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ORIGINAL PAPER

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Plastic analysis of multilayer sandwich beams with metal foam cores

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Abstract The plastic behavior of fully clamped multilayer sandwich beams with metal foam cores is investigated. A plastic yield criterion for multilayer sandwich structures with metal foam cores is proposed. Analytical solutions are derived for the multilayer sandwich beams transversely loaded by a flat punch considering the interaction of bending and stretching induced by large deflections. Numerical calculations are carried out, and good agreement is achieved between the analytical solutions and numerical results. The effects of the multilayer, punch size and core strength are discussed. Using the analytical formulae, optimal design charts are constructed to maximize the load-carrying capacity of the multilayer sandwich beams for a given mass. It is demonstrated that the present analytical model can reasonably predict the post-yield behavior of the multilayer sandwich beams with metal foam cores.

1 Introduction

Lightweight structures have been widely used in engineering, such as aerospace, aircraft, high speed trains, ships. Sandwich structures with two stiff and strong face sheets separated by a lightweight core are typical lightweight structures, due to various advantages. Several kinds of cores have been developed, for example metal foam, lattice material and woven material [1–11]. Compared with the monolayer sandwich structures, the multilayer sandwich structures have different performances and will provide much more choices for the design of structures [12,13].

The plastic behavior of monolayer sandwich structures has been investigated by using experimental, numerical, and theoretical methods. Ashby et al. [3], Bart-Smith et al. [14], McCormack et al. [15], and Yu et al. [16] studied the deformation and failure mechanisms of monolayer sandwich beams with aluminum foam cores under three- and four-point bending, respectively. Tagarielli and Fleck [17] investigated the collapse responses of fully clamped and simply supported monolayer metal sandwich beams with a foam core. Qin and Wang [18] derived a yield criterion for monolayer metal sandwich structures considering the effect of core strength and obtained analytical solutions for the large deflection of slender monolayer metal foam core sandwich beams transversely loaded by a flat punch.

Investigations on the quasi-static bending and compressive behaviors of multilayer sandwich structures were few, and multilayer metallic sandwich structures have been shown to be effective at benefit relative to monolayer ones, such as energy absorption ability. Capela et al. [19] studied the influence of the benefit on

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the bending behavior of sandwich panels promoted by using multilayer core foams. Wang [12] analyzed the compressive behavior of multilayer corrugated sandwich structures and found that the energy absorption of multilayer corrugated sandwich structures is significantly larger than the monolayer one, and the multilayer corrugated sandwich structures have compression resistance capability for repetitious shock. Fan et al. [20] conducted quasi-static compression experiments to study the energy absorption mechanism of the multilayer woven textile sandwich panels and indicated that multilayer panels manufactured by stacking thin monolayer panels can improve the energy absorption ability of the woven textile sandwich panels. Sha and Yip [21] investigated deformation and damage mechanisms of multilayer sandwich beams with aluminum foam core under four-point bending in situ by surface displacement analysis software (SDA) and observed the failure modes, such as indentation and core shear failure modes. Hou et al. [22] experimentally and numerically investigated the compressive behavior of multilayered corrugated sandwich panels under the quasi-static crushing loading and found that the sandwich configuration and number of layers play important roles in the failure mechanism and energy absorption. Arbaoui et al. [23] investigated the effect of core thickness and intermediate layers on the mechanical properties of a polypropylene honeycomb core/composite facing multilayer sandwich structure under three-point bending and found that the mechanical properties increase with increasing core thickness and intermediate layers. Xiong et al. [24] investigated the failure modes and deformation mechanisms of carbon fiber composite two-layer sandwich panels with pyramidal truss core under uniform quasi-static compression and low-velocity concentrated impact.

Although the yield criteria for the monolayer sandwich beams [18,27,28] were obtained, there is little work on the yield criterion and plastic analysis of multilayer sandwich structures. The study on the yield criterion of the multilayer sandwich structure is more complex than that of monolayer one. The bending moment, axial force and distributions of the strain and stresses on the cross section of a multilayer sandwich beam are different from those of the monolayer one, and more cases need to be considered compared with the monolayer one. Increasing interlayer sheets and foam cores increase the difficulty of the problem on the yield criteria and plastic analysis of multilayer sandwich beams.

The objective of this work is to investigate the yield criterion and plastic behavior of multilayer slender metal sandwich beams transversely loaded by a flat punch. The paper is organized as follows. In Sect. 2, elastic solutions for the bending response of clamped multilayer sandwich beams under transverse loading are given. In Sect. 3, a plastic yield criterion considering the effect of the core strength is proposed for multilayer sandwich structures. In Sect. 4, an analytical solution is derived to predict the plastic behavior of a multilayer sandwich beam transversely loaded by a flat punch at midspan. In Sect. 5, finite element analysis is performed. In Sect. 6, numerical results are compared with the analytical predictions. The effects of the multilayer, punch size and core strength on the structural response of the multilayer sandwich beams are discussed. In Sect. 7, an optimal design is performed on the basis of the present analytical solutions to maximize the load-carrying capacity of multilayer sandwich beams for a given mass. The concluding remarks are presented in Sect. 8.

2 Elastic solutions for the bending response of clamped multilayer sandwich beams

Consider a fully clamped slender multilayer metal foam core sandwich beam with rectangular cross section of width b and span $2L$ under transverse loading by a flat punch of length $2a$ at midspan, as shown in Fig. 1. Two identical top and bottom face sheets of thickness h and interlayer sheet of thickness h_m are assumed to be perfectly bonded to the identical top and bottom foam cores of thickness c. The top and bottom face sheets have the yield strength σ_f and Young's modulus E_f. The isotropic metal foam has the compressive strength σ_c, shear modulus G_c and Young's modulus E_c. To describe the relation between top and bottom face sheets and interlayer sheet, we define a multilayer factor

$$\gamma = \frac{h_m}{2h}. \tag{1}$$

For the case of $0 < \gamma < \frac{1}{2}$, the top/bottom face sheet is thicker than the interlayer sheet. If $\gamma > \frac{1}{2}$, the sandwich structure has a thinner top/bottom face sheet compared with the interlayer sheet. If $\gamma = \frac{1}{2}$, the top/bottom face sheet and interlayer sheet are identical. For the limit cases $\gamma \to 0$, the sandwich structure is reduced to a monolayer sandwich beam.

Elasticity solutions for monolayer sandwich beams under transverse loading have been presented [1,17]. The deflection W_0 at midspan of the clamped multilayer sandwich beam under loading P is given by

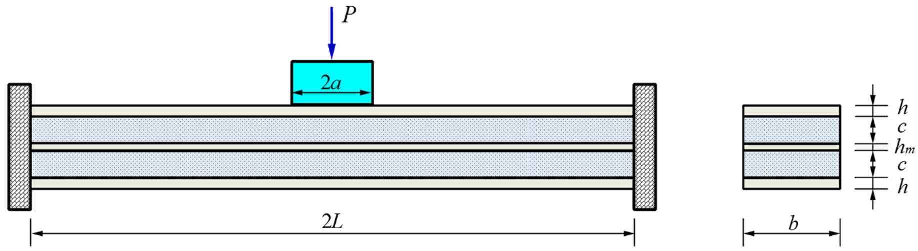


Fig. 1 Sketch of a fully clamped multilayer metal sandwich beam with metal foam cores transversely loaded by a flat punch

$$W_0 = \frac{\mathrm{PL}^3}{24(\mathrm{EI})_{\mathrm{eq}}} + \frac{\mathrm{PL}}{2(\mathrm{AG})_{\mathrm{eq}}}, \tag{2}$$

The equivalent flexural rigidity $(\mathrm{EI})_{\mathrm{eq}}$ of the multilayer sandwich beam is

$$(\mathrm{EI})_{\mathrm{eq}} = \frac{E_f bhd^2}{2} + \frac{E_f bh^3}{6} + \frac{E_f bh_m^3}{12} + \frac{E_c bc}{6}\left[3\left(h_m + c\right)^2 + c^2\right] \tag{3}$$

where

$$d = 2c + h + h_m.$$

In the case of weak core, i.e., $E_c << E_f$, Eq. (3) reduces to

$$(\mathrm{EI})_{\mathrm{eq}} = \frac{E_f bhd^2}{2} + \frac{E_f bh^3}{6} + \frac{E_f bh_m^3}{12}. \tag{4}$$

The equivalent shear rigidity is given by

$$(\mathrm{AG})_{\mathrm{eq}} = \frac{bd^2}{2c + h_m} G_c, \tag{5}$$

3 Yield criterion for a multilayer sandwich cross section

The classical yield criterion for a sandwich structure [25] is highly accurate for a monolayer symmetric sandwich beam with thin and strong face sheets, thick and weak core, while it becomes less accurate as the sandwich cross section approaches the monolithic limit [26]. Qin and Wang [18] developed a yield criterion for the monolayer symmetric sandwich structure considering the effect of core strength, which is not only accurate for the sandwich structure with a weak core, but also for that with a strong core. Subsequently, the yield criteria for the monolayer geometrically and plastically asymmetric sandwich structures were obtained also [27,28].

The previous studies [29] have demonstrated that the effect of shear force can be negligible compared with bending moment and axial force in large deflections. Moreover, the interaction of bending and axial stretching should be included in the analysis. It is reasonable to assume that the plastic behavior of a multilayer sandwich beam in the post-yield regime is determined by the bending moment and longitudinal axial force.

It is assumed that the face sheets obey the rigid–perfectly plastic (r-p-p) law with yield strength σ_f. The cores are made from the rigid–perfectly plastic locking (r-p-p-l) material with yield strength σ_c and densification strain ε_D. The distributions of the strains and stresses in the multilayer sandwich cross section are shown in Fig. 2. Assume that the sandwich cross section has a fully plastic stress distribution resulting from a combination of an axial force and a bending moment. Define the distance between the plastic neutral surface and the bottom surface of the bottom face sheet as $H = \zeta\,(2h + 2c + h_m)$, and $\zeta = 0$, 1/2 and 1 correspond to the states for fully plastic axial tension, bending and compression, respectively. Then, the resultants of the axial force$N$ and the bending moment M for different values of ζ can be expressed as

$$N = \int_A \sigma \mathrm{d}A = \begin{cases} \sigma_f b\left[2h + h_m - 2\zeta\left(2h + 2c + h_m\right)\right] + 2\sigma_c bc, & 0 \le \zeta \le \frac{h}{2h+2c+h_m} \\ \sigma_f bh_m + 2\sigma_c b\left[c + h - \zeta\left(2h + 2c + h_m\right)\right], & \frac{h}{2h+2c+h_m} \le \zeta \le \frac{h+c}{2h+2c+h_m} \\ \sigma_f b\left(2h + 2c + h_m\right)\left(1 - 2\zeta\right), & \frac{h+c}{2h+2c+h_m} \le \zeta \le \frac{1}{2} \end{cases} \tag{6}$$

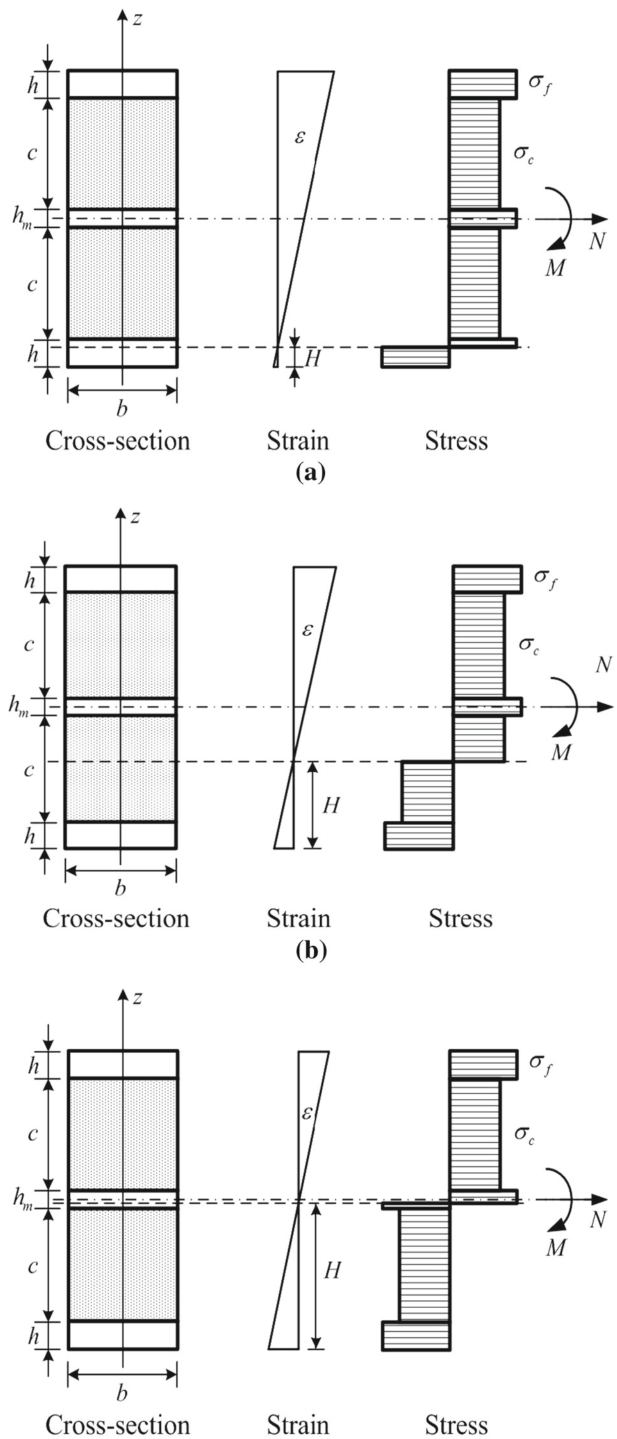


Fig. 2 Distributions of the strain and stresses on the cross section of a multilayer sandwich beam with metal foam cores. **a** $0 \le \zeta \le h/(2h + 2c + h_m)$, **b** $h/(2h + 2c + h_m) \le \zeta \le (h + c)/(2h + 2c + h_m)$ and **c** $(h + c)/(2h + 2c + h_m) \le \zeta \le 1/2$

and

$$M=\int_A \sigma z\mathrm{d}A=\begin{cases}\sigma_f b\,(2h+2c+h_m)^2\left(\zeta-\zeta^2\right), & 0\le\zeta\le\frac{h}{2h+2c+h_m}\\ \sigma_f bh\,(h+2c+h_m)+\sigma_c b\left[(c+h_m/2)^2-(\zeta-1/2)^2\,(2h+2c+h_m)^2\right], & \frac{h}{2h+2c+h_m}\le\zeta\le\frac{h+c}{2h+2c+h_m}\\ \sigma_f bh\,(h+2c+h_m)+\sigma_c bc\,(c+h_m)+\sigma_f b\left[h_m^2/4-(2h+2c+h_m)^2\,(\zeta-1/2)^2\right], & \frac{h+c}{2h+2c+h_m}\le\zeta\le\frac{1}{2}\end{cases}\tag{7}$$

in Fig. 2a–c. For the limit cases of $\zeta=0$ and 1/2, the fully plastic axial force N_p and bending moment M_p are

$$N_p=\sigma_f b\,(2h+h_m)+2\sigma_c bc\tag{8}$$

and

$$M_p=\sigma_f b\,(h+h_m/2)^2+2\sigma_f bhc+\sigma_c bc\,(h_m+c)\,.\tag{9}$$

From Eqs. (6) to (9), we have

$$\frac{N}{N_p}=\begin{cases}1-\frac{2(1+2\bar h)}{\bar\sigma+2\bar h}\zeta, & 0\le\zeta\le\frac{\bar h}{(1+\gamma)(1+2\bar h)}\\ \frac{2\gamma\bar h/(1+\gamma)+\bar\sigma\left[1+2\bar h/(1+\gamma)-2\zeta(1+2\bar h)\right]}{\bar\sigma+2\bar h}, & \frac{\bar h}{(1+\gamma)(1+2\bar h)}\le\zeta\le\frac{1+\gamma+2\bar h}{2(1+\gamma)(1+2\bar h)}\\ \frac{(1+2\bar h)}{\bar\sigma+2\bar h}\,(1-2\zeta)\,, & \frac{1+\gamma+2\bar h}{2(1+\gamma)(1+2\bar h)}\le\zeta\le\frac{1}{2}\end{cases}\tag{10}$$

and

$$\frac{M}{M_p}=\begin{cases}\frac{1}{A}\left(2+4\bar h\right)^2\left(\zeta-\zeta^2\right), & 0\le\zeta\le\frac{\bar h}{(1+\gamma)(1+2\bar h)}\\ \frac{2\bar h}{A(1+\gamma)}\left[\frac{2\bar h(1+2\gamma)}{1+\gamma}+2\right]+\frac{\bar\sigma}{A}\left[\left(\frac{2\gamma\bar h}{1+\gamma}+1\right)^2-\left(\zeta-\frac{1}{2}\right)^2\left(2+4\bar h\right)^2\right], & \frac{\bar h}{(1+\gamma)(1+2\bar h)}\le\zeta\le\frac{1+\gamma+2\bar h}{2(1+\gamma)(1+2\bar h)}\\ \frac{1}{A}\left[\frac{4\bar h}{1+\gamma}\left(\frac{\bar h}{1+\gamma}+\frac{2\gamma\bar h}{1+\gamma}+1\right)+\bar\sigma\left(\frac{4\gamma\bar h}{1+\gamma}+1\right)+\frac{4\gamma^2\bar h^2}{(1+\gamma)^2}-4\left(1+2\bar h\right)^2\left(\zeta-\frac{1}{2}\right)^2\right], & \frac{1+\gamma+2\bar h}{2(1+\gamma)(1+2\bar h)}\le\zeta\le\frac{1}{2}\end{cases}\tag{11}$$

where

$$\bar\sigma=\frac{\sigma_c}{\sigma_f},$$
$$\bar h=\frac{h_m+2h}{4c},$$
$$A=4\bar h^2+\bar\sigma+4\bar h(1+\gamma\bar\sigma)/(1+\gamma).$$

The bending moment, axial force and distributions of the strain and stresses on the cross section of a multilayer sandwich beam are different from those of monolayer ones [18,27,28].

Eliminating the parameter ζ from Eqs. (10) and (11), we can obtain the yield criterion for the multilayer sandwich section,

$$\begin{cases}|m|+\frac{\left(2\bar h+\bar\sigma\right)^2n^2}{A}-B=0, & 0\le|n|\le\frac{2\gamma\bar h}{(2\bar h+\bar\sigma)(1+\gamma)}\\ |m|+\frac{1}{\bar\sigma A}\left[\left(2\bar h+\bar\sigma\right)|n|-\frac{2\bar h\gamma(1-\bar\sigma)}{1+\gamma}\right]^2-C=0, & \frac{2\gamma\bar h}{(2\bar h+\bar\sigma)(1+\gamma)}\le|n|\le\frac{2\gamma\bar h+\bar\sigma(1+\gamma)}{(2\bar h+\bar\sigma)(1+\gamma)}\\ |m|+\frac{(|n|-1)\left(2\bar h+\bar\sigma\right)}{A}\left[\left(2\bar h+\bar\sigma\right)|n|+2\bar h-\bar\sigma+2\right]=0, & \frac{2\gamma\bar h+\bar\sigma(1+\gamma)}{(2\bar h+\bar\sigma)(1+\gamma)}\le|n|\le 1\end{cases}\tag{12}$$

where

$$m=\frac{M}{M_p},$$
$$n=\frac{N}{N_p},$$
$$B=\frac{4\bar h}{A\,(1+\gamma)}\left(\frac{\bar h}{1+\gamma}+\frac{2\gamma\bar h}{1+\gamma}+1\right)+\frac{\bar\sigma}{A}\left(\frac{4\bar h\gamma}{1+\gamma}+1\right)+\frac{4\gamma^2\bar h^2}{A\,(1+\gamma)^2},$$

$$C=\frac{4\bar{h}}{A\left(1+\gamma\right)}\left(\frac{\bar{h}\left(1+2\gamma\right)}{1+\gamma}+1\right)+\frac{\bar{\sigma}}{A}\left(\frac{2\gamma\bar{h}}{1+\gamma}+1\right)^{2}.$$

When $\gamma = 0$, the yield criterion Eq. (12) is reduced to the following solution for a monolayer sandwich structure [18]:

$$\begin{cases} |m|+\frac{(\bar{\sigma}+2\bar{h})^2}{4\bar{\sigma}\bar{h}(1+\bar{h})+\bar{\sigma}^2}n^2=1, & 0\leq|n|\leq\frac{\bar{\sigma}}{\bar{\sigma}+2\bar{h}} \\ |m|+\frac{(\bar{\sigma}+2\bar{h})\left[(\bar{\sigma}+2\bar{h})|n|+2\bar{h}-\bar{\sigma}+2\right](|n|-1)}{4\bar{h}(1+\bar{h})+\bar{\sigma}}=0, & \frac{\bar{\sigma}}{\bar{\sigma}+2\bar{h}}\leq|n|\leq 1 \end{cases}. \tag{13}$$

The predicted yield surfaces N/N_p, M/M_p for the multilayer metal sandwich cross sections are shown in Fig. 3a, b. It is seen that the present yield criterion can be applied to sandwich structures with different core strengths, face sheet materials and geometries, and can reduce to the yield criterion for monolayer sandwich cross sections [18]. Also, the present method can be expanded to study the yield criteria for the multilayer sandwich structures with any layers. In what follows, using the yield criterion, we give the analytical solutions for the large deflection of fully clamped multilayer sandwich beams with metal foam cores, in which the interaction of bending and stretching is considered.

4 Analytical solutions for large deflections of clamped multilayer sandwich beams

It is assumed that the multilayer sandwich beam is enough slender and deforms in a global manner without local denting beneath the central punch. Thus, the sandwich cross sections keep the original shape, and the global deflection profile is the same as that of the fully clamped solid monolithic beam under transverse loading by a flat punch, as shown in Fig. 4a.

There are four plastic hinges at the fully clamped ends and the points adjacent to the central punch. If the bottom face sheet has the maximum transverse deflection W_0 under loading P, then the left-hand portion $(L-a)$ of the multilayer sandwich beam has a total extension e,

$$e=e_1+e_2 \tag{14}$$

where e_1 and e_2 are the axial extensions concentrated at the end of the left-hand portion and the point adjacent to the central punch, as shown in Fig. 4a. The total elongation and the angular rotation of the left-hand part can be expressed as

$$e\approx\frac{W_0^2}{2(L-a)} \tag{15}$$

and

$$\psi\approx\frac{W_0}{L-a}. \tag{16}$$

The moment equilibrium equation for the left-hand portion, as shown in Fig. 4b, can be written as

$$2\left(M+M_m\right)-P\left(L-a\right)+2FW_0=0 \tag{17}$$

where $F\approx N$ for moderate deflection, M and M_m are the bending moment at fully clamped ends and adjacent to the central punch and $M=M_m$.

When $N=0$, $M=M_m=M_p$ and $W_0=0$, the global deformations of the sandwich beam occur, and the initial collapse load $\bar{P}_c$ can be calculated from Eq. (17),

$$\bar{P}_c=\frac{4M_p}{L-a}. \tag{18}$$

From the associated flow rule of yield criteria Eq. (12), the following relations at fully clamped ends and the points adjacent to the central punch are

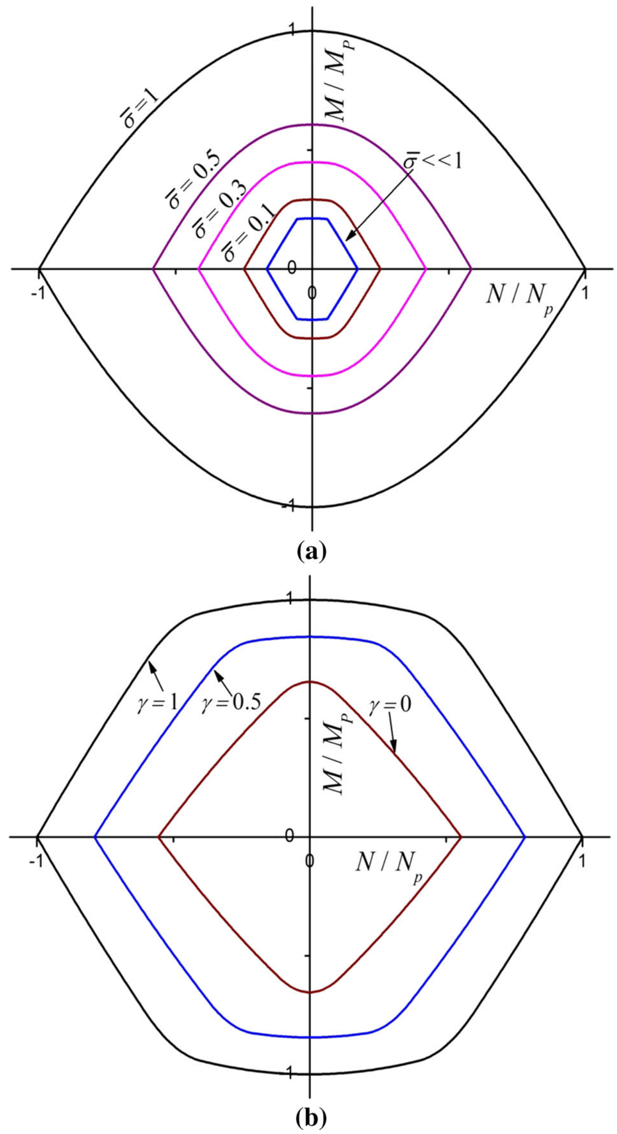


Fig. 3 Yield surfaces for the multilayer sandwich cross section with **a** core strength ($\gamma = 0.5, \bar{h} = 0.1$) and **b** multilayer factor ($\bar{\sigma} = 0.1, \bar{h} = 0.2$)

$$\frac{\dot{e}_1}{\dot{\psi}} = \frac{\dot{e}_2}{\dot{\psi}} = \begin{cases} \left(\bar{\sigma} + 2\bar{h}\right) c \left|n\right|, & 0 \le |n| \le \frac{2\gamma\bar{h}}{(2\bar{h}+\bar{\sigma})(1+\gamma)} \\ \frac{1}{\bar{\sigma}}\left[\left(\bar{\sigma} + 2\bar{h}\right)|n| - \frac{2\gamma\bar{h}(1-\bar{\sigma})}{1+\gamma}\right]c, & \frac{2\gamma\bar{h}}{(2\bar{h}+\bar{\sigma})(1+\gamma)} \le |n| \le \frac{2\gamma\bar{h}+\bar{\sigma}(1+\gamma)}{(2\bar{h}+\bar{\sigma})(1+\gamma)} \\ \left[\left(\bar{\sigma} + 2\bar{h}\right)|n| - \bar{\sigma} + 1\right]c, & \frac{2\gamma\bar{h}+\bar{\sigma}(1+\gamma)}{(2\bar{h}+\bar{\sigma})(1+\gamma)} \le |n| \le 1 \end{cases} . \tag{19}$$

Substituting Eqs. (14) to (16) into Eq. (19), the relation between normalized deflection W_0^* and non-dimensional axial force n can be given by

$$W_0^* = \begin{cases} \frac{(\bar{\sigma}+2\bar{h})|n|}{1+2\bar{h}}, & 0 \le |n| \le \frac{2\gamma\bar{h}}{\left(2\bar{h}+\bar{\sigma}\right)(1+\gamma)} \\ \frac{1}{\bar{\sigma}(1+2\bar{h})}\left[\left(\bar{\sigma} + 2\bar{h}\right)|n| - \frac{2\gamma\bar{h}(1-\bar{\sigma})}{1+\gamma}\right], & \frac{2\gamma\bar{h}}{(2\bar{h}+\bar{\sigma})(1+\gamma)} \le |n| \le \frac{2\gamma\bar{h}+\bar{\sigma}(1+\gamma)}{(2\bar{h}+\bar{\sigma})(1+\gamma)} \\ \frac{(\bar{\sigma}+2\bar{h})|n|-\bar{\sigma}+1}{1+2\bar{h}}, & \frac{2\gamma\bar{h}+\bar{\sigma}(1+\gamma)}{(2\bar{h}+\bar{\sigma})(1+\gamma)} \le |n| \le 1 \end{cases} \tag{20}$$

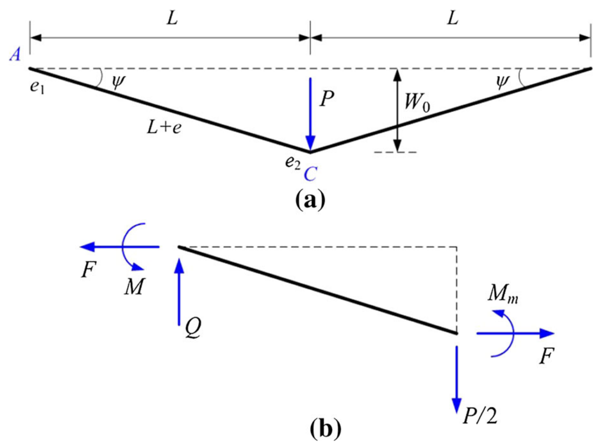


Fig. 4 Overall bending deformation pattern for the neutral surface of the fully clamped multilayer sandwich beam with metal foam cores transversely loaded by a flat punch. **a** Transverse deflection profile and **b** forces and moments

where

$$W_0^* = \frac{W_0}{2h + 2c + h_m}.$$

From Eqs. (12), (17) and (20), the solutions of the relation between the normalized load P^* and normalized deflection W_0^* can be expressed as

$$P^* = \frac{1}{1-\bar{a}}\left[\frac{\left(1+2\bar{h}\right)^2}{A} W_0^{*2} + B\right], \qquad 0 \le W_0^* \le \frac{2\gamma\bar{h}}{(1+\gamma)\left(1+2\bar{h}\right)}, \tag{21}$$

$$P^* = \frac{1}{1-\bar{a}}\left[\frac{\bar{\sigma}\left(1+2\bar{h}\right)^2}{A} W_0^{*2} + \frac{4\bar{h}\gamma(1-\bar{\sigma})\left(1+2\bar{h}\right)}{A(1+\gamma)} W_0^* + C\right],$$
$$\frac{2\gamma\bar{h}}{(1+\gamma)\left(1+2\bar{h}\right)} \le W_0^* \le \frac{1+\gamma+2\gamma\bar{h}}{(1+\gamma)\left(1+2\bar{h}\right)} \tag{22}$$

and

$$P^* = \frac{1+2\bar{h}}{(1-\bar{a})A}\left[\left(1+2\bar{h}\right)W_0^{*2} + 2(\bar{\sigma}-1)W_0^* + 2\bar{h} + 1\right], \frac{1+\gamma+2\gamma\bar{h}}{(1+\gamma)\left(1+2\bar{h}\right)} \le W_0^* \le 1 \tag{23}$$

where

$$P^* = \frac{P}{P_c},$$
$$P_c = \frac{4M_p}{L}\ \bar{a} = \frac{a}{L}.$$

When $W_0^* \ge 1$, $N = N_p (n = 1)$, the effect of bending in the multilayer sandwich beam diminishes, and only the membrane forces dominate the behavior of the overall bending. Then, we have

$$P^* = \frac{2\left(\bar{\sigma}+2\bar{h}\right)\left(1+2\bar{h}\right)}{(1-\bar{a})A} W_0^*. \tag{24}$$

For the limit case of a fully clamped monolayer sandwich beam transversely loaded at midspan, i.e., $\gamma = 0$, Eqs. (21) to (24) are reduced to [18]

$$P^* = \begin{cases} \frac{1}{1-\bar{a}}\left[\frac{\bar{\sigma}\left(1+2\bar{h}\right)^2}{4\bar{h}\left(1+\bar{h}\right)+\bar{\sigma}} W_0^{*2} + 1\right], & 0 \le W_0^* \le \frac{1}{1+2\bar{h}} \\ \frac{\left(1+2\bar{h}\right)\left[\left(1+2\bar{h}\right)\left(W_0^{*2}+1\right)+2(\bar{\sigma}-1)W_0^*\right]}{(1-\bar{a})\left[4\bar{h}\left(1+\bar{h}\right)+\bar{\sigma}\right]}, & \frac{1}{1+2\bar{h}} \le W_0^* \le 1 \\ \frac{2\left(\bar{\sigma}+2\bar{h}\right)\left(1+2\bar{h}\right)}{(1-\bar{a})\left[4\bar{h}\left(1+\bar{h}\right)+\bar{\sigma}\right]} W_0^*, & W_0^* \ge 1. \end{cases} \tag{25}$$

During the process of overall deformation of the sandwich beam, the external work will be dissipated by the overall deformation of the sandwich beam, and the absorbed plastic deformation energy U of the sandwich beam can be calculated as

$$U = \int_0^{W_0} P(W_0)\, \mathrm{d}W_0, \tag{26}$$

and the normalized plastic energy is defined as $U^* = U/[P_c(2h + 2h_m + c)]$. Substituting Eqs. (21) to (24) into Eq. (26), we can easily obtain the dissipated plastic energy. The formulae are not reported here for the sake of brevity. Using the present method, the plastic behavior of the multilayer sandwich beam can be given. The plastic behavior of the multilayer sandwich beam is different from that of monolayer ones [18,27,28]. Also, the present method can be expanded to study the plastic behavior of the multilayer sandwich beam with any layers.

5 Finite element analysis

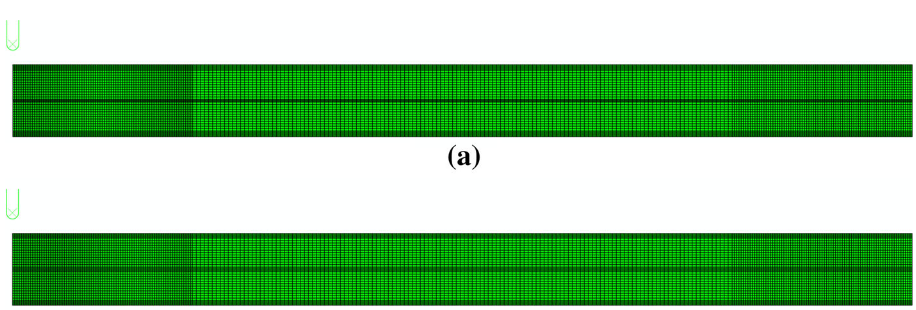
The finite element (FE) calculations are conducted to investigate the large deflection of fully clamped multilayer sandwich beams with metal foam cores using ABAQUS/Standard code. The face sheets, interlayer sheets and foam cores are modeled using four node and bilinear plane strain quadrilateral elements (Type CPE4 in ABAQUS) with full integration. The displacement loading is applied to a rigid roller. Half of the sandwich beam is considered in calculations due to the symmetry of the problem. The vertical, horizontal and rotational displacements of nodes at the ends of the beam are zero. Frictionless contact is employed between the rigid roller and the outer surface of the top face sheet. The typical samples of the multilayer sandwich beams are considered as follows.

Half span of the sandwich beam and the height of the foam core are $L = 750$ mm and $c = 24$ mm, and the sum of thicknesses of the top and bottom face sheets and interlayer sheet is $2h + h_m = 12$ mm, i.e., $\bar{c} = 2c/L = 0.064$ and $\bar{h} = 0.125$, respectively. Two kinds of the multilayer factor are considered: (i) $\gamma = 1/4$ and (ii) $\gamma = 1/2$. Radius of the loading roller is $R = 5$ mm. For the case of $\gamma = 1/4$, there are 12,600 elements in the FE model, 315 elements in the half span, and 6, 4, and 12 elements in the thickness direction of face sheets, the interlayer sheet, and foam cores, respectively. For the case of $\gamma = 1/2$, there are 12285 elements in the FE model, 315 elements in the half span, and 5, 5 and 12 elements in the thickness direction of the face sheets, the interlayer sheet and foam cores, respectively, as shown in Fig. 5. Appropriate mesh refinement near the clamped end and the location of the loading point is performed. A mesh sensitivity checked in calculations revealed that additional mesh refinement did not change the results appreciably.

The face sheets obey J_2 flow theory of plasticity. The top and bottom face sheets and interlayer sheets are made of aluminum with yield strength $\sigma_f = 30$ MPa, Young's modulus $E_f = 70$ GPa, and elastic Poisson's ratio $\nu_{\mathrm{ef}} = 0.3$, respectively. It is assumed that the face sheets and interlayer sheets obey a linear hardening law with tangent modulus $E_{\mathrm{tf}} = 5 \times 10^{-4} E_f$ and have sufficient ductility to sustain deformation without fracture.

The Deshpande–Fleck constitutive model [30] is used to model the plastic crushable behavior of the metal foam implemented in ABAQUS, which allows the shape change of the yield surface due to differential hardening along the hydrostatic and deviatoric axes. The yield function for the foam core is

$$\phi = \hat{\sigma} - \sigma_c = 0 \tag{27}$$



(a)

(b)

Fig. 5 Finite element model for the multilayer sandwich beams. **a** $\gamma = 1/4$ and **b** $\gamma = 1/2$

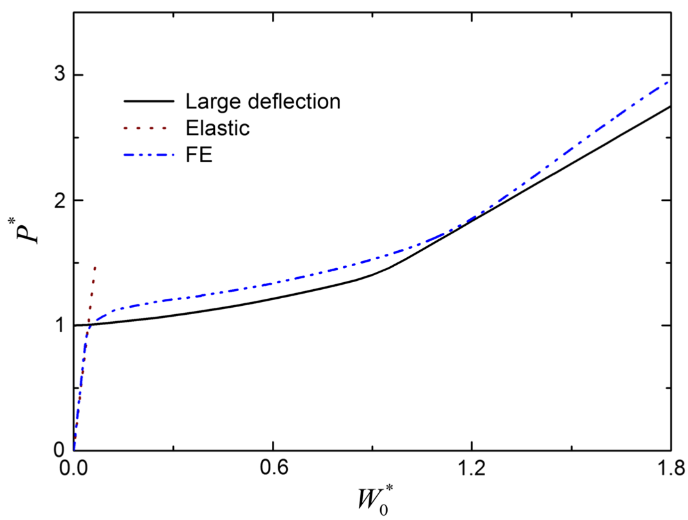
where

$$\hat{\sigma}^2 \equiv \frac{1}{1+(\alpha/3)^2}\left(\sigma_e^2+\alpha^2\sigma_m^2\right) \tag{28}$$

with $\sigma_e \equiv \sqrt{3s_{ij}s_{ij}/2}$ being von Mises effective stress, s_{ij} the deviatoric stress, $\sigma_m \equiv \sigma_{kk}/3$ the mean stress, and α the shape factor of the yield surface. The associated plastic flow rule is adopted, and the plastic Poisson's υ_p is calculated by

$$\nu_p = -\frac{\dot{\varepsilon}_{22}^p}{\dot{\varepsilon}_{11}^p} = \frac{1/2-(\alpha/3)^2}{1+(\alpha/3)^2}. \tag{29}$$

The isotropic foam cores are made of aluminum foam with yield strength $\sigma_c = 3\,\mathrm{MPa}$, Young's modulus $E_c = 1\,\mathrm{GPa}$, shear modulus $G_c = E_c/2\,(1+\nu_{\mathrm{ec}}) = 0.385\,\mathrm{GPa}$ with elastic Poission's ratio $\nu_{\mathrm{ec}} = 0.3$, and plastic Poission's ratio $\nu_p = 0$, relative density $\bar{\rho} = \rho_c/\rho_f = 0.05$, in which ρf and ρc are densities of face sheets and foam, respectively. The metal foam has a long plateau stress platform σ_c that continues up to densification strain $\varepsilon_D = 0.5$. Beyond densification, we assume that the metal foam obeys a linear hardening law with tangent modulus $E_{\mathrm{ct}} = 0.2E_f$.



(a)

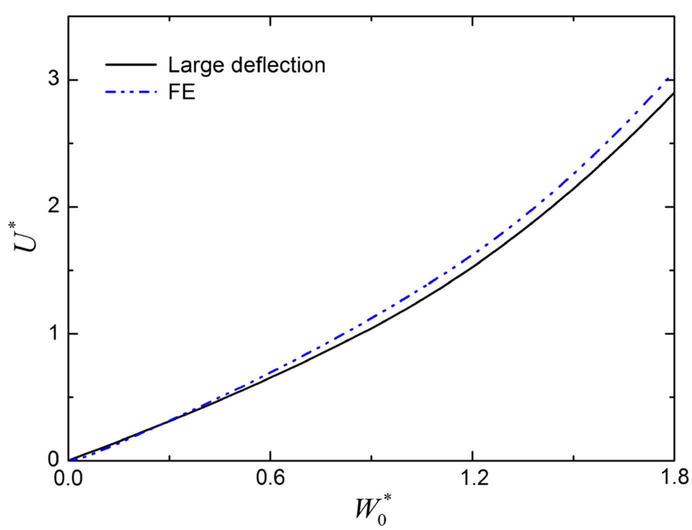


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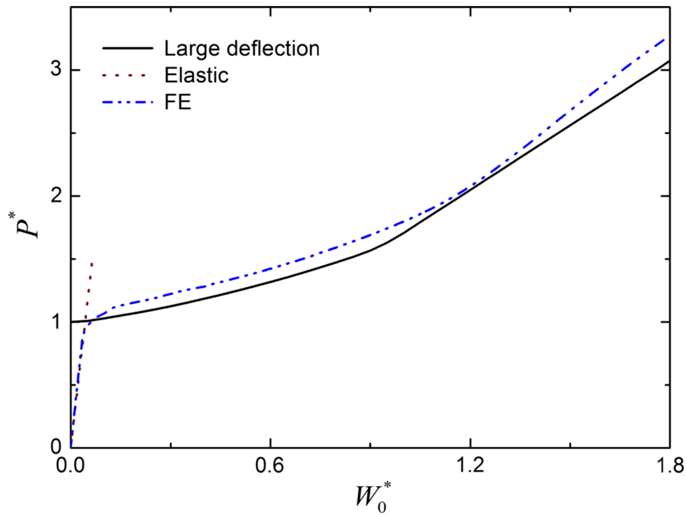
Fig. 6 Analytical and numerical results for the bending response of the fully clamped multilayer sandwich beam ($\gamma = 1/4$) with metal foam cores. **a** Normalized load versus normalized deflection, and **b** normalized plastic energy versus normalized deflection



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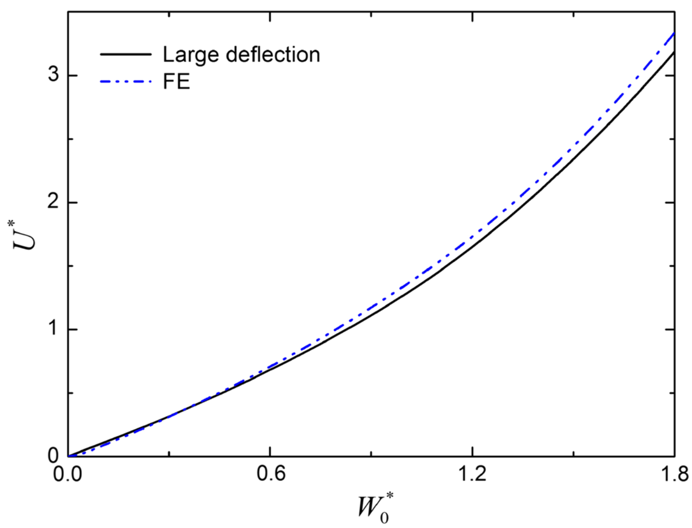


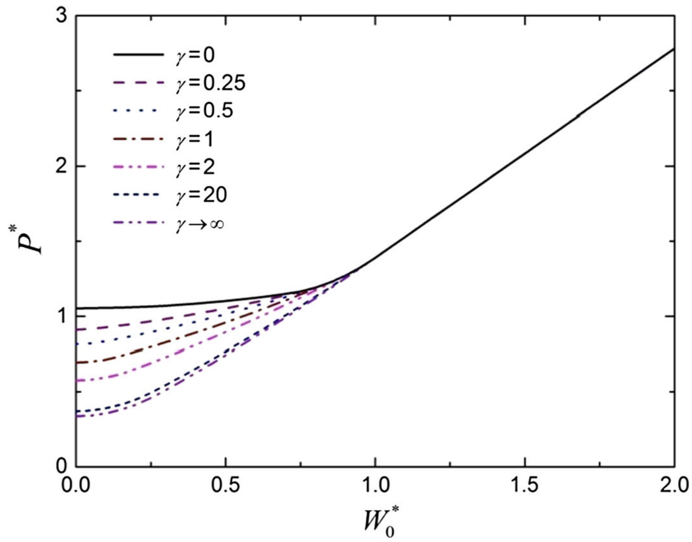
(b)

Fig. 7 Analytical and numerical results for the bending response of the fully clamped multilayer sandwich beam ($\gamma = 1/2$) with metal foam cores. **a** Normalized load versus normalized deflection, and **b** normalized plastic energy versus normalized deflection

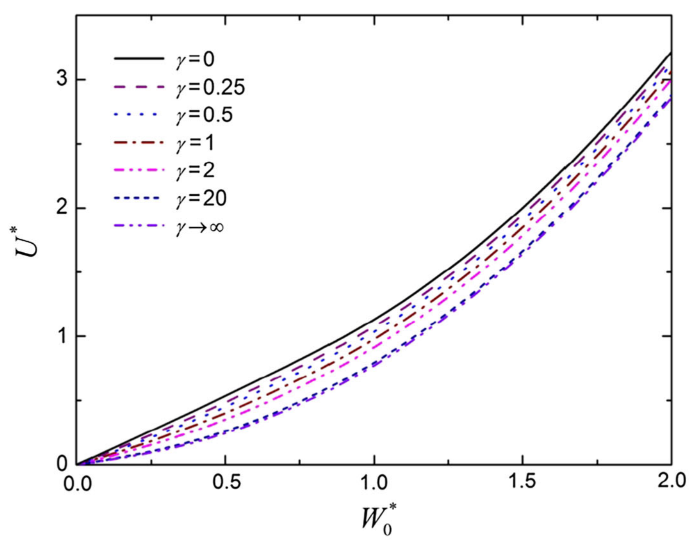
6 Results and discussion

Figures 6 and 7 show the present analytical predictions and numerical results for load P^* and energy U^* versus deflection W_0^* of the multilayer sandwich beams with $\gamma = 1/4$ and $\gamma = 1/2$ subjected to a concentrated loading ($\bar{a} \to 0$), respectively. It is readily seen that the large deflection solutions are in good agreement with FE results in the post-yield phase, and the elastic solutions agree well with FE results in the elastic phase, as shown in Figs. 6 and 7. Moreover, the discrepancy in the case of the small deflection may be due to neglecting the elastic deformation in theoretical analysis.

The effect of a multilayer on the relation of load P^* versus deflection W_0^* and energy U^* versus deflection W_0^* for the fully clamped multilayer sandwich beams under transverse loading by a flat punch is shown in Fig. 8a, b, respectively. When the deflection does not exceed the depth of the sandwich beams, the bigger the effect of multilayer factor γ is, the more the decrease of the load-carrying capacity of the multilayer sandwich beams is. Also, the load-carrying capacity of multilayer sandwich beams is the same as that of monolayer ones as the deflection exceeds the depth of the sandwich beams. The difference in the plastic energy absorption among the multilayer sandwich beams is almost a constant when $W_0^* \geq 1$, as shown in Fig. 8a, b. For the case



(a)

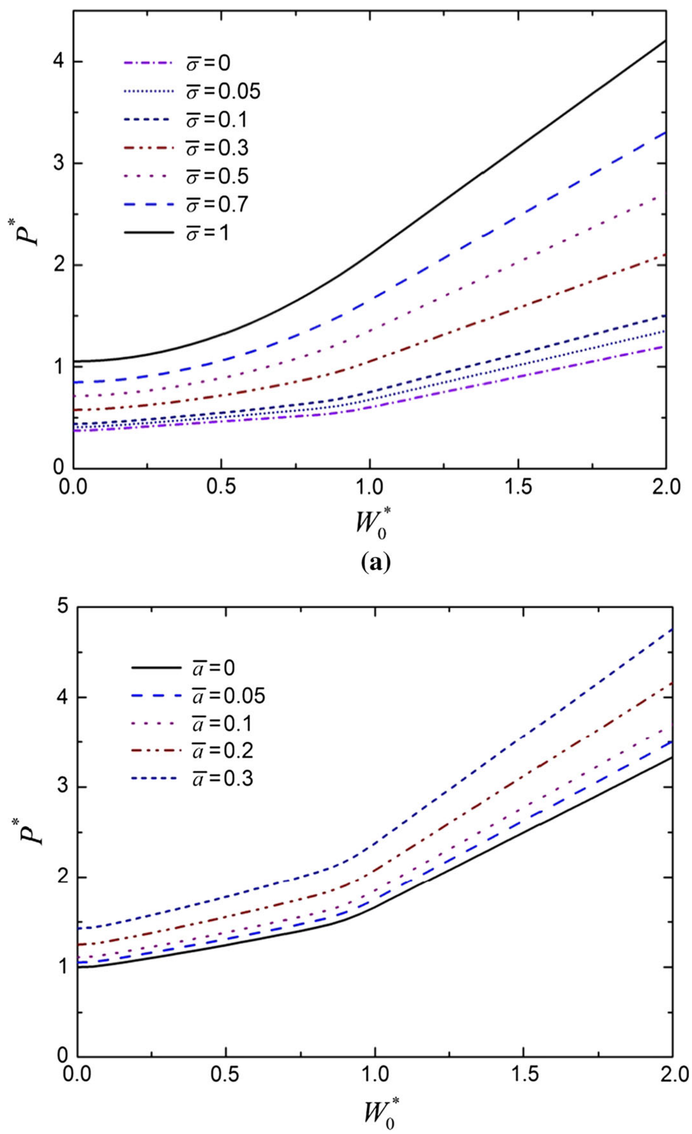


(b)

Fig. 8 Effect of multilayer factor on the load-carrying capacity and energy absorption of the fully clamped multilayer metal sandwich beams with $\bar{\sigma} = 0.1$, $\bar{h} = 0.2$ and $\bar{a} = 0.05$. **a** Normalized load versus normalized deflection, and **b** normalized plastic energy versus normalized deflection

of $\gamma = 20$, the load-carrying and energy absorption capacities of the multilayer sandwich beam are almost equivalent to those as $\gamma \to \infty$.

The effect of core strength on the load-carrying capacity for the fully clamped multilayer sandwich beams ($\gamma = 1/2$, $\bar{h} = 0.2$ and $\bar{a} = 0.05$) under transverse loading by a flat punch is shown in Fig. 9a. Also, the effect of punch size on the load versus deflection for the multilayer sandwich beams ($\gamma = 1/2$, $\bar{h} = 0.2$ and $\bar{\sigma} = 0.05$) is shown in Fig. 9b. It is readily seen that the core strength and punch size have significant effect on the load-carrying capacity of multilayer sandwich beams. For a fixed value of deflection, the load increases with the increase of core strength/punch size, which becomes more significant in large deflections.



(b)

Fig. 9 Effects of **a** core strength and **b** punch size on the load-carrying capacity of the fully clamped multilayer metal sandwich beams

7 Minimum mass design

As a preliminary application, the minimum mass design of multilayer sandwich beams is carried out with a constraint of an allowable maximum deflection at midspan. The aforementioned formulae for the load versus deflection of the multilayer sandwich beam are used in the minimum mass designs.

In order to construct the minimum mass design chart $(\bar{c}, \bar{h})$, Eqs. (21) to (24) for the normalized load $\bar{P}$ versus the normalized deflection $\bar{W}_0$ are rewritten, in which $\bar{P} = \frac{P}{bL\sigma_f}$ and $\bar{W}_0 = \frac{W_0}{L}$.

The normalized mass of the multilayer sandwich beam is defined as

$$\bar{M} = \frac{M}{bL^2\rho_f} = 2\bar{c}\left(\bar{\rho} + 2\bar{h}\right) \tag{30}$$

where M is the mass of the sandwich beam.

The optimal mass designs could be obtained in terms of the multilayer sandwich beam geometries $\bar{h}$ and $\bar{c}$, as shown in Fig. 10, which maximizes the normalized load $\bar{P}$ for the given normalized mass $\bar{M}$, the given

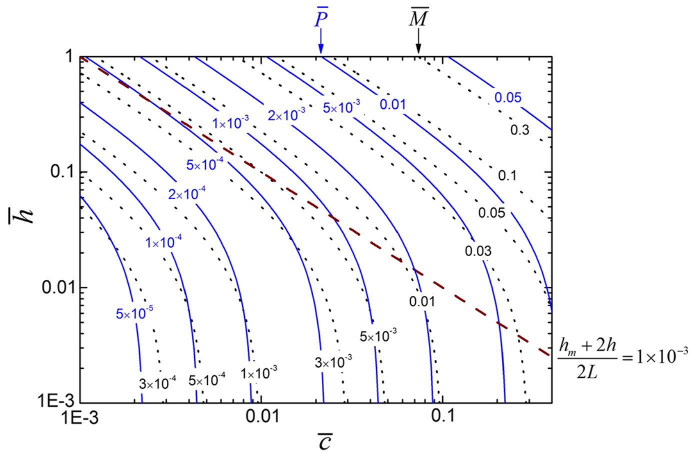


Fig. 10 Contours of the dimensionless mass $\bar{M}$ and the load index $\bar{P}$ for multilayer sandwich beams subjected to the allowable maximum deflection $\bar{W}_0 = 0.1$. The *dash line* $\frac{h_m+2h}{2L} = 1 \times 10^{-3}$ traces the path of the minimum mass designs

materials and allowable maximum central deflection $\bar{W}_0 = 0.1$. Figure 10 shows that multilayer sandwich beams have the maximum load $\bar{P}$ when $\bar{h} \to 0$ at almost constant $\bar{c}$ for a given mass $\bar{M}$, i.e., $\frac{h_m+2h}{2L} \to 0$. In engineering applications, the cases of $\frac{h_m+2h}{2L} \to 0$ do not satisfy other standards, and a minimum sum of thicknesses $h_m + 2h$ of face sheets and the interlayer sheet should be considered. Thus, similar to the constraint [26], the constraint of a normalized thicknesses of face sheet $\frac{h_m+2h}{2L} \geq 1 \times 10^{-3}$ is introduced. It means that the structural designs of the sandwich beam need to satisfy the maximum load-carrying capacity for a given mass and traces the path of the line $\frac{h_m+2h}{2L} = 1 \times 10^{-3}$, as shown in Fig. 10. The present method for the optimal designs can be used to study the optimal designs for monolayer sandwich structures also.

8 Concluding remarks

A plastic yield criterion and plastic behavior of fully clamped multilayer sandwich beams with metal foam cores were investigated. A yield criterion was proposed for multilayer sandwich structures, and an analytical solution was obtained for the large deflection of a fully clamped slender multilayer sandwich beam transversely loaded by a flat punch considering the interaction of bending and axial stretching. The analytical solutions are in good agreement with the FE results. Using the analytical formulae, the optimal designs were constructed to maximize the load-carrying capacity of multilayer sandwich beams for a given mass. It is shown that the core strength and loading punch size have significant effects on the structural response of the multilayer sandwich beam. The effect of a multilayer can be neglected as the deflection is larger than the depth of the sandwich beam. The axial stretching induced by large deflection plays an important role in the post-yield regime of the multilayer sandwich beam.

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