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ORIGINAL PAPER

Soniya Chaudhary · Sanjeev A. Sahu · Abhinav Singhal

Analytic model for Rayleigh wave propagation in piezoelectric layer overlaid orthotropic substratum

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Abstract An analytical approach is adopted to investigate Rayleigh waves in a layered composite structure with corrugated boundaries. The structure of the model has been taken in such a way that the pre-stressed piezoelectric layer with rotation is lying over a pre-stressed, rotating, gravitational orthotropic substrate. The frequency equations of the considered wave have been obtained in the form of a determinant for both electrically open and short cases. Notable effects of various parameters (piezoelectric constant, initial stress, rotation, undulation parameter and position parameter) on Rayleigh wave velocity have been observed. Numerical computation and graphical demonstration have been carried out. The obtained results are matched with existing results, under certain conditions. Also, the analytical solution of the problem is matched and found in good agreement with the solution obtained by the finite element method. The outcomes are widely useful for the development and characterization of rotation sensors and SAW devices.

1 Introduction

Around the beginning of twentieth century, the study of wave propagation in composites containing piezoelectric materials has attracted much attention due to the possible engineering applications. Dynamic or transient behaviour of piezoelectric materials are the primary concern in design as well as in performance. Piezoelectric materials have the ability to produce electrical energy from mechanical energy. For instance, they can convert mechanical vibrations into electricity. Such devices are commonly referred to as energy harvesters and can be used in certain applications like wireless transmission, technology for colour television sets, cell phones and Global Positioning Systems. Bluestein [1], Cheng [2], Fang [3] and Wang [4] have discussed numerous problems of surface wave propagation in piezoelectric media. To prevent the piezoelectric material from showing brittle fracture, the layered piezoelectric structures are usually pre-stressed during the manufacturing process. As initial stresses are inseparable in surface acoustic wave devices and resonators, analysis of such effects has been done using different approaches. Wave propagation in pre-stressed piezoelectric structures has been studied by several authors; some of them are Chai and Wu [5], Simionescu [6], Liu [7], Singh [8] and Son [9]. Yang [10] has explained various problems of surface wave propagation in piezoelectric media. Du et al. [11] have studied the Love wave propagation in a functionally graded piezoelectric material layer. Cao

S. Chaudhary · S. A. Sahu (✉) · A. Singhal
Department of Applied Mathematics, Indian School of Mines, Dhanbad 826004, India
E-mail: ism.sanjeev@gmail.com

S. Chaudhary
E-mail: soniya.ism14@gmail.com

A. Singhal
E-mail: ism.abhinav@gmail.com

et al. [12] have discussed Rayleigh waves in a piezoelectric wafer with subsurface damage. The propagation of elastic surface waves in a layered structure consisting of a semi-infinite solid substrate in contact with a finite layer of different material properties has been of interest due to its many applications in geophysics, material science, non-destructive evaluation and acoustic devices. Propagation of Rayleigh-type surface waves in a transversely isotropic piezoelectric layer over a piezomagnetic half-space has been discussed by Pang et al. [13]. Excitation and propagation of shear horizontal waves in a piezoelectric layer imperfectly bonded to a metal or elastic substrate have been materialized by Li and Jin [14]. Propagation of Rayleigh waves in an elastic half-space of orthotropic material has been studied by Abd-Alla [15]. He has investigated the generation of waves in an orthotropic elastic solid medium under the influence of initial stress and gravity field. Effects of rotation, magnetic field, initial stress and gravity on Rayleigh waves in a homogeneous orthotropic elastic half-space have been shown by Abd-Alla et al. [16]. Clarke and Burdness [17] have studied exclusively the effect of rotation on Rayleigh waves. They found that the rotation in an isotropic medium does not increase the number of waves but it affects their speeds significantly. Sensors are usually a layered structure involving configuration of a thin layer over a substrate. Among different layered structures, a piezoelectric structure is considered to be more competent as it enables the electric excitation of Rayleigh waves. In many realistic models, it is believed that the boundaries of the layer of the material medium are not perfectly plane but rather undulated in nature. The presence of the undulated nature of the irregular boundaries in the material properties generally produces a significant influence on the elastic waves propagating through the medium. Moreover, the shape of the irregularity may be arbitrary. In order to get a practical insight into the problem involving these irregular boundaries, an attempt has been made by considering corrugated boundary surfaces in the present problem. These non-planar boundaries in the material medium must be taken into account for a more accurate modelling procedures and interpretation of the results. Hurd [18] has investigated the propagation of electromagnetic waves along an infinite corrugated surface. Glass and Maradudin [19] have studied the leaky surface-elastic waves on both, flat and strongly corrugated surfaces for isotropic, non-dissipative media. Recently, Singh et al. [20] discussed the Love-type waves in a corrugated piezoelectric structure.

Usually, surface acoustic wave (SAW) devices and sensors adopt a layer/substrate model to achieve high performance. However, due to the mismatch of material properties, there exists a residual stress during the manufacturing process of piezoelectric surface acoustic wave devices. The study of Rayleigh wave propagation in this type of material provides outstanding results for characteristics of such type of wave. Orthotropic composites are sometimes used as the substrate of layered piezoelectric devices to enhance their mechanical performance. These facts altogether set the motivation for the present study. The objective of this paper is to investigate the effect of piezoelectric constant, initial stress, rotation, undulation parameter and position parameter on the Rayleigh wave velocity. The present study provides a theoretical framework for designing and development of devices involving piezoelectric composites. It is observed that different parameters of the medium have a significant effect on the Rayleigh wave velocity.

2 Formulation of the problem

We consider a gravitational orthotropic substrate with initial stress and rotation covered with pre-stressed piezoelectric layer with rotation of thickness H as shown in Fig. 1. A corrugated interface is taken on the boundary of the two media. We choose a Cartesian coordinate system in such a way that the x-axis is in the direction of wave propagation and the z axis pointing vertically downwards. The corrugation parameters $\zeta_1(x)$ and $\zeta_2(x)$ are continuous functions of x, independent of y.

The functions $\zeta_j(x)$ can be taken as periodic in the nature, and their Fourier series expansions are given by Singh [21] as

$$\zeta_m(x) = \sum_{n=1}^{\infty} \left(\zeta_n^m e^{inpx} + \zeta_{-n}^m e^{-inpx} \right), \quad m = 1, 2.$$

Here $\zeta_n(x)$ and $\zeta_{-n}(x)$ are Fourier series expansion coefficients with wave number (p), wave length $\left(\frac{2\pi}{p}\right)$ and n is the order of series. Amplitude of irregular boundaries surface assumed to be very small compared to the wavelength.

We establish the constants a_1, a_2, U_n^1, V_n^1 such that

$$\zeta_{\pm1}^1 = \frac{a_1}{2}, \quad \zeta_{\pm2}^1 = \frac{a_2}{2}, \quad \zeta_{\pm1}^1 = \frac{U_n^m \mp i V_n^m}{2}, \quad m = 1, 2 \text{ and } n = 2, 3, \ldots$$

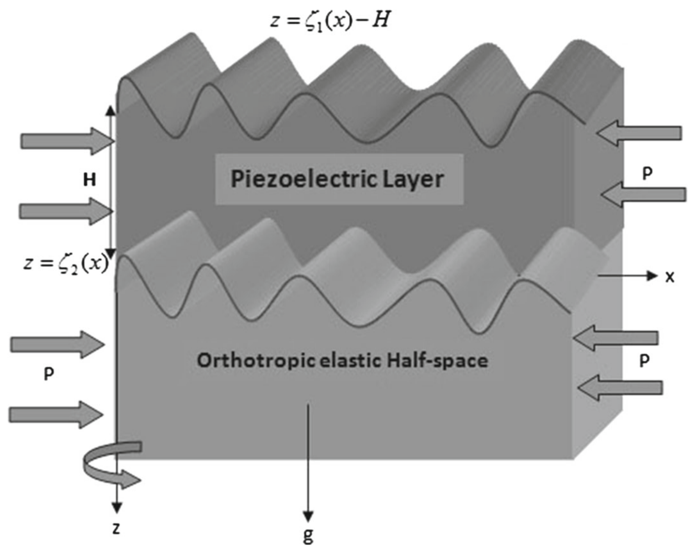

Fig. 1 Configuration of piezoelectric-layered structure and its coordinate

Using the above relations, the series may be converted to

$$\zeta_1 = a_1 \cos bx + \sum_{n=2}^{\infty} \left[U_n^1 \cos nbx + V_n^1 \sin nbx\right],$$
$$\zeta_2 = a_2 \cos bx + \sum_{n=2}^{\infty} \left[U_n^2 \cos nbx + V_n^2 \sin nbx\right],$$

where U_n^m, V_n^m are cosine and sine Fourier coefficients, respectively.

Now, we consider the following:

$$\zeta_{\pm n}^m = \begin{cases} 0 & \text{if } n \neq 1, \\ \frac{a_1}{2} & \text{if } n = 1, m = 1, \\ \frac{a_2}{2} & \text{if } n = 1, m = 2. \end{cases}$$

Also, we have considered the upper corrugated boundary surface as $\zeta_1 = a_1 \cos bx$ and the corrugated interface between layer and half-space as $\zeta_2 = a_2 \cos bx$, where a_1 and a_2 are corresponding amplitudes of the corrugated boundary surfaces.

2.1 Governing equation for piezoelectric layer

The equation of motion and the charge equation for a piezoelectric layer with initial stress are

$$\left(\sigma_{ij} + u_{j,k}\sigma_{ik}^1\right)_{,i} + \rho_1 f_i = \rho_1 \left[\ddot{\vec{u}} + \left(\vec{\Omega} \times \vec{\Omega} \times \vec{u}\right) + \left(2\vec{\Omega} \times \dot{\vec{u}}\right)\right]_{,j}, \tag{1}$$

$$D_{i,i} = 0, \tag{2}$$

where D_i and σ are increments of electric displacement and electric charge density due to a dynamic disturbance superposed on the initial state, u_i are components of the displacement vector, f_i is the body force per unit mass, σ_{ik}^1 is the stress tensor referring to initial stress, ρ_1 is the mass density of the layer and $\left(\vec{\Omega} \times \vec{\Omega} \times \vec{u}\right)$ and $\left(2\vec{\Omega} \times \dot{\vec{u}}\right)$ are the centripetal and Coriolis acceleration, respectively.

The constitutive equations of the piezoelectric medium can be expressed as

$$\begin{aligned}\sigma_{ij} &= c_{ijkl}\varepsilon_{kl} - e_{mij}E_m,\\ D_m &= e_{imj}\varepsilon_{kl} + p_{im}E_m,\\ \varepsilon_{kl} &= \frac{1}{2}\left(u_{i,j} + u_{j,i}\right), \quad E_m = -\phi_{,m},\end{aligned} \tag{3}$$

where c_{ijkl}, e_{mij} and ε_{kl} are the elastic, piezoelectric and dielectric constants, respectively, and ϕ is the scalar electric potential.

Hence, the equation of motion for a linear transversely isotropic piezoelectric medium without body force can be written as

$$\begin{aligned}&\left(c_{11} + \sigma_{11}^1\right)\frac{\partial^2 u_1}{\partial x^2} + (c_{31} + c_{44})\frac{\partial^2 w_1}{\partial x \partial z} + \left(c_{44} + \sigma_{33}^1\right)\frac{\partial^2 u_1}{\partial z^2} + (e_{31} + e_{15})\frac{\partial^2 \phi_1}{\partial x \partial z}\\ &= \rho_1\left(\frac{\partial^2 u_1}{\partial t^2} - \Omega_1^2 u_1 + 2\Omega_1\frac{\partial w_1}{\partial t}\right),\end{aligned} \tag{4}$$

$$\begin{aligned}&\left(c_{44} + \sigma_{11}^1\right)\frac{\partial^2 w_1}{\partial x^2} + (c_{31} + c_{44})\frac{\partial^2 u_1}{\partial x \partial z} + \left(c_{33} + \sigma_{33}^1\right)\frac{\partial^2 w_1}{\partial z^2} + e_{15}\frac{\partial^2 \phi_1}{\partial x^2} + e_{33}\frac{\partial^2 \phi_1}{\partial z^2}\\ &= \rho_1\left(\frac{\partial^2 w_1}{\partial t^2} - \Omega_1^2 w_1 + 2\Omega_1\frac{\partial u_1}{\partial t}\right),\end{aligned} \tag{5}$$

$$e_{15}\frac{\partial^2 w_1}{\partial x^2} + (e_{15} + e_{31})\frac{\partial^2 u_1}{\partial x \partial z} + e_{33}\frac{\partial^2 w_1}{\partial z^2} - \varepsilon_{11}\frac{\partial^2 \phi_1}{\partial x^2} - \varepsilon_{33}\frac{\partial^2 \phi_1}{\partial z^2} = 0. \tag{6}$$

2.2 Governing equation for half-space

The components of body forces are $X = 0$, $Z = -g$ (where g is acceleration due to gravity). Considering that the initial compression stress field due to gravity field is hydrostatic (Datta [22]), the state of initial stress τ_{ij} becomes

$$\tau_{11} = \tau_{33} = \tau, \quad \tau_{13} = 0. \tag{7}$$

The equilibrium conditions of the initial stress field are

$$\frac{\partial \tau}{\partial x} = 0, \quad \frac{\partial \tau}{\partial z} - \rho g = 0. \tag{8}$$

Substituting Eqs. (7) and (8) into the three-dimensional form of the dynamical equations of an elastic medium under initial compression stress P in the x-direction, considering Lorentz's body forces F, we establish

$$\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{12}}{\partial y} + \frac{\partial \tau_{13}}{\partial z} + P\left(\frac{\partial \omega_{21}}{\partial y} - \frac{\partial \omega_{13}}{\partial z}\right) - \rho_2 g\frac{\partial \omega}{\partial x} = \rho_2\left(\frac{\partial^2 u_2}{\partial t^2} - \Omega_2^2 u_2 + 2\Omega_2\frac{\partial w_2}{\partial t}\right), \tag{9}$$

$$\frac{\partial \tau_{12}}{\partial x} + \frac{\partial \tau_{22}}{\partial y} + \frac{\partial \tau_{23}}{\partial z} + P\frac{\partial \omega_{12}}{\partial x} = \rho_2\frac{\partial^2 v_2}{\partial t^2}, \tag{10}$$

$$\frac{\partial \tau_{13}}{\partial x} + \frac{\partial \tau_{23}}{\partial y} + \frac{\partial \tau_{33}}{\partial z} - P\frac{\partial \omega_{13}}{\partial x} + \rho_2 g\frac{\partial u_2}{\partial x} = \rho_2\left(\frac{\partial^2 w_2}{\partial t^2} - \Omega_2^2 w_2 - 2\Omega_2\frac{\partial u_2}{\partial t}\right), \tag{11}$$

where

$$\omega_{ij} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} - \frac{\partial u_j}{\partial x_i}\right).$$

Due to the characteristic of Rayleigh waves, the equation of motion reduces to

$$\frac{\partial \tau_{11}}{\partial x} + \frac{\partial \tau_{13}}{\partial z} - P\frac{\partial \omega_{13}}{\partial z} - \rho_2 g\frac{\partial \omega}{\partial x} = \rho_2\left(\frac{\partial^2 u_2}{\partial t^2} - \Omega_2^2 u_2 + 2\Omega_2\frac{\partial w_2}{\partial t}\right), \tag{12}$$

$$\frac{\partial \tau_{13}}{\partial x} + \frac{\partial \tau_{33}}{\partial z} - P\frac{\partial \omega_{13}}{\partial x} + \rho_2 g\frac{\partial u}{\partial x} = \rho_2\left(\frac{\partial^2 w_2}{\partial t^2} - \Omega_2^2 w_2 - 2\Omega_2\frac{\partial u_2}{\partial t}\right), \tag{13}$$

where

$$\tau_{11} = (c_{11} + P)\,\frac{\partial u_2}{\partial x} + (c_{13} + P)\,\frac{\partial w_2}{\partial z}, \tag{14}$$

$$\tau_{13} = c_{44}\left(\frac{\partial u_2}{\partial z} + \frac{\partial w_2}{\partial x}\right), \tag{15}$$

$$\tau_{33} = c_{13}\frac{\partial u_2}{\partial x} + c_{33}\frac{\partial w_2}{\partial z}. \tag{16}$$

Introducing Eqs. (14), (15) and (16) into Eqs. (12) and (13) with assumption that $c_{44} = \frac{1}{2}\,(c_{11} - c_{13})$, we obtain

$$\begin{aligned}
&(c_{11} + P)\left(2\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial z^2} + \frac{\partial^2 w_2}{\partial x \partial z}\right) + c_{13}\left(\frac{\partial^2 w_2}{\partial x \partial z} - \frac{\partial^2 u_2}{\partial z^2}\right) - 2\rho_2 g\frac{\partial w_2}{\partial x}\\
&\quad = 2\rho_2\left(\frac{\partial^2 u_2}{\partial t^2} - \Omega_2^2 u_2 + 2\Omega_2\frac{\partial w_2}{\partial t}\right),
\end{aligned} \tag{17}$$

$$\begin{aligned}
&c_{11}\left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial x \partial z}\right) + (c_{13} + P)\left(\frac{\partial^2 u_2}{\partial x \partial z} - \frac{\partial^2 w_2}{\partial x^2}\right) + 2\rho_2 g\frac{\partial u_2}{\partial x} + 2c_{33}\frac{\partial^2 w_2}{\partial z^2}\\
&\quad = 2\rho_2\left(\frac{\partial^2 w_2}{\partial t^2} - \Omega_2^2 w_2 - 2\Omega_2\frac{\partial u_2}{\partial t}\right).
\end{aligned} \tag{18}$$

Assuming that displacement components are derivable from the displacement potentials $\phi\,(x, z, t)$ and $\psi\,(x, z, t)$ by the relations

$$u_2 = \frac{\partial \phi}{\partial x} - \frac{\partial \psi}{\partial z} \quad \text{and} \quad w_2 = \frac{\partial \phi}{\partial z} + \frac{\partial \psi}{\partial x}. \tag{19}$$

From Eqs. (17), (18) and (19), we have

$$(c_{11} + P)\,\nabla^2\phi - \rho_2 g\frac{\partial \psi}{\partial x} = \rho_2\left(\frac{\partial^2 \phi}{\partial t^2} - \Omega_2^2\phi + 2\Omega_2\frac{\partial \psi}{\partial t}\right), \tag{20}$$

$$(c_{11} + P - c_{13})\,\nabla^2\psi + 2\rho_2 g\frac{\partial \phi}{\partial x} = 2\rho_2\left(\frac{\partial^2 \psi}{\partial t^2} - \Omega_2^2\phi + 2\Omega_2\frac{\partial \psi}{\partial t}\right), \tag{21}$$

and

$$c_{11}\left(\frac{\partial^2 \psi}{\partial x^2} - \frac{\partial^2 \psi}{\partial z^2}\right) - (c_{13} + P)\,\nabla^2\psi + 2\rho g\frac{\partial \phi}{\partial x} + 2c_{33}\frac{\partial^2 \psi}{\partial z^2} = 2\rho_2\left(\frac{\partial^2 \psi}{\partial t^2} - \Omega_2^2\phi + 2\Omega_2\frac{\partial \psi}{\partial t}\right), \tag{22}$$

$$c_{11}\frac{\partial^2 \phi}{\partial x^2} + c_{33}\frac{\partial^2 \phi}{\partial z^2} - \rho_2 g\frac{\partial \psi}{\partial x} = \rho_2\left(\frac{\partial^2 \phi}{\partial t^2} - \Omega_2^2\phi + 2\Omega_2\frac{\partial \psi}{\partial t}\right), \tag{23}$$

where

$$\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial z^2}.$$

Since the initial compressive wave has been taken in the direction of x only, the velocities of the body waves are different in the (x, z) directions. Equations (20) and (23) represent the compressive wave along the x and z directions, whereas Eqs. (21) and (22) represent the shear wave along the z and x directions, respectively. Hence we solve only Eqs. (20) and (22).

3 Analytical solutions

3.1 Solution for piezoelectric Layer

We seek the solution of Eqs. (4), (5) and (6) as

$$u_1 = U_1 e^{ik(x-ct)}, \quad w_1 = W_1 e^{ik(x-ct)} \quad \text{and} \quad \phi_1 = \overline{\phi_1} e^{ik(x-ct)}. \tag{24}$$

Using Eq. (24) in Eqs. (4), (5) and (6), we obtain

$$\begin{aligned}
&\left(c_{44}+\sigma_{33}^1\right) U_1'' + \left(k^2\rho_1 c^2 - k^2\left(c_{11}+\sigma_{11}^1\right) + \rho_1 \Omega_1^2\right) U_1 + ik\left(c_{13}+c_{44}\right) W_1' \\
&\quad + 2\Omega_1 \rho_1 ikc W_1 + ik\left(e_{31}+e_{15}\right)\overline{\phi}_1' = 0, && (25)\\
&\left(c_{33}+\sigma_{33}^1\right) W_1'' + \left(k^2\rho_1 c^2 - k^2\left(c_{44}+\sigma_{11}^1\right) + \Omega_1^2\rho_1\right) W_1 + ik\left(c_{13}+c_{44}\right) U_1' \\
&\quad + 2\rho_1 kci\Omega_1 U_1 + e_{33}\overline{\phi}_1'' - e_{15}k^2\overline{\phi}_1 = 0, && (26)\\
&e_{33} W_1'' - e_{15}k^2 W_1 + ik\left(e_{15}+e_{31}\right) U_1' - \varepsilon_{33}\overline{\phi}_1'' + \varepsilon_{11}k^2\overline{\phi_1} = 0. && (27)
\end{aligned}$$

On solving Eqs. (25), (26) and (27), we get

$$\left(D^6\chi_1 + \chi_2 D^4 + \chi_3 D^2 + \chi_4\right)\left(U_1, W_1, \overline{\phi_1}\right) = 0, \tag{28}$$

where

$$\begin{aligned}
\chi_1 &= \frac{-\left(\varepsilon_{33}\left(c_{33}+\sigma_{33}^1\right) + e_{33}^2\right)}{e_{33}\left(c_{33}+\sigma_{33}^1\right)},\\
\chi_2 &= \frac{2e_{15}}{\left(c_{33}+\sigma_{33}^1\right)} + \frac{e_{33}}{\left(c_{33}+\sigma_{33}^1\right)} + \frac{\varepsilon_{33}}{e_{33}}\left(\upsilon_1+\upsilon_2\right) + \frac{\varepsilon_{11}}{e_{33}} - \frac{\varepsilon_{33}\left(c_{13}+c_{44}\right)^2}{e_{33}\left(c_{33}+\sigma_{33}^1\right)\left(c_{44}+\sigma_{33}^1\right)}\\
&\quad - \frac{\left(e_{15}+e_{31}\right)\left(c_{13}+c_{44}\right)}{e_{33}\left(c_{44}+\sigma_{33}^1\right)} + \frac{\left(e_{15}+e_{31}\right)^2}{e_{33}\left(c_{44}+\sigma_{33}^1\right)},\\
\chi_3 &= \frac{\varepsilon_{11}}{e_{33}}\left(\upsilon_1+\upsilon_2\right) - \frac{\left(e_{15}\right)^2}{e_{33}\left(c_{13}+c_{44}\right)} - \frac{2e_{15}a_1}{\left(c_{33}+\sigma_{33}^1\right)} - \frac{e_{33}a_1a_2}{\left(c_{33}+\sigma_{33}^1\right)} + \frac{\varepsilon_{11}\left(c_{13}+c_{44}\right)^2}{e_{33}\left(c_{33}+\sigma_{33}^1\right)\left(c_{44}+\sigma_{33}^1\right)}\\
&\quad + \frac{2\left(e_{15}+e_{31}\right)\left(c_{13}+c_{44}\right)e_{15}}{e_{33}\left(c_{44}+\sigma_{33}^1\right)\left(c_{33}+\sigma_{33}^1\right)} - \frac{\left(e_{15}+e_{31}\right)^2\upsilon_2}{e_{33}\left(c_{44}+\sigma_{33}^1\right)} + \frac{4\rho_1^2c^2\varepsilon_{33}\Omega_1^2}{e_{33}\left(c_{44}+\sigma_{33}^1\right)\left(c_{33}+\sigma_{33}^1\right)\omega^2},\\
\chi_4 &= \frac{\varepsilon_{11}\upsilon_1\upsilon_2}{e_{33}} + \frac{\left(e_{15}\right)^2\upsilon_1}{e_{33}\left(c_{33}+\sigma_{33}^1\right)} - \frac{4\rho_1^2c^4\varepsilon_{11}\Omega_1^2}{e_{33}\left(c_{44}+\sigma_{33}^1\right)\left(c_{33}+\sigma_{33}^1\right)\omega^2},\\
\upsilon_1 &= \frac{\left(c_{11}+\sigma_{11}^1\right)}{\left(c_{44}+\sigma_{33}^1\right)} - \frac{\rho_1\Omega_1^2}{\left(c_{44}+\sigma_{33}^1\right)}\left(1-\frac{\Omega_1^2}{\omega^2}\right), \upsilon_2 = \frac{\left(c_{44}+\sigma_{33}^1\right)}{\left(c_{33}+\sigma_{33}^1\right)} - \frac{\rho_1\Omega_1^2}{\left(c_{33}+\sigma_{33}^1\right)}\left(1-\frac{\Omega_1^2}{\omega^2}\right).
\end{aligned}$$

Hence, the mechanical displacement and electric potential for the piezoelectric layer are

$$u_1 = \left(Ae^{-ik\lambda_1 z} + Be^{-ik\lambda_2 z} + Ce^{-ik\lambda_3 z} + A'e^{ik\lambda_1 z} + B'e^{ik\lambda_2 z} + C'e^{ik\lambda_3 z}\right)e^{ik(x-ct)}, \tag{29}$$

$$w_1 = \left(\overline{A}e^{-ik\lambda_1 z} + \overline{B}e^{-ik\lambda_2 z} + \overline{C}e^{-ik\lambda_3 z} + \overline{A}'e^{ik\lambda_1 z} + \overline{B}'e^{ik\lambda_2 z} + \overline{C}'e^{ik\lambda_3 z}\right)e^{ik(x-ct)}, \tag{30}$$

$$\overline{\phi}_1 = \left(\overline{\overline{A}}e^{-ik\lambda_1 z} + \overline{\overline{B}}e^{-ik\lambda_2 z} + \overline{\overline{C}}e^{-ik\lambda_3 z} + \overline{\overline{A}}'e^{ik\lambda_1 z} + \overline{\overline{B}}'e^{ik\lambda_2 z} + \overline{\overline{C}}'e^{ik\lambda_3 z}\right)e^{ik(x-ct)}, \tag{31}$$

where $A, B, C, A', B', C', \overline{A}, \overline{B}, \overline{B}, \overline{A}', \overline{B}', \overline{C}', \overline{\overline{A, B, C, A', B', C'}},$ are constants and are defined as $\overline{A} = P'A, \overline{B} = Q'B, \overline{C} = R'C, \overline{A}' = SA', \overline{B}' = TB', \overline{C}' = VC', \overline{\overline{A}} = P''A, \overline{\overline{B}} = Q''B, \overline{\overline{C}} = R''C, \overline{\overline{A}}' =$

$S'A', \overline{\overline{B}}' = T'B', \overline{\overline{C}}' = V'C'$. And

$$P' = \frac{-P''}{(s_{11}+S)}, \quad Q' = \frac{-Q''}{(m_{11}+T)}, \quad R' = \frac{-R''}{(n_{11}+V)},$$

$$P'' = \frac{\left(a_{12}-a_{11}k^2\lambda_1^2\right)\left(a_{22}-a_{21}k^2\lambda_1^2\right)+\left(a_{24}-ik\lambda_1 a_{23}\right)\left(a_{14}+k^2\lambda_1^2 a_{13}\right)}{i\lambda_1 k a_{15}\left(a_{22}-a_{21}k^2\lambda_1^2\right)+\left(e_{33}k^2+k^2\lambda_1^2 e_{33}\right)\left(a_{14}+k^2\lambda_1^2 a_{13}\right)},$$

$$Q'' = \frac{\left(a_{12}-a_{11}k^2\lambda_2^2\right)\left(a_{22}-a_{21}k^2\lambda_2^2\right)+\left(a_{24}-ik\lambda_2 a_{23}\right)\left(a_{14}+k^2\lambda_2^2 a_{13}\right)}{i\lambda_2 k a_{15}\left(a_{22}-a_{21}k^2\lambda_2^2\right)+\left(e_{33}k^2+k^2\lambda_2^2 e_{33}\right)\left(a_{14}+k^2\lambda_2^2 a_{13}\right)},$$

$$R'' = \frac{\left(a_{12}-a_{11}k^2\lambda_3^2\right)\left(a_{22}-a_{21}k^2\lambda_3^2\right)+\left(a_{24}-ik\lambda_3 a_{23}\right)\left(a_{14}+k^2\lambda_3^2 a_{13}\right)}{i\lambda_3 k a_{15}\left(a_{22}-a_{21}k^2\lambda_3^2\right)+\left(e_{33}k^2+k^2\lambda_3^2 e_{33}\right)\left(a_{14}+k^2\lambda_3^2 a_{13}\right)},$$

$$S = \frac{-k^2\lambda_1\left(e_{15}+e_{31}\right)\left(e_{15}k^2+k^2\lambda_1^2 e_{33}\right)+\left(\varepsilon'_{11}k^2+k^2\lambda_1^2\varepsilon_{11}\right)\left(a_{24}+ik\lambda_1 a_{23}\right)}{\left(a_{22}-k^2\lambda_1^2 a_{22}\right)\left(\varepsilon'_{11}k^2+k^2\lambda_1^2\varepsilon_{11}\right)-\left(e_{15}k^2+k^2\lambda_1^2 e_{33}\right)\left(e_{15}k^2+k^2\lambda_1^2 e_{33}\right)},$$

$$T = \frac{-k^2\lambda_2\left(e_{15}+e_{31}\right)\left(e_{15}k^2+k^2\lambda_2^2 e_{33}\right)+\left(\varepsilon'_{11}k^2+k^2\lambda_2^2\varepsilon_{11}\right)\left(a_{24}+ik\lambda_2 a_{23}\right)}{\left(a_{22}-k^2\lambda_2^2 a_{22}\right)\left(\varepsilon'_{11}k^2+k^2\lambda_2^2\varepsilon_{11}\right)-\left(e_{15}k^2+k^2\lambda_2^2 e_{33}\right)\left(e_{15}k^2+k^2\lambda_2^2 e_{33}\right)},$$

$$V = \frac{-k^2\lambda_3\left(e_{15}+e_{31}\right)\left(e_{15}k^2+k^2\lambda_3^2 e_{33}\right)+\left(\varepsilon'_{11}k^2+k^2\lambda_3^2\varepsilon_{11}\right)\left(a_{24}+ik\lambda_3 a_{23}\right)}{\left(a_{22}-k^2\lambda_3^2 a_{22}\right)\left(\varepsilon'_{11}k^2+k^2\lambda_3^2\varepsilon_{11}\right)-\left(e_{15}k^2+k^2\lambda_3^2 e_{33}\right)\left(e_{15}k^2+k^2\lambda_3^2 e_{33}\right)}.$$

3.2 Solution for half-space

The solutions for Eqs. (20) and (22) are considered as

$$\phi = f(z)\,e^{ik(x-ct)}, \tag{32}$$

$$\psi = h(z)\,e^{ik(x-ct)}. \tag{33}$$

Substituting Eqs. (32), (33) into Eqs. (20) and (22), we have

$$f''(z) + \beta_2^2 f(z) - \frac{i\rho_2\alpha_1 k}{\alpha^2}h(z) = 0, \tag{34}$$

$$h''(z) + \beta_3^2 h(z) + \frac{2i\rho_2\alpha_2 k}{\beta^2}f(z) = 0, \tag{35}$$

where

$$\beta_2 = \frac{c^2\rho_2 k^2 + \rho_2\Omega_2^2}{\alpha^2} - k^2, \quad \beta_3 = \frac{2\rho_0 c^2 k^2 + \left(c'_{13}+P\right)k^2 - c'_{11}k^2 + 2\rho_2\Omega_2^2}{\beta^2}, \quad \alpha_1 = g + 2\Omega_2 c,$$

$$\alpha_2 = g - 2\Omega_2 c, \quad \alpha = c_{11} + P, \quad \beta^2 = 2c'_{33} - \left(c'_{11} - c'_{13} - P\right).$$

For simplification, Eqs. (34) and (35) may be written as

$$\left[\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}+\lambda_4^2\right)\left(\frac{\mathrm{d}^2}{\mathrm{d}z^2}+\lambda_5^2\right)\right](f,h) = 0, \tag{36}$$

where

$$\lambda_4^2 + \lambda_5^2 = \beta_2^2 + \beta_3^2 \text{ and } \lambda_4^2\lambda_5^2 = \beta_2^2\beta_3^2 - \left[\frac{\sqrt{2\alpha_1\alpha_2}\rho_2 k}{\alpha_1\beta}\right]^2.$$

Now we assume that the solution of Eq. (36) has the form

$$f(z) = Ae^{-i\lambda_4 z} + Be^{i\lambda_4 z} + Ce^{-i\lambda_4 z} + De^{i\lambda_5 z}, \tag{37}$$

$$h(z) = A'e^{-i\lambda_4 z} + B'e^{i\lambda_4 z} + C'e^{-i\lambda_4 z} + D'e^{i\lambda_5 z}, \tag{38}$$

where A, B, C, D, A', B', C' and D' are constants.

We consider the appropriate solution in view of the condition $f(z) \to 0$, $h(z) \to 0$ as $z \to \infty$:

$$\phi = \left(Ae^{-i\lambda_4 z} + Ce^{-i\lambda_5 z}\right) e^{ik(x-ct)}, \tag{39}$$

$$\psi = \left(A'e^{-i\lambda_4 z} + C'e^{-i\lambda_5 z}\right) e^{ik(x-ct)}, \tag{40}$$

where

$$A' = iAL_1 \quad \text{and} \quad C' = iCL_2$$

such that

$$L_1 = \frac{\alpha^2\left(\lambda_4^2 - \beta_2^2\right)}{\rho_2\alpha_1 k} \quad \text{and} \quad L_2 = \frac{\alpha^2\left(\lambda_5^2 - \beta_2^2\right)}{\rho_2\alpha_2 k}.$$

Hence, the mechanical displacement and electric potential for the orthotropic half-space are

$$u_2 = \left[Ae^{-i\lambda_4 z}\left(ik - \lambda_4 L_1\right) + Ce^{-i\lambda_5 z}\left(ik - \lambda_5 L_2\right)\right] e^{ik(x-ct)}, \tag{41}$$

$$w_2 = \left[Ae^{-i\lambda_4 z}\left(-i\lambda_4 - kL_1\right) + Ce^{-i\lambda_5 z}\left(-i\lambda_5 - kL_2\right)\right] e^{ik(x-ct)}. \tag{42}$$

4 Boundary conditions

For Rayleigh wave propagation in piezoelectric-layered structure, mechanical displacements and electrical potentials satisfy the following boundary and interface continuity conditions. It may be noted that two kinds of electrical boundary conditions, i.e. electrical open and short conditions, are to considered in this study.

1. Traction free conditions at the free surface are given by

$$\text{(a)} \quad \left(\tau_{zx} - \zeta_1'\tau_{xx}\right) = 0 \tag{43}$$

$$\text{(b)} \quad \left(\tau_{zz} - \zeta_1'\tau_{zx}\right) = 0 \tag{44}$$

2. Electrical boundary condition at the free surface is expressed as

$$\phi_1\left(\zeta - h, y\right) = 0 \quad \text{(Electrically short condition)} \tag{45}$$

3. Electrical boundary conditions at the free surface may be written as

$$D_1\left(\zeta - h, y\right) = 0 \quad \text{(Electrically open condition)} \tag{46}$$

4. Along the interfaces between piezoelectric layer and orthotropic half-space, the stresses, mechanical displacements, electrical potentials are all continuous:

$$\text{(a)} \quad u_1 = u_2, \tag{47}$$

$$\text{(b)} \quad w_1 = w_2, \tag{48}$$

$$\text{(c)} \quad \phi_1 = \phi_2, \tag{49}$$

$$\text{(d)} \quad \left(\tau_{zx} - \zeta_2'\tau_{xx}\right)_1 = \left(\tau_{zx} - \zeta_2'\tau_{xx}\right)_2, \tag{50}$$

$$\text{(e)} \quad \left(\tau_{zz} - \zeta_2'\tau_{zx}\right)_1 = \left(\tau_{zz} - \zeta_2'\tau_{zx}\right)_2, \tag{51}$$

where $\zeta_1' = \frac{\partial \zeta_1}{\partial x}$ and $\zeta_2' = \frac{\partial \zeta_2}{\partial x}$.

The subscripts "1" and "2" are used for upper corrugated piezoelectric layer and lower corrugated orthotropic half-space, respectively.

5 Frequency equation

From Eq. (43), we get

$$\begin{aligned}
&A\left[\left(-c_{44}ik\lambda_1+aikc_{44}+a'ike_{15}\right)-\zeta_1'\left(ikc_{11}-ik\lambda_1ac_{13}+a'e_{31}\right)\right]e^{-ik\lambda_1(\zeta_1-H)}\\
&\quad+B\left[\left(-c_{44}ik\lambda_2+bikc_{44}+b'ike_{15}\right)-\zeta_1'\left(ikc_{11}-ik\lambda_2bc_{13}+b'e_{31}\right)\right]e^{-ik\lambda_2(\zeta_1-H)}\\
&\quad+C\left[\left(-c_{44}ik\lambda_3+cikC_{44}+c'ike_{15}\right)-\zeta_1'\left(ikc_{11}-ik\lambda_3cc_{13}+c'e_{31}\right)\right]e^{-ik\lambda_3(\zeta_1-H)}\\
&\quad+A'\left[\left(c_{44}ik\lambda_1+dikc_{44}+d'ike_{15}\right)-\zeta_1'\left(ikc_{11}+ik\lambda_1dc_{13}+d'e_{31}\right)\right]e^{ik\lambda_1(\zeta_1-H)}\\
&\quad+B'\left[\left(c_{44}ik\lambda_2+eikc_{44}+e'ike_{15}\right)-\zeta_1'\left(ikc_{11}+ik\lambda_2ec_{13}+e'e_{31}\right)\right]e^{ik\lambda_2(\zeta_1-H)}\\
&\quad+C'\left[\left(c_{44}ik\lambda_3+fikc_{44}+f'ike_{15}\right)-\zeta_1'\left(ikc_{11}+ik\lambda_3fc_{13}+f'e_{31}\right)\right]e^{ik\lambda_3(\zeta_1-H)}=0.
\end{aligned}\tag{52}$$

Equation (44) yields

$$\begin{aligned}
&A\left[\left(c_{13}ik-\lambda_1aikc_{33}-a'ik\lambda_1e_{33}\right)-\zeta_1'\left(aikc_{44}-ik\lambda_1c_{44}+a'ike_{15}\right)\right]e^{-ik\lambda_1(\zeta_1-H)}\\
&\quad+B\left[\left(c_{13}ik-\lambda_2bikc_{33}-b'ik\lambda_2e_{33}\right)-\zeta_1'\left(bikc_{44}-ik\lambda_2c_{44}+b'ike_{15}\right)\right]e^{-ik\lambda_2(\zeta_1-H)}\\
&\quad+C\left[\left(c_{13}ik-\lambda_3cikc_{33}-c'ik\lambda_3e_{33}\right)-\zeta_1'\left(cikc_{44}-ik\lambda_3c_{44}+c'ike_{15}\right)\right]e^{-ik\lambda_3(\zeta_1-H)}\\
&\quad+A'\left[\left(c_{13}ik+dik\lambda_1c_{33}+d'ik\lambda_1e_{33}\right)-\zeta_1'\left(ikc_{44}\lambda_1+ikdc_{44}+d'ike_{15}\right)\right]e^{ik\lambda_1(\zeta_1-H)}\\
&\quad+B'\left[\left(c_{13}ik+eik\lambda_2c_{33}+e'ik\lambda_2e_{33}\right)-\zeta_1'\left(ikc_{44}\lambda_2+ikec_{44}+e'ike_{15}\right)\right]e^{ik\lambda_2(\zeta_1-H)}\\
&\quad+C'\left[\left(c_{13}ik+fik\lambda_3c_{33}+f'ik\lambda_3e_{33}\right)-\zeta_1'\left(ikc_{44}\lambda_3+ikfc_{44}+f'ike_{15}\right)\right]e^{ik\lambda_3(\zeta_1-H)}=0.
\end{aligned}\tag{53}$$

Using Eq. (45), we obtain

$$\begin{aligned}
&a'Ae^{-ik\lambda_1(\zeta_1-H)}+b'Be^{-ik\lambda_2(\zeta_1-H)}+c'Ce^{-ik\lambda_3(\zeta_1-H)}+d'De^{ik\lambda_1(\zeta_1-H)}\\
&\quad+e'Ee^{ik\lambda_2(\zeta_1-H)}+f'Fe^{ik\lambda_3(\zeta_1-H)}=0.
\end{aligned}\tag{54}$$

In view of Eq. (46), we establish

$$\begin{aligned}
&A\left[e_{15}\left(-ik\lambda_1+ika\right)+ika'\varepsilon_{11}\right]e^{-ik\lambda_1(\zeta_1-H)}+B\left[e_{15}\left(-ik\lambda_2+ikb\right)+ikb'\varepsilon_{11}\right]e^{-ik\lambda_2(\zeta_1-H)}\\
&\quad+C\left[e_{15}\left(-ik\lambda_3+ikc\right)+ikc'\varepsilon_{11}\right]e^{-ik\lambda_3(\zeta_1-H)}+D\left[e_{15}\left(ik\lambda_1+ikd\right)+ikd'\varepsilon_{11}\right]e^{ik\lambda_1(\zeta_1-H)}\\
&\quad+E\left[e_{15}\left(ik\lambda_2+ike\right)+ike'\varepsilon_{11}\right]e^{ik\lambda_2(\zeta_1-H)}+F\left[e_{15}\left(ik\lambda_3+ikf\right)+ikf'\varepsilon_{11}\right]e^{ik\lambda_3(\zeta_1-H)}=0.
\end{aligned}\tag{55}$$

Form Eq. (47), we have

$$Ae^{-ik\lambda_1\zeta_2}+Be^{-ik\lambda_2\zeta_2}+Ce^{-ik\lambda_3\zeta_2}+De^{ik\lambda_1\zeta_2}+Ee^{ik\lambda_2\zeta_2}+Fe^{ik\lambda_3\zeta_2}+LX_1e^{-ik\lambda_4\zeta_2}+MX_2e^{-ik\lambda_5\zeta_2}=0.\tag{56}$$

Equation (48) leads to

$$\begin{aligned}
&Aae^{-ik\lambda_1\zeta_2}+Bbe^{-ik\lambda_2\zeta_2}+Cce^{-ik\lambda_3\zeta_2}+Dde^{ik\lambda_1\zeta_2}+Eee^{ik\lambda_2\zeta_2}\\
&\quad+Ffe^{ik\lambda_3\zeta_2}+LY_1e^{-ik\lambda_4\zeta_2}+MY_2e^{-ik\lambda_5\zeta_2}=0.
\end{aligned}\tag{57}$$

From Eq. (49), we find

$$Aa'e^{-ik\lambda_1\zeta_2}+Bb'e^{-ik\lambda_2\zeta_2}+Cc'e^{-ik\lambda_3\zeta_2}+Dd'e^{ik\lambda_1\zeta_2}+Ee'e^{ik\lambda_2\zeta_2}+Ff'e^{ik\lambda_3\zeta_2}=0.\tag{58}$$

Using Eq. (50), we obtain

$$\begin{aligned}
&A\left[\left(c_{13}ik-ik\lambda_1ac_{33}-ik\lambda_1a'e_{33}\right)-\zeta_2'\left(-c_{44}ik\lambda_1+ikac_{44}+a'ike_{15}\right)\right]e^{-ik\lambda_1\zeta_2}\\
&\quad+B\left[\left(c_{13}ik-ik\lambda_2bc_{33}-ik\lambda_2b'e_{33}\right)-\zeta_2'\left(-c_{44}ik\lambda_2+ikbc_{44}+b'ike_{15}\right)\right]e^{-ik\lambda_2\zeta_2}\\
&\quad+C\left[\left(c_{13}ik-ik\lambda_3cc_{33}-ik\lambda_3c'e_{33}\right)-\zeta_2'\left(-c_{44}ik\lambda_3+ikcc_{44}+c'ike_{15}\right)\right]e^{-ik\lambda_3\zeta_2}\\
&\quad+D\left[\left(c_{13}ik+ik\lambda_1dc_{33}+ik\lambda_1d'e_{33}\right)-\zeta_2'\left(c_{44}ik\lambda_1+ikdc_{44}+d'ike_{15}\right)\right]e^{ik\lambda_1\zeta_2}\\
&\quad+E\left[\left(c_{13}ik+ik\lambda_2ec_{33}+ik\lambda_2e'e_{33}\right)-\zeta_2'\left(c_{44}ik\lambda_2+ikec_{44}+e'ike_{15}\right)\right]e^{ik\lambda_2\zeta_2}
\end{aligned}$$

$$+ F\left[\left(c_{13}ik + ik\lambda_3 f c_{33} + ik\lambda_3 f' e_{33}\right) - \zeta_2'\left(c_{44}ik\lambda_3 + ikfc_{44} + f'ike_{15}\right)\right]e^{ik\lambda_3\zeta_2}$$
$$- L\left[\left(c_{13}'ikX_1 - c_{33}'ikY_1\lambda_4\right) - \zeta_2'\left(c_{44}'ikY_1 - c_{44}'ikX_1\lambda_4\right)\right]e^{-k\lambda_4\zeta_2}$$
$$- M\left[\left(c_{13}'ikX_2 - c_{33}'ikY_2\lambda_5\right) - \zeta_2'\left(c_{44}'ikY_2 - c_{44}'ikX_1\lambda_5\right)\right]e^{-k\lambda_5\zeta_2} = 0. \tag{59}$$

From Eq. (51), we get

$$A\left[\left(-c_{44}\lambda_1 + ac_{44} + a'e_{15}\right)ik - \zeta_2'\left(ikc_{11} - ik\lambda_1 ac_{13} + e_{31}a'\right)\right]e^{-ik\lambda_1\zeta_2}$$
$$+ B\left[\left(-c_{44}\lambda_2 + bc_{44} + b'e_{15}\right)ik - \zeta_2'\left(ikc_{11} - ik\lambda_2 bc_{13} + e_{31}b'\right)\right]e^{-ik\lambda_2\zeta_2}$$
$$+ C\left[\left(-c_{44}\lambda_3 + cc_{44} + c'e_{15}\right)ik - \zeta_2'\left(ikc_{11} - ik\lambda_3 cc_{13} + e_{31}c'\right)\right]e^{-ik\lambda_3\zeta_2}$$
$$+ D\left[\left(c_{44}\lambda_1 + dc_{44} + d'e_{15}\right)ik - \zeta_2'\left(ikc_{11} + ik\lambda_1 dc_{13} + e_{31}d'\right)\right]e^{ik\lambda_1\zeta_2}$$
$$+ E\left[\left(c_{44}\lambda_2 + ec_{44} + e'e_{15}\right)ik - \zeta_2'\left(ikc_{11} + ik\lambda_2 ec_{13} + e_{31}e'\right)\right]e^{ik\lambda_2\zeta_2}$$
$$+ F\left[\left(c_{44}\lambda_3 + fc_{44} + f'e_{15}\right)ik - \zeta_2'\left(ikc_{11} + ik\lambda_3 fc_{13} + e_{31}f'\right)\right]e^{ik\lambda_3\zeta_2}$$
$$- L\left[\left(c_{44}'ikY_1 - c_{44}'ikX_1\lambda_4\right) - \zeta_2'\left(\left(c_{11}' + P\right)ikX_1 - \left(c_{13}' + P\right)ikY_1\lambda_4\right)\right]e^{-ik\lambda_4\zeta_2}$$
$$- M\left[\left(c_{44}'ikY_2 - c_{44}'ikX_2\lambda_5\right) - \zeta_2'\left(\left(c_{11}' + P\right)ikX_2 - \left(c_{13}' + P\right)ikY_2\lambda_5\right)\right]e^{-ik\lambda_5\zeta_2} = 0. \tag{60}$$

5.1 Frequency equation for electrically open case

By eliminating the constants from Eqs. (52), (53) and (55)–(60) and applying the necessary and sufficient condition for the existence of non-trivial solutions, we have

$$\left|A_{ij}\right|_{8\times 8} = 0, \tag{61}$$

where

$$A_{11} = \left[\left(-c_{44}ik\lambda_1 + aikc_{44} + a'ike_{15}\right) - \zeta_1'\left(ikc_{11} - ik\lambda_1 ac_{13} + a'e_{31}\right)\right]e^{-ik\lambda_1(\zeta_1 - H)},$$
$$A_{12} = \left[\left(-c_{44}ik\lambda_2 + bikc_{44} + b'ike_{15}\right) - \zeta_1'\left(ikc_{11} - ik\lambda_2 bc_{13} + b'e_{31}\right)\right]e^{-ik\lambda_2(\zeta_1 - H)},$$
$$A_{13} = \left[\left(-c_{44}ik\lambda_3 + cikc_{44} + c'ike_{15}\right) - \zeta_1'\left(ikc_{11} - ik\lambda_3 cc_{13} + c'e_{31}\right)\right]e^{-ik\lambda_3(\zeta_1 - H)},$$
$$A_{14} = \left[\left(c_{44}ik\lambda_1 + dikc_{44} + d'ike_{15}\right) - \zeta_1'\left(ikC_{11} + ik\lambda_1 dc_{13} + d'e_{31}\right)\right]e^{ik\lambda_1(\zeta_1 - H)},$$
$$A_{15} = \left[\left(c_{44}ik\lambda_2 + eikc_{44} + e'ike_{15}\right) - \zeta_1'\left(ikc_{11} + ik\lambda_2 ec_{13} + e'e_{31}\right)\right]e^{ik\lambda_2(\zeta_1 - H)},$$
$$A_{16} = \left[\left(c_{44}ik\lambda_3 + fikc_{44} + f'ike_{15}\right) - \zeta_1'\left(ikc_{11} + ik\lambda_3 fc_{13} + f'e_{31}\right)\right]e^{ik\lambda_3(\zeta_1 - H)},$$
$$A_{17} = 0 = A_{18},$$
$$A_{21} = \left[\left(C_{13}ik - \lambda_1 aikC_{33} - a'ik\lambda_1 e_{33}\right) - \zeta_1'\left(aikC_{44} - ik\lambda_1 C_{44} + a'ike_{15}\right)\right]e^{-ik\lambda_1(\zeta_1 - H)},$$
$$A_{22} = \left[\left(C_{13}ik - \lambda_2 bikC_{33} - b'ik\lambda_2 e_{33}\right) - \zeta_1'\left(bikC_{44} - ik\lambda_2 C_{44} + b'ike_{15}\right)\right]e^{-ik\lambda_2(\zeta_1 - H)},$$
$$A_{23} = \left[\left(C_{13}ik - \lambda_3 cikC_{33} - c'ik\lambda_3 e_{33}\right) - \zeta_1'\left(cikC_{44} - ik\lambda_3 C_{44} + c'ike_{15}\right)\right]e^{-ik\lambda_3(\zeta_1 - H)},$$
$$A_{24} = \left[\left(C_{13}ik + dik\lambda_1 C_{33} + d'ik\lambda_1 e_{33}\right) - \zeta_1'\left(ikC_{44}\lambda_1 + ikdC_{44} + d'ike_{15}\right)\right]e^{ik\lambda_1(\zeta_1 - H)},$$
$$A_{25} = \left[\left(C_{13}ik + eik\lambda_2 C_{33} + e'ik\lambda_2 e_{33}\right) - \zeta_1'\left(ikC_{44}\lambda_2 + ikeC_{44} + e'ike_{15}\right)\right]e^{ik\lambda_2(\zeta_1 - H)}$$
$$A_{26} = \left[\left(C_{13}ik + fik\lambda_3 C_{33} + f'ik\lambda_3 e_{33}\right) - \zeta_1'\left(ikC_{44}\lambda_3 + ikfC_{44} + f'ike_{15}\right)\right]e^{ik\lambda_3(\zeta_1 - H)},$$
$$A_{27} = 0 = A_{28},\ A_{31} = \left[e_{15}\left(-ik\lambda_1 + ika\right) + ika'\varepsilon_{11}\right]e^{-ik\lambda_1(\zeta_1 - H)},$$
$$A_{32} = \left[e_{15}\left(-ik\lambda_2 + ikb\right) + ikb'\varepsilon_{11}\right]e^{-ik\lambda_2(\zeta_1 - H)},$$
$$A_{33} = \left[e_{15}\left(-ik\lambda_3 + ikc\right) + ikc'\varepsilon_{11}\right]e^{-ik\lambda_3(\zeta_1 - H)},\ A_{34} = \left[e_{15}\left(ik\lambda_1 + ikd\right) + ikd'\varepsilon_{11}\right]e^{ik\lambda_1(\zeta_1 - H)},$$
$$A_{35} = \left[e_{15}\left(ik\lambda_2 + ike\right) + ike'\varepsilon_{11}\right]e^{ik\lambda_2(\zeta_1 - H)},\ A_{36} = F\left[e_{15}\left(ik\lambda_3 + ikf\right) + ikf'\varepsilon_{11}\right]e^{ik\lambda_3(\zeta_1 - H)},$$
$$A_{37} = 0 = A_{38},$$
$$A_{41} = e^{-ik\lambda_1\zeta_2},\ A_{42} = e^{-ik\lambda_2\zeta_2},\ A_{43} = Ce^{-ik\lambda_3\zeta_2},\ A_{44} = De^{ik\lambda_1\zeta_2},\ A_{45} = Ee^{ik\lambda_2\zeta_2},\ A_{46} = Fe^{ik\lambda_3\zeta_2},$$
$$A_{47} = X_1 e^{-ik\lambda_4\zeta_2},\ A_{48} = X_2 e^{-ik\lambda_5\zeta_2},$$

$A_{51}=ae^{-ik\lambda_1\zeta_2}$, $A_{52}=be^{-ik\lambda_2\zeta_2}$, $A_{53}=ce^{-ik\lambda_3\zeta_2}$, $A_{54}=de^{ik\lambda_1\zeta_2}$, $A_{55}=ee^{ik\lambda_2\zeta_2}$, $A_{56}=fe^{ik\lambda_3\zeta_2}$,
$A_{57}=Y_1e^{-ik\lambda_4\zeta_2}$, $A_{58}=Y_2e^{-ik\lambda_5\zeta_2}$, $A_{61}=a'e^{-ik\lambda_1\zeta_2}$, $A_{62}=b'e^{-ik\lambda_2\zeta_2}$, $A_{63}=c'e^{-ik\lambda_3\zeta_2}$, $A_{64}=d'e^{ik\lambda_1\zeta_2}$,
$A_{65}=e'e^{ik\lambda_2\zeta_2}$, $A_{66}=f'e^{ik\lambda_3\zeta_2}$, $A_{67}=0=A_{68}$,

$$A_{71}=\left[\left(c_{13}ik-ik\lambda_1ac_{33}-ik\lambda_1a'e_{33}\right)-\zeta_2'\left(-c_{44}ik\lambda_1+ikac_{44}+a'ike_{15}\right)\right]e^{-ik\lambda_1\zeta_2},$$
$$A_{72}=\left[\left(c_{13}ik-ik\lambda_2bc_{33}-ik\lambda_2b'e_{33}\right)-\zeta_2'\left(-c_{44}ik\lambda_2+ikbc_{44}+b'ike_{15}\right)\right]e^{-ik\lambda_2\zeta_2},$$
$$A_{73}=\left[\left(c_{13}ik-ik\lambda_3cc_{33}-ik\lambda_3c'e_{33}\right)-\zeta_2'\left(-c_{44}ik\lambda_3+ikcc_{44}+c'ike_{15}\right)\right]e^{-ik\lambda_3\zeta_2},$$
$$A_{74}=\left[\left(c_{13}ik+ik\lambda_1dc_{33}+ik\lambda_1d'e_{33}\right)-\zeta_2'\left(c_{44}ik\lambda_1+ikdc_{44}+d'ike_{15}\right)\right]e^{ik\lambda_1\zeta_2},$$
$$A_{81}=\left[\left(-c_{44}\lambda_1+ac_{44}+a'e_{15}\right)ik-\zeta_2'\left(ikc_{11}-ik\lambda_1ac_{13}+e_{31}a'\right)\right]e^{-ik\lambda_1\zeta_2},$$
$$A_{82}=\left[\left(-c_{44}\lambda_2+bc_{44}+b'e_{15}\right)ik-\zeta_2'\left(ikc_{11}-ik\lambda_2bc_{13}+e_{31}b'\right)\right]e^{-ik\lambda_2\zeta_2},$$
$$A_{83}=\left[\left(-c_{44}\lambda_3+cc_{44}+c'e_{15}\right)ik-\zeta_2'\left(ikc_{11}-ik\lambda_3cc_{13}+e_{31}c'\right)\right]e^{-ik\lambda_3\zeta_2},$$
$$A_{75}=\left[\left(c_{13}ik+ik\lambda_2ec_{33}+ik\lambda_2e'e_{33}\right)-\zeta_2'\left(c_{44}ik\lambda_2+ikec_{44}+e'ike_{15}\right)\right]e^{ik\lambda_2\zeta_2},$$
$$A_{76}=\left[\left(c_{13}ik+ik\lambda_3fc_{33}+ik\lambda_3f'e_{33}\right)-\zeta_2'\left(c_{44}ik\lambda_3+ikfc_{44}+f'ike_{15}\right)\right]e^{ik\lambda_3\zeta_2},$$
$$A_{77}=\left[\left(c_{13}'ikX_1-c_{33}'ikY_1\lambda_4\right)-\zeta_2'\left(c_{44}'ikY_1-c_{44}'ikX_1\lambda_4\right)\right]e^{-k\lambda_4\zeta_2},$$
$$A_{78}=\left[\left(c_{13}'ikX_2-c_{33}'ikY_2\lambda_5\right)-\zeta_2'\left(c_{44}'ikY_2-c_{44}'ikX_1\lambda_5\right)\right]e^{-k\lambda_5\zeta_2},$$
$$A_{84}=\left[\left(c_{44}\lambda_1+dc_{44}+d'e_{15}\right)ik-\zeta_2'\left(ikc_{11}+ik\lambda_1dc_{13}+e_{31}d'\right)\right]e^{ik\lambda_1\zeta_2},$$
$$A_{85}=\left[\left(c_{44}\lambda_2+ec_{44}+e'e_{15}\right)ik-\zeta_2'\left(ikc_{11}+ik\lambda_2ec_{13}+e_{31}e'\right)\right]e^{ik\lambda_2\zeta_2},$$
$$A_{86}=\left[\left(c_{44}\lambda_3+fc_{44}+f'e_{15}\right)ik-\zeta_2'\left(ikc_{11}+ik\lambda_3fc_{13}+e_{31}f'\right)\right]e^{ik\lambda_3\zeta_2},$$
$$A_{87}=\left[\left(c_{44}'ikY_1-c_{44}'ikX_1\lambda_4\right)-\zeta_2'\left(\left(c_{11}'+P\right)ikX_1-\left(c_{13}'+P\right)ikY_1\lambda_4\right)\right]e^{-ik\lambda_4\zeta_2},$$
$$A_{88}=\left[\left(c_{44}'ikY_2-c_{44}'ikX_2\lambda_5\right)-\zeta_2'\left(\left(c_{13}'+P\right)ikX_2-\left(c_{13}'+P\right)ikY_2\lambda_5\right)\right]e^{-ik\lambda_5\zeta_2}.$$

Equation (61) gives the frequency equation for Rayleigh waves propagating in the corrugated piezoelectric layer with initial stress, rotation over-lying a gravitational orthotropic half-space with corrugated boundary, initial stress and rotation for the electrically open case.

5.2 Frequency equation for electrically short case

By eliminating the constants from Eqs. (52), (53), (54) and (56)–(60), we obtain the frequency equation for the electrically the short case as

$$\left|A_{ij}\right|_{8\times 8}=0. \tag{62}$$

Equation (62) gives the frequency equation for Rayleigh waves propagating in the corrugated piezoelectric layer with initial stress and rotation over-lying a gravitational orthotropic half-space with corrugated boundary, initial stress and rotation for the electrically short case. Here

$A_{41}=a'e^{-ik\lambda_1(\zeta_1-H)}$, $A_{42}=b'e^{-ik\lambda_2(\zeta_1-H)}$, $A_{43}=c'e^{-ik\lambda_3(\zeta_1-H)}$, $A_{44}=d'e^{ik\lambda_1(\zeta_1-H)}$, $A_{45}=e'e^{ik\lambda_2(\zeta_1-H)}$,
$A_{46}=f'e^{ik\lambda_3(\zeta_1-H)}$

and all other entries remain same as in Eq. (61).

6 Particular cases

6.1 Particular case for electrically open case

Case 1
When the upper layer has a plane surface $z=-H$, i.e. $\zeta_1=0$, and the intermediate surface has a periodic corrugated delineation by $\zeta_2=a\cos(bx)$, then Eq. (61) becomes

$$\left|B_{ij}\right|_{8\times 8}=0. \tag{63}$$

Values of all B_{ij} are given in Appendix 1.

Equation (63) gives the frequency equation for Rayleigh waves in a piezoelectric layer with free surface lying over a corrugated boundary of orthotropic half-space with initial stress and rotation for the electrically open case.

Case 2

When the upper piezoelectric layer is bounded by a corrugated surface $\zeta_1 = a\cos(bx)$ and the common interface between layer and half-space is planar, i.e. $\zeta_2 = 0$, so $\zeta_1' = -ab\sin(bx)$ and $\zeta_2' = 0$, then Eq. (61) yields

$$\left|D_{ij}\right|_{8\times 8}=0. \tag{64}$$

Values of all D_{ij} are given in Appendix 2.

Equation (64) gives the frequency equation for the propagation of Rayleigh waves in a corrugated piezoelectric layer lying over an orthotropic half-space with initial stress and rotation for the electrically open case.

Case 3

When the corrugated boundary of the piezoelectric layer and the common interface are continuous and periodic in nature, with $\zeta_1 = a_1\cos(bx)$, $\zeta_2 = a_2\cos(bx)$, $\zeta_2' = -a_2 b\sin(bx)$, $\zeta_1' = -a_1 b\sin(bx)$, then Eq. (61) becomes

$$\left|E_{ij}\right|_{8\times 8}=0. \tag{65}$$

Values of all E_{ij} are given in Appendix 3.

Equation (65) gives the frequency equation for propagation of Rayleigh waves in corrugated piezoelectric layer bounded by corrugated surfaces, lying over an orthotropic half-space for electrically open case.

Case 4

When the medium is consists of a half-space only $(H=0)$ with plane boundary, i.e. $\zeta_2 = 0$, then Eq. (65) becomes

$$\begin{vmatrix} \left(c_{13}' ikX_1 - c_{33}' ikY_1\lambda_4\right) & \left(c_{13}' ikX_2 - c_{33}' ikY_2\lambda_5\right) \\ \left(c_{44}' ikY_1 - c_{44}' ikX_1\lambda_4\right) & \left(c_{44}' ikY_2 - c_{44}' ikX_2\lambda_5\right) \end{vmatrix} = 0. \tag{66}$$

Equation (66) gives the frequency equation for Rayleigh wave propagation in an orthotropic half-space with initial stress, rotation and gravity given by Abd-Alla et al. [16].

Case 5

When the medium consists of a half-space only $(H=0)$ with plane boundary, i.e. $\zeta_2 = 0$, and the half-space is free of rotation, i.e. $\Omega_2 = 0$, then Eq. (61) reduces to

$$\begin{vmatrix} \left(c_{13}' ikX_1' - c_{33}' ikY_1'\lambda_4'\right) & \left(c_{13}' ikX_2' - c_{33}' ikY_2'\lambda_5'\right) \\ \left(c_{44}' ikY_1' - c_{44}' ikX_1'\lambda_4'\right) & \left(c_{44}' ikY_2 - c_{44}' ikX_2'\lambda_5'\right) \end{vmatrix} = 0, \tag{67}$$

where

$$\lambda_4'^2 + \lambda_5'^2 = \beta_2^2 + \beta_3^2 \quad \text{and} \quad \lambda_4'^2\lambda_5'^2 = \beta_2^2\beta_3^2 - \left[\frac{\sqrt{2\alpha_1\alpha_2}\rho_2 k}{\alpha_1\beta}\right]^2,$$

$$\beta_2 = \frac{c^2\rho_2 k^2}{\alpha^2} - k^2, \quad \beta_3 = \frac{2\rho_0 c^2 k^2 + \left(c_{13}' + P\right)k^2 - c_{11}' k^2}{\beta^2}, \quad \alpha_1 = g,$$

$$\alpha_2 = g, \quad \alpha = c_{11} + P, \quad \beta^2 = 2c_{33}' - \left(c_{11}' - c_{13}' - P\right).$$

Equation (67) gives the frequency equation for Rayleigh wave propagation in an orthotropic half-space with initial stress and gravity given by Abd-Alla et al. [15].

Case 6

When the medium consists of a half-space only $(H=0)$ with plane boundary, i.e. $\zeta_2 = 0$, it is rotation free half-space, i.e. $\Omega_2 = 0$, and the half-space is isotropic, i.e. $\frac{g}{c^2 k} << 1$, then Eq. (61) becomes

$$\left(2 - \frac{c^2}{\sigma_2^2}\right)^2 - 4\left[\left(\frac{c^2}{\sigma_1^2} - 1\right)\left(\frac{c^2}{\sigma_2^2} - 1\right)\right]^{\frac{1}{2}}$$

$$+\frac{4g}{c^2(\sigma_1^2-\sigma_2^2)k}\left[\left(1-\frac{c^2}{\sigma_2^2}\right)^{\frac{1}{2}}(\sigma_1^2+\sigma_2^2-c^2)-\left(1-\frac{c^2}{\sigma_1^2}\right)^{\frac{1}{2}}\left\{(\sigma_1^2+\sigma_2^2)-\frac{\sigma_1^2c^2}{\sigma_2^2}\right\}\right]=0, \quad (68)$$

where

$$\sigma_1^2=\frac{\lambda+2\mu+P}{\rho_2}, \quad \sigma_2^2=\frac{\mu-\frac{P}{2}}{\rho_2}.$$

Equation (68) represents that the Rayleigh surface waves in isotropic medium under the influence of initial stress and gravity.

Case 7

When the medium is consists of half-space only ($H=0$) with plane boundary, i.e. $\zeta_2=0$, such that the half-space is rotation free, i.e. $\Omega_2=0$, isotropic $\left(\frac{g}{c^2k}<<1\right)$ and without initial stress and gravity, then Eq. (61) reduces to

$$\left(2-\frac{c^2}{\gamma_2^2}\right)^2=4\sqrt{\left[\left(\frac{c^2}{\gamma_1^2}-1\right)\left(\frac{c^2}{\gamma_2^2}-1\right)\right]}, \quad (69)$$

where

$$\gamma_1^2=\frac{\lambda+2\mu}{\rho} \quad \text{and} \quad \gamma_2^2=\frac{\mu}{\rho}.$$

Equation (69) is the classical Rayleigh wave equation given by Rayleigh [24].

6.2 Particular case for electrically short case

Case 1

When the upper layer is bounded by a plane surface $z=-H$, i.e. $\zeta_1=0$, and the intermediate surface is corrugated (periodic nature) by $\zeta_2=a\cos(bx)$, $\zeta_1'=0$ and $\zeta_2'=-ab\sin(bx)$, then Eq. (62) becomes

$$\left|B_{ij}\right|_{8\times 8}=0. \quad (70)$$

Equation (70) gives the frequency equation for the propagation of Rayleigh waves in a piezoelectric layer lying over a corrugated orthotropic half-space with initial stress and rotation in the electrically short case, where all the terms are same as in Eq. (63) except the following terms:

$$B_{41}=a'e^{ik\lambda_1H}, \quad B_{42}=b'e^{ik\lambda_2H}, \quad B_{43}=c'e^{ik\lambda_3H}, \quad B_{44}=d'e^{-ik\lambda_1H},$$
$$B_{45}=e'e^{-ik\lambda_2H}, \quad B_{46}=f'e^{-ik\lambda_3H}.$$

Case 2

When the corrugation of the upper piezoelectric layer is given by $\zeta_1=a\cos(bx)$ and the common interface between the layer and the half-space is planar, i.e. $\zeta_2=0$, then Eq. (62) yields

$$\left|D_{ij}\right|_{8\times 8}=0. \quad (71)$$

Equation (71) gives the frequency equation for the propagation of Rayleigh wave in a corrugated piezoelectric layer lying over an orthotropic half-space with initial stress and rotation for the electrically short case, where all the terms are the same as in Eq. (64) except the following terms:

$$D_{41}=a'e^{-ik\lambda_1(a\cos(bx)-H)}, \quad D_{42}=b'e^{-ik\lambda_2(a\cos(bx)-H)}, \quad D_{43}=c'e^{-ik\lambda_3(a\cos(bx)-H)},$$
$$D_{44}=d'e^{ik\lambda_1(a\cos(bx)-H)}, \quad D_{45}=e'e^{ik\lambda_2(a\cos(bx)-H)}, \quad D_{46}=f'e^{ik\lambda_3(a\cos(bx)-H)}.$$

Case 3

When the corrugated boundary of piezoelectric layer and common interface of layer and half-space are continuous and periodic in nature, then we have

$$\zeta_1=a_1\cos(bx), \quad \zeta_2=a_2\cos(bx), \quad \zeta_2'=-a_2b\sin(bx), \quad \zeta_1'=-a_1b\sin(bx),$$

Table 1 Piezoelectric and elastic constants for piezoelectric layer and orthotropic substrate

Parameters	Piezoelectric layer (PZT-5H)	Orthotropic elastic half-space
$c_{11}\ (10^{10} N/m^2)$	12.1	2.694
$c_{33}\ (10^{10} N/m^2)$	11.7	2.363
$c_{44}\ (10^{10} N/m^2)$	2.34	–
$c_{13}\ (10^{10} N/m^2)$	8.41	0.661
$e_{33}\ (c/m^2)$	23.3	–
$e_{13}\ (c/m^2)$	−6.5	–
$e_{15}\ (c/m^2)$	17	–
ε_{33}	3400	–
ε_{11}	3100	–
$\rho\ (10^3 kg/m^3)$	7.7	2.7

and Eq. (62) becomes

$$\left|E_{ij}\right|_{8\times 8} = 0, \tag{72}$$

where all the terms remain the same as in Eq. (65) except the terms

$$E_{41} = a' e^{-ik\lambda_1(a_1\cos(bx)-H)},\quad E_{42} = b' e^{-ik\lambda_2(a_1\cos(bx)-H)},\quad E_{43} = c' e^{-ik\lambda_3(a_1\cos(bx)-H)},$$
$$E_{44} = d' e^{ik\lambda_1(a_1\cos(bx)-H)},\quad E_{45} = e' e^{ik\lambda_2(a_1\cos(bx)-H)},\quad E_{46} = f' e^{ik\lambda_3(a_1\cos(bx)-H)}.$$

Equation (72) is the required frequency equation for the propagation of Rayleigh wave in a corrugated piezoelectric layer bounded by periodic surfaces, lying over an orthotropic half-space in the electrically short case.

7 Numerical example and discussion

An analytical solution for the propagation of Rayleigh waves in an initially stressed piezoelectric layer with rotation and gravitational orthotropic elastic substrate with initial stress and rotation has been obtained. The obtained analytical solution is matched with the solution using the finite element method (FEM) and is discussed separately. Furthermore, to show the effect of elastic parameters (viz. initial stress, rotation and corrugation) on the frequency equation of Rayleigh waves, we have considered the structure made up of a PZT-5H ceramic layer over the orthotropic elastic substrate. The elastic and piezoelectric constants of the PZT-5H ceramic layer and orthotropic substrate are taken as given in Table 1 [23,25].

7.1 Effect of undulation parameter on frequency equation of Rayleigh Wave

The effect of the undulation parameter on the Rayleigh wave velocity is taken into account. Figures 2 and 3 represent the dispersion curves of Rayleigh waves for different values of undulation parameters for electrically open and short cases, respectively. The graphs indicate that the Rayleigh wave velocity decreases with increasing values of the undulation parameter for both the electrically open and short case.

7.2 Effect of position parameter on frequency equation

To show the effect of the position parameter on the Rayleigh wave velocity in electrically open and short cases, Figs. 4 and 5 are plotted. The obtained curves indicate that the Rayleigh wave velocity decreases as we increases the values of the position parameter for both the electrically open and short case.

7.3 Effect of corrugation of upper boundary surface

Figures 6, 7, 8 and 9 represent the effect of corrugation on the Rayleigh wave velocity. Figures 6 and 7 are plotted for the case when the interface of the layer is planar but the upper surface has corrugation, for electrically

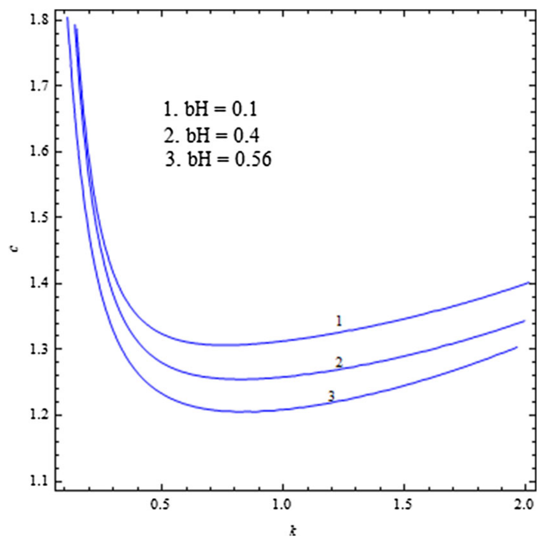

Fig. 2 Variation of Rayleigh wave velocity c with respect to wave number k for different values of undulation parameter bH for electrically open condition

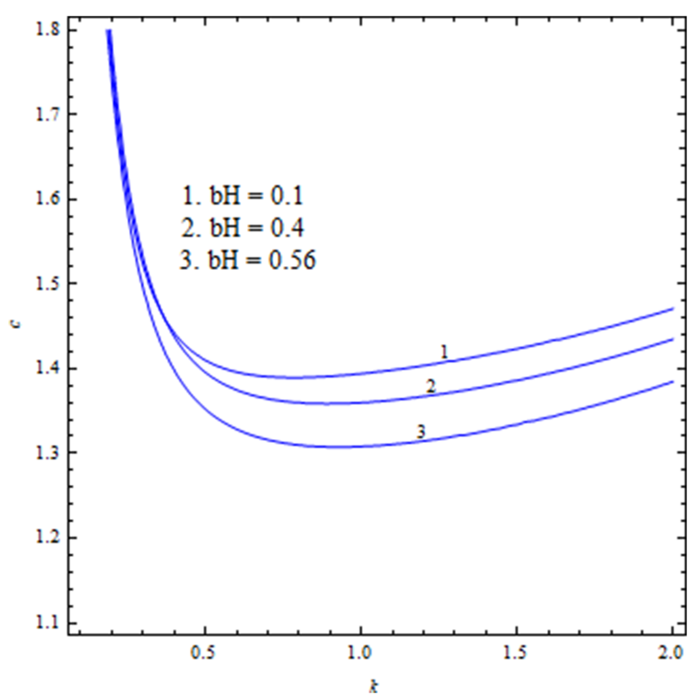

Fig. 3 Variation of Rayleigh wave velocity c with respect to wave number k for different values of undulation parameter (bH) for electrically short condition

open and short case, respectively. Figures 8 and 9 show the variation of Rayleigh wave velocity with wave number when both the upper surface and intermediate boundary are of corrugated type. It is observed that the Rayleigh wave velocity is significantly influenced by corrugation of boundary surface. In particular, the Rayleigh wave velocity increases with increasing values of ζ_1 for both the electrically open and short case.

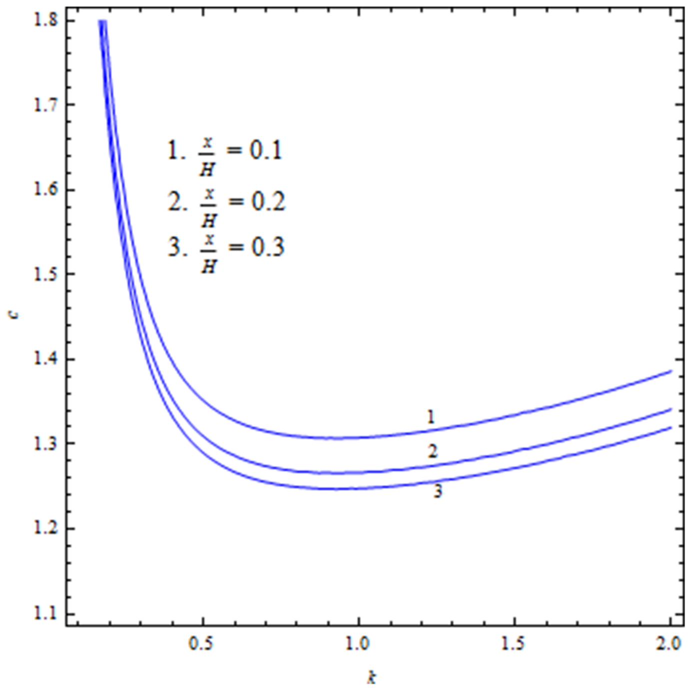

Fig. 4 Variation of Rayleigh wave velocity c with respect to wave number k for different values of position parameter $\frac{x}{H}$ for electrically open case

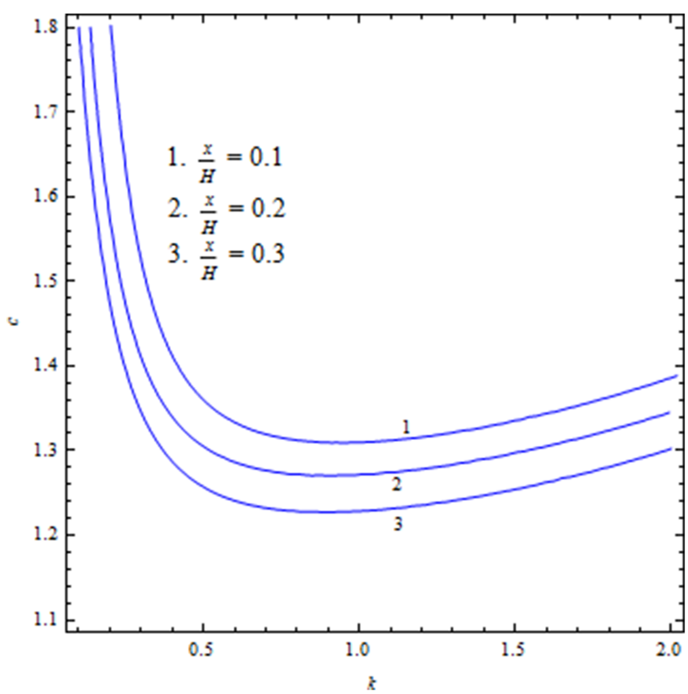

Fig. 5 Variation of Rayleigh wave velocity c with respect to wave number k for different values of position parameter $\left(\frac{x}{H}\right)$ for electrically short case

7.4 Effect of corrugation of interface on frequency equation

Figures 10 and 11 depict the prominent influence of corrugation of the common interface on the dispersion curves when the upper boundary surface of the layer is planar. It is conveyed from Figs. 10 and 11 that an

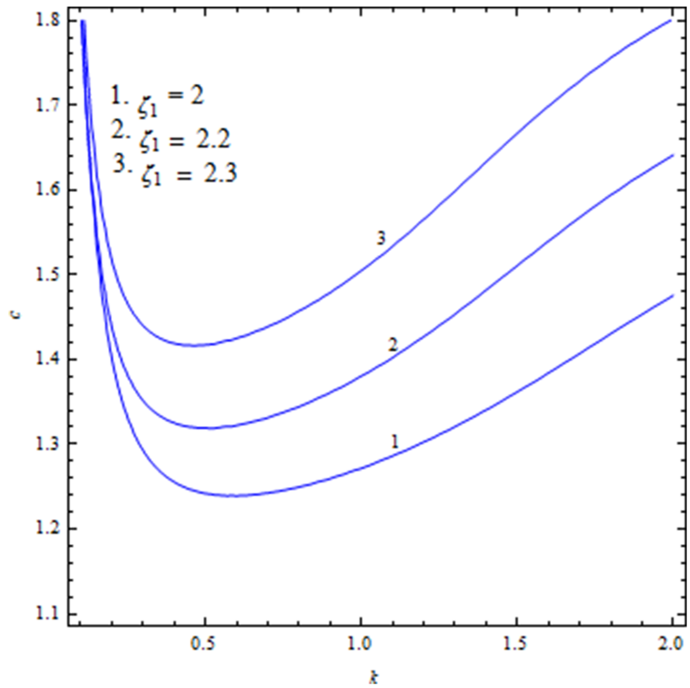

Fig. 6 Variation of Rayleigh wave velocity c with respect to wave number k for different values of corrugation ζ_1 of upper boundary surface when $\zeta_2 = 0$ for electrically open condition

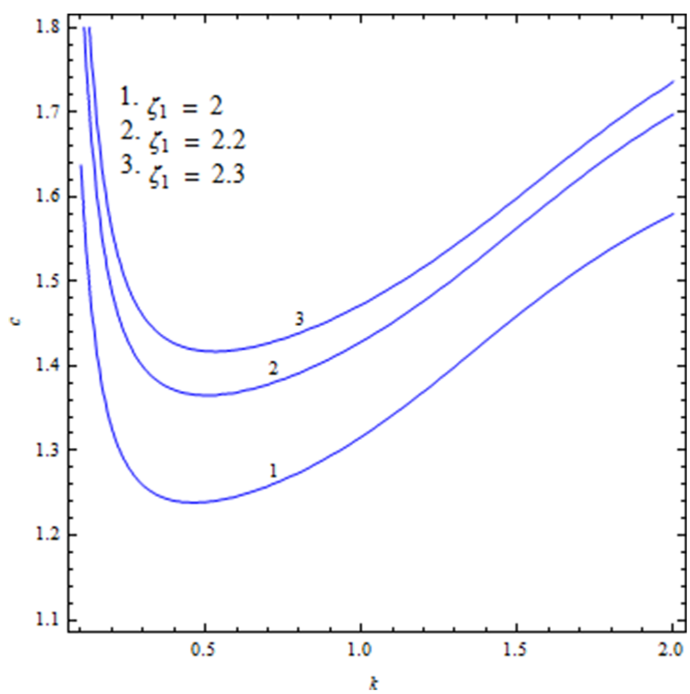

Fig. 7 Variation of Rayleigh wave velocity c with respect to wave number k for different values of corrugation (ζ_1) of upper boundary surface when $\zeta_2 = 0$ for electrically short condition

increment in the value of corrugation of the interface decreases the Rayleigh wave velocity in the electrically open case but increase the Rayleigh wave velocity in the electrically short case.

Furthermore, the substantial impact of the corrugated boundary of the common interface between layer and substrate on the frequency curves has been obtained and shown in Figs. 12 and 13. It is

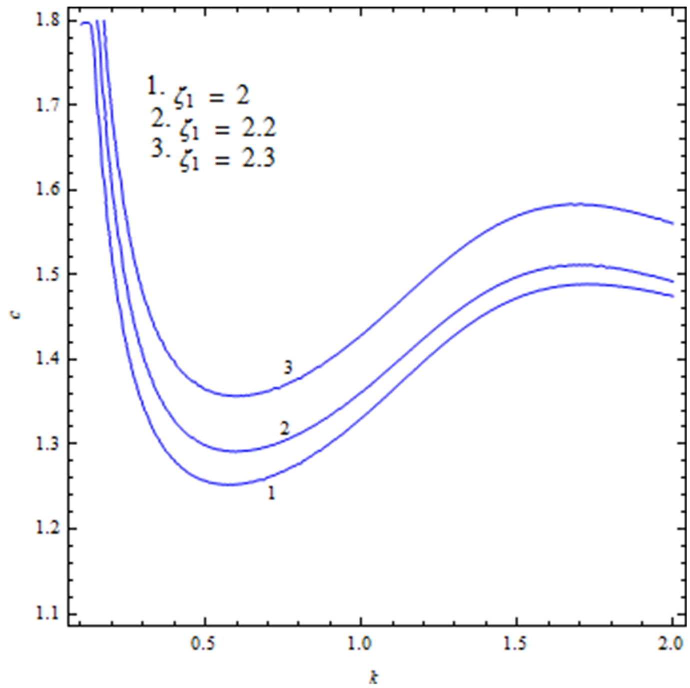

Fig. 8 Variation of Rayleigh wave velocity c with respect to wave number k for different values of corrugation ζ_1 of upper boundary surface when $\zeta_2 \neq 0$ for electrically open condition

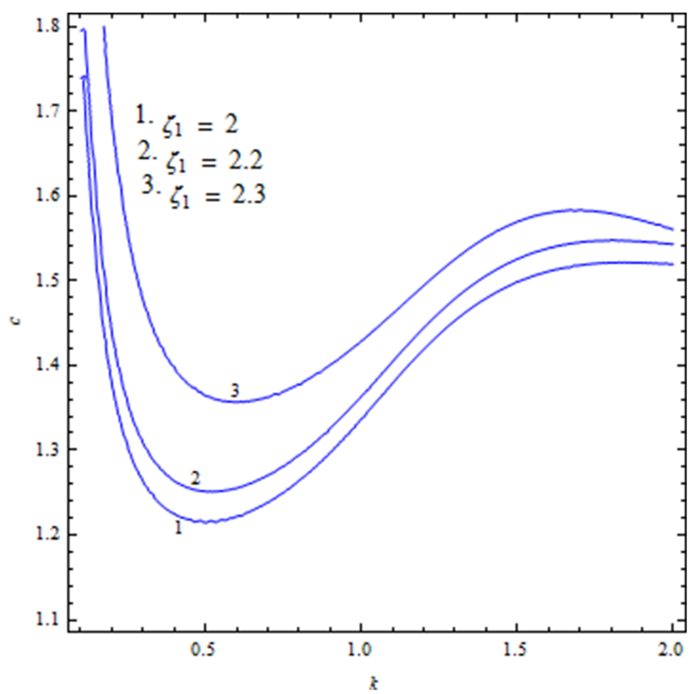

Fig. 9 Variation of Rayleigh wave velocity c with respect to wave number k for different values of corrugation (ζ_1) of upper boundary surface when $\zeta_2 \neq 0$ for electrically short condition

noticed that the Rayleigh wave velocity decreases with increasing value of corrugation of the common interface when the upper surface is corrugated, for both the electrically open and short case.

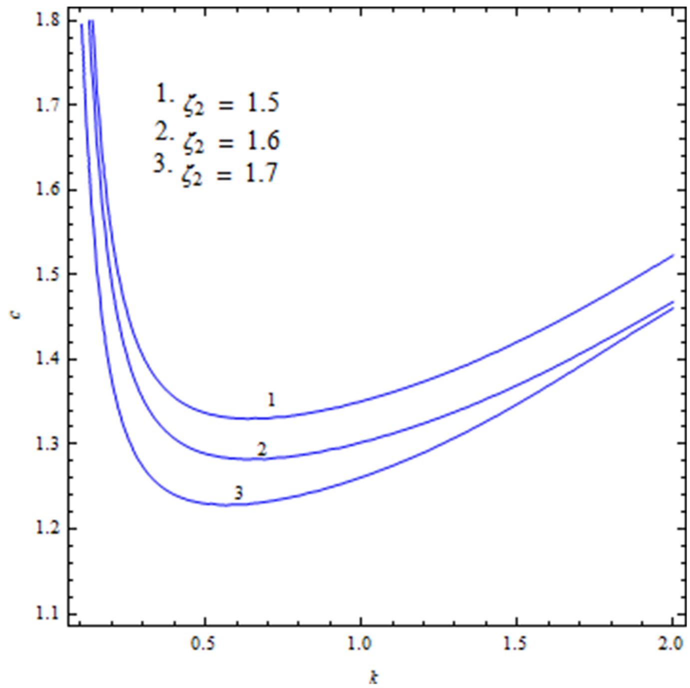

Fig. 10 Variation of Rayleigh wave velocity c with respect to wave number k for different values of corrugation ζ_2 of the interface when $\zeta_1 = 0$ for electrically open condition

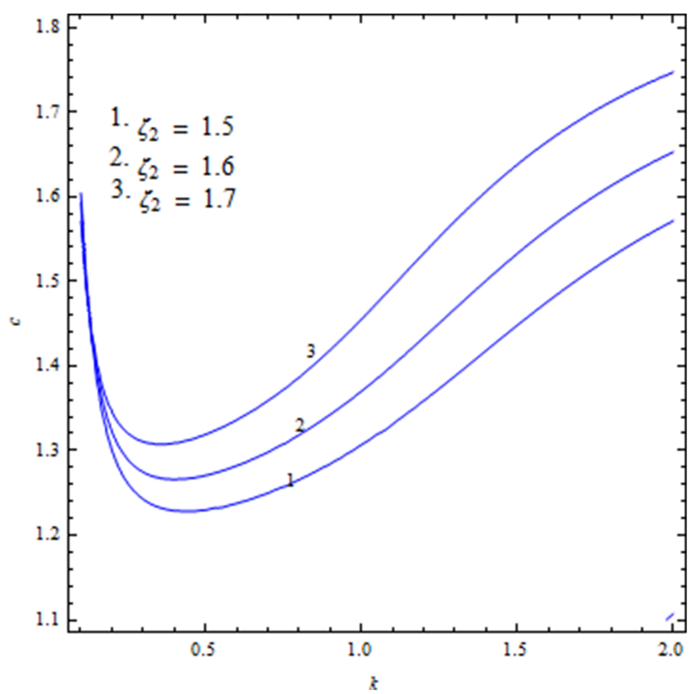

Fig. 11 Variation of Rayleigh wave velocity c with respect to wave number k for different values of corrugation (ζ_2) of interface when $\zeta_1 = 0$ for electrically short condition

7.5 Effect of rotation parameters on frequency equation

The substantial effect of the rotation parameters of the piezoelectric layer and orthotropic substrate on the frequency curves is demonstrated through Figs. 14, 15, 16 and 17. Figures 14 and 15 distinctly reveal the

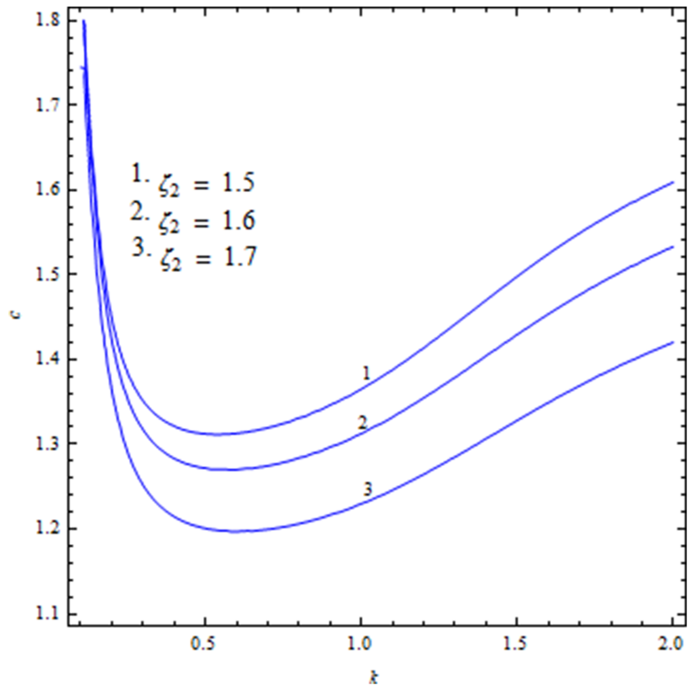

Fig. 12 Variation of Rayleigh wave velocity c with respect to wave number k for different values of corrugation ζ_2 of interface when $\zeta_1 \neq 0$ for electrically open condition

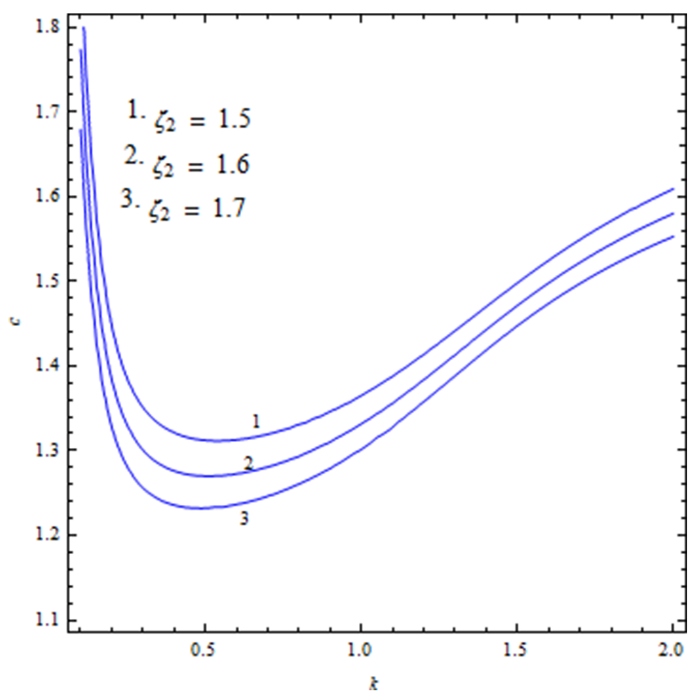

Fig. 13 Variation of Rayleigh wave velocity c with respect to wave number k for different values of corrugation ζ_2 of interface when $\zeta_1 \neq 0$ for electrically short condition

effect of the rotation parameter of the piezoelectric layer for the electrically open and short case, respectively. Moreover, Figs. 16 and 17 represents the effect of the rotation parameter of the orthotropic substrate for the electrically open and short case, respectively. These four figures establish that the rotation parameter of either medium decreases the Rayleigh wave velocity remarkably.

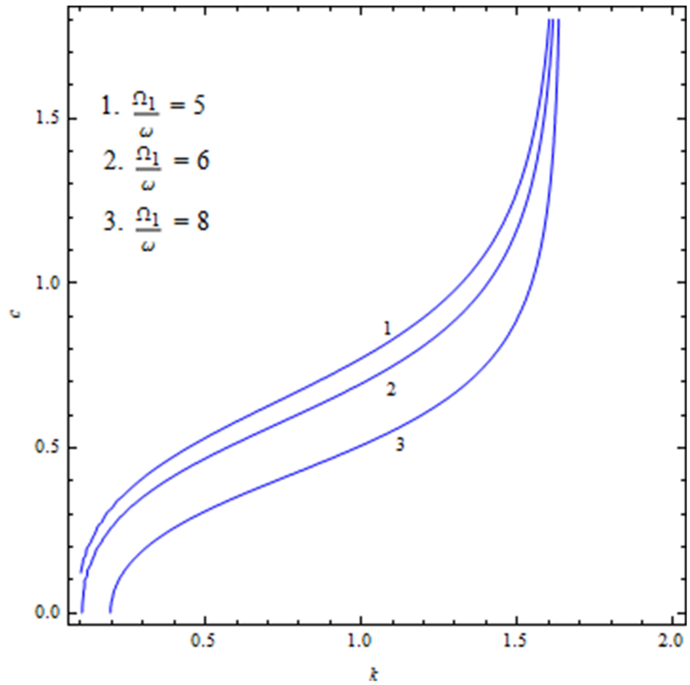

Fig. 14 Variation of Rayleigh wave velocity c with respect to wave number k for different values of rotation parameter of corrugated piezoelectric layer for electrically open condition

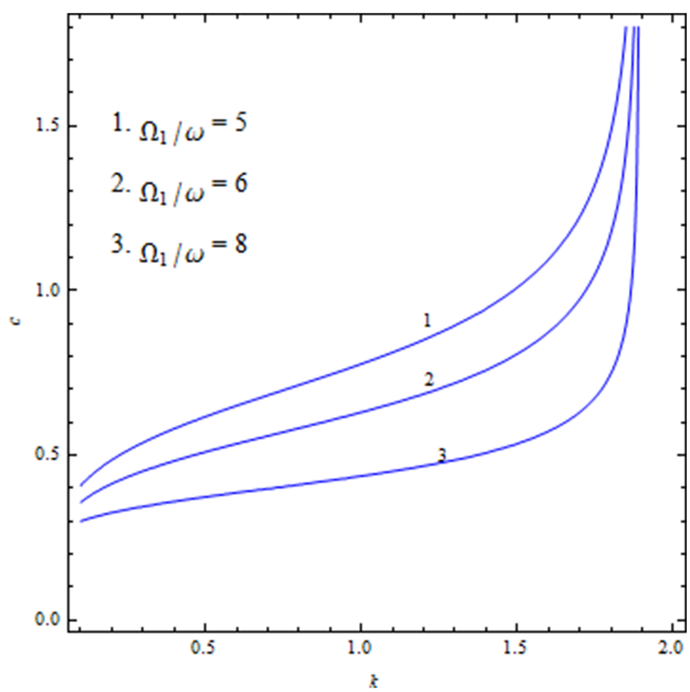

Fig. 15 Variation of Rayleigh wave velocity c with respect to wave number k for different values of rotation parameter for corrugated piezoelectric layer for electrically short condition

7.6 Effect of initial stress on frequency equation

Figures 18, 19, 20 and 21 manifest that profound effect of initial stress of both piezoelectric layer and orthotropic half-space on the frequency curves. The effect of initial stress of the corrugated piezoelectric layer is represented

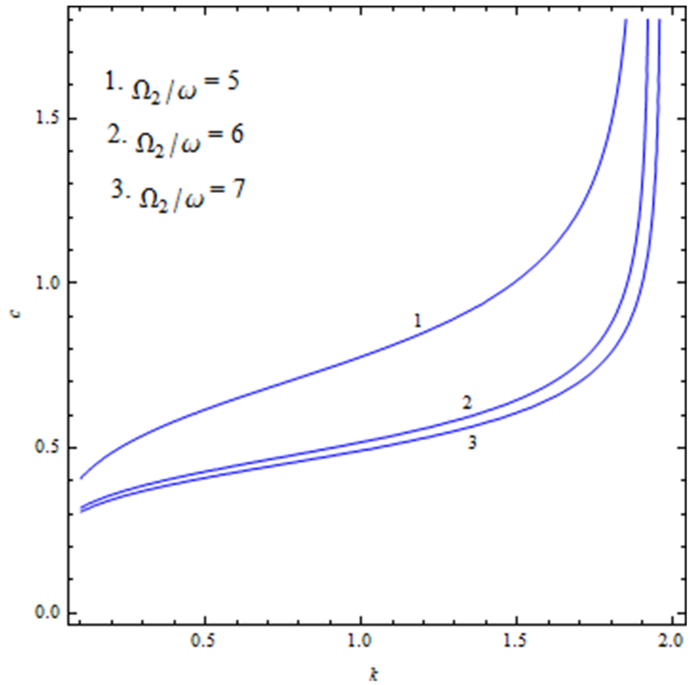

Fig. 16 Variation of Rayleigh wave velocity c with respect to wave number k for different values of rotation parameter of corrugated substrate for electrically open condition

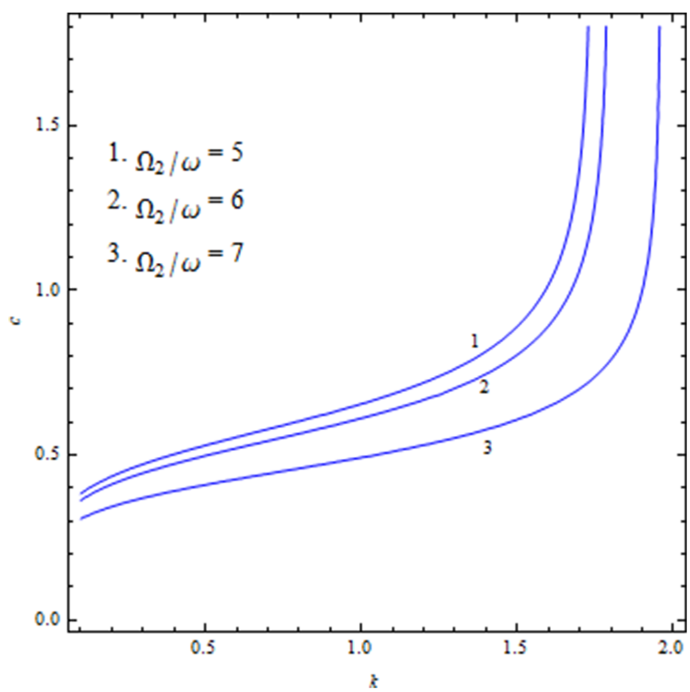

Fig. 17 Variation of Rayleigh wave velocity c with respect to wave number k for different values of rotation parameter of corrugated substrate for electrically short condition

through Figs. 18 and 19 for the electrically open and short case, respectively. Both Figs. 18 and 19 reveal that the Rayleigh wave velocity decreases with an increase in the value of initial stress. Figures 20 and 21 distinctly study the effect of initial stress of the orthotropic substrate on the frequency curves for the electrically open and short case, respectively. It is established that for the electrically open case the Rayleigh

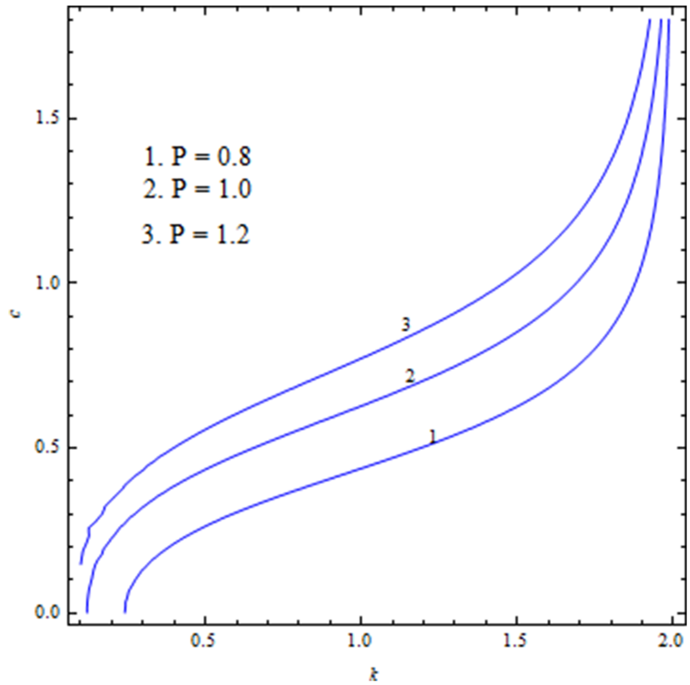

Fig. 18 Variation of Rayleigh wave velocity c with respect to wave number k for different values of initial stress of corrugated substrate for electrically open condition

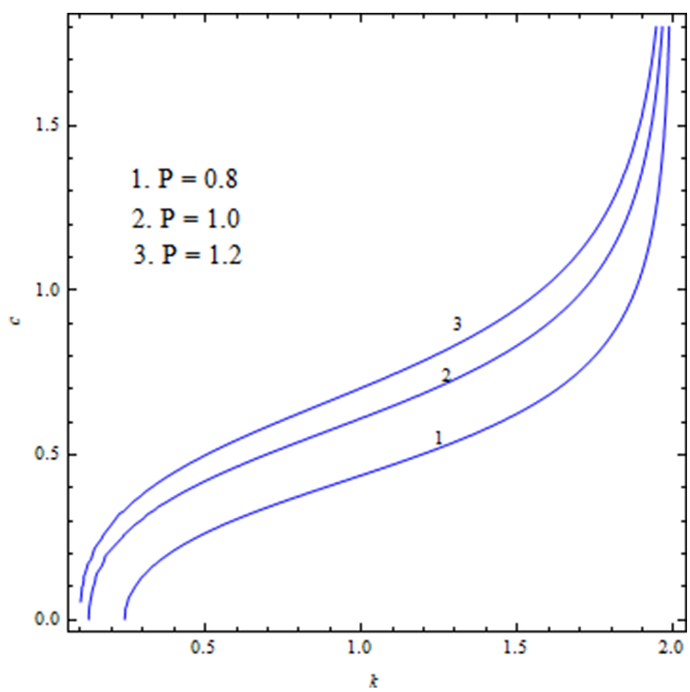

Fig. 19 Variation of Rayleigh wave velocity c with respect to wave number k for different values of initial stress of the corrugated substrate for electrically short condition

wave velocity decreases with increasing value of initial stress, but it has a reverse effect in the electrically short case.

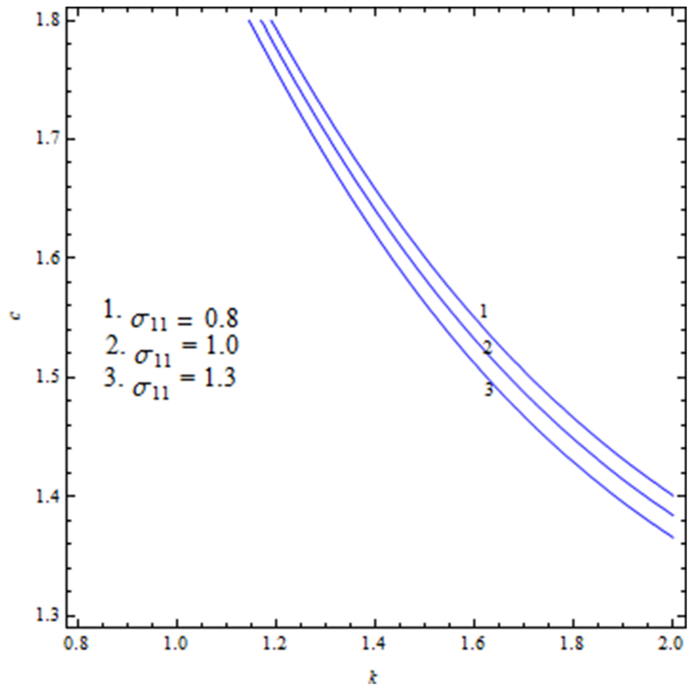

Fig. 20 Variation of Rayleigh wave velocity c with respect to wave number k for different values of initial stress of corrugated piezoelectric layer for electrically open condition

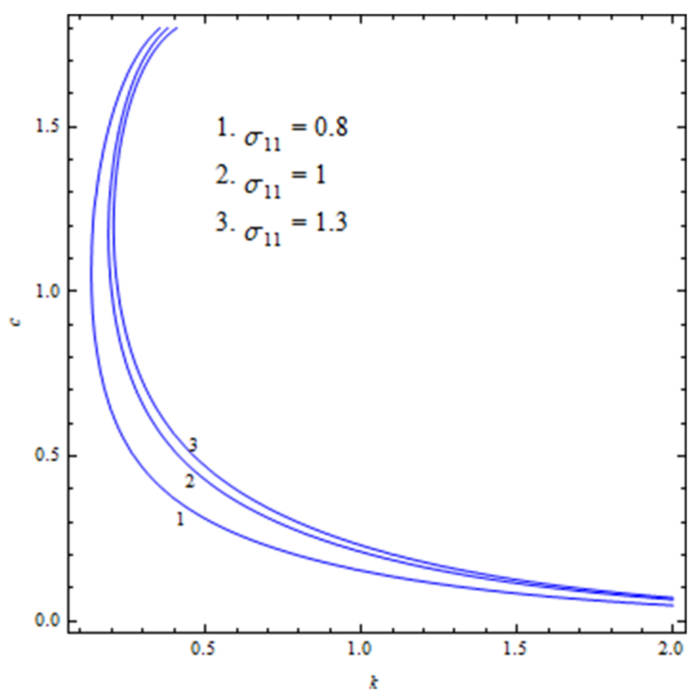

Fig. 21 Variation of Rayleigh wave velocity c with respect to wave number k for different values of initial stress $\left(\sigma_{11}^{1}\right)$ of the corrugated layer for electrically short condition

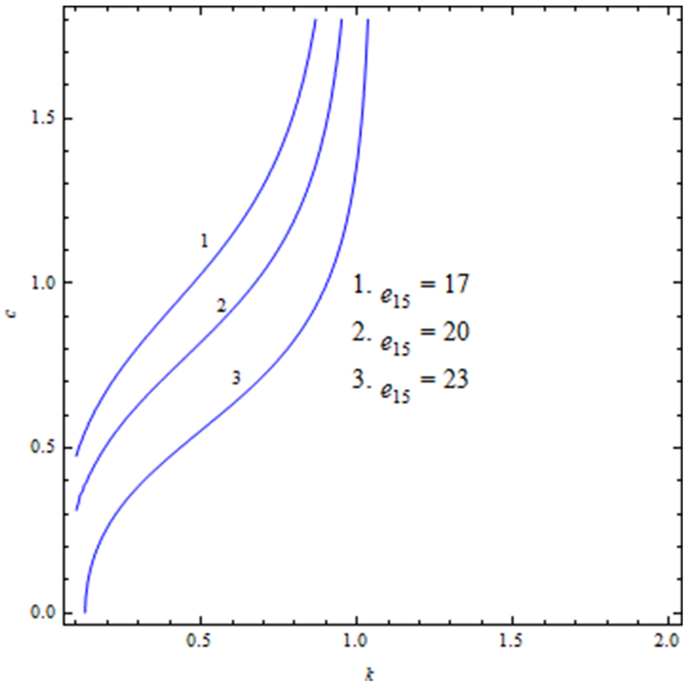

Fig. 22 Variation of Rayleigh wave velocity c with respect to wave number k for different values of piezoelectric constant (e_{15}) of the corrugated piezoelectric layer for electrically open condition

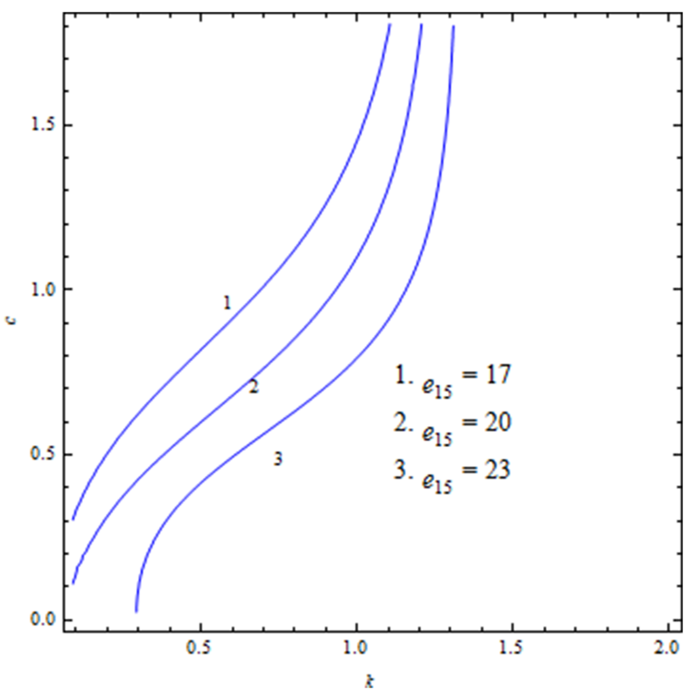

Fig. 23 Variation of Rayleigh wave velocity c with respect to wave number k for different values of piezoelectric constant (e_{15}) of the corrugated layer for electrically short condition

7.7 Effect of piezoelectricity on frequency equation

To exhibit the effect of piezoelectricity on the Rayleigh wave velocity, Figs. 22 and 23 have been plotted for the electricity open and short case, respectively. These graphs indicate that an increase in the value of the piezoelectric constant decreases the Rayleigh wave velocity in both the electric open and short conditions.

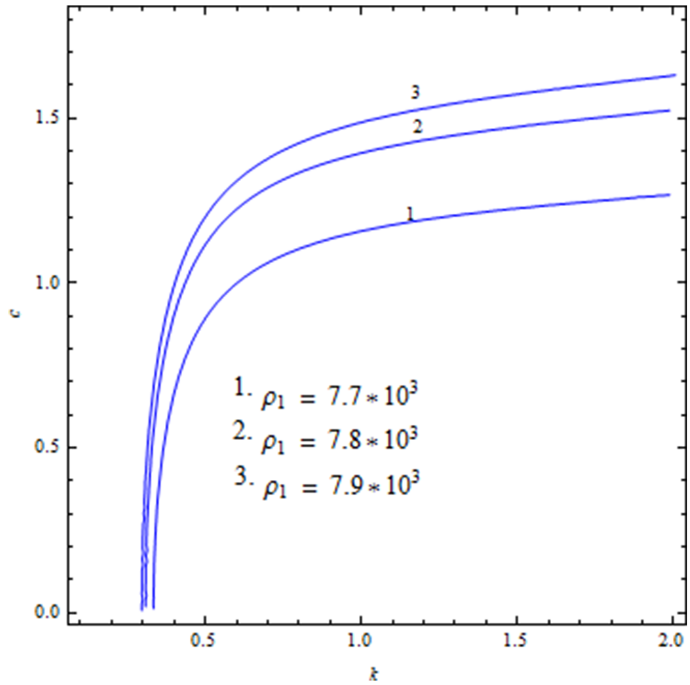

Fig. 24 Variation of Rayleigh wave velocity c with respect to wave number k for different values of density (ρ_1) of corrugated piezoelectric layer for electrically open condition

7.8 Effect of density on frequency equation

Figures 24, 25, 26 and 27 are plotted to display the effect of the density parameter of the piezoelectric layer and orthotropic substrate for both the electrically open and short case. The effect of the density parameter of the piezoelectric layer on the frequency curve is represented in Figs. 24 and 25 for electrically open and short case, respectively. It is seen from Figs. 24 and 25 that as the density of the layer increases, the Rayleigh wave velocity increases. Moreover, Figs. 26 and 27 represent the effect of substrate density for the electrically open and short case, respectively. It is concluded that an increase in the density has a reverse effect on the Rayleigh wave velocity in both the electrically open and short case.

7.9 Validation of analytical solution through finite element method (FEM)

To validate the analytical solution, we have applied the Finite Element Method and the comparison has been shown by graph. In the finite element method, the problem domain is discrietized into smaller regions called elements which are connected at specific points called nodes (Fig. 28). The detailed theory about FEM can be found in Bathe [26]. The elements of the domain are usually polygons with three or four corners, but elements with curved sides can be introduced in order to follow curved domain boundaries [27]. There is always a node at each corner of the element, and often one or more nodes equally spaced along the sides. In this paper a triangular element is used with element size 0.08 cm approx with three degrees of freedom per node (i.e., two components of displacement and the electric potential). The values of the field variables at any other arbitrary position on the element are given by a linear combination of polynomial interpolation functions with nodal point values of the quantities as the coefficients [28]. The choice of shape functions or interpolation functions determines or approximates how the field varies across a single-element domain. Normally, a polynomial function is chosen as a shape function and the number of nodes assigned to a particular element defines the order of the polynomial. For each of the three nodes in the considered element, a shape function (N_i) will be defined, such that it is unity at node i and is zero at all other nodes as well as at outside of the element. Also, the sum of all the shape functions will be at unity anywhere within an element. The displacements in each element is assumed to be a function of displacement in the n nodes of the element where interpolation functions N_1 and N_2 take care of the mapping from the displacement in the nodal point to the displacement at an arbitrary point in an element. Thus, in the finite element method, the displacement (u) and electric potential (ϕ) can be expressed in terms of the corresponding nodal values of the element $\{u^e\}$ and $\{\phi^e\}$ as

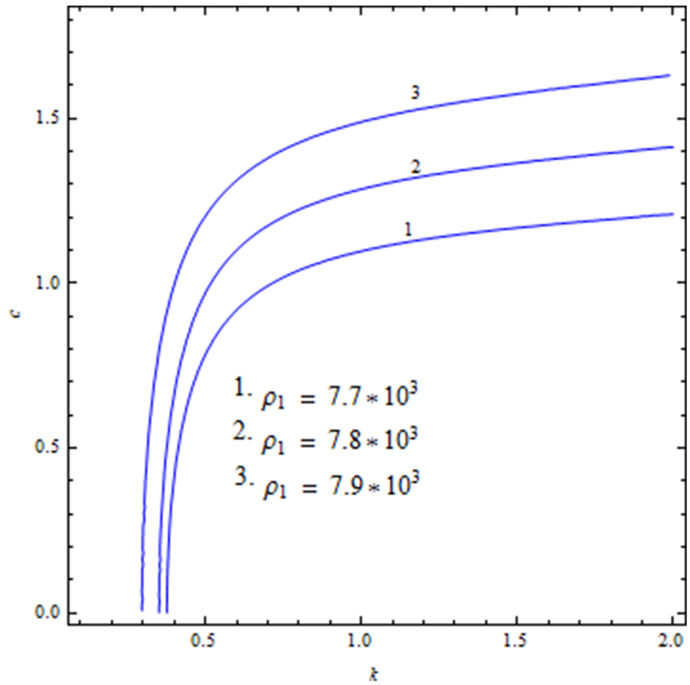

Fig. 25 Variation of Rayleigh wave velocity c with respect to wave number k for different values of density (ρ_1) of corrugated piezoelectric layer for electrically short condition

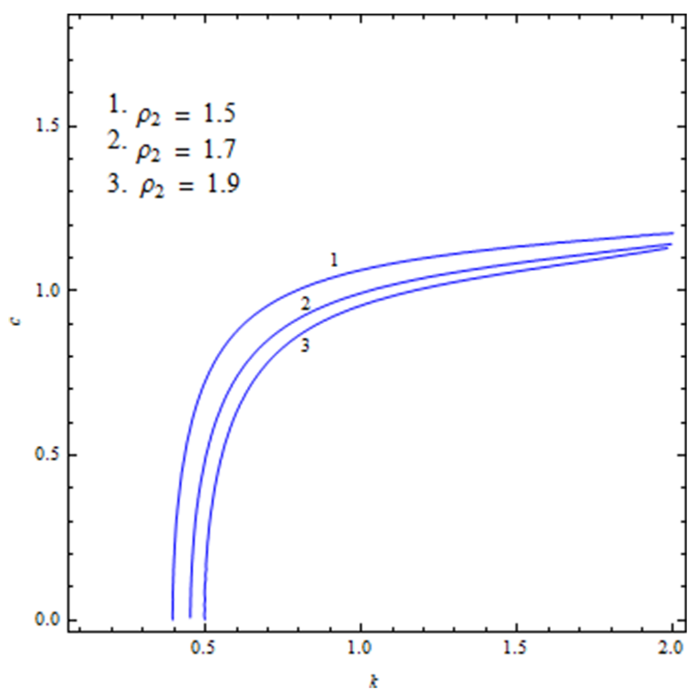

Fig. 26 Variation of Rayleigh wave velocity c with respect to wave number k for different values of density (ρ_2) of corrugated half-space for electrically open condition

$$\{u\} = [N_1^e]\left\{u^e\right\} \quad \text{and} \quad \{\phi\} = \left\{N_2^e\right\}^T \left\{\phi^e\right\}, \tag{73}$$

where

$$\left[N_1^e\right] = \begin{bmatrix} N_1 & 0 & N_2 & 0 & \dots & N_q & 0 \\ 0 & N_1 & 0 & N_2 & \dots & 0 & N_q \end{bmatrix}$$

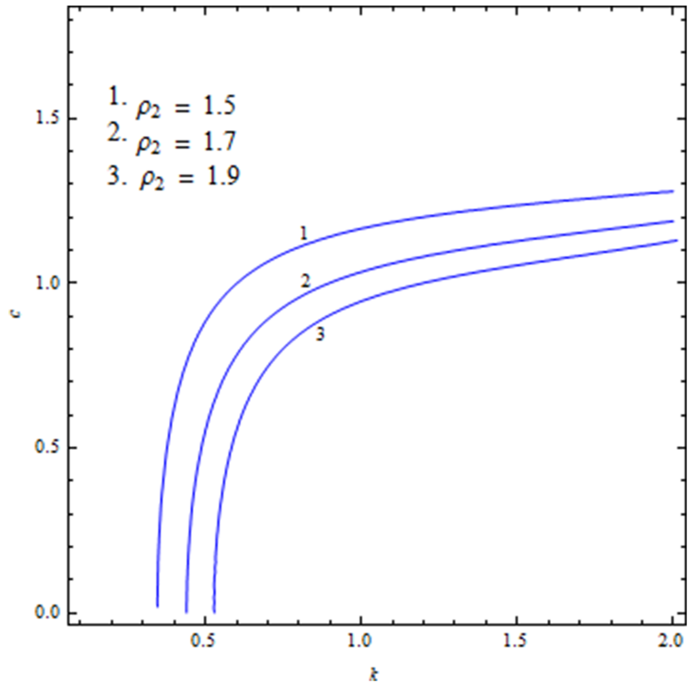

Fig. 27 Variation of Rayleigh wave velocity c with respect to wave number k for different values of density (ρ_2) of corrugated half-space for electrically short condition

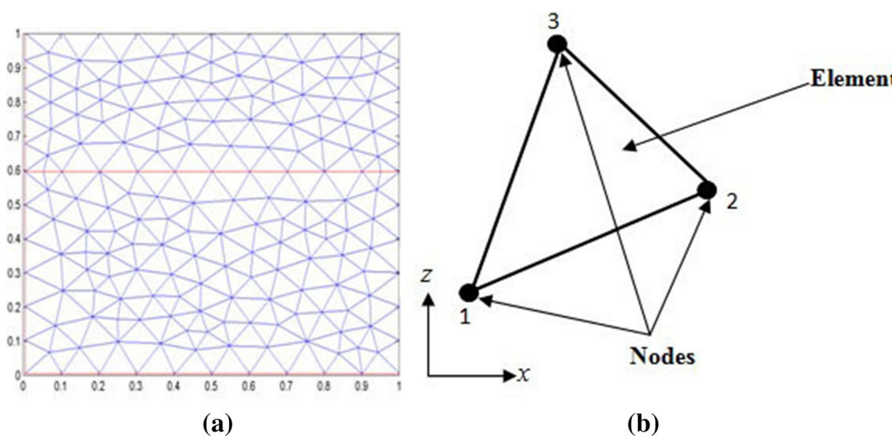

Fig. 28 Discretization of the problem domain in FEM. **a** Mesh, **b** Nodal placements in a triangular element

and

$$\left\{N_2^e\right\}^T = \left\{N_1 \quad N_2 \quad \ldots \quad N_q\right\},$$

where q denotes the number of nodes in the grid and the polynomial degree of the shape functions is one. We know that the electric potential is scalar, so their shape functions are the same.

Now, let us rewrite Eq. (3) in vector form as

$$\begin{aligned} \{\sigma\} &= [c]\{\varepsilon\} - [e]\{E\}, \\ \{D\} &= [e]^T\{\varepsilon\} + [p]\{E\}. \end{aligned} \tag{74}$$

According to the following geometrical equations

$$\varepsilon_{kl} = \frac{1}{2}\left(u_{i,j} + u_{j,i}\right) \quad \text{and} \quad E_m = -\phi_{,m}, \tag{75}$$

we can have

$$\{\varepsilon\} = [S_1]\left\{u^e\right\} \text{ and } \{E\} = [S_2]\left\{\phi^e\right\}, \tag{76}$$

where S_1 is the strain matrix.

The coordinates are considered in x and z direction, so $[S_1]$ and $[S_2]$ have the following form:

$$[S_1] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & 0 & \frac{\partial N_2}{\partial x} & 0 & \ldots & \frac{\partial N_n}{\partial x} & 0 \\ 0 & \frac{\partial N_1}{\partial z} & 0 & \frac{\partial N_2}{\partial z} & \ldots & 0 & \frac{\partial N_n}{\partial z} \\ \frac{\partial N_1}{\partial z} & \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial z} & \frac{\partial N_2}{\partial x} & \ldots & \frac{\partial N_n}{\partial z} & \frac{\partial N_n}{\partial x} \end{bmatrix},$$

$$[S_2] = \begin{bmatrix} \frac{\partial N_1}{\partial x} & \frac{\partial N_2}{\partial x} & \ldots & \frac{\partial N_n}{\partial x} \\ \frac{\partial N_1}{\partial z} & \frac{\partial N_2}{\partial z} & \ldots & \frac{\partial N_n}{\partial z} \end{bmatrix}.$$

The variational form of Eq. (76) can be written as

$$\delta\left(\varepsilon\right) = [S_1]\,\delta\left(u^e\right), \quad \delta\left(E\right) = -[S_2]\,\delta\left(\phi^e\right). \tag{77}$$

Considering the body force $\{f\}$, the virtual displacement principle can be written as

$$\int_\Omega \left[\delta\left(\varepsilon\right)^T\{\sigma\} - \delta\left(E\right)^T\{D\}\right]\mathrm{d}\Omega = \int_\Omega \left[\delta\left(u\right)^T\left(\{f\} - \rho\{\ddot{u}\}\right)\right]\mathrm{d}\Omega$$
$$+ \int_{A\sigma}\left[\delta\left(u\right)^T\left\{\bar{T}\right\}\right]\mathrm{d}A + \int_{Aw}\delta\phi\bar{w}\,\mathrm{d}A, \tag{78}$$

where $\bar{T}$ denotes the components of the traction vector.

With the help of Eqs. (74) and (78), we get

$$\int_\Omega \delta\{\varepsilon\}^T\{\sigma\}\,\mathrm{d}\Omega = \delta\left\{u^e\right\}^T\int_\Omega [S_1]^T\left([C][S_1]\left\{u^e\right\} - [e]\left(-[S_2]\left\{\phi^e\right\}\right)\right)\mathrm{d}\Omega$$
$$= \delta\left\{u^e\right\}^T\left(\left[K_{mm}^e\right]\left\{u^e\right\} + \left[K_{me}^e\right]\left\{\phi^e\right\}\right), \tag{79}$$

$$\int_\Omega -\delta\{E\}^T\{D\}\,\mathrm{d}\Omega = \delta\left\{\phi^e\right\}^T\int_\Omega [S_2]^T\left([e]^T[S_1]\left\{u^e\right\} + [p]\left(-[S_2]\left\{\phi^e\right\}\right)\right)\mathrm{d}\Omega$$
$$= \delta\left\{\phi^e\right\}^T\left(\left[K_{em}^e\right]\left\{u^e\right\} - \left[K_{ee}^e\right]\left\{\phi^e\right\}\right), \tag{80}$$

$$\int_\Omega \delta\{u\}^T\left(\{f\} - \rho\{\ddot{u}\}\right)\mathrm{d}\Omega = \delta\left\{u^e\right\}^T\int_\Omega \left[N_1^e\right]^T\left(\{f\} - \rho\{\ddot{u}\}\left[N_1^e\right]\right)\mathrm{d}\Omega$$
$$= \delta\left\{u^e\right\}^T\left(\left\{f_m^e\right\} - \left[M_{mm}^e\right]\left\{\ddot{u}^e\right\}\right), \tag{81}$$

$$\int_{Aw}\delta\phi\bar{w}\,\mathrm{d}A = \delta\left\{\phi^e\right\}^T\int_{Aw}\left[N_2^e\right]^T\bar{w}\,\mathrm{d}A = \delta\left\{\phi^e\right\}^T\left\{T_e^e\right\}, \tag{82}$$

$$\int_{A\sigma}\delta\{u\}^T\left\{\bar{T}\right\}\mathrm{d}A = \delta\left\{u^e\right\}^T\int_{A\sigma}\left[N_1^e\right]^T\left\{\bar{T}\right\}\mathrm{d}A = \delta\left\{u^e\right\}^T\left\{T_m^e\right\}. \tag{83}$$

From Eqs. (79) to (83), we conclude

$$[M_{mm}]\,\ddot{u} + [C_{mm}]\,\dot{u} + [K_{mm}]\,u + [K_{me}]\,\phi = F$$

and

$$[K_{me}]^t\,u + [K_{ee}]\,\phi = Q,$$

where u, F, ϕ, Q are global field quantities and M, C, K are the global matrices and all the values are given in Appendix 3.

The above equations can be written in matrix form as

$$\begin{bmatrix} M_{mm} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \ddot{u} \\ \ddot{\phi} \end{bmatrix} + \begin{bmatrix} C_{mm} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{\phi} \end{bmatrix} + \begin{bmatrix} K_{mm} & K_{me}^t \\ K_{me} & K_{ee} \end{bmatrix} \begin{bmatrix} u \\ \phi \end{bmatrix} = \begin{pmatrix} F \\ Q \end{pmatrix}, \tag{84}$$

where the subscript m denotes the displacement vector whereas ϕ refers to the electric potential vector. C_{mm} is the structural damping matrix, K_{mm} is the structural stiffness matrix, K_{me} is the finite element equivalent of the material piezoelectric matrix e and K_{ee} is the finite element equivalent of the capacitance matrix ε.

Equation (84) can be written as

$$[M]\{\ddot{u}\} + [C]\{\dot{u}\} + [K]\{u\} = \{L\}, \tag{85}$$

where

$$M = \begin{bmatrix} M_{mm} & 0 \\ 0 & 0 \end{bmatrix}, \quad C = \begin{bmatrix} C_{mm} & 0 \\ 0 & 0 \end{bmatrix}, \quad K = \begin{bmatrix} K_{mm} & K_{me}^t \\ K_{me} & K_{ee} \end{bmatrix} \quad \text{and} \quad L = \begin{pmatrix} F \\ Q \end{pmatrix}.$$

Equation (85) is the set of ordinary differential equations whose solution provides the values at each node in the finite element mesh. Equation (85) is solved directly in time domain by direct integration method, the Newmark method. Direct integration means that Eq. (85) is directly integrated through a numerical step-by-step procedure without transforming the equations to other forms. Though the integration methods which follow work equally well for non uniform intervals, but we assume that the intervals are uniform and are equal to Δt. The algorithms are to determine the solutions at $\Delta t, 2\Delta t, \ldots, t - \Delta t, t, t + \Delta t, \ldots, N\Delta t$. The solutions at $t + \Delta t = (i + 1)\,\Delta t$ are based on those, obtained for previous steps up to $t = i\,\Delta t$. Furthermore, the boundary condition for the displacement and electric potential are applied by directly inserting the prescribed values in the vectors u and ϕ [29]. Similarly we can set up the formulation for the orthotropic elastic substrate following the same procedure as done for the piezoelectric layer. On the interface the boundary conditions are given by continuity of the traction vector and the displacement vector u_i (Reinen and Berg [30]).

The Newmark method

To derive the algorithm of the Newmark method, we start with the Taylor series expansion of the displacement and its time derivative keeping terms up to the third-order derivative,

$$u_{t+\Delta t} = u_t + \dot{u}_t \Delta t + \frac{1}{2}\ddot{u}_t \Delta t^2 + \frac{1}{6}\left(\ddot{u}_{t+\Delta t} - \dot{u}_t\right)\Delta t^2, \tag{86}$$

$$\dot{u}_{t+\Delta t} = \dot{u}_t + \ddot{u}_t \Delta t + \frac{1}{2}\left(\ddot{u}_{t+\Delta t} - \dot{u}_t\right)\Delta t, \tag{87}$$

where the third time derivative has been replaced with the difference of the second time derivatives under the assumption of linear acceleration. The Newmark integration scheme extends the above equations by changing the numerical coefficients $1/6$ in Eq. (86) and $1/2$ in Eq. (87), to somewhat arbitrary parameters ξ and χ, respectively, where we can chose ξ and χ accordingly:

$$u_{t+\Delta t} = u_t + \dot{u}_t \Delta t + \left[\left(\frac{1}{2} - \xi\right)\ddot{u}_t + \xi \ddot{u}_{t+\Delta t}\right]\Delta t^2, \tag{88}$$

$$\dot{u}_{t+\Delta t} = \dot{u}_t + \ddot{u}_t \Delta t + \left[(1-\chi)\,\ddot{u}_t + \chi \ddot{u}_{t+\Delta t}\right]\Delta t. \tag{89}$$

From Eq. (88), we have

$$\ddot{u}_{t+\Delta t} = \frac{1}{\xi \Delta t^2}\left(u_{t+\Delta t} - u_t\right) + \frac{1}{\xi \Delta t}\dot{u}_t + \left(1 - \frac{1}{2\xi}\right)\ddot{u}_t. \tag{90}$$

Introducing Eq. (90) in Eq. (89), we get

$$\dot{u}_{t+\Delta t} = \frac{\chi}{\xi \Delta t}\left(u_{t+\Delta t} - u_t\right) + \left(1 - \frac{\chi}{\xi}\right)\dot{u}_t + \left(1 - \frac{1}{2\xi}\right)\ddot{u}_t \Delta t. \tag{91}$$

At $t + \Delta t$ the system equations have the following form:

$$M\ddot{u}_{t+\Delta t} + C\dot{u}_{t+\Delta t} + K u_{t+\Delta t} = L_{t+\Delta t}. \tag{92}$$

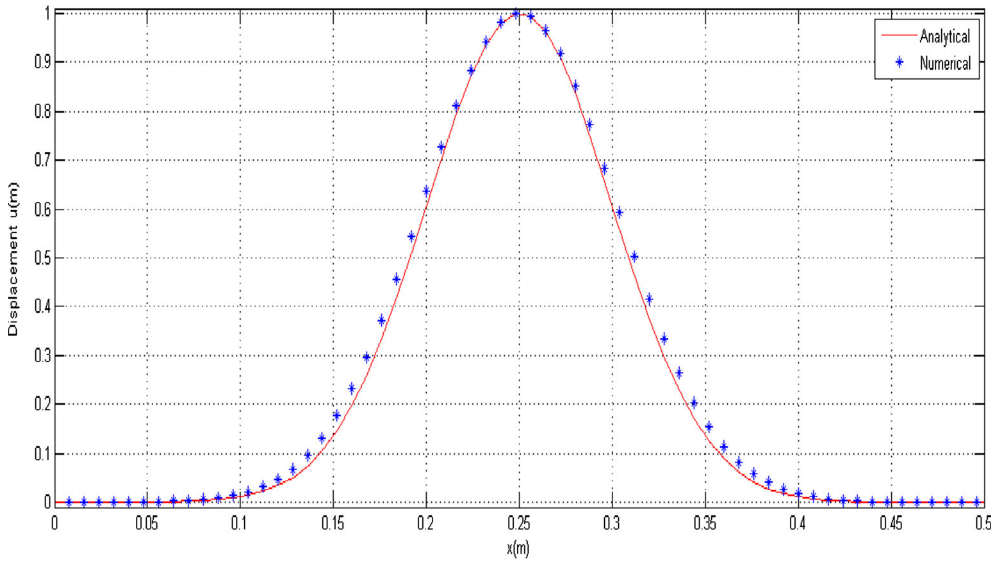

Fig. 29 Comparison between analytical solution and result by finite element method

Substituting Eqs. (90) and (91) in Eq. (92), we obtain

$$\left(\frac{1}{\xi\Delta t^2}M+\frac{\chi}{\xi\Delta t}C+K\right)u_{t+\Delta t}=L_{t+\Delta t}+\left(\frac{1}{\xi\Delta t^2}M+\frac{\chi}{\xi\Delta t}C\right)u_t+\left[\frac{1}{\xi\Delta t}M+\left(1-\frac{\chi}{\xi}\right)C\right]\dot{u}_t$$
$$+\left[\left(\frac{1}{2\xi}-1\right)M-\left(1-\frac{\chi}{2\xi}\right)C\Delta t\right]\ddot{u}_t. \tag{93}$$

It is seen that the Newmark integration scheme is a single step implicit method. No starting procedure is needed for the time stepping. It can be shown that this method is unconditionally stable for suitably chosen ξ and χ. This means that a larger time step can be used if desired. Appearance of the term involving K on the left side of Eq. (93) requires the computationally intensive matrix inversion procedure. In the used finite element method, the region under consideration is $\Omega = \{(x,z) : 0 < x < l, 0 < z < h\}$. Also we consider $l = h = 1m$ and the Newmark parameters have been taken as $\xi = 0.5$ and $\chi = 0.25$. The displacement variation along the x-axis at $t = 1s$ has been shown in Fig. 29.

The above graph displays the matching of the obtained analytical solution with that of the numerical method (FEM). It is pretty clear that the analytical solution is in high agreement with the numerical results, which validates the present study.

8 Conclusions

An analytical approach is used to investigate the Rayleigh wave propagation in rotating, pre-stressed piezoelectric layer overlying a gravitational orthotropic substratum with initial stress and rotation. Upper boundary surface of the piezoelectric layer and interface between layer and substrate both are taken with corrugation. Frequency equations have been obtained in determinant form for electrically open and short cases. The obtained results are validated by matching the findings with the classical Rayleigh wave equation and other existing results. Also the analytical solution is matched with numerical solution using finite element method (FEM). It is noticed that the Rayleigh wave velocity is affected significantly by different parameters (corrugation, piezoelectric, undulation, position parameter, rotation, initial stress etc.). The results may be used to improve the efficiency and life of certain seismic devices. Finally, we may conclude with the following points:

1. For the electrically open case, the Rayleigh wave velocity decreases with an increment in the undulation parameter and initial stress whereas it has a reverse effect for the electrically short case.
2. The Rayleigh wave velocity increases with increasing value of the position parameter for the electrically open case and it shows the reverse effect for the electrically short case.

3. For the electrically open case, the Rayleigh wave velocity decreases with increasing value of the corrugation parameter of the interface between layer and half-space whereas it has a reverse effect for the electrically short case.
4. The Rayleigh wave velocity decreases for increasing values of the rotation parameter (of either of the two media) for both electrically open and short case.
5. The obtained results are found in agreement with the results established by Abd-Alla et al. [16], Abd-Alla [15] and Rayleigh [24].
6. The obtained analytical results are compared with the results obtained by the finite element method (FEM) and are found to be in good agreement.
7. The obtained results may be helpful in further experimental works related to surface wave propagation in piezoelectric materials and may also be used for filters, resonators, oscillators, sensors and signal processing elements. This can be achieved by obtaining the desired propagation of Rayleigh wave by selecting the appropriate material, elastic constants and other boundary conditions. This work may be relevant to the analysis and design of various acoustic surface wave devices constructed from piezoelectric materials combined with elastic substrates.

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Appendix 1

$$B_{11}=\left(-c_{44}ik\lambda_1+aikc_{44}+a'ike_{15}\right)e^{ik\lambda_1 H},\ B_{12}=\left(-c_{44}ik\lambda_2+bikc_{44}+b'ike_{15}\right)e^{ik\lambda_2 H},$$
$$B_{13}=\left(-c_{44}ik\lambda_3+cikc_{44}+c'ike_{15}\right)e^{ik\lambda_3 H},\ B_{14}=\left(c_{44}ik\lambda_1+dikc_{44}+d'ike_{15}\right)e^{-ik\lambda_1 H},$$
$$B_{15}=\left(c_{44}ik\lambda_2+eikc_{44}+e'ike_{15}\right)e^{-ik\lambda_2 H},\ B_{16}=\left(c_{44}ik\lambda_3+fikc_{44}+f'ike_{15}\right)e^{-ik\lambda_3 H},\ B_{17}=0=A_{18}.$$
$$B_{21}=\left[\left(C_{13}ik-\lambda_1 aikC_{33}-a'ik\lambda_1 e_{33}\right)\right]e^{ik\lambda_1 H},\ B_{22}=\left[\left(C_{13}ik-\lambda_2 bikC_{33}-b'ik\lambda_2 e_{33}\right)\right]e^{ik\lambda_2 H},$$
$$B_{23}=\left[\left(C_{13}ik-\lambda_3 cikC_{33}-c'ik\lambda_3 e_{33}\right)\right]e^{ik\lambda_3 H},\ B_{24}=\left[\left(C_{13}ik+dik\lambda_1 C_{33}+d'ik\lambda_1 e_{33}\right)\right]e^{-ik\lambda_1 H},$$
$$B_{25}=\left[\left(C_{13}ik+eik\lambda_2 C_{33}+e'ik\lambda_2 e_{33}\right)\right]e^{-ik\lambda_2 H}B_{26}=\left[\left(C_{13}ik+fik\lambda_3 C_{33}+f'ik\lambda_3 e_{33}\right)\right]e^{-ik\lambda_3 H},$$
$$B_{27}=0=A_{28},\ A_{31}=\left[e_{15}\left(-ik\lambda_1+ika\right)+ika'\varepsilon_{11}\right]e^{ik\lambda_1 H},\ B_{32}=\left[e_{15}\left(-ik\lambda_2+ikb\right)+ikb'\varepsilon_{11}\right]e^{ik\lambda_2 H},$$
$$B_{33}=\left[e_{15}\left(-ik\lambda_3+ikc\right)+ikc'\varepsilon_{11}\right]e^{ik\lambda_3 H},\ B_{34}=\left[e_{15}\left(ik\lambda_1+ikd\right)+ikd'\varepsilon_{11}\right]e^{-ik\lambda_1 H},$$
$$B_{35}=\left[e_{15}\left(ik\lambda_2+ike\right)+ike'\varepsilon_{11}\right]e^{-ik\lambda_2 H},\ B_{35}=F\left[e_{15}\left(ik\lambda_3+ikf\right)+ikf'\varepsilon_{11}\right]e^{-ik\lambda_3 H}.$$
$$B_{41}=e^{-ik\lambda_1\zeta_2},\ B_{42}=e^{-ik\lambda_2\zeta_2},\ B_{43}=Ce^{-ik\lambda_3\zeta_2},\ B_{44}=De^{ik\lambda_1\zeta_2},\ B_{45}=Ee^{ik\lambda_2\zeta_2},\ B_{46}=Fe^{ik\lambda_3\zeta_2}$$
$$B_{47}=X_1 e^{-ik\lambda_4\zeta_2},\ B_{48}=X_2 e^{-ik\lambda_5\zeta_2}$$
$$B_{51}=ae^{-ik\lambda_1 a\cos(bx)},\ B_{52}=be^{-ik\lambda_2 a\cos(bx)},\ B_{53}=ce^{-ik\lambda_3 a\cos(bx)},\ B_{54}=de^{ik\lambda_1 a\cos(bx)},$$
$$B_{55}=ee^{ik\lambda_2 a\cos(bx)},$$
$$B_{56}=fe^{ik\lambda_3 a\cos(bx)},\ B_{57}=Y_1 e^{-ik\lambda_4 a\cos(bx)},\ B_{58}=Y_2 e^{-ik\lambda_5 a\cos(bx)},\ B_{61}=a'e^{-ik\lambda_1 a\cos(bx)},$$
$$B_{62}=b'e^{-ik\lambda_2 a\cos(bx)},$$
$$B_{63}=c'e^{-ik\lambda_3 a\cos(bx)},\ B_{64}=d'e^{ik\lambda_1 a\cos(bx)},\ B_{65}=e'e^{ik\lambda_2 a\cos(bx)},\ B_{66}=f'e^{ik\lambda_3 a\cos(bx)}$$
$$B_{71}=\left[\left(c_{13}ik-ik\lambda_1 ac_{33}-ik\lambda_1 a'e_{33}\right)+ab\sin(bx)\left(-c_{44}ik\lambda_1+ikac_{44}+a'ike_{15}\right)\right]e^{-ik\lambda_1 a\cos(bx)},$$
$$B_{72}=\left[\left(c_{13}ik-ik\lambda_2 bc_{33}-ik\lambda_2 b'e_{33}\right)+ab\sin(bx)\left(-c_{44}ik\lambda_2+ikbc_{44}+b'ike_{15}\right)\right]e^{-ik\lambda_2 a\cos(bx)},$$
$$B_{73}=\left[\left(c_{13}ik-ik\lambda_3 cc_{33}-ik\lambda_3 c'e_{33}\right)+ab\sin(bx)\left(-c_{44}ik\lambda_3+ikcc_{44}+c'ike_{15}\right)\right]e^{-ik\lambda_3 a\cos(bx)},$$
$$B_{75}=\left[\left(c_{13}ik+ik\lambda_2 ec_{33}+ik\lambda_2 e'e_{33}\right)+ab\sin(bx)\left(c_{44}ik\lambda_2+ikec_{44}+e'ike_{15}\right)\right]e^{ik\lambda_2 a\cos(bx)},$$
$$B_{76}=\left[\left(c_{13}ik+ik\lambda_3 fc_{33}+ik\lambda_3 f'e_{33}\right)+ab\sin(bx)\left(c_{44}ik\lambda_3+ikfc_{44}+f'ike_{15}\right)\right]e^{ik\lambda_3 a\cos(bx)},$$
$$B_{77}=\left[\left(c'_{13}ikX_1-c'_{33}ikY_1\lambda_4\right)+ab\sin(bx)\left(c'_{44}ikY_1-c'_{44}ikX_1\lambda_4\right)\right]e^{-k\lambda_4 a\cos(bx)},$$
$$B_{78}=\left[\left(c'_{13}ikX_2-c'_{33}ikY_2\lambda_5\right)+ab\sin(bx)\left(c'_{44}ikY_2-c'_{44}ikX_1\lambda_5\right)\right]e^{-k\lambda_5 a\cos(bx)},$$

$B_{81} = \left[\left(-c_{44}\lambda_1 + ac_{44} + a'e_{15}\right) ik + ab\sin(bx)\left(ikc_{11} - ik\lambda_1 ac_{13} + e_{31}a'\right)\right] e^{-ik\lambda_1 a\cos(bx)},$
$B_{82} = \left[\left(-c_{44}\lambda_2 + bc_{44} + b'e_{15}\right) ik + ab\sin(bx)\left(ikc_{11} - ik\lambda_2 bc_{13} + e_{31}b'\right)\right] e^{-ik\lambda_2 a\cos(bx)},$
$B_{83} = \left[\left(-c_{44}\lambda_3 + cc_{44} + c'e_{15}\right) ik + ab\sin(bx)\left(ikc_{11} - ik\lambda_3 cc_{13} + e_{31}c'\right)\right] e^{-ik\lambda_3 a\cos(bx)},$
$B_{84} = \left[\left(c_{44}\lambda_1 + dc_{44} + d'e_{15}\right) ik + ab\sin(bx)\left(ikc_{11} + ik\lambda_1 dc_{13} + e_{31}d'\right)\right] e^{ik\lambda_1 a\cos(bx)},$
$B_{85} = \left[\left(c_{44}\lambda_2 + ec_{44} + e'e_{15}\right) ik + ab\sin(bx)\left(ikc_{11} + ik\lambda_2 ec_{13} + e_{31}e'\right)\right] e^{ik\lambda_2 a\cos(bx)},$
$B_{86} = \left[\left(c_{44}\lambda_3 + fc_{44} + f'e_{15}\right) ik + ab\sin(bx)\left(ikc_{11} + ik\lambda_3 fc_{13} + e_{31}f'\right)\right] e^{ik\lambda_3 a\cos(bx)},$
$B_{87} = \left[\left(c'_{44}ikY_1 - c'_{44}ikX_1\lambda_4\right) + ab\sin(bx)\left(\left(c'_{11} + P\right) ikX_1 - \left(c'_{13} + P\right) ikY_1\lambda_4\right)\right] e^{-ik\lambda_4 a\cos(bx)},$
$B_{88} = \left[\left(c'_{44}ikY_2 - c'_{44}ikX_2\lambda_5\right) + ab\sin(bx)\left(\left(c'_{11} + P\right) ikX_2 - \left(c'_{13} + P\right) ikY_2\lambda_5\right)\right] e^{-ik\lambda_5 a\cos(bx)}.$

Appendix 2

$D_{11} = \left[\left(-c_{44}ik\lambda_1 + aikc_{44} + a'ike_{15}\right) + ab\sin(bx)\left(ikc_{11} - ik\lambda_1 ac_{13} + a'e_{31}\right)\right] e^{-ik\lambda_1(a\cos(bx)-H)},$
$D_{12} = \left[\left(-c_{44}ik\lambda_2 + bikc_{44} + b'ike_{15}\right) + ab\sin(bx)\left(ikc_{11} - ik\lambda_2 bc_{13} + b'e_{31}\right)\right] e^{-ik\lambda_2(a\cos(bx)-H)},$
$D_{13} = \left[\left(-c_{44}ik\lambda_3 + cikc_{44} + c'ike_{15}\right) + ab\sin(bx)\left(ikc_{11} - ik\lambda_3 cc_{13} + c'e_{31}\right)\right] e^{-ik\lambda_3(a\cos(bx)-H)},$
$D_{14} = \left[\left(c_{44}ik\lambda_1 + dikc_{44} + d'ike_{15}\right) + ab\sin(bx)\left(ikC_{11} + ik\lambda_1 dc_{13} + d'e_{31}\right)\right] e^{ik\lambda_1(a\cos(bx)-H)},$
$D_{15} = \left[\left(c_{44}ik\lambda_2 + eikc_{44} + e'ike_{15}\right) + ab\sin(bx)\left(ikc_{11} + ik\lambda_2 ec_{13} + e'e_{31}\right)\right] e^{ik\lambda_2(a\cos(bx)-H)},$
$D_{16} = \left[\left(c_{44}ik\lambda_3 + fikc_{44} + f'ike_{15}\right) + ab\sin(bx)\left(ikc_{11} + ik\lambda_3 fc_{13} + f'e_{31}\right)\right] e^{ik\lambda_3(a\cos(bx)-H)},$
$D_{17} = 0 = D_{18}, D_{41} = 1 = D_{42} = D_{43} = D_{44} = D_{45} = D_{46}, D_{47} = X_1, A_{48} = X_2,$
$D_{51} = a, D_{52} = b, D_{53} = c, D_{54} = d, D_{55} = e, D_{56} = f, D_{57} = Y_1, D_{58} = Y_2, D_{61} = a', D_{62} = b',$
$D_{63} = c', D_{64} = d', D_{65} = e', D_{66} = f',$
$D_{71} = \left[\left(c_{13}ik - ik\lambda_1 ac_{33} - ik\lambda_1 a'e_{33}\right)\right], D_{72} = \left[\left(c_{13}ik - ik\lambda_2 bc_{33} - ik\lambda_2 b'e_{33}\right)\right],$
$D_{73} = \left[\left(c_{13}ik - ik\lambda_3 cc_{33} - ik\lambda_3 c'e_{33}\right)\right], A_{74} = \left[\left(c_{13}ik + ik\lambda_1 dc_{33} + ik\lambda_1 d'e_{33}\right)\right],$
$A_{75} = \left[\left(c_{13}ik + ik\lambda_2 ec_{33} + ik\lambda_2 e'e_{33}\right)\right], A_{76} = \left[\left(c_{13}ik + ik\lambda_3 fc_{33} + ik\lambda_3 f'e_{33}\right)\right],$
$A_{77} = \left[\left(c'_{13}ikX_1 - c'_{33}ikY_1\lambda_4\right)\right], A_{78} = \left[\left(c'_{13}ikX_2 - c'_{33}ikY_2\lambda_5\right)\right],$
$D_{21} = \left[\left(C_{13}ik - \lambda_1 aikC_{33} - a'ik\lambda_1 e_{33}\right) + ab\sin(bx)\left(aikC_{44} - ik\lambda_1 C_{44} + a'ike_{15}\right)\right] e^{-ik\lambda_1(a\cos(bx)-H)},$
$D_{22} = \left[\left(C_{13}ik - \lambda_2 bikC_{33} - b'ik\lambda_2 e_{33}\right) + ab\sin(bx)\left(bikC_{44} - ik\lambda_2 C_{44} + b'ike_{15}\right)\right] e^{-ik\lambda_2(a\cos(bx)-H)},$
$D_{23} = \left[\left(C_{13}ik - \lambda_3 cikC_{33} - c'ik\lambda_3 e_{33}\right) + ab\sin(bx)\left(cikC_{44} - ik\lambda_3 C_{44} + c'ike_{15}\right)\right] e^{-ik\lambda_3(a\cos(bx)-H)},$
$D_{24} = \left[\left(C_{13}ik + dik\lambda_1 C_{33} + d'ik\lambda_1 e_{33}\right) + ab\sin(bx)\left(ikC_{44}\lambda_1 + ikdC_{44} + d'ike_{15}\right)\right] e^{ik\lambda_1(a\cos(bx)-H)},$
$D_{25} = \left[\left(C_{13}ik + eik\lambda_2 C_{33} + e'ik\lambda_2 e_{33}\right) + ab\sin(bx)\left(ikC_{44}\lambda_2 + ikeC_{44} + e'ike_{15}\right)\right] e^{ik\lambda_2(a\cos(bx)-H)}$
$D_{26} = \left[\left(C_{13}ik + fik\lambda_3 C_{33} + f'ik\lambda_3 e_{33}\right) + ab\sin(bx)\left(ikC_{44}\lambda_3 + ikfC_{44} + f'ike_{15}\right)\right] e^{ik\lambda_3(a\cos(bx)-H)},$
$D_{81} = \left[\left(-c_{44}\lambda_1 + ac_{44} + a'e_{15}\right) ik\right], D_{82} = \left[\left(-c_{44}\lambda_2 + bc_{44} + b'e_{15}\right) ik\right],$
$D_{83} = \left[\left(-c_{44}\lambda_3 + cc_{44} + c'e_{15}\right) ik\right], D_{84} = \left[\left(c_{44}\lambda_1 + dc_{44} + d'e_{15}\right) ik\right],$
$D_{85} = \left[\left(c_{44}\lambda_2 + ec_{44} + e'e_{15}\right) ik\right], D_{86} = \left[\left(c_{44}\lambda_3 + fc_{44} + f'e_{15}\right) ik\right],$
$D_{87} = \left[\left(c'_{44}ikY_1 - c'_{44}ikX_1\lambda_4\right)\right], D_{88} = \left[\left(c'_{44}ikY_2 - c'_{44}ikX_2\lambda_5\right)\right],$
$D_{27} = 0 = A_{28}, A_{31} = \left[e_{15}\left(-ik\lambda_1 + ika\right) + ika'\varepsilon_{11}\right] e^{-ik\lambda_1(a\cos(bx)-H)},$
$D_{32} = \left[e_{15}\left(-ik\lambda_2 + ikb\right) + ikb'\varepsilon_{11}\right] e^{-ik\lambda_2(a\cos(bx)-H)},$
$D_{33} = \left[e_{15}\left(-ik\lambda_3 + ikc\right) + ikc'\varepsilon_{11}\right] e^{-ik\lambda_3(a\cos(bx)-H)},$
$D_{34} = \left[e_{15}\left(ik\lambda_1 + ikd\right) + ikd'\varepsilon_{11}\right] e^{ik\lambda_1(a\cos(bx)-H)}, D_{35} = \left[e_{15}\left(ik\lambda_2 + ike\right) + ike'\varepsilon_{11}\right] e^{ik\lambda_2(a\cos(bx)-H)},$
$D_{35} = F\left[e_{15}\left(ik\lambda_3 + ikf\right) + ikf'\varepsilon_{11}\right] e^{ik\lambda_3(a\cos(bx)-H)}.$

Appendix 3

$$E_{11}=\left[\left(-c_{44}ik\lambda_1+aikc_{44}+a'ike_{15}\right)+a_1b\sin(bx)\left(ikc_{11}-ik\lambda_1ac_{13}+a'e_{31}\right)\right]e^{-ik\lambda_1(a_1\cos(bx)-H)},$$
$$E_{12}=\left[\left(-c_{44}ik\lambda_2+bikc_{44}+b'ike_{15}\right)+a_1b\sin(bx)\left(ikc_{11}-ik\lambda_2bc_{13}+b'e_{31}\right)\right]e^{-ik\lambda_2(a_1\cos(bx)-H)},$$
$$E_{13}=\left[\left(-c_{44}ik\lambda_3+cikc_{44}+c'ike_{15}\right)+a_1b\sin(bx)\left(ikc_{11}-ik\lambda_3cc_{13}+c'e_{31}\right)\right]e^{-ik\lambda_3(a_1\cos(bx)-H)},$$
$$E_{14}=\left[\left(c_{44}ik\lambda_1+dikc_{44}+d'ike_{15}\right)+a_1b\sin(bx)\left(ikC_{11}+ik\lambda_1dc_{13}+d'e_{31}\right)\right]e^{ik\lambda_1(a_1\cos(bx)-H)},$$
$$E_{21}=\left[\left(C_{13}ik-\lambda_1aikC_{33}-a'ik\lambda_1e_{33}\right)+a_1b\sin(bx)\left(aikC_{44}-ik\lambda_1C_{44}+a'ike_{15}\right)\right]e^{-ik\lambda_1(a_1\cos(bx)-H)},$$
$$E_{22}=\left[\left(C_{13}ik-\lambda_2bikC_{33}-b'ik\lambda_2e_{33}\right)+a_1b\sin(bx)\left(bikC_{44}-ik\lambda_2C_{44}+b'ike_{15}\right)\right]e^{-ik\lambda_2(a_1\cos(bx)-H)},$$
$$E_{23}=\left[\left(C_{13}ik-\lambda_3cikC_{33}-c'ik\lambda_3e_{33}\right)+a_1b\sin(bx)\left(cikC_{44}-ik\lambda_3C_{44}+c'ike_{15}\right)\right]e^{-ik\lambda_3(a_1\cos(bx)-H)},$$
$$E_{24}=\left[\left(C_{13}ik+dik\lambda_1C_{33}+d'ik\lambda_1e_{33}\right)+a_1b\sin(bx)\left(ikC_{44}\lambda_1+ikdC_{44}+d'ike_{15}\right)\right]e^{ik\lambda_1(a_1\cos(bx)-H)},$$
$$E_{15}=\left[\left(c_{44}ik\lambda_2+eikc_{44}+e'ike_{15}\right)+a_1b\sin(bx)\left(ikc_{11}+ik\lambda_2ec_{13}+e'e_{31}\right)\right]e^{ik\lambda_2(a_1\cos(bx)-H)},$$
$$E_{16}=\left[\left(c_{44}ik\lambda_3+fikc_{44}+f'ike_{15}\right)+a_1b\sin(bx)\left(ikc_{11}+ik\lambda_3fc_{13}+f'e_{31}\right)\right]e^{ik\lambda_3(a_1\cos(bx)-H)},$$
$$E_{17}=0=E_{18}.$$
$$E_{25}=\left[\left(C_{13}ik+eik\lambda_2C_{33}+e'ik\lambda_2e_{33}\right)+a_1b\sin(bx)\left(ikC_{44}\lambda_2+ikeC_{44}+e'ike_{15}\right)\right]e^{ik\lambda_2(a_1\cos(bx)-H)}$$
$$E_{26}=\left[\left(C_{13}ik+fik\lambda_3C_{33}+f'ik\lambda_3e_{33}\right)+a_1b\sin(bx)\left(ikC_{44}\lambda_3+ikfC_{44}+f'ike_{15}\right)\right]e^{ik\lambda_3(a_1\cos(bx)-H)},$$
$$E_{27}=0=A_{28},\ A_{31}=\left[e_{15}\left(-ik\lambda_1+ika\right)+ika'\varepsilon_{11}\right]e^{-ik\lambda_1(a_1\cos(bx)-H)},$$
$$E_{32}=\left[e_{15}\left(-ik\lambda_2+ikb\right)+ikb'\varepsilon_{11}\right]e^{-ik\lambda_2(a_1\cos(bx)-H)},$$
$$E_{33}=\left[e_{15}\left(-ik\lambda_3+ikc\right)+ikc'\varepsilon_{11}\right]e^{-ik\lambda_3(a_1\cos(bx)-H)},\ E_{34}=\left[e_{15}\left(ik\lambda_1+ikd\right)+ikd'\varepsilon_{11}\right]e^{ik\lambda_1(a_1\cos(bx)-H)},$$
$$E_{35}=\left[e_{15}\left(ik\lambda_2+ike\right)+ike'\varepsilon_{11}\right]e^{ik\lambda_2(a_1\cos(bx)-H)},\ E_{36}=F\left[e_{15}\left(ik\lambda_3+ikf\right)+ikf'\varepsilon_{11}\right]e^{ik\lambda_3(a_1\cos(bx)-H)},$$
$$E_{37}=0=E_{38}.$$
$$E_{41}=e^{-ik\lambda_1a_2\cos(bx)},\ E_{42}=e^{-ik\lambda_2a_2\cos(bx)},\ E_{43}=Ce^{-ik\lambda_3a_2\cos(bx)},\ E_{44}=De^{ik\lambda_1a_2\cos(bx)},$$
$$E_{45}=Ee^{ik\lambda_2a_2\cos(bx)},\ E_{46}=Fe^{ik\lambda_3a_2\cos(bx)},\ E_{47}=X_1e^{-ik\lambda_4a_2\cos(bx)},\ E_{48}=X_2e^{-ik\lambda_5a_2\cos(bx)}$$
$$E_{51}=ae^{-ik\lambda_1a_2\cos(bx)},\ E_{52}=be^{-ik\lambda_2a_2\cos(bx)},\ E_{53}=ce^{-ik\lambda_3a_2\cos(bx)},\ E_{54}=de^{ik\lambda_1a_2\cos(bx)},$$
$$E_{55}=ee^{ik\lambda_2a_2\cos(bx)},\ E_{56}=fe^{ik\lambda_3a_2\cos(bx)},\ E_{57}=Y_1e^{-ik\lambda_4a_2\cos(bx)},$$
$$E_{58}=Y_2e^{-ik\lambda_5a_2\cos(bx)},\ E_{61}=a'e^{-ik\lambda_1a_2\cos(bx)},\ E_{62}=b'e^{-ik\lambda_2a_2\cos(bx)},$$
$$E_{63}=c'e^{-ik\lambda_3a_2\cos(bx)},\ E_{64}=d'e^{ik\lambda_1a_2\cos(bx)},\ E_{65}=e'e^{ik\lambda_2a_2\cos(bx)},\ E_{66}=f'e^{ik\lambda_3a_2\cos(bx)},\ E_{67}=0=E_{68}.$$
$$E_{71}=\left[\left(c_{13}ik-ik\lambda_1ac_{33}-ik\lambda_1a'e_{33}\right)+a_2b\sin(bx)\left(-c_{44}ik\lambda_1+ikac_{44}+a'ike_{15}\right)\right]e^{-ik\lambda_1a_2\cos(bx)},$$
$$E_{72}=\left[\left(c_{13}ik-ik\lambda_2bc_{33}-ik\lambda_2b'e_{33}\right)+a_2b\sin(bx)\left(-c_{44}ik\lambda_2+ikbc_{44}+b'ike_{15}\right)\right]e^{-ik\lambda_2a_2\cos(bx)},$$
$$E_{73}=\left[\left(c_{13}ik-ik\lambda_3cc_{33}-ik\lambda_3c'e_{33}\right)+a_2b\sin(bx)\left(-c_{44}ik\lambda_3+ikcc_{44}+c'ike_{15}\right)\right]e^{-ik\lambda_3a_2\cos(bx)},$$
$$E_{74}=\left[\left(c_{13}ik+ik\lambda_1dc_{33}+ik\lambda_1d'e_{33}\right)+a_2b\sin(bx)\left(c_{44}ik\lambda_1+ikdc_{44}+d'ike_{15}\right)\right]e^{ik\lambda_1a_2b\cos(bx)},$$
$$E_{81}=\left[\left(-c_{44}\lambda_1+ac_{44}+a'e_{15}\right)ik+a_2b\sin(bx)\left(ikc_{11}-ik\lambda_1ac_{13}+e_{31}a'\right)\right]e^{-ik\lambda_1a_2\cos(bx)},$$
$$E_{82}=\left[\left(-c_{44}\lambda_2+bc_{44}+b'e_{15}\right)ik+a_2b\sin(bx)\left(ikc_{11}-ik\lambda_2bc_{13}+e_{31}b'\right)\right]e^{-ik\lambda_2a_2\cos(bx)},$$
$$E_{83}=\left[\left(-c_{44}\lambda_3+cc_{44}+c'e_{15}\right)ik+a_2b\sin(bx)\left(ikc_{11}-ik\lambda_3cc_{13}+e_{31}c'\right)\right]e^{-ik\lambda_3a_2\cos(bx)},$$
$$E_{75}=\left[\left(c_{13}ik+ik\lambda_2ec_{33}+ik\lambda_2e'e_{33}\right)+a_2b\sin(bx)\left(c_{44}ik\lambda_2+ikec_{44}+e'ike_{15}\right)\right]e^{ik\lambda_2a_2\cos(bx)},$$
$$E_{76}=\left[\left(c_{13}ik+ik\lambda_3fc_{33}+ik\lambda_3f'e_{33}\right)+a_2b\sin(bx)\left(c_{44}ik\lambda_3+ikfc_{44}+f'ike_{15}\right)\right]e^{ik\lambda_3a_2\cos(bx)},$$
$$E_{77}=\left[\left(c'_{13}ikX_1-c'_{33}ikY_1\lambda_4\right)+a_2b\sin(bx)\left(c'_{44}ikY_1-c'_{44}ikX_1\lambda_4\right)\right]e^{-k\lambda_4a_2\cos(bx)},$$
$$E_{78}=\left[\left(c'_{13}ikX_2-c'_{33}ikY_2\lambda_5\right)+a_2b\sin(bx)\left(c'_{44}ikY_2-c'_{44}ikX_1\lambda_5\right)\right]e^{-k\lambda_5a_2\cos(bx)},$$
$$E_{84}=\left[\left(c_{44}\lambda_1+dc_{44}+d'e_{15}\right)ik+a_2b\sin(bx)\left(ikc_{11}+ik\lambda_1dc_{13}+e_{31}d'\right)\right]e^{ik\lambda_1a_2b\cos(bx)},$$
$$E_{85}=\left[\left(c_{44}\lambda_2+ec_{44}+e'e_{15}\right)ik+a_2b\sin(bx)\left(ikc_{11}+ik\lambda_2ec_{13}+e_{31}e'\right)\right]e^{ik\lambda_2a_2b\cos(bx)},$$
$$E_{86}=\left[\left(c_{44}\lambda_3+fc_{44}+f'e_{15}\right)ik+a_2b\sin(bx)\left(ikc_{11}+ik\lambda_3fc_{13}+e_{31}f'\right)\right]e^{ik\lambda_3a_2b\cos(bx)},$$

$$E_{87} = \left[\left(c'_{44}ikY_1 - c'_{44}ikX_1\lambda_4\right) + a_2b\sin(bx)\left(\left(c'_{11}+P\right)ikX_1 - \left(c'_{13}+P\right)ikY_1\lambda_4\right)\right]e^{-ik\lambda_4 a_2 b\cos(bx)},$$
$$E_{88} = \left[\left(c'_{44}ikY_2 - c'_{44}ikX_2\lambda_5\right) + a_2b\sin(bx)\left(\left(c'_{11}+P\right)ikX_2 - \left(c'_{13}+P\right)ikY_2\lambda_5\right)\right]e^{-ik\lambda_5 a_2 b\cos(bx)}.$$
$$M_{mm} = \int\limits_{\Omega}\left[N_1^e\right]^T\rho\left[N_1^e\right]\mathrm{d}\Omega,\ K_{mm} = \int\limits_{\Omega}[S_1]^T[C][S_1]\,\mathrm{d}\Omega,\ K_{me} = \int\limits_{\Omega}[S_2]^T[e][S_1]\,\mathrm{d}\Omega,$$
$$K_{ee} = \int\limits_{\Omega}[S_2]^T[p][S_2]\,\mathrm{d}\Omega,\ F = \int\limits_{A_\sigma}\left[N_1^e\right]^T\{T\}\mathrm{d}A_\sigma,$$
$$Q = \int\limits_{A_w}\left[N_2^e\right]\varpi\,\mathrm{d}A_w,\ F = \int\limits_{\Omega}\left[N_1^e\right]^T\{f\}\,\mathrm{d}\Omega,\ Q = \int\limits_{Aw}\left[N_2^e\right]^T\bar{w}\mathrm{d}A.$$

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