveillance, planetary exploration, and search and rescue, has received most attention in the past few years [1, 2]. Micro planes are usually divided into three classes, namely micro air vehicle (MAV), nano air vehicle (NAV), and pico air vehicle (PAV) [3]. MAV airplanes are those micro planes usually with a length smaller than 500 mm and a weight lower than 500 g [4]. These MAVs can be grouped into four categories: fixed wings, vertical takeoff and landing (VTOLs), flapping wings, and rotary wings [3]. Depending on the flight mission of the MAV, the size and the type of installed equipment are different. The consolidated size of MAVs, compared to UAVs, provides a broader performance range. According to the mentioned characteristics, MAV benefits from the potential to perform the variety of operations including reconnaissance, patrolling, protection, and transportation of small

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loads <10 g, such as sensors to mark specific locations [5,6]. In the past decade, due to the quick advances in microtechnology, MAVs have drawn a great deal of attention, as a result in subsequent years, several research investigations have been carried out on the micro drones. Recently, few studies have been aimed at inventing MAVs smaller than 15 cm with the capability to carry out reconnaissance and rescue missions [7,8]. In addition to their small sizes, these types of drones are capable of flying with low speeds and in low Reynolds numbers [9].

The design of flapping wing MAVs is inspired from birds, PAV flapping wings are inspired from insects, and NAV flapping wings are inspired from organisms between very small birds and huge insects, such as hummingbirds and dragonflies. The research on flapping wings has shown that these types of air vehicles have more complexities compared with fixed and rotary wings mainly due to their complex aerodynamics [10]. As a result, there are few relevant research studies that are available in this field. Biological inspiration indicates that flying by flapping wings presents unique maneuverability advantages [11]. There are fundamental challenges for fixed and rotary wings to fly reliably when their sizes are reduced. When the wing area is reduced, a flow transition to low Reynolds number occurs which reduces the aerodynamic wing efficiency. A flapping wing has the potential to benefit from the advantages of other MAV types and eliminates their disadvantages [1,12]. For example, the merits for insect-like flapping wings are numerous. The hovering ability of insects coupled with the ability for a quick transition to forward flight provides an ideal drone for search and rescue and other applications [13].

To design bio-inspired flapping wing drones, some methods have been used [14–25]. In general, these methods are based on empirical formulae. These formulae have been established based on allometrical data extracted from biological avian flight. The pioneers of these researches include Greenewalt [15], Pennycuik [16,17], Rayner [18], Norburg and Rayner [19], Tucker [20,21], Lighthill [22,23], and Spedding [24]. Their empirical formulae have related the sizing parameters of flapping wings, such as wing area, weight, and wing loading to the flapping frequency, flight speed, and required power for flight. In addition to that, these formulae have related the geometry of the wing including the area and wingspan to the weight of the FWMAV. These empirical formulae have been used for sizing of FWMAVs by some researchers, such as Beng [25] and Beasley [26]. In his design, Beasley [26] has utilized the biological mimicry for sizing the flapping wing. Indeed, by using the geometric scaling factors for Passeriformes [27], the fixed span, weight, flapping frequency, wing area, and aspect ratio of the MAV have been determined from the logarithmic relationships [26]. Other methods based on statistical and experimental sizing and testing have been applied. As an example, Gerrard and Ward [28] have designed their flapping wing MAV based on existing FWMAVs, such as Luna and DelFly. Moreover, there are other methods which have been utilized for sizing of NAV and PAV flapping wings. For instance, Whitney and Wood [10] proposed a conceptual design process for insect-sized flapping wings, with a primary focus on hovering flight. Many assumptions have been considered in their method including linear and lumped representations to model the dynamics of the vehicle and blade element method to model the aerodynamic forces. In their method, after developing a dynamic model for the flapping wings, they used energy methods to determine the fractions of the actuation mechanism and weight of the battery. Combining this sizing methodology with derived limits on wing structural–inertial efficiency, the range of feasible designs and the limits of performance of the flapping wing PAVs have been specified [10].

Most of the mentioned sizing methods are based on allometric formulae extracted from natural birds and insects which have been applied directly for sizing of artificial flapping wings without taking into account the impacts of other parameters including the used materials for the wing membranes. Using the empirical formulae of natural birds and insects, non-optimized micro drones will be designed. Therefore, these empirical formulae should be revisited and probably some correction factors are needed. In this work, using theoretical, statistical, and revised allometrical methods, a comprehensive sizing methodology is proposed which finds solutions for the drawbacks of previous methods. This sizing process gives guidelines for the design of efficient bio-inspired FWMAVs. The rest of this study is organized as follows: In Sect. 2, the needed steps for the sizing of efficient flapping wings are presented and discussed. These steps include the definition of the mission, the determination of the planform and aspect ratio of the FWMAV, the selection of the flight modes, the constraints analysis for optimum wing and thrust loadings, and the estimation of the structural and electrical weights. In Sect. 3, other important parameters for efficient sizing of flapping wings are studied, such as flapping frequency and upstroke and downstroke angles. In Sect. 4, using the proposed methodology for designing efficient FWMAVs, the sizing process, fabrication, and flight test of a flapping wing MAV named Thunder I are presented and discussed. Summary and conclusions are shown in Sect. 5.

2 Needed steps for sizing of efficient flapping wings

In this study, our proposed sizing methodology of bio-inspired flapping wings is composed of five steps: (i) defining the mission, (ii) setting the flight mode, (iii) determining the wing shape and aspect ratio, (iv) constraint analysis, and (v) weight estimation. In the definition of the mission, the analysis of the route is conducted resulting in the determination of the flight time, cruise speed, and turning speed. After that, the determination of the flight modes, shape of the wing, and its aspect ratio are determined based on the type of mission. Then, to determine the appropriate wing loading of the flapping wing, a constraint analysis is used in which the kinematic and dynamic equations of the flight are simulated. Along with the four mentioned steps, a statistical method for weight estimation is introduced and employed. The result of this process is the determination of the geometry and dimensions of the flapping wing MAVs and also the calculation of the parameters, such as frequency and angles of flapping. In Fig. 1, a flowchart for the flapping wings sizing process and the relation between different steps is presented. Next, all steps of flapping wings sizing will be defined and discussed.

2.1 Defining the flight mission and determining the flight modes

For executing a defined mission, at first, the kind of flight mission (indoor or outdoor) should be decided. Then, based on this mission, the flight class of the flapping wing (MAV, NAV, or PAV) should be pinpointed. After determining the class of the flapping wing vehicle, the mission is analyzed including extracting the atmosphere features of the flight zone and preparing the plan. By preparing the flight plan, the distances and flight time should be determined. After that, by dividing these parameters, the estimated cruise speed can be calculated. Other parameters that need to be determined can be calculated using latitude and altitude values. The gravity acceleration and air density are the two most important environmental qualities for the flight calculations of the flapping wings. The effects of the altitude and latitude on gravity in this defined relationship are combined by Helmert. Helmert relationship [29,30] is a polynomial expression in which the acceleration of gravity (g) is stated based on a function of latitude (L_0) and altitude (h_0) which is given by:

$$g = 9.80616 - [0.025928 \cos(2L_0)] + [0.000069 \cos^2(2L_0)] - [(3.086 \times 10^{-6})h_0]. \quad (1)$$

The air temperature (T_a), the air density (ρ), the kinematic viscosity (ν), and the air pressure (p) are related through these expressions [29]:

$$T_a = 15 - 0.0065h_0, \quad (2)$$

$$\rho = 1.226(p/1013) [288/(T_a + 273)], \quad (3)$$

$$\nu = 1.466 + 95.07h_0 + 10470h_0^2, \quad (4)$$

$$p = 1013 \left[1 - (2.26 \times 10^{-5}) h_0 \right]^{5.256}. \quad (5)$$

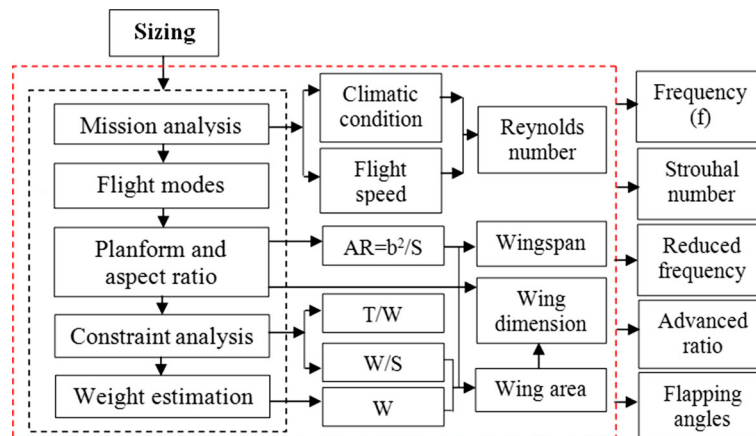


Fig. 1 Overall process of flapping wings' sizing. In this flowchart, T , W , S , AR , and b , respectively, denote the thrust, weight, wing area, aspect ratio, and wingspan

It should be mentioned that in Eqs. (1), (2), (4), and (5) the unit of h_0 is meter, and in Eq. (4), the unit of v is $10^{-5} \text{ m}^2/\text{s}$, and T_a is in $^{\circ}\text{C}$.

One of the important steps during the sizing process is the determination of the flight modes (second step), which include flapping, gliding, hovering, soaring, and bounding [31]. Depending on the defined mission, the needed flight modes of the drone will be determined. As an example, flapping wing MAVs benefit from flapping and gliding [32], whereas PAVs benefit from flapping and hovering. Generally, the flight modes are composed of different steps including takeoff and landing, cruise, turning, climb, and descent.

2.2 Selecting the planform and aspect ratio

After defining the flight mission and determining the flight modes, the third step in the proposed sizing methodology can be achieved by selecting the best shape of the wing and its aspect ratio. The planform and aspect ratio for the flapping wing can be selected either by patterning the birds' wing shape or combining the geometric shapes. For instance, all birds follow one of the six basic shapes of flying [33]: (i) short, broad, and cupped wings for quick takeoff and fly for a short distance, (ii) short and wide wings with cracked primary feathers for soaring, (iii) high flat, thin, and triangular wings for flying with high speed and maneuverability, (iv) large and arched wings for flapping flight, (v) tipped, flat, high, and thin wings for gliding and long distances, and (vi) tipped and rollback wings for hovering or motionless flights. In Fig. 2, we present the possible planforms that are used in flapping wings which are extracted from patterning the birds' wings shapes.

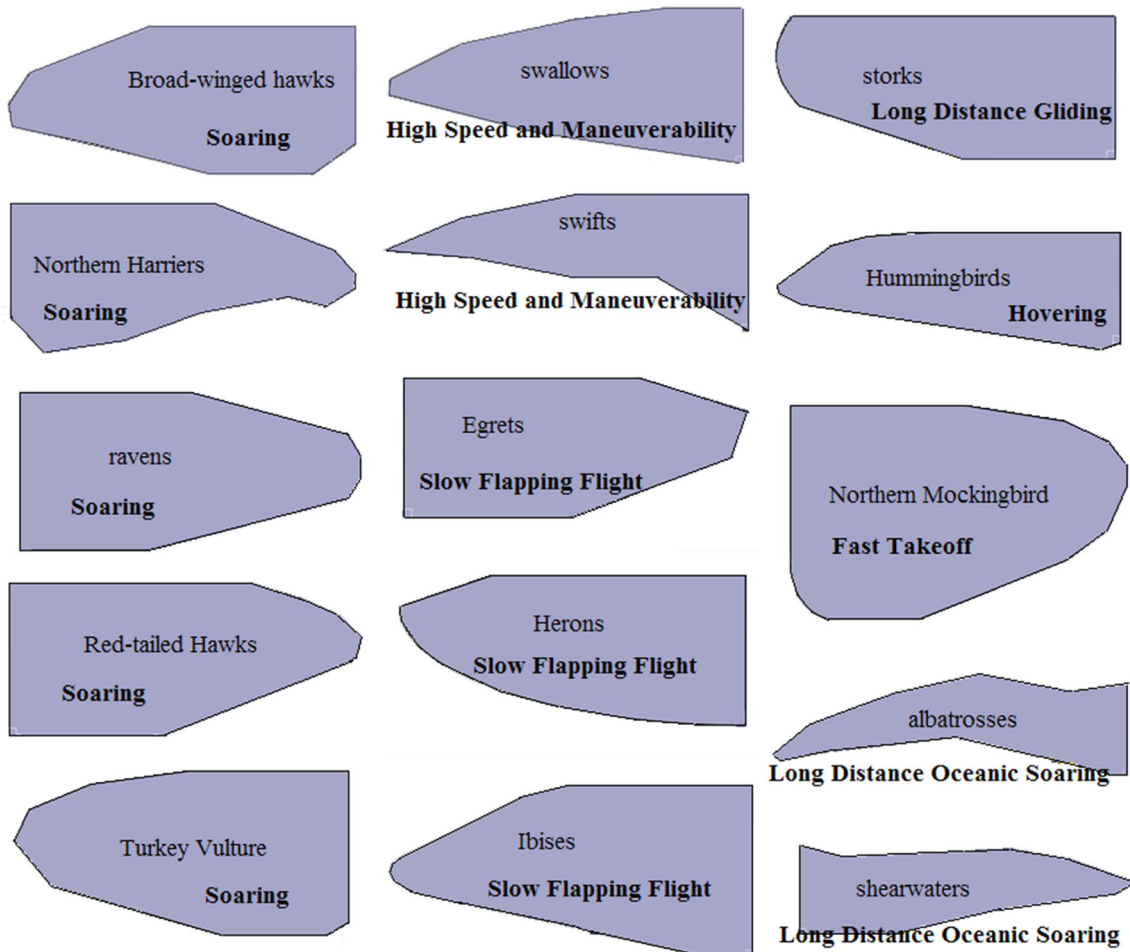


Fig. 2 Extracted planforms from birds' wings shape

2.3 Determining the parameter of wing loading

The fourth step in the sizing of flapping wings is the determination of the wing loading (W/S) parameter. To this end, a constraint analysis should be performed. Regarding a flapping wing at the state of flying, the inserted forces are lift, drag, thrust, and weight. Based on the energy balance equation, a relation between these existing forces is given by Nam [34]:

$$\frac{T - D}{W} = \frac{1}{U} \frac{d}{dt} \left(h + \frac{U^2}{2g} \right) \quad (6)$$

where T stands for thrust, D for drag, U for flight speed, W for weight of FWMAV, and h for flight height. In Eq. (6), the sum of the potential energy and kinetic energy is called the head of the energy.

Generally, in any aerial vehicle, the momentarily thrust and sea level thrust relationship is given by the following relation; $T = \alpha T_{SL}$. The relationship between the momentarily weight and the weight at the time of takeoff is defined by $W = \beta W_{TO}$. However, since the driving force of flapping wing is obtained from the battery, the weight during flying does not change, and β is equal to 1. Considering the above relations, Eq. (6) can be rewritten as:

$$\frac{T_{SL}}{W_{TO}} = \frac{1}{\alpha} \left\{ \frac{D}{W_{TO}} + \frac{1}{U} \frac{dZ}{dt} \right\} \quad (7)$$

where $Z = h + U^2/(2g)$, and α is defined by Roskam [35]:

$$\alpha = \frac{T_a + 273.16}{T_a + 273.16 - 0.001981h_0} \left\{ 1 - \frac{0.001981h_0}{288.16} \right\}^{5.256} \quad (8)$$

To make sure that the designed FWMAV tolerates its weight, the lift force should be always larger than the weight of the MAV. Generally, the lift force is constantly equal to a coefficient (n) of the weight of the flapping wing ($L = nW$). Furthermore, the lift force for any flying object is given by:

$$L = 0.5\rho U^2 S C_L \quad (9)$$

where ρ and C_L denote, respectively, the air density and lift coefficient. Since the lift force is changing in flapping wings due to flapping up and down, a mean lift can be considered.

The total drag force on a bird is equal to the sum of the induced and parasite drag in the way that parasite drag includes the two drags of profile and shape. The induced drag for a lift distribution for flapping wings is determined by Harmon [36], Hedenstrom and Liechti [37]:

$$D_i = \frac{2L^2}{e\pi b^2 \rho U_{ref}^2} = \frac{2(Wn)^2}{e\pi b^2 \rho U_{ref}^2} \quad (10)$$

where e is the Oswald number which determines the ellipticalness of the lift distribution. The parasite drag is determined by using the Tucker method. This method starts with determining the coefficient of the frictional drag (C_f) of a flat sheet during turbulence by applying the Prandtl equation [36]:

$$C_f = 0.455 (\log_{10} Re)^{-2.58} \quad (11)$$

where Re denotes the Reynolds number. Next, we define $\Psi = C_{DP}/C_f$ which is the ratio of the parasite drag coefficient of flapping wing (C_{DP}) to the frictional drag coefficient (C_f) for a flat sheet. This coefficient (Ψ) is changing from 2 to 4.4 for most birds [36]. The parasite drag is calculated by utilizing Ψ , and the drag coefficient for a wetted sheet which is given by:

$$D_p = \frac{\rho U_{ref}^2 S_{wet} C_{DP}}{2} = \frac{\rho U_{ref}^2 S_{wet} \Psi C_f}{2} \quad (12)$$

where S_{wet} is the sum of the wings' wetted surfaces. Considering $S_{wet} = 2S$, the total drag force can be expressed as:

$$D_{tot} = \frac{2(Wn)^2}{e\pi AR\rho U_{ref}^2 S} + \frac{2\rho U_{ref}^2 S\Psi C_f}{2} = qS \left\{ k_1 \left(\frac{n}{q} \frac{W}{S} \right)^2 + 2\Psi C_f \right\} \quad (13)$$

where $k_1 = 1/\pi eAR$ and $q = 0.5\rho U_{ref}^2$. Substituting Eq. (13) into Eq. (7), one obtains:

$$\frac{T_{SL}}{W} = \frac{1}{\alpha} \left\{ \frac{qS}{W} \left[k_1 \left(\frac{n}{q} \frac{W}{S} \right)^2 + 2\psi C_f \right] + \frac{1}{U} \frac{dZ}{dt} \right\}. \quad (14)$$

Equation (14) states the relationship between the wing loading and thrust loading. This equation is simulated for any flapping wings MAVs with electric engine in constant cruise speed, constant climb speed, constant turning altitude/speed, horizontal acceleration, accelerated climb, and flapping wing launching situations. In the six mentioned flight scenarios, Eq. (14) of the thrust loading (T/W) has been represented as a function of wing loading (W/S). Therefore, with drawing the related curves of the six corresponding equations, a bounden space for the determination of a design point ($W/S, T/W$) can be obtained. The bounden space is specified by the drawn curves. It should be mentioned that the selected point in the bounden space should satisfy all of the constraints according to the defined mission.

2.4 Estimating the electrical and structural weights of flapping wing MAVs

The fifth step in the process of the sizing of FWMAVs is the weight estimation. Since FWMAVs have low and modest weight, it is required to carefully estimate the weight with a minimum error. Various methods have been used to estimate the weight of MAVs before their construction which are mostly based on guessing or statistical data [38]. The one used in this study divides the weights into the weight of the structure and the weight of the components and then estimates the weight of each one separately. Defining the weight of the structure by W_{Str} and the weight of the equipment by W_{Eq} , the total weight of the flapping wing MAV can be expressed as:

$$W_{TO} = W_{Eq} + W_{Str}. \quad (15)$$

The weight of the electrical components can be written as follows:

$$W_{Eq} = W_B + W_{PL} + W_{AV} + W_{PP} \quad (16)$$

where W_B denotes the weight of the battery, W_{PL} represents the weight of payload, such as the loads, sensors, cameras, and other similar weights, W_{AV} is the weight of the avionic system including the servo motors, receivers, and navigation systems, such as autopilot, and W_{PP} denotes the weight of the power plant which consists of the motor and speed controller. Among these electrical components, the motor and battery are the most useful in the estimation of the weight of the flapping wings' components. It should be noted that the main criteria in the selection of the motors are low weight, high torque, and sufficient weight-to-power ratio. The main criterion for battery selection is high energy density. In the estimation of the weight of the applied devices in flapping wings, a list of different types of them should be made, and then score them based on the less weight, features, and links in order to select the best ones.

The other part of the weight of an FWMAV is the weight of the structure, W_{Str} , which can be expressed as:

$$W_{Str} = W_{Wing} + W_{Tail} + W_{Fuselage} + W_{Mechanism} + W_{Other}. \quad (17)$$

The sum of the weights of the wing structure, the wing membrane, and their links represents the weight of the wing (W_{Wing}). The weight of the wing structure is the sum of the leading edge spars, diagonal spars, and ribs weights. The weight of the tail (W_{Tail}) includes the weights of the tails structure, tail membrane, and their links. It should be noted that the weight of the tail varies according to the type of the used tail. $W_{Fuselage}$ is the sum of the flapping wing's body, flapping wing's cape, and their links weights. As for the weight of the mechanism ($W_{Mechanism}$), it involves the weights of the gearbox system, linking bars, crankshaft, joints, and external parts linked to the flapping wing. Depending on its type, the weight of the mechanisms can vary. Furthermore, it should be mentioned that the weights of other parts including landing gear and protective guards are not included. The total weight of each component and the sum of the separate components weight are determined and expressed as a percentage of the total weight.

To estimate the flapping wing structures, in our proposed sizing methodology, a statistical method is utilized. In general, as noted above, the total weight of an FWMAV includes W_{PP} , W_B , W_{PL} , W_{AV} , and W_{Str} . The weight of the used electrical components in the flapping wing can be estimated, and the only remaining unknown is the weight of the structure which can be calculated by using statistical data. In this method, the weight parts (W_{PP} , W_{PL} , W_B , W_{AV} , and W_{Str}) of many flapping wings have been extracted from different references [39–41]. Three

Table 1 Percentage of the weight of the constituents of flapping wings for the three weight classes

Weight range (g)	W_{PP} (%)	W_{PL} (%)	W_B (%)	W_{AV} (%)	W_{Str} (%)
<100	23	2	24	13	38
Between 100 and 400	16	1	14	9	60
Between 400 and 800	12	0	12	4	72

distinct categories are considered depending on the weight of the FWMAV ($m < 100$ g, 100 g $< m < 400$ g, and 400 g $< m < 800$ g), as shown in Table 1. In this Table, the approximate percentage of each constituent of the flapping wing for these three considered weight classes is presented.

By referring to the statistical data presented in Table 1, it can be concluded that $x(x = W_{Str}/W_{TO})$ is 38% for flapping wings weighing <100 g, 60% for flapping wings weighing between 100 and 400 g, and 72% for flapping wings weighing between 400 and 800 g. It should be mentioned that most of the available and considered flapping wings in the weight range of 400–800 g are commercial ones which are not able to carry payloads. Thus, W_{PL} for them is around zero. The shown percentages in Table 1 will be considered when fabricating our prototype Thunder I.

3 Specifying other parameters in sizing of flapping wings

There are various parameters which can determine the geometry and physics of flapping wings MAVs. Among them, we can point to the flapping frequency, angles of upward and downward flapping, aspect ratio, wing surface, wingspan, root chord length of the wing, wing loading parameter, dynamic twist angle, angle of attack, and wing dihedral angle. With the implementation of the fivefold process of sizing, the initial values of parameters of wing loading, weight, planform, and aspect ratio are determined. Thus, the surface of the wing and its span can be calculated as:

$$W/S, \quad W \rightarrow S \quad \text{and} \quad AR = b^2/S \rightarrow b = \sqrt{AR \times S}. \quad (18)$$

The mean chord which has a unique value for the wing of a FWMAV is equal to the ratio of the wing surface to the wingspan: ($C_m = S/b$). In the next Subsections, a particular focus is given to the determination of the flapping frequency, time fractions, and flapping angles of downstrokes and upstrokes.

3.1 Calculating the flapping frequency of an FWMAV

For birds, the flapping frequency is the complete number of flapping cycles per second which varies according to the type of the birds. The flapping frequency should be sufficient to allow the birds to provide the required lift and thrust during the steady level flight. Pennycuik [42] performed many observations for various samples with low frequencies (<13 Hz). Using 47 samples of low frequency birds [42], a formula which introduces an estimation for the flapping frequency is then developed:

$$f = m^{\frac{3}{8}} g^{\frac{1}{2}} b^{-\frac{23}{24}} S^{-\frac{1}{3}} \rho^{-\frac{3}{8}}. \quad (19)$$

In Table 2, both flapping frequencies of the real commercial FWMAV and the estimated one from the relationship frequency of Pennycuik [Eq. (19)] are determined. It follows from this Table that for lower flapping frequency of the commercial FWMAVs, Eq. (19) can be used with a smaller correction factor. When the flapping frequency is increased, higher correction factors are needed. For instance, the commercial flapping wing Park Hawk [25] has a wingspan of 120 cm and weight of 425 g with 6 Hz frequency. If Eq. (19) is used, the estimated frequency will be 2.7 Hz while this flapping wing should flap 2.2 times more than birds' wing due to the membrane wing, as shown in Table 2. Indeed, the membrane wings are often made from flexible and flat materials. These materials can either be thin plastic film or thin cloth. Nowadays, because of their lightweight, they become so popular especially for small flapping wing applications. However, the disadvantage of these materials is their poor efficiencies. Thus, the obtained frequency should be multiplied by a correction factor. To estimate the ratio between the actual and estimated frequencies, the frequency ratios of most of the used flapping wings in the literature are addressed in Table 2. Clearly, the ratio between the actual and estimated

Table 2 Comparison between actual and estimated flapping frequencies

Flapping wing	m (g)	b (cm)	S (m ²)	Actual frequency f (Hz)	Estimated frequency f (Hz)	Frequency ratio
UA-74 [41]	248	74	0.0991	6	4.95	1.21
Slowhawk2 [43]	420	122	0.359	3.6	2.43	1.48
Adelaide3 [28]	20.09	34.768	0.07	7.2	4.47	1.60
Adelaide2 [28]	16.09	35	0.07	6.93	4	1.73
Adelaide1 [28]	25	38.66	0.08	7.65	4.22	1.80
Tadbir [44]	230	80	0.107	8	4.36	1.83
Adelaide4 [28]	23	35.3	0.072	9.48	4.6	2.06
Park Hawk [25]	425	120	0.07	6	2.7	2.2
UA-25 [41]	32.4	25	0.0137	13	5.8	2.24
Adelaide7 [28]	12.75	33.84	0.07	8.8	3.86	2.28
Adelaide8 [28]	34.06	48.83	0.07	9.5	3.9	2.43
Konkuk2 [45]	30.6	28	0.028	24	8.85	2.71
Konkuk1 [45]	50	36	0.0432	20	7.14	2.8
Adelaide6 [28]	23.44	37	0.07	14.03	4.46	3.14
Konkuk4 [45]	4.32	10	0.06	35	8.74	4.00
Konkuk3 [45]	8.7	15	0.085	30	6.85	4.38
Adelaide5 [28]	73.34	89.4	0.08	13.25	2.8	4.73

Table 3 Time ratio between downstroke and upstroke flapping

Bird type	Downstroke/upstroke ratio	Insect type [48]	Downstroke/upstroke ratio
Cockatie [49]	0.88	Honeybee	1.3
Albatross [50]	1.06	June beetle	1.5
Cockatoo [49]	1.18	Housefly	1.5
Bat (fast) [47]	1.31	Fruit fly	1.7
Vulture [47]	1.4	Locust	1.9
Bat (slow) [50]	2.1	Dragonfly	2.37

frequencies should be ranged between 1.2 and 4.7. To this end, in our proposed methodology, Eq. (19) is adjusted as follows:

$$f = \xi m^{\frac{3}{8}} g^{\frac{1}{2}} b^{-\frac{23}{24}} S^{-\frac{1}{3}} \rho^{-\frac{3}{8}} \quad (20)$$

where ξ is the correction factor which depends on the type of the FWMAV.

3.2 Determining the time fractions of flapping wings

In general, flapping wings are designed with sinusoidal motions [46]. For instance, the movement of the tandem wing tip and insects like bees is much similar to simple harmonic motion. In hovering flight, a basic assumption about flapping wings in insect size is that the movements of wings are harmonic and sinusoidal. In forward flight, it was reported in previous studies [47,48] that upward and downward flapping motions are not necessarily symmetric in flapping time. Indeed, Alexander [48] showed that the downstroke flapping time of some dragonflies can be 2.37 times larger than their upstroke flapping time, as presented in Table 3.

It should be mentioned that the time fraction for downstroke-to-upstroke ratio of artificial flapping wings is near one, and often mechanisms which provide for them symmetric flapping movements are used [47,51].

3.3 Determining the upstroke and downstroke angles

Any flapping wing has a cycle of alternative flapping including downstroke flapping to produce positive lift and thrust forces and upstroke flapping to produce mainly positive thrust and negative lift force [52]. The sum of the upstroke and downstroke flapping angles is called flapping angle, as shown in Fig. 3.

Upstroke and downstroke angles are input parameters for the mechanical design that should be considered in the design of the flapping wing mechanism. For estimating the initial values of the flapping angle, we can benefit from upward and downward movements of the wing. From Fig. 3, for flapping angles calculation, we have:

$$\sin \varphi_{Up} = \frac{2h_a}{b} \rightarrow \varphi_{Up} = \sin^{-1} \left(\frac{2h_a}{b} \right) \quad (21)$$

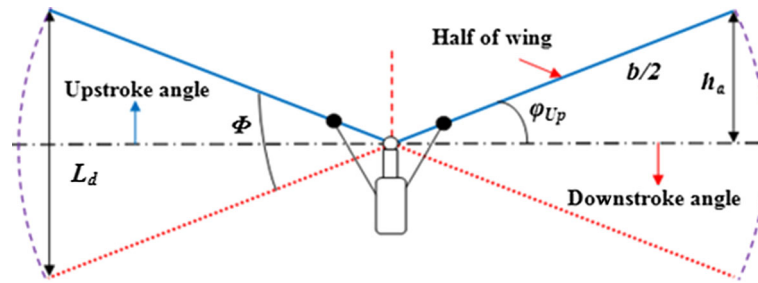


Fig. 3 Upstroke and downstroke angles, ϕ is the total flapping angle of an FWMAV

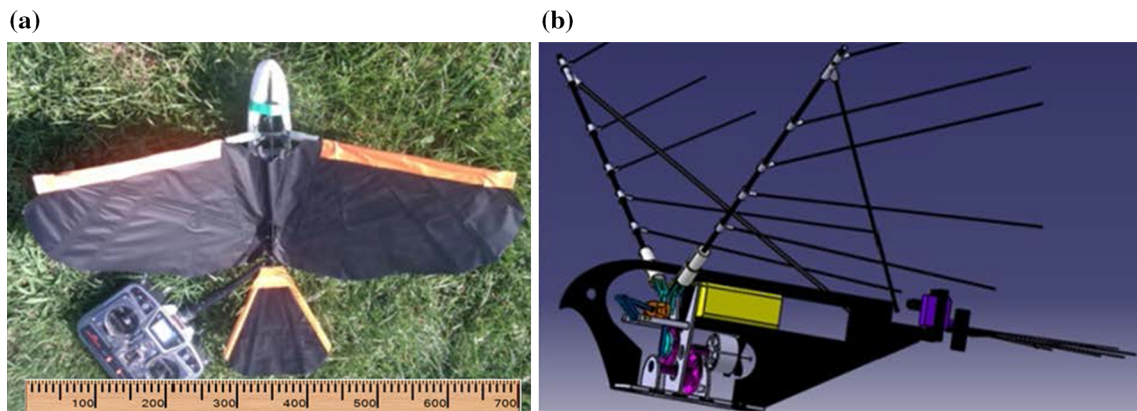


Fig. 4 a General view of the Thunder I and b skeleton of Thunder I

where h_a is the domain of upstroke of the flapping wing. For estimating the value of h_a , the Strouhal number can be used, which is one of the important non-dimensional parameters and can be obtained by dividing the flapping frequency (f) and the vertical domain of the wing tip (L_d) by the forward speed (U) of the FWMAV as [32]:

$$St = \frac{f L_d}{U} = \frac{2 f h_a}{U}. \tag{22}$$

It was demonstrated that high propulsive efficiencies (almost 70%) are obtained when the Strouhal number is in the range of 0.2–0.4. It was demonstrated that a peak efficiency is achieved when the Strouhal number is set equal to 0.3 [53,54]. Considering this value, h_a can be estimated by having the values of the frequencies and forward speeds.

4 Sizing an FWMAV: Thunder I prototype and experiments

Based on the aforementioned procedures and techniques, an FWMAV called Thunder I is designed and constructed, as presented in Fig. 4a, b.

As mentioned in Sect. 2.1, the first step in sizing of an FWMAV is the mission analysis and the preparation of the flight plan. In Fig. 5, the flight route of this fabricated MAV is depicted. In place (1), the FWMAV should be hand launched, and after flying over route (1), it should detect the first black square. With flying over route (2), the MAV should arrive at the start point, and after that it should detect two other black squares when crossing the route (3). Starting the flight performance, the flapping wing should fly as many laps as possible in the mission around two poles (red circles) in route (4). After doing the flight performance mission, the MAV should detect the other two black squares when crossing route (5) and then fly between two aligned arches [route (6)] without going out of the corridor landing. Finally, the flapping wing should land when crossing route (7). The estimated flight route of the fabricated MAV is considered 6000 m. The flight endurance of Thunder I is considered almost equal to 10 min. Hence, the approximate cruise speed for this MAV is 10 m/s.

Since this flapping wing’s flight tests are carried out in Isfahan, the latitude (L_0) and altitude (h_0) are, respectively, considered equal to 32.42° and 1631 m. Then, regarding these values and considering Eqs. (1–5)

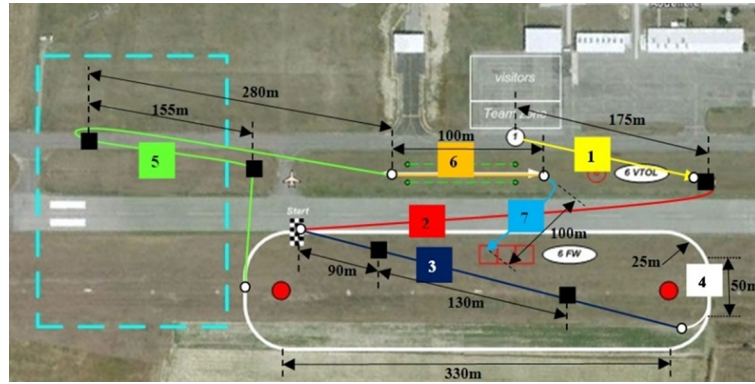


Fig. 5 Defined flight route for the flapping wing in outdoor place

Table 4 Physical parameters and flight atmosphere for the Thunder I flapping wing

Parameter	Symbol	Value
Gravity acceleration (m/s ²)	G	9.79
Air pressure (hPa)	P	831.5
Kinematic viscosity (m ² /s)	ν	1.649×10^{-5}
Air density (kg/m ³)	ρ	1.01
Air temperature (°C)	T_a	25

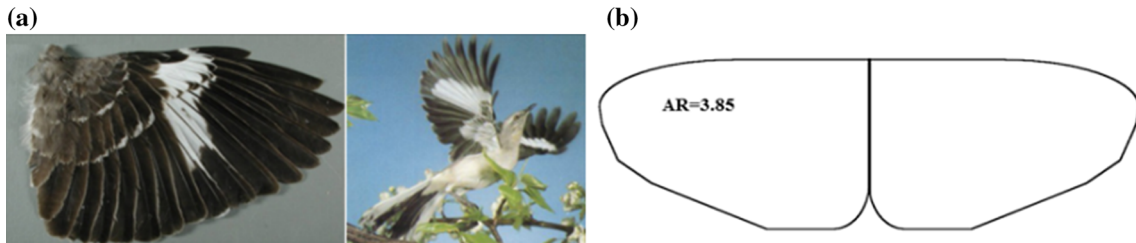


Fig. 6 a Wing shape of northern mockingbird and b extracted planform of Thunder I from northern mockingbird

Table 5 Used input data to carry out the constraint analysis

Parameter	Symbol	Value	Parameter	Symbol	Value
Aspect ratio	AR	3.85	Climbing speed	U_C	7 m/s
Oswald numbers [50]	E	0.8	Horizontal acceleration	a_H	1 m/s ²
Cruise speed	U	10 m/s	Turning radius	R_c	10 m
Climbing rate	C_R	1 m/s	Launch speed	V_{HL}	6 m/s
Climbing acceleration	a_C	0.5 m/s ²	Turning speed	V_T	7 m/s

given in Sect. 2.1, the air pressure, density, temperature, and viscosity as well as gravity acceleration are determined, as presented in Table 4.

The designed flapping wing should be able to execute different flight modes including takeoff (launch), landing, cruise, turning, climbing, and descending. According to what was discussed in Sect. 2.1 and based on the defined mission, the flight of the FWMAV is only composed of flapping and gliding modes. It should be mentioned that the hovering mode is often used for flapping wings with small size (NAV and PAV). Based on what was mentioned in Sect. 2.2, it is better to consider a short wing with low aspect ratio. In addition to that, it is better that the fabricated flapping wing could takeoff rapidly in order to consume less energy. Consequently, the selected planform and aspect ratio for Thunder I are imitated from the northern mockingbird wing shape, as presented in Fig. 6a, b.

Based on what was discussed in Sect. 2.3, for an estimation of the wing loading parameter, the constraint analysis process is used. The input data for such an analysis are shown in Table 5.

One of the parameters that should be estimated when applying the constraint analysis for the prototype flapping wing is the maximum lift coefficient (C_{Lmax}). In Fig. 7, the range of the maximum lift coefficient is shown for different birds when the Reynolds number is varied from 10^2 to 10^7 [55]. Clearly, in steady flow configurations, most of the flapping wings have maximum lift coefficient higher than 0.6 and lower than 2. According to the Reynolds number of artificial flapping wings, which usually is in the order of 10^5 [56], it can be concluded that the proper range for estimating this coefficient (C_{Lmax}), in the constraint analysis process for flapping wings MAVs, is between 1.4 and 1.9 [55].

For this prototype, the maximum lift coefficient is considered equal to 1.8 which is in the defined range and is verified with other flapping wings [57–59]. It should be noted that the determination of an accurate value of C_{Lmax} requires experimental tests, which should be done in a wind tunnel after fabrication. The parasite drag coefficient (C_{DP}) is considered equal to 0.03 for this prototype flapping wing. In Fig. 8, the obtained results from the simulations of the constraint analysis are presented.

As shown in Fig. 8, for the six flight scenarios consisting of constant cruise speed, constant climb speed, constant turning altitude/speed, horizontal acceleration, accelerated climb, and bird takeoff, the thrust loading (T/W) is represented as a function of the wing loading (W/S). It should be mentioned that the selected point in the bounden space should satisfy all of the constraints according to the defined mission. Therefore, a point which can provide the maximum possible wing loading and minimum thrust loading is pinpointed. It follows

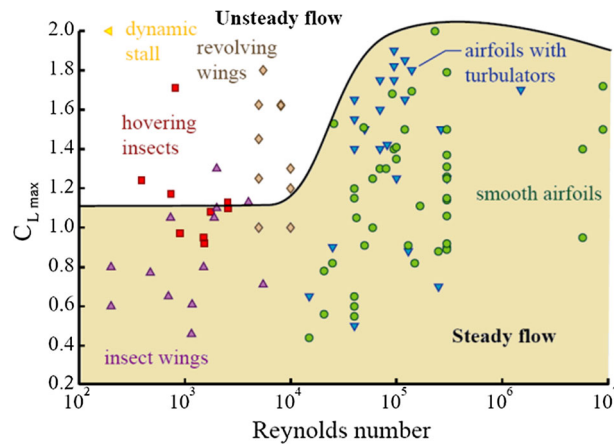


Fig. 7 Variations of the maximum lift coefficient as a function of the Reynolds number for various flapping wings [55]

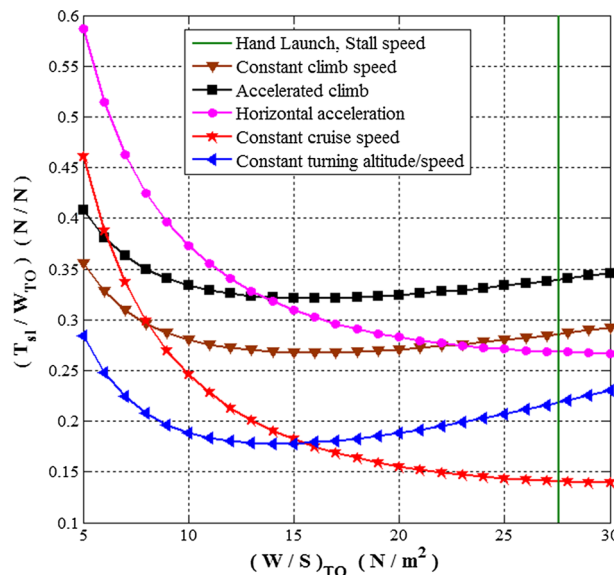


Fig. 8 Results of the constraint analysis

from these plots that the maximum possible wing loading value that should be considered for this prototype is 26 N/m².

Based on what was mentioned in Sect. 2.4, for the estimation of the weight of the FWMAV, its total weight is divided into two parts, namely structure and equipment weights. Considering the maximum thrust loading (T_{SL}/W) shown in Fig. 8, the weights of the electrical components are estimated using engineering designing [26,60] and presented in Table 6.

It follows from Table 6 that the weight of the equipment is about 135 g. Based on Table 1 and the estimated weight of the electrical components, the weight range of the flapping wing is from 100 to 400 g. Thus, the weight of the structural components comprises about 60% of the total weight. Using the proposed statistical method, W_{TO} is estimated to be equal to 337.5 g. For the fabricated prototype, the total weight is considered 350 g.

By specifying the types of the planform, aspect ratio, flapping wing weight, and wing loading parameter, the flapping wing micro air vehicle's dimensions are calculated as follows:

$$\frac{W}{S} = \frac{0.35 \text{ g}}{S} = 26 \text{ N/m}^2 \rightarrow S = 0.132 \text{ m}^2, \tag{23}$$

$$\text{AR} = \frac{b^2}{S} \rightarrow b = \sqrt{3.85 \times 0.132} = 0.713 \approx 70 \text{ cm}, \tag{24}$$

Using a scaling procedure for the chosen planform, the final dimensions of Thunder I are presented in detail in Fig 9.

The extracted mean chord length of Thunder I is $C_m = S/b = 18.14$ cm. After the specification of the flapping wing geometry, some of the physical and aerodynamic parameters can be estimated. After determining the wing geometry, we deal with the calculation of the Reynolds number during the flapping by having parameters like speed, mean chord, viscosity, and density according to the following equation:

$$Re = \frac{VC_m}{\nu} = \frac{10 \times 0.1814}{1.649 \times 10^{-5}} = 1.1 \times 10^5. \tag{25}$$

Then, Eq. (20) is used to estimate the flapping frequency of this MAV, as presented in Table 7. Based on what was discussed in Sect. 3.1, the calculated frequency which is obtained from the Pennycuik formula

Table 6 Estimated weights of the electrical components

Equipment	Estimated weight (g)
Motor + linking wire	30
Speed controller + linking wire	15
Servo motors + two linking wires	20
Receiver + linking wire	10
Battery + linking wire	60
Total weight	135

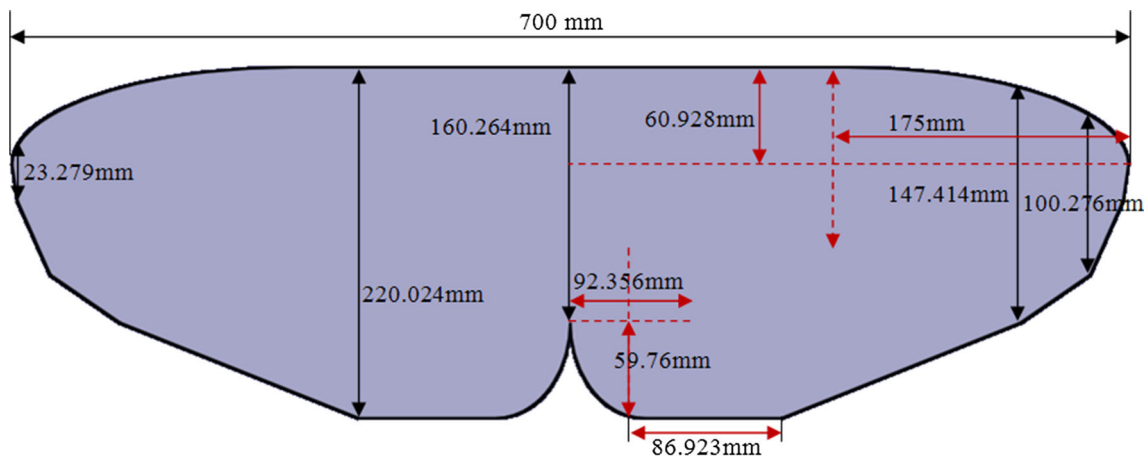


Fig. 9 Dimensions of Thunder I wing with northern mockingbird planform

should be multiplied by a correction factor which is between 1.2 and 4.7 depending on the type of the FWMAV. According to the fabricated FWMAV with the parameters given in Table 7, the correction factor is considered equal to 1.53 in order to get a flapping frequency of 9Hz. This correction factor is estimated by averaging the correction factors of two flapping wings (UA-74 and Tadbir) with similar wingspan to our prototype, as shown in Table 2. In fact, according to Greenwalt formula [61], the flapping frequency is directly related to the wingspan of the MAV. Based on statistical data, Greenewalt [62] proposed the correlation between wing flapping frequency f (Hz) and wingspan b (cm) as shown in Eq. (26),

$$fb^{1.15} = 3.54. \tag{26}$$

Based on Sect. 3.2, the time fraction for downstroke-to-upstroke ratio of this prototype is considered one, and hence, symmetric flapping movements are considered. As reported in Sect. 3.3, the Strouhal number should be considered equal to 0.3 because the peak efficiency is achieved at this value [53,54]. Having the values of the frequency and cruise speed, the domain of upstroke and downstroke movements of the wing (h_a) will be 16.66 cm. To obtain the upstroke and downstroke angles, we use Eq. (21) as:

$$\sin \varphi = \frac{2h_a}{b} = \frac{2 \times 0.166}{0.70} = 0.476 \rightarrow \varphi = \sin^{-1}(0.476) = 28.42^\circ \approx 30^\circ. \tag{27}$$

Thus, the upstroke and downstroke angles are considered equal to 30° .

In Fig. 10, the time history of the lift force in a flapping cycle is plotted, which has been obtained from Thunder I's wing simulation in FlapSIM software [63]. It follows from Fig. 10 that the mean value of the lift force is 3.435 N which tolerates the weight of the Thunder I flapping wing ($0.35 \times 9.8 = 3.43$ N).

The final designs of Thunder I flapping wing and its wing are given in Figs. 11 and 12.

After designing and fabricating the Thunder I FWMAV, various flight tests were then performed. In Table 8, the used equipment is presented.

The first test of Thunder I was performed to check the launching and safe takeoff. The second test was to check that an adequate thrust can be produced from the wing. Then, a turning test when using a constant

Table 7 Thunder I flapping frequency

Parameter	Value
Weight (m , kg)	0.350
Gravity acceleration (g , m/s^2)	9.79
Wingspan (b , m)	0.7
Wing surface (S , m^2)	0.127
Air density (ρ , kg/m^3)	1.01
Estimated frequency (f , Hz)	5.88

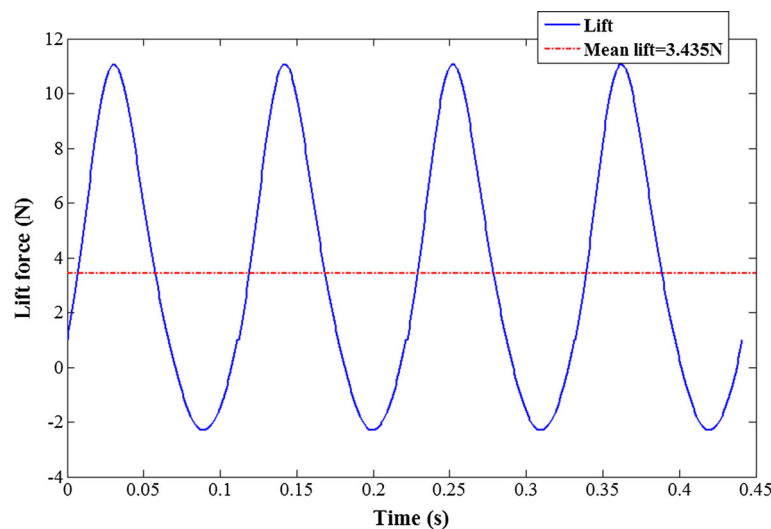


Fig. 10 Time history of the lift force for Thunder I's wing



Fig. 11 Wing of the Thunder I MAV



Fig. 12 Thunder I FWMAV

Table 8 Thunder I flapping equipment

Equipment	Model
Battery	Lipo 3 cell 800 mAh
Motor	DualSky XM300ES
Receiver	Spektrum DX6
Servo motors	Futaba S3114
Speed controller	DualSky XC1210BA

Table 9 Launching tests of Thunder I FWMAV

Number	Wind speed (m/s)	Result	Reason	Correction
1	≈1	Crashing	Nose up fly	Moving the center of gravity forward about 1cm with battery relocation
2	≈0	Crashing	Below launch speed	Launching with more speed
3	≈2	Takeoff	–	–
4	≈1	Takeoff	–	–

altitude was performed. After that, a gliding test was investigated. The tests of the launching and safe takeoff are presented in Table 9.

In the launching process of the flapping wing, four tests were performed. As presented in Table 9, the second test was unsuccessful. In this launching process, Thunder I did a nose up flight, which means that the flapping wing has not been stabilized longitudinally. It is concluded from these performed tests that there are some important parameters which significantly impact the vehicle takeoff. These parameters are the wind speed, wind direction, place of the center of gravity (CG), throttle, launching angle, launching speed, etc. It should be mentioned that in these launching tests the associated launch speed values are not exactly determined. In the takeoff cases, the launch speed is higher than the stall speed. Some pictures of the launching tests are given in Fig. 13.

After successful takeoff, Thunder I did undergo a thorough flight testing. With doing necessary adjustments for the CG position which was obtained with relocation of the battery position, Thunder I was then able to fly at a cruise speed of 10 m/s. The results of the powered flight tests were satisfactory after a number of attempts where the center of gravity and flapping frequency were varied during the tests. It is found that a flapping frequency of approximately 8–9 Hz is sufficient for the Thunder I to achieve powered flight, as estimated by our developed methodology. We should mention that during flight, Thunder I is stable and not perturbed. During these tests, our fabricated FWMAV can withstand against a wind disturbance up to 2 m/s and can perform a series of turns while maintaining its altitude. The flight endurance of this flapping wing is between 11 and 13 min at a moderate throttle setting without any payload. It should be noted that by making batteries with higher energy density endurance can be further improved.



Fig. 13 Thunder I FWMAV takeoff

In the final test, we tried to measure the gliding ability of this flapping wing in ceiling almost equal to 50 m, which was also satisfactory. It should be mentioned that all of these flight tests were done in heights below 50 m. In general, in the design and fabrication of any FWMAV, one of the most vital components which has important role in flight performance is the wing. With considering this point, it is concluded that sizing of the wing is one of the most important parts in the design of FWMAVs.

5 Conclusions

A reasonable and new sizing strategy based on theoretical and statistical analyses for designing efficient flapping wings MAVs has been introduced and tested. The sizing of these MAVs was composed of five main steps including (i) defining and analyzing the mission, (ii) determining the flight modes, (iii) determining the planform and aspect ratio of the wing, (iv) conducting constraint analysis based on the defined mission, and (v) estimating the weight. In defining the mission, the analysis of the flight mission as well as the calculation of the atmosphere parameters is performed. After completing these two steps, the wing shape (planform) and aspect ratio of the designed FWMAV were determined. The constraint analysis and the weight estimation steps were then performed. It was demonstrated that these two steps are related by the wing loading and surface of the wing. After working on all these steps, all wing dimensions and its flapping frequency were easily determined. To check the accuracy of our proposed method, a wing sizing procedure was performed for an FWMAV named Thunder I. Several flight tests were successfully performed to determine the impact of different parameters on the efficiency of Thunder I. The proposed methodology in this study can open new horizons for designing efficient bio-inspired flapping wings micro air vehicles.

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