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ORIGINAL PAPER

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Vibrating particles system algorithm for truss optimization with multiple natural frequency constraints

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Abstract In this paper, the recently developed physically inspired non-gradient algorithm is employed for structural optimization with frequency constraints. The algorithm being called vibrating particles system (VPS) mimics the free vibration of single degree of freedom systems with viscous damping. Truss optimization with frequency constraints has attracted substantial attention recently in order to enhance the dynamic performance of structures. These kinds of problems are believed to represent nonlinear and non-convex search spaces with several local optima and therefore are suitable for examining the capabilities of the new algorithms. A set of five truss design problems are considered for evaluating the VPS in this article. The numerical results demonstrate the efficiency and robustness of the new method and its competitive performance to other algorithms for structural optimization problems.

1 Introduction

Fundamental frequencies of a structure are important, easily obtained characteristics which allow the designer to keep out from the dangerous resonance phenomenon. When dynamic excitations are critical, these characteristics cannot be neglected [1]. Frequency responses are highly implicit, non-convex, and nonlinear with respect to the cross-sectional area of bar elements, so the search spaces normally contain multiple local minima [2] and call for a competent optimization algorithm in order to be appropriately addressed.

Structural optimization considering natural frequency constraints has been studied since the 1980s [3] and approached with mathematical programming and meta-heuristic algorithms. Lin et al. [4] studied the minimum weight design of structures under simultaneous static and dynamic constraints proposing a bi-factor algorithm based on the Kuhn–Tucker criteria. Konzelman [5] considered the problem using some dual methods and approximation concepts for structural optimization. Grandhi and Venkayya [6] utilized an optimality criterion based on uniform Lagrangian density for a resizing and scaling procedure to locate the constraint boundary. Wang et al. [7] proposed an optimality criteria on algorithm for combined sizing-layout optimization of a three-dimensional truss structure. In this method, the sensitivity analysis helps to determine the search direction, and the optimal solution is achieved gradually from an infeasible starting point with a minimum weight increment, and the structural weight is indirectly minimized. Sedaghati [8] utilized a new approach using combined mathematical programming based on the sequential quadratic programming (SQP) technique and a finite element solver based on the integrated force method. Lingyun et al. [9] combined the simplex search

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method and the niche genetic hybrid algorithm (NGHA) for mass minimization of structures with frequency constraints. Gomes [10] used the particle swarm optimization (PSO) algorithm to study simultaneous layout and sizing optimization of truss structures with multiple frequency constraints. Kaveh and Zolghadr [11] combined charged system search and big bang with trap recognition capability (CSS-BBBC) to solve layout and sizing optimization problems of truss structures with natural frequency constraints. Miguel and Fadel Miguel [12] employed harmony search (HS) and firefly algorithm (FA), to study simultaneous layout and sizing optimization of truss structures with multiple frequency constraints. A hybrid optimality criterion (OC) and genetic algorithm (GA) method was used by Zuo et al. [13] for truss optimization with frequency constraints. Kaveh and Javadi [14] utilized hybridization of harmony search, ray optimizer, and particle swarm optimization algorithm (HRPSO) for weight minimization of trusses under multiple natural frequency constraints. Kaveh and Ilchi Ghazaan [15] employed particle swarm optimization with an aging leader and challengers (ALC-PSO) and HALC-PSO that transplants a harmony search-based mechanism to ALC-PSO as a variable constraint handling approach to optimize truss structures with frequency constraints. Hosseinzadeh et al. [16] used a hybrid electromagnetism-like mechanism algorithm and migration strategy (EM–MS) for layout and size optimization of truss structures with multiple frequency constraints.

This paper proposes the application of the newly developed optimization algorithm so-called the vibrating particles system (VPS) for an optimum design of truss structures with frequency constraints. In this method, the solution candidates are considered as particles that gradually approach to their equilibrium positions. Equilibrium positions are achieved from current population and historically best position in order to have a proper balance between exploration and exploitation [17]. In order to evaluate the performance of the VPS, five truss structures are optimized for minimum weight so that the design variables are considered to be the cross-sectional areas of the members and/or the coordinates of some nodes. The truss examples have 10, 37, 72, 120 and 600 members, respectively. The numerical results indicate that the proposed algorithm is quite competitive with other state-of-the-art metaheuristic methods.

The remainder of this paper is organized as follows: In Sect. 2, the mathematical formulations of the structural optimization with frequency constraints are stated. The optimization algorithm is proposed after a brief overview of the free vibration of single degree of freedom systems with viscous damping in Sect. 3. Five structural design examples are studied in Sect. 4, and some concluding remarks are finally provided in Sect. 5.

2 Statement of the optimization problem

In this paper, the objective is to minimize the weight of the structure while satisfying some constraints on the natural frequencies. Each variable should be chosen within a permissible range. The mathematical formulation of these problems can be expressed as follows:

$$\begin{aligned} &\text{Find}\{X\} = [x_1, x_2, \ldots, x_{ng}] \\ &\text{to minimize } W(\{X\}) = \sum_{i=1}^{nm} \rho_i A_i L_i \\ &\text{subjected to: } \begin{cases} \omega_j \leq \omega_j^* \\ \omega_k \geq \omega_k^* \\ x_{i\,\min} \leq x_\mathrm{i} \leq x_{i\,\max} \end{cases} \end{aligned} \tag{1}$$

where $\{X\}$ is the vector containing the design variables; ng is the number of design variables; $W(\{X\})$ presents the weight of the structure; nm is the number of elements of the structure; ρ_i, A_i and L_i denote the material density, cross-sectional area, and the length of the ith member, respectively; ω_j is the jth natural frequency of the structure, and ω_j^* is its upper bound; ω_k is the kth natural frequency of the structure, and ω_k^* is its lower bound; $x_{i\,\min}$ and $x_{i\,\max}$ are the lower and upper bounds of the design variable x_i, respectively.

To handle the constraints, the well-known penalty approach is employed. Thus, the objective function is redefined as follows:

$$f_{\cos t}(\{X\}) = (1 + \varepsilon_1.\upsilon)^{\varepsilon_2} \times W(\{X\}), \quad \upsilon = \sum_{j=1}^{nc} \max[0, g_j(\{X\})] \tag{2}$$

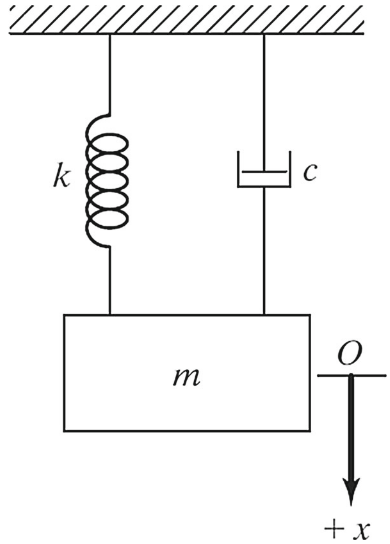

Fig. 1 Free vibration of a system with damping

where υ denotes the sum of the violations of the design constraints and nc is the number of the constraints. Here, ε_1 is set to unity, and ε_2 is calculated by

$$\varepsilon_2 = 1.5 + 1.5 \times \frac{\text{iter}}{\text{iter}_{\max}}. \tag{3}$$

Thus, in the first steps of the search process, ε_2 is set to 1.5 and ultimately increased to 3. Such a scheme penalizes the unfeasible solutions more severely as the optimization process proceeds. As a result, in the early stages the agents are free to explore the search space, but at the end they tend to choose solutions with no violation.

3 The vibrating particles system algorithm

This Section describes the VPS algorithm. First, a brief overview of the free vibration of single degree of freedom systems with viscous damping is provided, and then the proposed method is presented.

3.1 The physical background of the VPS algorithm

There are two general types of vibrations, namely free vibration and forced vibration. In free vibration, the motion is only maintained by the restoring forces, and in the forced vibration, a periodic force is applied to the system. The effects of friction in a vibrating system can be neglected resulting in an undamped vibration. However, all vibrations are actually damped to some degree by friction forces. These forces can be caused by dry friction, or Coulomb friction, between rigid bodies, by fluid friction when a rigid body moves in a fluid, or by internal friction between the molecules of a seemingly elastic body. In this Section, the free vibration of single degree of freedom systems with viscous damping is studied. The viscous damping is caused by fluid friction at low and moderate speeds. Viscous damping is characterized by the fact that the friction force is directly proportional and opposite to the velocity of the moving body [18].

Figure 1 shows the vibrating motion of a body or system of mass m having viscous damping. A spring of constant k and a dashpot are connected to the block. The effect of damping is provided by the dashpot, and the magnitude of the friction force exerted on the plunger by the surrounding fluid is equal to $c\dot{x}$ (c is the coefficient of viscous damping, and its value depends on the physical properties of the fluid and the construction of the dashpot). When the block is displaced a distance x from its position of stable equilibrium, the equation of motion can be expressed as:

$$m\ddot{x} + c\dot{x} + kx = 0. \tag{4}$$

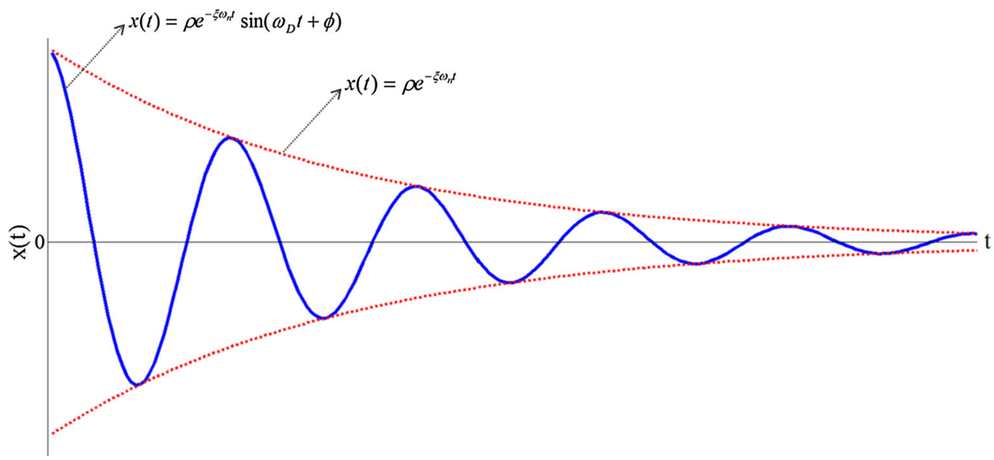

Fig. 2 Vibrating motion of under-damped system

Before presenting the solutions for this differential equation, we define the critical damping coefficient c_c as:

$$c_c = 2m\omega_n, \tag{5}$$

$$\omega_n = \sqrt{\frac{k}{m}} \tag{6}$$

where ω_n is the natural circular frequency of the vibration.

Depending on the value of the coefficient of viscous damping, three different cases of damping can be distinguished: (i) over-damped system ($c > c_c$), (ii) critically damped system ($c = c_c$) and (iii) under-damped system ($c < c_c$). The solutions of over-damped and critically damped system correspond to a non-vibratory motion. Therefore, the system only oscillates and returns to its equilibrium position when $c < c_c$.

The solution of Eq. (4) for an under-damped system is as follows:

$$x(t) = \rho e^{-\xi\omega_n t}\sin(\omega_D t + \phi), \tag{7}$$

$$\omega_D = \omega_n\sqrt{1 - \xi^2}, \tag{8}$$

$$\xi = \frac{c}{2m\omega_n} \tag{9}$$

where ρ and ϕ are constants generally determined from the initial conditions of the problem. ω_D and ξ are damped natural frequency and damping ratio, respectively. Equation (7) is shown in Fig. 2, and the effect of damping ratio on vibratory motion is illustrated in Fig. 3.

3.2 The VPS algorithm

The VPS is a population-based algorithm which simulates a free vibration of single degree of freedom systems with viscous damping [17]. Similar to other multi-agent methods, VPS has a number of individuals (or particles) consisting of the variables of the problem. The solution candidates gradually approach to their equilibrium positions that are achieved from current population and historically best position in order to have a proper balance between diversification and intensification. In VPS, the initial locations of particles are created randomly in an n-dimensional search space,

$$x_i^j = x_{\min} + \text{rand} \cdot (x_{\max} - x_{\min}), \tag{10}$$

where x_i^j is the jth variable of the particle i. $x_{\min}$ and $x_{\max}$ are the minimum and the maximum allowable variables vectors; rand is a random number uniformly distributed in the range of [0, 1].

For each particle, three equilibrium positions with different weights are defined, and during each generation, the particle position is updated by learning from them: (i) the historically best position of the entire population (HB), (ii) a good particle (GP), and (iii) a bad particle (BP). In order to select the GP and BP for each candidate

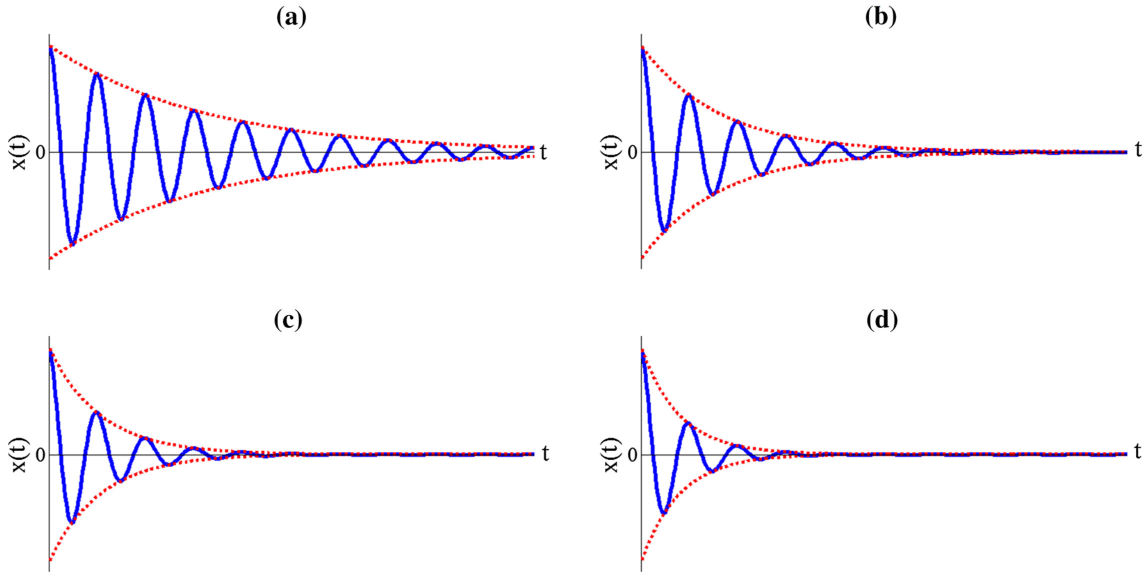

Fig. 3 Free vibration of systems with four levels of damping: **a** $\xi = 5\,\%$, **b** $\xi = 10\,\%$, **c** $\xi = 15\,\%$ and **d** $\xi = 20\,\%$

solution, the current population is sorted according to their objective function values in an increasing order, and then GP and BP are chosen randomly from the first and second half, respectively.

A descending function based on the number of iterations is proposed in VPS to model the effect of the damping level in the vibration that is depicted in Fig. 3.

$$D = \left(\frac{\text{iter}}{\text{iter}_{\max}}\right)^{\alpha} \tag{11}$$

where iter is the current iteration number and $\text{iter}_{\max}$ is the total number of iterations for the optimization process. α is a constant.

According to the above concepts, the update rules in the VPS are given by

$$x_i^j = w_1 \cdot [D \cdot A \cdot \text{rand1} + \text{HB}^j] + w_2 \cdot [D \cdot A \cdot \text{rand2} + \text{GP}^j] + w_3 \cdot [D \cdot A \cdot \text{rand3} + \text{BP}^j], \tag{12}$$

$$A = [w_1 \cdot (\text{HB}^j - x_i^j)] + [w_2 \cdot (\text{GP}^j - x_i^j)] + [w_3 \cdot (\text{BP}^j - x_i^j)], \tag{13}$$

$$w_1 + w_2 + w_3 = 1 \tag{14}$$

where x_i^j is the jth variable of the particle i. w_1, w_2, and w_3 are three parameters to measure the relative importance of HB, GP and BP, respectively. rand1, rand2, and rand3 are random numbers uniformly distributed in the range of [0, 1]. The effect of A and D parameters in Eq. (12) is similar to that of ρ and $e^{-\xi\omega_n t}$ in Eq. (7), respectively. Also, the value of $\sin(\omega_D t + \phi)$ is considered unity in Eq. (12) ($x(t) = \rho e^{-\xi\omega_n t}$ are shown in Fig. 2 by red lines).

In order to have a fast convergence in the VPS, the effect of BP is sometimes considered in updating the position formula. Therefore, for each particle, a parameter like p within (0, 1) is defined, and it is compared with rand (a random number uniformly distributed in the range of [0,1]) and if $p <$ rand, then $w_3 = 0$ and $w_2 = 1 - w_1$.

There is a possibility of boundary violation when a particle moves to its new position. In the proposed algorithm, for handling boundary constraints a harmony search-based approach is used [19]. In this technique, there is a possibility like harmony memory considering rate (HMCR) that specifies whether the violating component must be changed with the corresponding component of the historically best position of a random particle or it should be determined randomly in the search space. Moreover, if the component of a historically best position is selected, there is a possibility like pitch adjusting rate (PAR) that specifies whether this value should be changed with the neighboring value or not.

In this study, after the predefined maximum evaluation number, the optimization process is terminated. However, any terminating condition can be used. The flowchart of the VPS is illustrated in Fig. 4.

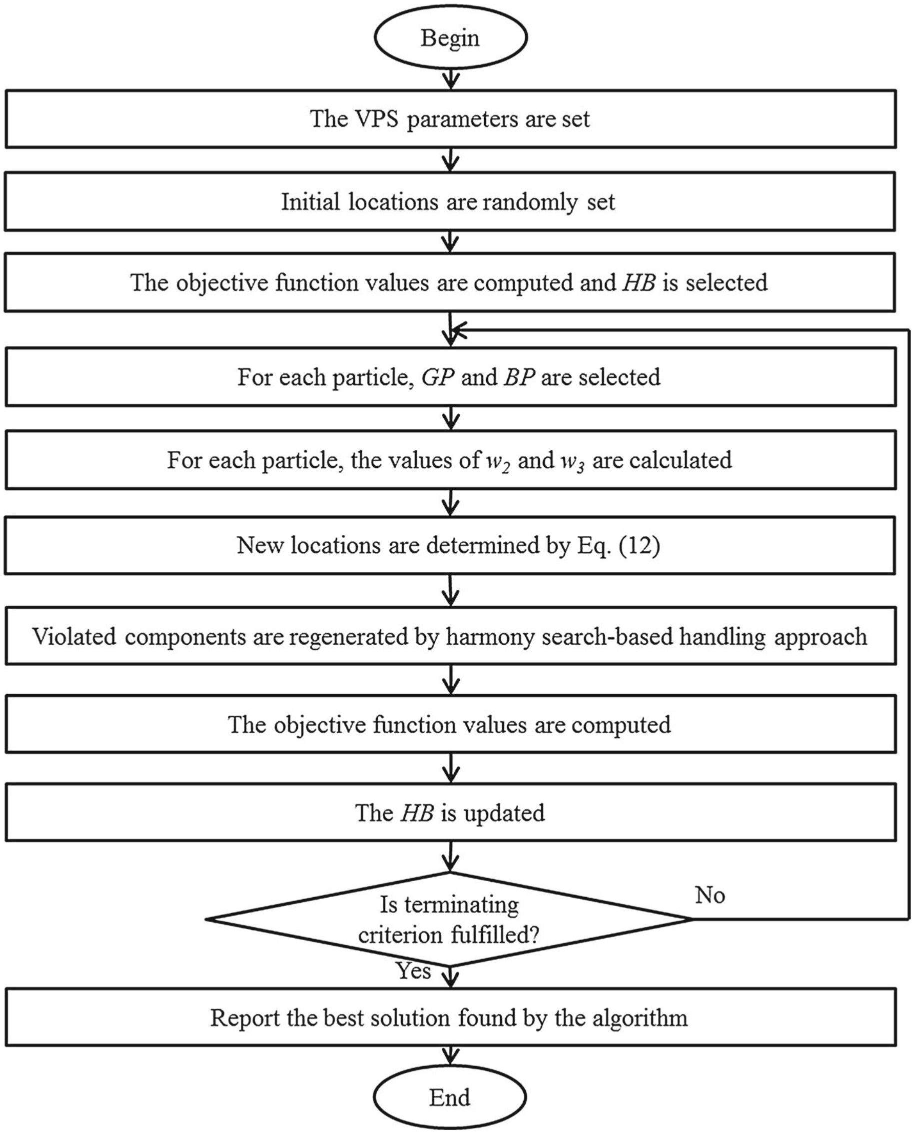

Fig. 4 Flowchart of the VPS algorithm

4 Test problems and optimization results

This Section discusses the computational examples used to investigate the performance of the proposed algorithm. The values of population size, the total number of iterations, α, p, w_1, and w_2 are set to 20, 1500, 0.05, 70 %, 0.3, and 0.3 for all examples, respectively. Sensitivity analyses of the VPS on these parameters are investigated in [17]. Twenty independent optimization runs are carried out for the first four considered examples, and the last example has been solved five times independently. The algorithm is coded in MATLAB, and the structures are analyzed using the direct stiffness method by our own codes.

4.1 A 10-bar plane truss

The 10-bar plane truss is a well-known benchmark problem, and Fig. 5 shows the topology, nodal and element numbering of this truss. The cross-sectional area of each of the members is considered to be an independent variable. The material density is 2767.99 kg/m^3, and the modulus of elasticity is 68.95 GPa for all elements. At each free node (1–4), a non-structural mass of 453.6 kg is attached. The range of cross-sectional area of all

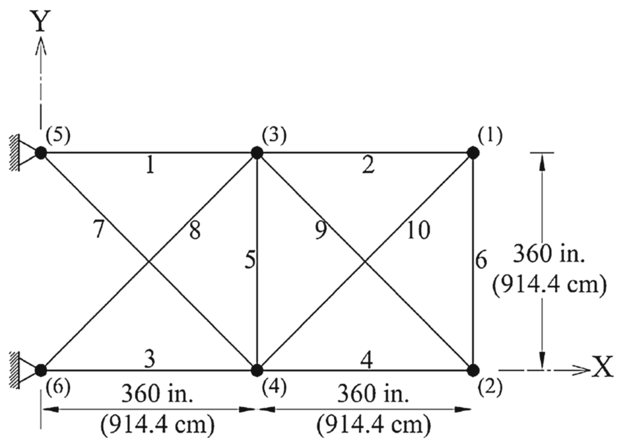

Fig. 5 Schematic of the 10-bar plane truss

Table 1 Comparison of optimized designs found for the 10-bar plane truss problem

Design variable	Areas (cm^2)				
	Wang et al. [7]	Lingyun et al. [9]	Gomes [10]	Miguel and Fadel Miguel [12]	Present work
1	32.456	42.234	37.712	36.198	35.1471
2	16.577	18.555	9.959	14.030	14.6668
3	32.456	38.851	40.265	34.754	35.6889
4	16.577	11.222	16.788	14.900	15.0929
5	2.115	4.783	11.576	0.654	0.6450
6	4.467	4.451	3.955	4.672	4.6221
7	22.810	21.049	25.308	23.467	23.5552
8	22.810	20.949	21.613	25.508	24.4680
9	17.490	10.257	11.576	12.707	12.7198
10	17.490	14.342	11.186	12.351	12.6845
Weight (kg)	553.8	542.75	537.98	531.28	530.77
Average optimized weight (kg)	N/A	552.447	540.89	535.07	535.64
Standard deviation on average weight (kg)	N/A	4.864	6.84	3.64	2.55

Table 2 Natural frequencies (Hz) evaluated at the optimum designs of the 10-bar plane truss problem

Frequency number	Natural frequencies (Hz)				
	Wang et al. [7]	Lingyun et al. [9]	Gomes [10]	Miguel and Fadel Miguel [12]	Present work
1	7.011	7.008	7.000	7.0002	7.0000
2	17.302	18.148	17.786	16.1640	16.1599
3	20.001	20.000	20.000	20.0029	20.0000
4	20.100	20.508	20.063	20.0221	20.0001
5	30.869	27.797	27.776	28.5428	28.6008
6	32.666	31.281	30.939	28.9220	29.0628
7	48.282	48.304	47.297	48.3538	48.4904
8	52.306	53.306	52.286	50.8004	51.0476

elements is from 0.645 to 50 cm^2. The first three natural frequencies of the structure must satisfy the following limitations: ($f_1 \geq 7$ Hz, $f_2 \geq 15$ Hz, and $f_3 \geq 20$ Hz.

Table 1 provides a comparison between some optimal design reported in the literature and the present work. It can be seen that the lightest design (i.e., 530.77 kg) and the best standard deviation on average (i.e., 2.55 kg) are obtained by the VPS. The firefly algorithm (FA) [12] achieved the best average optimized weight (i.e., 535.07 kg), and after that the VPS obtained 535.64 kg. Table 2 reports the natural frequencies of the optimized

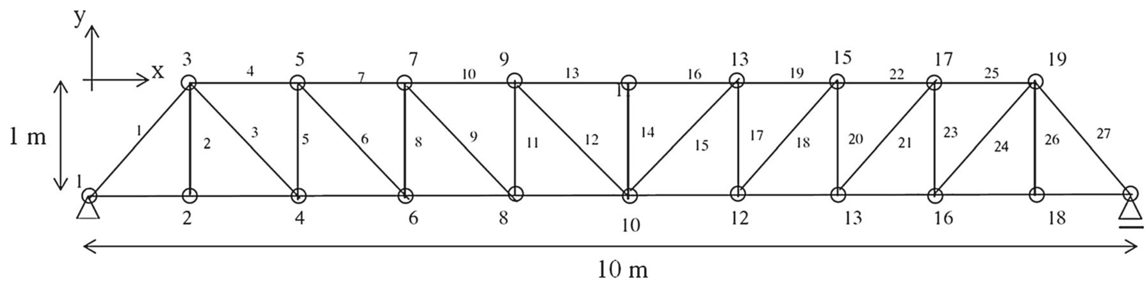

Fig. 6 Schematic of the simply supported 37-bar plane truss

structures, and it is clear that none of the frequency constraints are violated. The VPS converges to the optimum solution after 4620 analyses. The methods were utilized by Lingyun et al. [9], Gomes [10], and Miguel and Fadel Miguel [12] give the best result in 8000, 2000 and 50,000 analyses. However, the VPS achieves the best design of PSO [10] after 940 analyses.

4.2 A simply supported 37-bar plane truss

The 37-bar plane truss with initial configuration is shown in Fig. 6. Nodal coordinates in the upper chord and member areas are regarded as design variables. In the optimization process, nodes of the upper chord can be shifted vertically. In addition, nodal coordinates and member areas are linked to maintain the structural symmetry. Thus, only five layout variables and fourteen sizing variables will be redesigned for the optimization. All parts on the lower chord (numbers 28–37) are modeled as bar elements with constant rectangular cross-sectional areas of 4×10^{-3} m^2 and the other are modeled as bar elements with initial cross-sectional areas of 1×10^{-4} m^2. The material density is 7800 kg/m^3, and the modulus of elasticity is 210 GPa for all elements. A non-structural mass of 10kg is attached at each of the free nodes on the lower chord which remain fixed during the design process. The first three natural frequencies of the structure must satisfy the following limitations: $f_1 \geq 20$ Hz, $f_2 \geq 40$ Hz, and $f_3 \geq 60$ Hz.

This truss structure was previously optimized by Wang et al. [7] utilizing an evolutionary node shift method, Lingyun et al. [9] using a niche hybrid genetic algorithm, Gomes [10] employing particle swarm optimization algorithm, Miguel and Fadel Miguel [12] using firefly algorithm, and Kaveh and Ilchi Ghazaan [15] utilizing particle swarm optimization with an aging leader and challengers and harmony search-based side constraint handling approach. Table 3 presents a comparison between the results of the optimal designs reported in the literature and the present work. The best weight, average optimized weight and standard deviation on average weight obtained by VPS and HALC-PSO [15] are approximately identical although their designs are different. Table 4 shows the optimized structural frequencies (Hz) for various methods. None of the frequency constraints are violated. The proposed method requires 7940 structural analyses to find the optimum solution while NHGA [9], PSO [10], FA [12] and HALC-PSO [15] require 8000, 12,500, 50,000, and 10,000 structural analyses, respectively.

4.3 A 72-bar space truss

The 72-bar space truss is shown in Fig. 7 as the third design example. The elements are divided into sixteen groups, because of symmetry. The material density is 2767.99 kg/m^3, and the elastic modulus is 68.95 GPa for all members. Four non-structural masses of 2268 kg are attached to the nodes 1–4. The allowable minimum cross-sectional area of all elements is set to 0.645 cm^2. This example has two frequency constraints. The first frequency is required to be $f_1 = 4$ Hz, and the third frequency is required to be $f_3 \geq 6$ Hz.

Optimal structures found by Konzelman [5], Gomes [10], Kaveh and Zolghadr [11], Miguel and Fadel Miguel [12], Kaveh and Ilchi Ghazaan [15] and the proposed method are summarized in Table 5. The CSS-BBBC (hybridization of charged system search and big bang with trap recognition capability) [11] obtained the lightest design; however, the best designs of all methods are approximately identical. The average optimized weight and the standard deviation on average weight of the VPS are less than those of all other methods. Frequency constraints are satisfied by all methods (see Table 6). Figure 8 shows the comparison of best and

Table 3 Comparison of optimized designs found for the 37-bar truss problem

Design variable	*Y* coordinates (m) and areas (cm^2)					
	Wang et al. [7]	Lingyun et al. [9]	Gomes [10]	Miguel and Fadel Miguel [12]	Kaveh and Ilchi Ghazaan [15]	Present work
Y3, Y19 (m)	1.2086	1.1998	0.9637	0.9392	0.9750	0.9042
Y5, Y17 (m)	1.5788	1.6553	1.3978	1.3270	1.3577	1.2850
Y7, Y15 (m)	1.6719	1.9652	1.5929	1.5063	1.5520	1.5017
Y9, Y13 (m)	1.7703	2.0737	1.8812	1.6086	1.6920	1.6509
Y11 (m)	1.8502	2.3050	2.0856	1.6679	1.7688	1.7277
A1, A27 (cm^2)	3.2508	2.8932	2.6797	2.9838	2.9652	3.1306
A2, A26 (cm^2)	1.2364	1.1201	1.1568	1.1098	1.0114	1.0023
A3, A24 (cm^2)	1.0000	1.0000	2.3476	1.0091	1.0090	1.0001
A4, A25 (cm^2)	2.5386	1.8655	1.7182	2.5955	2.4601	2.5883
A5, A23 (cm^2)	1.3714	1.5962	1.2751	1.2610	1.2300	1.1119
A6, A21 (cm^2)	1.3681	1.2642	1.4819	1.1975	1.2064	1.2599
A7, A22 (cm^2)	2.4290	1.8254	4.6850	2.4264	2.4245	2.6743
A8, A20 (cm^2)	1.6522	2.0009	1.1246	1.3588	1.4618	1.3961
A9, A18 (cm^2)	1.8257	1.9526	2.1214	1.4771	1.4328	1.5036
A10, A19 (cm^2)	2.3022	1.9705	3.8600	2.5648	2.5000	2.4441
A11, A17 (cm^2)	1.3103	1.8294	2.9817	1.1295	1.2319	1.2977
A12, A15 (cm^2)	1.4067	1.2358	1.2021	1.3199	1.3669	1.3619
A13, A16 (cm^2)	2.1896	1.4049	1.2563	2.9217	2.2801	2.3500
A14 (cm^2)	1.0000	1.0000	3.3276	1.0004	1.0011	1.0000
Weight (kg)	366.5	368.84	377.20	360.05	359.93	359.94
Average optimized weight (kg)	N/A	378.8259	381.2	360.37	360.23	360.23
Standard deviation on average weight (kg)	N/A	9.0325	4.26	0.26	0.24	0.22

Table 4 Natural frequencies (Hz) evaluated at the optimum designs of the 37-bar truss problem

Frequency number	Natural frequencies (Hz)					
	Wang et al. [7]	Lingyun et al. [9]	Gomes [10]	Miguel and Fadel Miguel [12]	Kaveh and Ilchi Ghazaan [15]	Present work
1	20.0850	20.0013	20.0001	20.0024	20.0216	20.0002
2	42.0743	40.0305	40.0003	40.0019	40.0098	40.0005
3	62.9383	60.0000	60.0000	60.0043	60.0017	60.0000
4	74.4539	73.0444	73.0440	77.2153	76.7857	77.2124
5	90.0576	89.8244	89.8240	96.9900	96.3543	97.3173

average runs convergence histories for the proposed method. The VPS requires 4720 structural analyses to find the optimum solution, while PSO [10], FA [12] and HALC-PSO [15] require 42,840, 100,000, and 8000 structural analyses, respectively.

4.4 A 120-bar dome truss

Figure 9 shows the 120-bar dome truss. The members are categorized into seven groups because of symmetry. The material density is 7971.810 kg/m^3, and the modulus of elasticity is 210 GPa for all elements. Non-structural masses are attached to all free nodes as follows: 3000 kg at node one, 500 kg at nodes 2–13 and 100 kg at the remaining nodes. Element cross-sectional areas can vary between 1 and 129.3 cm^2. The frequency constraints are as follows: $f_1 \geq 9$ Hz and $f_2 \geq 11$ Hz.

The comparison of the results of the VPS algorithm with the outcomes of other algorithms is shown in Table 7. The present algorithm yields the least weight. The best weight of the VPS algorithm is 8888.74 kg, while it is 9046.34 kg for CSS-BBBC [11] and 8889.96 kg for the HALC-PSO [15]. Moreover, it can be seen that the lightest average optimized weight and the standard deviation on average weight are found by the

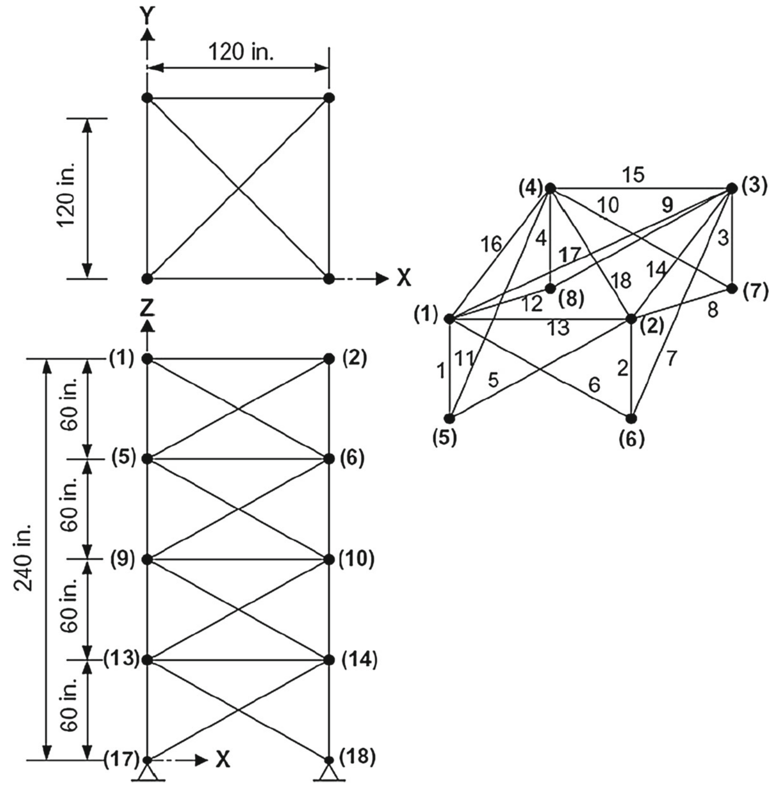

Fig. 7 Schematic of the spatial 72-bar truss

Table 5 Comparison of optimized designs obtained for the 72-bar truss problem

Design variable	Elements in the group	Areas (cm^2)					
		Konzelman [5]	Gomes [10]	Kaveh and Zolghadr [11]	Miguel and Fadel Miguel [12]	Kaveh and Ilchi Ghazaan [15]	Present work
1	1–4	3.499	2.987	2.854	3.3411	3.3437	3.5017
2	5–12	7.932	7.849	8.301	7.7587	7.8688	7.9340
3	13–16	0.645	0.645	0.645	0.6450	0.6450	0.6450
4	17–18	0.645	0.645	0.645	0.6450	0.6450	0.6450
5	19–22	8.056	8.765	8.202	9.0202	8.1626	8.0215
6	23–30	8.011	8.153	7.043	8.2567	7.9502	7.9826
7	31–34	0.645	0.645	0.645	0.6450	0.6452	0.6450
8	35–36	0.645	0.645	0.645	0.6450	0.6450	0.6450
9	37–40	12.812	13.450	16.328	12.0450	12.2668	12.8175
10	41–48	8.061	8.073	8.299	8.0401	8.1845	8.1129
11	49–52	0.645	0.645	0.645	0.6450	0.6451	0.6450
12	53–54	0.645	0.645	0.645	0.6450	0.6451	0.6450
13	55–58	17.279	16.684	15.048	17.3800	17.9632	17.3362
14	59–66	8.088	8.159	8.268	8.0561	8.1292	8.1010
15	67–70	0.645	0.645	0.645	0.6450	0.6450	0.6450
16	71–72	0.645	0.645	0.645	0.6450	0.6450	0.6450
Weight (kg)		327.605	328.823	327.507	327.691	327.77	327.649
Average optimized weight (kg)		N/A	332.24	N/A	329.89	327.99	327.670
Standard deviation on average weight (kg)		N/A	4.23	N/A	2.59	0.19	0.018

Table 6 Natural frequencies (Hz) evaluated at the optimum designs of the 72-bar truss problem

Frequency number	Natural frequencies (Hz)					
	Konzelman [5]	Gomes [10]	Kaveh and Zolghadr [11]	Miguel and Fadel Miguel [12]	Kaveh and Ilchi Ghazaan [15]	Present work
1	4.000	4.000	4.000	4.0000	4.000	4.0000
2	4.000	4.000	4.000	4.0000	4.000	4.0002
3	6.000	6.000	6.004	6.0000	6.000	6.0000
4	6.247	6.219	6.2491	6.2468	6.230	6.2428
5	9.074	8.976	8.9726	9.0380	9.041	9.0698

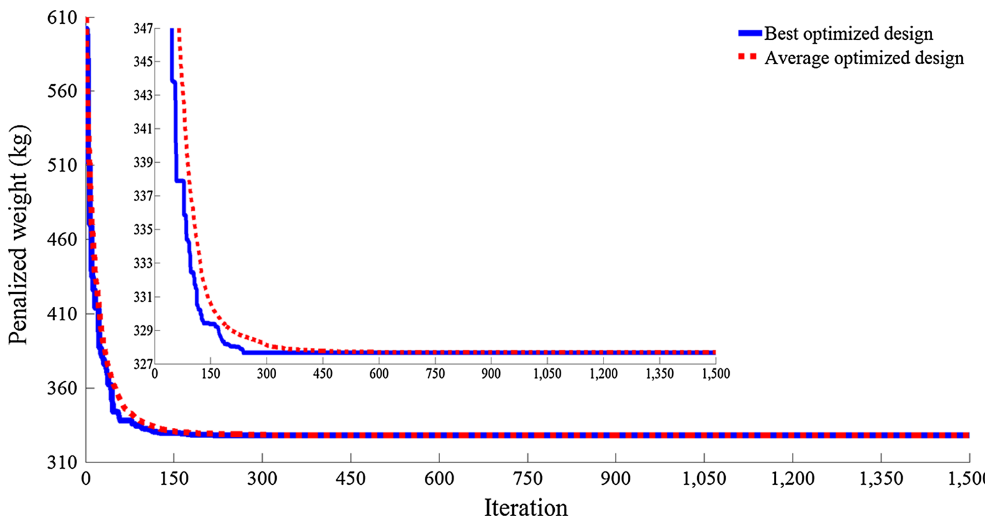

Fig. 8 Convergence curves obtained for the 72-bar truss

proposed method. Table 8 reports the natural frequencies of the optimized structures, and it is clear that none of the frequency constraints are violated. Figure 10 shows the comparison of convergence curves of the best and the average results obtained by the proposed method. The HALC-PSO [15] and VPS algorithms get the optimal solution after 17,000 and 6860 analyses, respectively.

4.5 A 600-bar single-layer dome truss

The 600-bar single-layer dome structure shown in Fig. 11 is considered as the last example. The entire structure is composed of 216 nodes and 600 elements. A substructure in more details for nodal numbering and coordinates is depicted in Fig. 12. Each of the elements of this substructure is considered as a design variable. Thus, this is a size optimization problem with 25 variables. The material density is 7850 kg/m^3, and the elastic modulus is 200 GPa for all members. A non-structural mass of 100 kg is attached to all free nodes. The minimum cross-sectional area of all parts is 1×10^{-4}, and the maximum cross-sectional area is taken as 100×10^{-4} m^2. The frequency constraints are as follows: $\omega_1 \geq 5$ Hz and $\omega_3 \geq 7$ Hz.

The optimized designs found by the ECBO [20] and VPS are compared in Table 9. It can be seen that the lightest design (i.e., 6133.02 kg) is obtained by the VPS, and this method performs better than ECBO in terms of average optimized weight and standard deviation on average weight. Table 10 reports the natural frequencies of the optimized structures, and it is clear that none of the frequency constraints are violated. The convergence rates of the best and average result found by the proposed method are provided in Fig. 13. The ECBO and VPS algorithms get the optimal solution after 19,020 and 19,740 analyses, respectively.

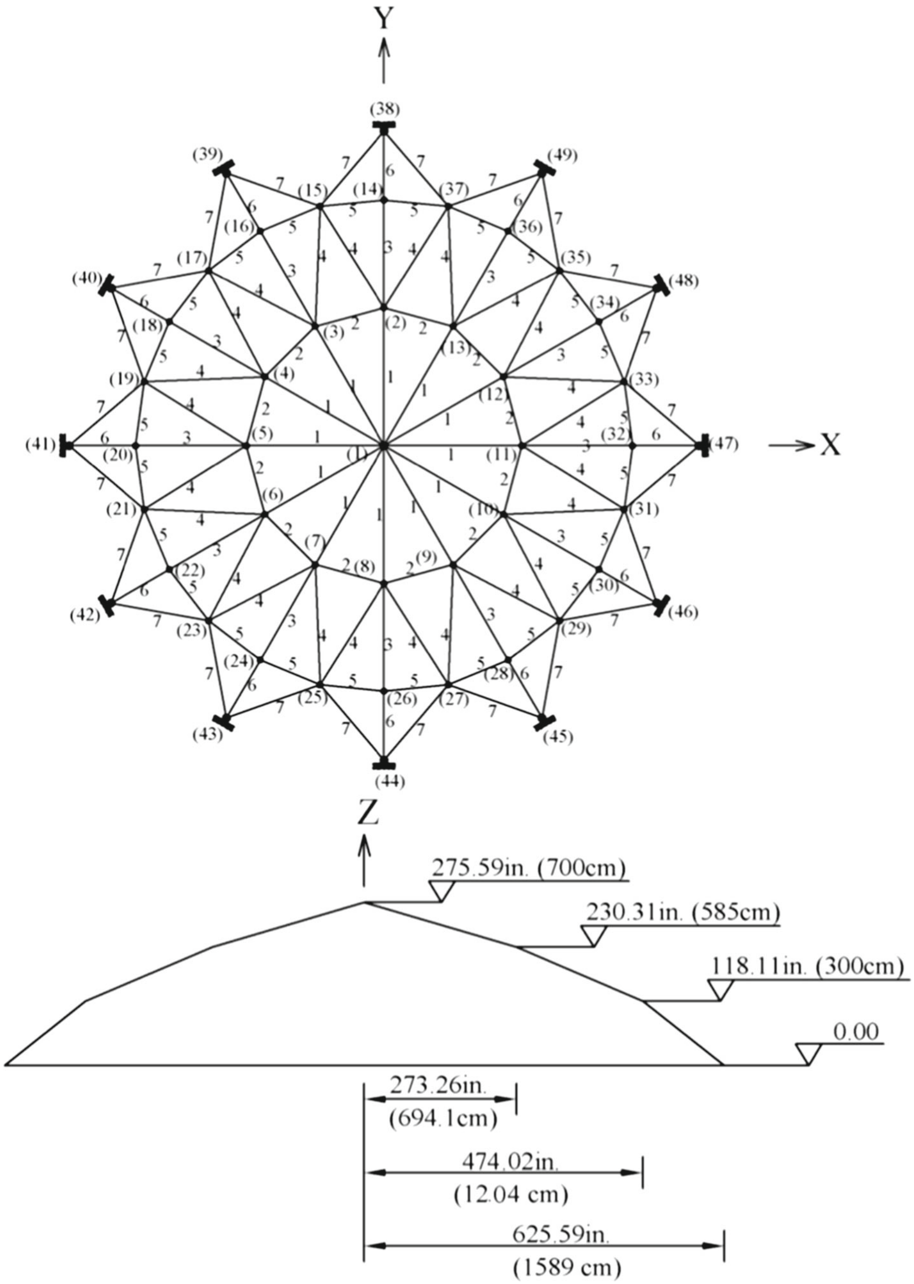

Fig. 9 Schematic of the spatial 120-bar dome truss

5 Concluding remarks

Structural optimization with multiple natural frequency constraints is a challenging class of optimization problems characterized by highly nonlinear and non-convex search spaces with numerous local optima. This paper presents a vibrating particles system for finding the optimum design of this kind of problems. The VPS has simple theoretical structure, and self-adaptation, cooperation and competition concepts are considered in its updating formula. The solution candidates gradually approach to HB, and any particle has the chance to have influence on the new position of the other one; therefore, the self-adaptation and cooperation between the particles are respectively provided. Moreover, since the influence of GP is more than that of BP in position updating, the competition is supplied. Five planar and spatial trusses are studied in this

Table 7 Comparison of optimized designs obtained for the 120-bar dome problem

Design variable	Areas (cm^2)		
	Kaveh and Zolghadr [11]	Kaveh and Ilchi Ghazaan [15]	Present work
1	17.478	19.8905	19.6836
2	49.076	40.4045	40.9581
3	12.365	11.2057	11.3325
4	21.979	21.3768	21.5387
5	11.190	9.8669	9.8867
6	12.590	12.7200	12.7116
7	13.585	15.2236	14.9330
Weight (kg)	9046.34	8889.96	8888.74
Average optimized weight (kg)	N/A	8900.39	8896.04
Standard deviation on average weight (kg)	N/A	6.38	6.65

Table 8 Natural frequencies (Hz) evaluated at the optimum designs of the 120-bar dome problem

Frequency number	Natural frequencies (Hz)		
	Kaveh and Zolghadr [11]	Kaveh and Ilchi Ghazaan [15]	Present work
1	9.000	9.000	9.0000
2	11.007	11.000	11.0000
3	11.018	11.000	11.0000
4	11.026	11.010	11.0096
5	11.048	11.050	11.0491

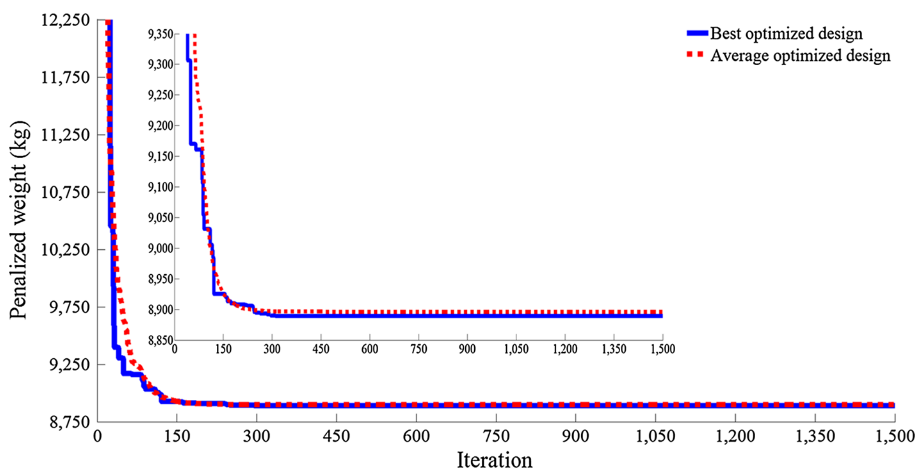

Fig. 10 Convergence curves obtained for the 120-bar dome truss

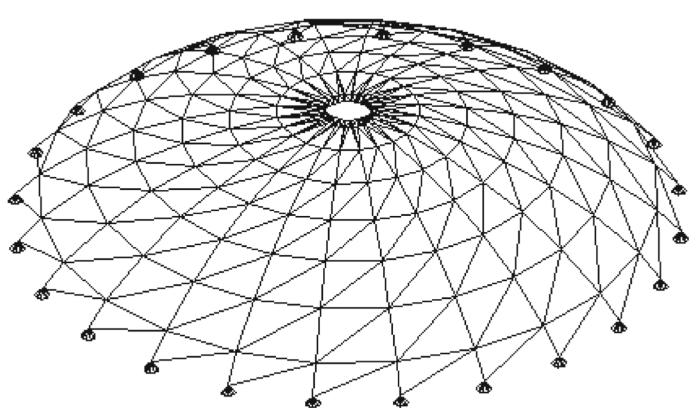

Fig. 11 Schematic of the 600-bar single-layer dome truss

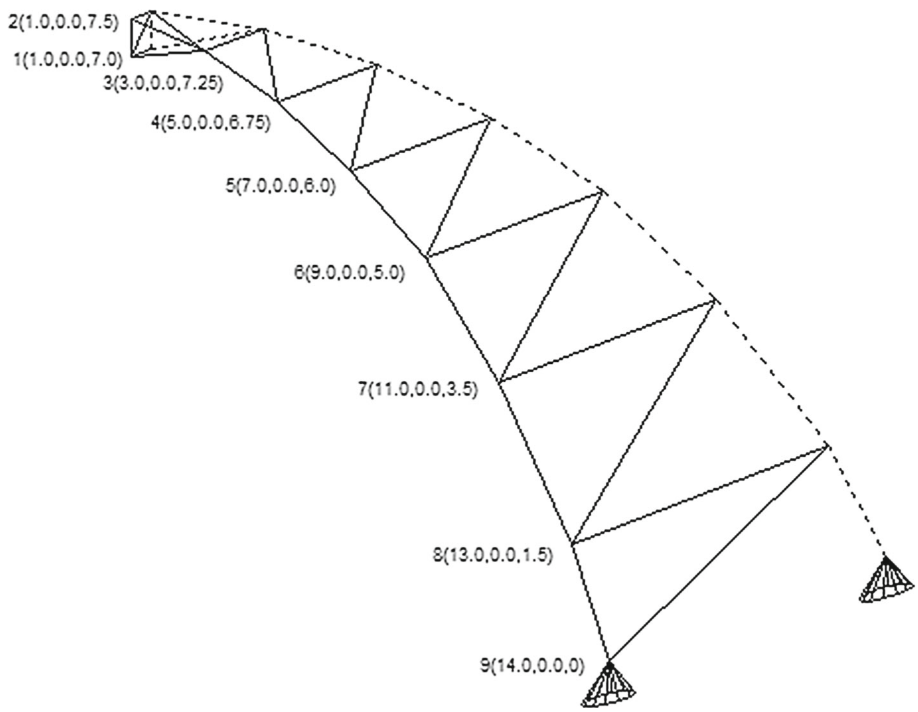

Fig. 12 Details of a substructure of the 600-bar single-layer dome truss

Table 9 Comparison of optimized designs obtained for the 600-bar single-layer dome truss problem

Design variable (nodes)	Areas (cm^2)	
	Kaveh and Ilchi Ghazaan [20]	Present work
1 (1–2)	1.4305	1.3030
2 (1–3)	1.3941	1.3998
3 (1–10)	5.5293	5.1072
4 (1–11)	1.0469	1.3882
5 (2–3)	16.9642	16.9217
6 (2–11)	35.1892	38.1432
7 (3–4)	12.2171	11.8319
8 (3–11)	16.7152	16.6149
9 (3–12)	12.5999	11.3403
10 (4–5)	9.5118	9.3865
11 (4–12)	8.9977	8.7692
12 (4–13)	9.4397	9.6682
13 (5–6)	6.8864	6.9826
14 (5–13)	4.2057	5.4445
15 (5–14)	7.2651	6.3247
16 (6–7)	6.1693	5.1349
17 (6–14)	3.9768	3.3991
18 (6–15)	8.3127	7.7911
19 (7–8)	4.1451	4.4147
20 (7–15)	2.4042	2.2755
21 (7–16)	4.3038	4.9974
22 (8–9)	3.2539	4.0145
23 (8–16)	1.8273	1.8388
24 (8–17)	4.8805	4.7965
25 (9–17)	1.5276	1.5551
Weight (kg)	6171.51	6133.02
Average optimized weight (kg)	6191.50	6142.03
Standard deviation on average weight (kg)	39.08	12.54

Table 10 Natural frequencies (Hz) evaluated at the optimum designs of the 600-bar single-layer dome truss problem

Frequency number	Natural frequencies (Hz)	
	Kaveh and Ilchi Ghazaan [20]	Present work
1	5.002	5.0000
2	5.003	5.0003
3	7.001	7.0000
4	7.001	7.0001
5	7.002	7.0002

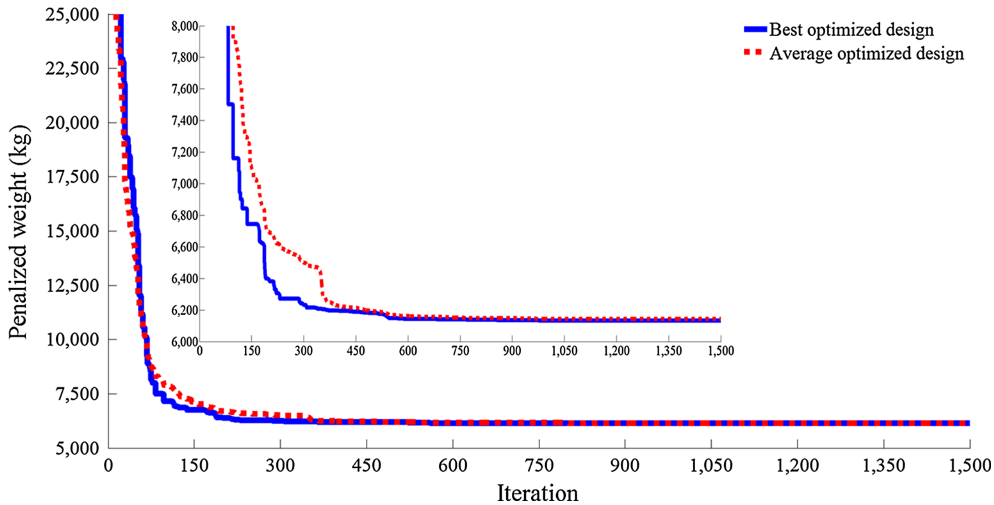

Fig. 13 Convergence curves obtained for the 600-bar single-layer dome truss

work to verify the proposed method. The numerical results of the investigated design examples bring out the advantages of the proposed method in terms of speed of convergence, stability, and optimality of the final solutions.

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