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Progressive damage identification using dual extended Kalman filter

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Abstract Existing Kalman filter-based parameter identification algorithms estimate the system parameters as either sole states or a subset of augmented states. While the former approach requires the measurement to be sufficiently clean, the latter is reported to have numerical stability issues. Since the parameters are estimated in both these approaches in an optimal sense, in the presence of a significant variation in parameters (due to damage), the estimates may often diverge. In this article, we propose an online health monitoring scheme powered by dual extended Kalman filtering technique to simultaneously estimate the system parameters along with the response states of a reduced-order system. To capacitate damage localization beyond sensor resolution, the proposed method employs location-based structural properties as system parameter. This reduces the dimensionality of the formulation from $4n^2$ elements in the state matrix to only a few physical parameters. Unnecessary estimation of a large number of unmeasured response states has been avoided by employing the system reduction technique and thus by describing the system using only measured DOFs. This in turn enables estimating a poorly observed system as a fully observed one. Two numerical experiments are performed on two degrading structures: an Euler–Bernoulli beam and a bridge truss to demonstrate the competency of the algorithm with reduced-order models.

1 Introduction

To ensure safety and serviceability of the existing infrastructure systems, the current health of the structure is necessary to be estimated frequently and any instance of damage should be reported without any delay. This creates the requirement of online health monitoring of important civil infrastructure systems to identify anomalies in structures ideally just when they occurred. For uninterrupted monitoring, the structures are generally equipped with a network of sensors placed at critical locations to record response signals on a continuous basis. A computationally inexpensive real-time algorithm then analyzes the measured response and the important health parameters of the structure are estimated to assess the current health or existence of damage indirectly.

Structural health monitoring (SHM) or damage identification based on measured vibration signals has always been an intensively researched topic. Different methods of damage identification (encompassing the four stages of damage detection discussed by Rytter [35]) are tried by several researchers [6,15–17]. Traditionally, the structural health is defined by some control parameters which are identified using online or offline optimization algorithms. There exist several time or frequency domain techniques to assess structural

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health [6,21,45]. In time domain algorithms, least-square estimation techniques are mostly employed to estimate parameters in real time [38–40]. Lately, recursive Bayesian filters have emerged as an efficient alternative for this objective owing to the simplicity in the estimation procedure [18,20,29,36,37]. Among the different types of Bayesian filters used for SHM, Kalman filter (KF) and its nonlinear variants, e.g., extended (EKF) [20] and unscented (UKF) [44] Kalman filter, Monte Carlo filter [9], and particle filter [46], are the most frequently employed techniques.

2 Kalman filter-based parameter identification

KF [22,23,43] is a recursive Bayesian filtering technique to obtain clean estimates of states from noise contaminated measured signals. At every time step, it defines a gain matrix using a set of predefined process and measurement noise covariance to incorporate the mismatch between measured and predicted output response as feedback into the system. This gain matrix in turn regulates the intermixing between prior belief and the information in the new data to predict the state estimate for the next time step. This algorithm when used for parameter identification either appends the parameters to an existing state vector (Augmented state KF (ASKF) or coupled approach) or considers parameters as the only states (decoupled approach). The system dynamics is thus described as:

$$\begin{aligned} \bar{\mathbf{x}}_k &= f(\bar{\mathbf{x}}_{k-1}) + \mathbf{v}_k; \\ \varepsilon_k &= \{\mathbf{y}_k - h(\bar{\mathbf{x}}_k)\} + \mathbf{w}_k, \end{aligned} \tag{1}$$

where $\bar{\mathbf{x}}_k = \{\mathbf{x}_k\ \theta_k\}^T$ (coupled approach) or $\bar{\mathbf{x}}_k = \theta_k$ (decoupled approach) with $\mathbf{x}_k$ and θ_k are the response state and system parameters, respectively. $f(\bar{\mathbf{x}}_{k-1})$ is the state transition function describing the temporal evolution of the system states $\bar{\mathbf{x}}_k$. $h(\bar{\mathbf{x}}_k)$ is a linear or nonlinear mapping of unobservable states to measurement. $\mathbf{v}_k$ and $\mathbf{w}_k$ are the zero-mean Gaussian process and measurement noise with constant covariance.

For a given noisy measurement signal recorded from a system, KF estimates the system states $\bar{\mathbf{x}}_k$ in an optimal sense. At any arbitrary time step k, KF uses Bayesian belief propagation technique recursively to update prior estimate of states $\hat{\bar{\mathbf{x}}}_{k-1}$ using the measurements $\mathbf{y}_{1:k}$. Since the noise $\mathbf{v}_k$ and $\mathbf{w}_k$ are considered to be uncorrelated Gaussian, Eq. (1) describes a Gauss–Markov process in $\bar{\mathbf{x}}_k$, and hence $\bar{\mathbf{x}}_k$ has the Markovian property. Since $\mathbf{y}_k$ is a mapping of $\bar{\mathbf{x}}_k$, it too can be shown to be Markovian satisfying the following conditions:

$$\begin{aligned} \rho\left(\bar{\mathbf{x}}_k|\bar{\mathbf{x}}_{1:k-1}, \mathbf{y}_{1:k-1}\right) &= \rho\left(\bar{\mathbf{x}}_k|\bar{\mathbf{x}}_{k-1}\right), \\ \rho\left(\mathbf{y}_k|\bar{\mathbf{x}}_{1:k}, \mathbf{y}_{1:k-1}\right) &= \rho\left(\mathbf{y}_k|\bar{\mathbf{x}}_k\right). \end{aligned} \tag{2}$$

Provided that the prior estimate of states $\rho(\bar{\mathbf{x}}_{k-1}|\mathbf{y}_{1:k-1})$ at any arbitrary time step $k-1$ is known, the prediction for the next time step k can be obtained using Chapman–Kolmogorov equation as:

$$\rho\left(\bar{\mathbf{x}}_k|\mathbf{y}_{1:k-1}\right) = \int \rho(\bar{\mathbf{x}}_k|\mathbf{x}_{k-1})\rho\left(\bar{\mathbf{x}}_{k-1}|\mathbf{y}_{1:k-1}\right) \mathrm{d}\mathbf{x}_{k-1}. \tag{3}$$

While the second part of the integrand describes the prior probability, the former demonstrates the temporal transition of states, i.e., process model.

Using recursion rule for Bayesian inferencing, in each step of filtering the updated (or posterior) probability density function $\rho(\bar{\mathbf{x}}_k|\mathbf{y}_{1:k})$ of the current state $\bar{\mathbf{x}}_k$ conditioned upon measurements $\mathbf{y}_{1:k}$ can be obtained as:

$$\rho(\bar{\mathbf{x}}_k|\mathbf{y}_{1:k}) = \alpha\rho(\bar{\mathbf{x}}_k|\mathbf{y}_{1:k-1})\rho(\mathbf{y}_k|\bar{\mathbf{x}}_k). \tag{4}$$

The densities $\rho(\bar{\mathbf{x}}_k|\mathbf{y}_{1:k-1})$ and $\rho(\bar{\mathbf{x}}_k|\mathbf{y}_{1:k})$ are the prior and posterior probabilities, respectively, of the states $\bar{\mathbf{x}}_k$, while $\rho(\mathbf{y}_k|\bar{\mathbf{x}}_k)$ signifies the likelihood estimate of the states for the given measurement $\mathbf{y}_k$. α is a normalization coefficient.

KF assumes Gaussian distribution for the initial state and the noise terms. Thus the maximum a posteriori (MAP) estimate corresponds to the mean of the posterior distribution conditioned on the measurements $\mathbf{y}_{1:k}$:

$$\hat{\bar{\mathbf{x}}}_{k|k} = \arg\max_{\bar{\mathbf{x}}_k} \rho\left(\bar{\mathbf{x}}_k|\mathbf{y}_{1:k}\right), \tag{5}$$

where $\hat{\bar{\mathbf{x}}}_{k|k}$ signifies the estimate of state $\bar{\mathbf{x}}_k$ for a given measurement information up to time step k.

However, either the process model $f(\cdot)$ (for the coupled approach) or the measurement model $h(\cdot)$ (for the decoupled approach) brings an unavoidable nonlinearity into the system description for parameter estimation problems. Thus, instead of KF (a linear estimator), its nonlinear variants (e.g., extended KF, unscented KF) are generally employed which perform a local linearization of the system around the current estimate of states.

Hoshiya and Saito [20] demonstrated the use of the Kalman filtering technique for structural parameter identification. This approach was later implemented with modifications on different problems by several others [18,29]. In their work, the researchers appended the control parameters below the regular states (generally system responses), and this extended state vector is identified online to estimate the current health of the structure. This dual estimation method is popularly used under the name of Joint estimation of state and parameters. Under joint estimation of states and parameters, Corigliano and Mariani [12] investigated the performance of EKF for time varying systems and found it to be not precise enough. Al-Hussein and Haldar [2] demonstrated the application of the UKF technique with the ASKF approach for structural health estimation which is later extended to systems with unknown input [1]. Mariani and Ghisi [28] compared the performance of UKF against EKF for parameter estimation. For limited instrumented structures, Das et al. [14] demonstrated an FE model-based EKF technique to localize damage using limited response measurements. Chatzi and Smyth [8] presented the PF method for estimating states along with model parameters of a Bouc–Wen model, while Azam and Mariani [5] described a coupled extended Kalman PF approach in order to estimate a partially observed system.

With parameters appended to the state vector, the major problem with dual estimation that has been identified by Ljung [25], Ljung and Söderström [26] and Nelson and Stear [32] is convergence. Ljung [25] and Ljung and Söderström [26] attribute this drawback to a simplification in the gradient calculation of the process function with respect to keeping parameter states constant and thus by avoiding recursive derivation of states, while Nelson and Stear [32] blame the crude linearization of higher-order coupling between states and parameters. The issue of convergence can however be avoided through decoupling the parameters and estimating them as the only states (decoupled approach). Nevertheless, it demands a sufficiently clean measured signal [19] while the reality seldom conforms to this idealization.

In both these approaches (coupled or decoupled), the parameters are considered as time-invariant system states evolving over time through an identity state transition matrix. KF is then employed to estimate an optimal set of time-invariant system parameters which minimizes the measurement prediction error. On the contrary, real systems hardly remain time invariant, and for the case of a progressive damage scenario, the system can drastically change over a small time interval. Since KF-based parameter identification algorithms are model based, it is very likely that on the occurrence of significant change in a parameter, the estimation may diverge and can even leave the zone of optimal solution [12]. In order to cater for this possible time variance of the system, simultaneous estimation of system states and parameters is necessary. Chen [10] demonstrated the differences between joint and dual estimation methods along with the benefits of the dual estimation strategies.

2.1 Present work

We employ the dual extended Kalman filtering technique [41,42] for SHM application with location-based structural parameters as parameter states of the system which are estimated simultaneously with response states. In each time step, the DEKF estimates the system states based on the current estimates of parameters and then proceeds to estimate parameters based on the current estimates of states. This enables estimation of the time-varying system parameters using filtered system states.

Since instrumenting a structure is a costly affair due to both initial and recurring maintenance cost, it should be maintained at minimum required level. Employing location-based structural parameters of an appropriate phenomenological model of the system as parameter states capacitate health estimation beyond sensor resolution and thus by reducing the demand of dense instrumentation. However, detailed estimation necessitates primary model to be discretized accordingly with large numbers of DOFs, while in practice only a few DOFs are instrumented for the response measurement purpose. This renders the system to be a poorly observed system with a paltry fraction of all states measured. Estimation of the unmeasured response quantities is usually redundant yet required only for state prediction purpose. In this article, we therefore employ system reduction approach in order to remove all these unmeasured DOFs from the estimation algorithm.

Two numerical case studies, performed on a simply supported concrete beam and a truss bridge, are presented to validate the proposed method's competence to monitor structural systems efficiently using few channels of measured signal.

3 Dual extended Kalman filter

DEKF was first introduced by Wan and Nelson [41,42] as a nonlinear extension of Nelson and Stear's dual Kalman filtering technique [32]. This development employs two concurrent EKF to simultaneously estimate states and parameters of a nonlinear speech recognition problem. DEKF alternates between estimating states based on current estimate of parameters and estimating parameters based on current estimate of states.

Wan and Nelson [42] detailed the basic philosophy of this algorithm in their paper [19]. The probabilistic aspect of the algorithm is presented in the following. With the DEKF approach, the process and measurement equations of the system dynamics presented in Eq. (1) can be described as:

$$\begin{aligned} \text{Process equation 1:} \quad \mathbf{x}_k &= f(\mathbf{x}_{k-1}, \theta_{k-1}) + \mathbf{v}_k^x; \\ \text{Process equation 2:} \quad \theta_k &= \theta_{k-1} + \mathbf{v}_k^\theta; \\ \text{Measurement equation:} \quad \varepsilon_k &= \{\mathbf{y}_k - h(\mathbf{x}_k, \theta_k)\} + \mathbf{w}_k^x. \end{aligned} \tag{6}$$

The process and measurement noise terms $\mathbf{v}_k^x$, $\mathbf{v}_k^\theta$ and $\mathbf{w}_k^x$ in the above equation are characterized by zero-mean Gaussian noise with constant covariance matrices $\mathbf{Q}^x$, $\mathbf{Q}^\theta$ and $\mathbf{R}^x$ respectively.

To employ Bayesian estimation to estimate the system states conditioned on the current estimates of parameters, the DEKF algorithm expands Eq. (5) as:

$$\{\hat{\mathbf{x}}_{k|k}, \hat{\theta}_{k|k}\} = \underset{\{\mathbf{x}_k, \theta_k\}}{\arg\max} \rho(\mathbf{x}_k|\theta_k, \mathbf{y}_{1:k})\rho(\theta_k|\mathbf{y}_{1:k}). \tag{7}$$

In each time step, DEKF adopts two separate estimation schemes for the states and the parameters. This can be described as follows:

$$\hat{\mathbf{x}}_{k|k} = \underset{\mathbf{x}_k}{\arg\max} \rho(\mathbf{x}_k|\theta_k, \mathbf{y}_{1:k}); \quad \hat{\theta}_{k|k} = \underset{\theta_k}{\arg\max} \rho(\theta_k|\mathbf{y}_{1:k}). \tag{8}$$

Subsequently, two concurrent EKFs are employed to estimate the states and the parameters using the noisy measurement response. The algorithmic approach is detailed in the following.

The estimation initiates with an initially assumed Gaussian states and parameters as:

$$\begin{aligned} \hat{\mathbf{x}}_0 &= E(\mathbf{x}_0) \quad \text{with covariance} \quad \mathbf{P}_{\mathbf{x}_0} = E\left[\left(\mathbf{x}_0 - \hat{\mathbf{x}}_0\right)\left(\mathbf{x}_0 - \hat{\mathbf{x}}_0\right)^T\right], \\ \hat{\theta}_0 &= E(\theta_0) \quad \text{with covariance} \quad \mathbf{P}_{\theta_0} = E\left[\left(\theta_0 - \hat{\theta}_0\right)\left(\theta_0 - \hat{\theta}_0\right)^T\right]. \end{aligned} \tag{9}$$

In the following, one-step-ahead prediction of the mean of the states $\tilde{\mathbf{x}}_{k|k-1}$ and corresponding state error covariance $\tilde{\mathbf{P}}_{\mathbf{x}_{k|k-1}}$ are obtained by running the system model with current estimates of states and parameters, i.e., $\hat{\mathbf{x}}_{k-1|k-1}$ and $\hat{\theta}_{k-1|k-1}$ and linearly propagating the prior covariance estimate $\hat{\mathbf{P}}_{\mathbf{x}_{k-1|k-1}}$ as:

$$\tilde{\mathbf{x}}_{k|k-1} = f(\hat{\mathbf{x}}_{k-1|k-1}, \hat{\theta}_{k-1|k-1}); \quad \tilde{\mathbf{P}}_{\mathbf{x}_{k|k-1}} = \mathbf{A}_k \hat{\mathbf{P}}_{\mathbf{x}_{k-1|k-1}} \mathbf{A}_k^T + \mathbf{Q}^x, \tag{10}$$

where the state transition function $f(\cdot)$ is linearized around the current state estimate $\hat{\mathbf{x}}_{k-1|k-1}$ as:

$$\mathbf{A}_k = \nabla_x f\left(\hat{\mathbf{x}}_{k-1|k-1}, \hat{\theta}_{k-1|k-1}\right)|_{k-1}. \tag{11}$$

The temporal evolution model for parameters is, however, considered to be time invariant and only perturbed by Gaussian noise. The time update of parameter statistics thus can be obtained as:

$$\tilde{\theta}_{k|k-1} = \hat{\theta}_{k-1|k-1}; \quad \tilde{\mathbf{P}}_{\theta_{k|k-1}} = \hat{\mathbf{P}}_{\theta_{k-1|k-1}} + \mathbf{Q}^\theta. \tag{12}$$

This constitutes the prediction phase of the algorithm. In the following the predicted estimates are corrected sequentially using the current measurement $\mathbf{y}_k$. The error in the measurement is therefore calculated as:

$$\varepsilon_k = \mathbf{y}_k - \mathbf{C}\tilde{\mathbf{x}}_{k|k-1}. \tag{13}$$

where $\mathbf{C}$ is a Boolean matrix to select the states corresponding to the measured DOFs. The measurement update of states is quite straight forward. The Kalman gain matrix for the state update can be obtained as:

$$\mathbf{K}_k^x = \tilde{\mathbf{P}}_{\mathbf{x}_{k|k-1}} \mathbf{C}^T \left(\mathbf{C} \tilde{\mathbf{P}}_{\mathbf{x}_{k|k-1}} \mathbf{C}^T + \mathbf{R}^x \right)^{-1}, \tag{14}$$

which can subsequently be employed for updating the predicted state estimates as:

$$\hat{\mathbf{x}}_{k|k} = \tilde{\mathbf{x}}_{k|k-1} + \mathbf{K}_k^x \varepsilon_k; \quad \hat{\mathbf{P}}_{\mathbf{x}_{k|k}} = \left(\mathbf{I} - \mathbf{K}_k^x \mathbf{C} \right) \tilde{\mathbf{P}}_{\mathbf{x}_{k|k-1}}. \tag{15}$$

The measurement update of the parameter is, however, a little complex since the relation between error statistics and parameter is not straight forward. The measurement matrix $\mathbf{C}_k^\theta$ for the parameter filter is therefore calculated as:

$$\mathbf{C}_k^\theta = -\nabla_\theta \left(\mathbf{y}_k - \mathbf{C} \hat{\mathbf{x}}_{k|k} \right) |_{\theta = \tilde{\theta}_{k|k-1}} = \mathbf{C} \nabla_\theta \tilde{\mathbf{x}}_{k|k-1} |_{\theta = \tilde{\theta}_{k|k-1}}. \tag{16}$$

The Kalman gain for the parameter update is subsequently obtained as:

$$\mathbf{K}_k^\theta = \tilde{\mathbf{P}}_{\theta_{k|k-1}} \mathbf{C}_k^\theta \left(\mathbf{C}_k^\theta \tilde{\mathbf{P}}_{\theta_{k|k-1}} \mathbf{C}_k^{\theta\,T} + \mathbf{R}^\theta \right)^{-1}. \tag{17}$$

The measurement update of the parameter estimate can then be obtained as:

$$\hat{\theta}_{k|k} = \tilde{\theta}_{k|k-1} + \mathbf{K}_k^\theta \varepsilon_k; \quad \hat{\mathbf{P}}_{\theta_{k|k}} = \left(\mathbf{I} - \mathbf{K}_k^\theta \mathbf{C}_k^\theta \right) \tilde{\mathbf{P}}_{\theta_{k|k-1}}. \tag{18}$$

4 Existing application of DEKF

There exists sufficient evidence of the competency of this algorithm tested on battery management, vehicular motion control, reservoir monitoring, speech recognition problems etc. [11,24,31,34,41] in the literature. However, application of this powerful tool for structural health monitoring is not widely explored. Mariani and Corigliano [27] and Corigliano et al. [13] implemented the DEKF algorithm in order to detect composite layer delamination by estimating the constitutive parameters of a softening model. Recently Azam et al. [4] demonstrated the application of the DKF method to simultaneous estimation of states and input forces.

The above-described DEKF algorithm can be successfully employed considering the elements of the state transition matrix as parameters of the system. In this attempt the process equations can be described as:

$$\begin{aligned} \mathbf{x}_k &= \mathbf{A}_k \mathbf{x}_{k-1} + \mathbf{v}^x, \text{ where } \mathbf{x}_k = \{\mathbf{x}_d \quad \mathbf{x}_v\}^T, \\ \mathbf{A}_k^{ij} &= \mathbf{A}_{k-1}^{ij} + \mathbf{v}^\theta, \end{aligned} \tag{19}$$

where $\mathbf{x}_d$ and $\mathbf{x}_v$ are the array of displacements and velocity response states, respectively, with the elements of the state transition matrix $\mathbf{A}_k^{ij}$ being the parameters of this formulation.

This approach is, however, model independent and thus limited for systems with collocated measurements with damage localization capability restrained by the available measurement resolution. For non-collocated measurements, the application of phenomenological models (e.g., FE model) becomes necessary in order to incorporate the essential correlations between different states for identifying the system as a whole. However, the order of calculation increases with the model order at the rate of $4n^2$, where n is the physical space model order (i.e., numbers of sensors placed on the structure). Thus, unless the number of sensors is small, the computational demand for this algorithm becomes so high that it loses practicality.

Additionally, since general civil engineering systems are rather complicated, damage interpretation from the elements of state matrix is not always easy. We, therefore, considered location-based structural parameters as the system parameters to be estimated using limited numbers of signal channels. The modification is detailed in the following sections.

5 Modified DEKF algorithm

Unlike in the previously described approach (see Eq. (19)), in the modified approach, we define the location-based health parameters as system parameters θ_k. The state transition function $f(\cdot)$ in Eq. (6) thus uses a set of physical structural parameters in the FE modeling environment to define the time evolution of the state estimate. This strategy helps in three ways:

1. It preserves the physics of the system through the FE model, which is not guaranteed in case of identification of each element of the state matrix $\mathbf{A}_k$,
2. Updating every element of $\mathbf{A}_k$ without using any constraints may result in an infeasible estimation especially when the model order is large,
3. It drastically reduces the order of calculation from $4n^2$ to the few physical parameters intended to be identified.

Detection of system deterioration through time variance of its elemental property is physically more interpretable than through the elements of the state transition matrix. It should also be noted that with the former algorithm, the order of the identified state matrix $\mathbf{A}_k$ is $2n$ for which n is the number of channels in the measured signal describing the physical system with n DOFs. Thus, with this approach, the resolution of damage localization is limited by the instrumentation density which is in turn constrained by cost. In contrast the proposed method considers parameters of phenomenological models as parameter states of the formulation. Thus, for limited instrumented structure, precise damage localization can still be achieved beyond sensor resolution using higher-order models at the expense of computation time.

Nevertheless, in reality only a small fraction of all the DOFs of the system model are instrumented. Thus, complete state estimation of such poorly observed system often increases the ill-posedness of the problem. Estimation of the complete system states (including the unmeasured states) is mostly unnecessary, yet it increases the computational burden. In addition to that, while the proposed method puts more importance on the model calibration than the state filtering by putting more belief in the measurement signal compared to the estimated response, any poor estimate of the unmeasured states may often cause a significant divergence in the parameter estimates [4,47]. In order to avoid such a scenario, the unmeasured states are decided to be kept out of the parameter estimation.

With a similar objective, in a very recent research article Capellari et al. [7] (an extension of [3]) proposed particle Kalman filter-based estimation of a reduced-order system employing proper orthogonal modes (POM) for system reduction. However, their method is near real time in nature, and in order to construct a snapshot matrix, it requires response data for a particular time span during which either the system must behave as time invariant or a recursive updating of the POMs becomes necessary. Our development, on the other hand, aims at real-time estimation of health parameters of systems for which stiffness deterioration is ongoing, so that it can be applied to non-stationary signals as well. In this modified approach the non-obligatory estimation of unmeasured states is dropped through employing the Structural Equivalent Reduction Expansion Programme (SEREP) [33] and is characteristically a real-time algorithm.

In this modified DEKF approach, the response state transition function $f(\cdot)$ is defined as an envelop of several functions which uses current estimate of state and parameters as input for one-step-ahead prediction of the system state (at measured DOFs only). Within this function, using the current parameter estimates, a sufficiently large-order FE model of the system along with the stiffness, mass and damping matrices (i.e., $\mathbf{K}$, $\mathbf{M}$, $\mathbf{D}$) involving all required location-based parameters, is prepared. Subsequently, the full-order model is reduced to the instrumented DOFs only using SEREP and a corresponding reduced-order state space model is prepared:

$$\begin{aligned} \text{System reduction:} \quad & \left[\mathbf{K}^r, \mathbf{M}^r, \mathbf{D}^r\right] = \text{SEREP}(\mathbf{K}, \mathbf{M}, \mathbf{D}), \\ \text{State space model formulation:} \quad & \mathbf{A}_c^r = \begin{bmatrix} \mathbb{Z}^r & \mathbb{I}^r \\ -\mathbf{M}^{r\,-1}\mathbf{K}^r & -\mathbf{M}^{r\,-1}\mathbf{D}^r \end{bmatrix}, \end{aligned} \tag{20}$$

where the superscript r signifies a reduced-order model involving r measured DOFs only, and $\mathbb{Z}^r$ and $\mathbb{I}^r$ are the rth order null and identity matrices, respectively. $\mathbf{A}_c^r$ is the continuous time state matrix. Thus, this model order reduction converts the partially observed system to a fully observed system which is then estimated using the DEKF algorithm.

However, since the filtering is performed using sampled data, the continuous time state space model is required to be transformed accordingly using the Zero Order Hold (ZOH) technique as:

$$\mathbf{A}_d^r = \text{ZOH}(\mathbf{A}_c^r), \tag{21}$$

where $\mathbf{A}_d^r$ is the discrete time state matrix of the system. Using this discrete time state matrix, the prior estimate of the state is propagated to predict the state for the next time step as:

$$\mathbf{x}_k^r = \mathbf{A}_d^r \mathbf{x}_{k-1}^r, \tag{22}$$

where $\mathbf{x}_k^r$ signifies the reduced-order system states.

6 Numerical validation

The proposed method is validated through two numerical experiments performed on two basic structure types: an Euler–Bernoulli beam and a bridge truss. In both these structures, the progressive damage is represented by systematically degrading the material or geometric parameters for some selected zone. A white noise excitation is applied to simulate the system response which is subsequently used for online identification of the parameters. Details of each of these experiments are given in the next two sections.

6.1 Euler–Bernoulli beam

An Euler–Bernoulli beam is considered for the first numerical experiment. Details of the assumed geometric and material properties are listed in Table 1. The model is excited with a white noise sequence and a 20 s acceleration signal is simulated at a sampling frequency of 1000 Hz using the Newmark-Beta algorithm. A 2 % (signal-to-noise or SNR) Gaussian uncorrelated white noise has been added to the numerical signal in order to replicate the measurement noise in reality. Damage is induced in the 16th and 28th element and the rate of degradation is considered to be following a sigmoid curve shown in Fig. 1 along with its equation. The assumed damage rate is considered to be following the same equation with $t_s = 8$ s, $t_e = 12$ s, $p = 50\,\%$ for 16th and $p = 75\,\%$ for 28th element. Thus the progressive damage is induced at the 8th second of simulation and within next 4 s the damaged elements are degraded up to their final value, i.e., 50 and 75 %. Furthermore it is considered that the structure is instrumented with only vertical translational accelerometers at some selected locations and therefore only the acceleration response corresponding to those DOFs is considered for the identification.

Two case studies are undertaken, each using a different number of signal channels to identify damage. For each case study, the number of accelerometers and their respective locations are listed in Table 1. The contaminated signal obtained from the degrading structure is used in the proposed identification algorithm. Elasticity of all 40 elements of the beam is considered as the control parameters through which damage is interpreted as degradation in element elasticity. DEKF is then employed to filter the response states in parallel to estimating element elasticities. Identification results corresponding to all case studies are presented in Fig. 2.

It has been observed that with a 20 channel measured signal the estimation runs perfectly localizing the damage without much difficulty. 10 and 5 channels of recorded signal are therefore considered for demonstration of the efficacy of the algorithm. Figure 2a, b describes the identified elasticity values of each element of the beam along with its actual values for 10 and 5 channels of recorded signal, respectively. This is evident from Fig. 2 that denser instrumentation increases the efficiency of the algorithm which is obvious as well. However, Fig. 2b demonstrates that even with considerable reduction of instrumentation the parameter filter yields sufficiently good result which is beneficial from an economic view point. Eventually, there should be a compromise between added cost and desired precision level to achieve overall efficiency of the monitoring system.

Table 1 Details of the FE model of the beam

Dimension	6 m × 0.2 m × 0.25 m
Elasticity	3.2×10^9 N/mm^2
Poissson's ratio	0.3
Number of elements	40
Damping ratio	2 %
Damaged elements and damage percentage	75 % at 16th and 50 % at 28th
Instrumented locations	Case 1: nodes 3, 7, 11, 15, 19, 23, 27, 31, 35 and 39
	Case 2: nodes 5, 13, 21, 29 and 37

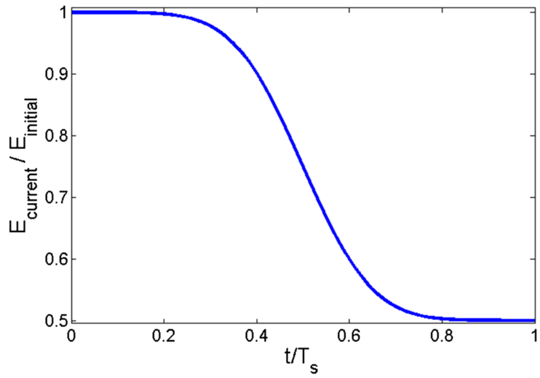

Fig. 1 Equation for assumed degradation curve: $\theta_{\text{current}} = \theta_{\text{initial}} \left(1 - \frac{0.01p}{2\text{erf}(3)}\right) \left(\text{erf}\left\{\frac{6(t-t_s)}{(t_e-t_s)-3}\right\} - \text{erf}(-3)\right)$ t_s, t_e and t signifies the damage initiation, completion time and current time respectively. θ is the parameter of consideration. Above figure presents a sample parameter degradation with $t_s = 0$ s, $t_e = 1$ s and $p = 50\,\%$ for a time span of 1 s

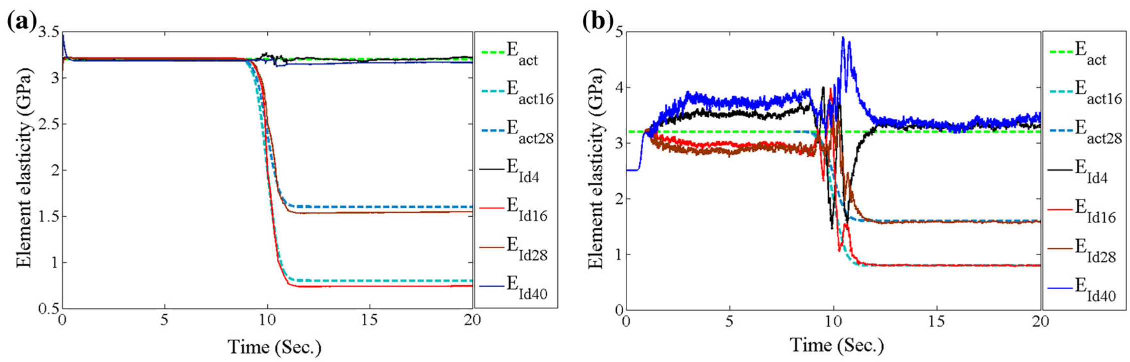

Fig. 2 Progressive damage identification of beam with different numbers of response signal channels. **a** Estimation result for 10 channel measured signal, **b** estimation result for 5 channel measured signal

6.2 Bridge truss

In the second numerical experiment, a bridge truss is taken for which the geometry, boundary condition and section details are adopted with minor alterations from a paper titled "Damage detection of truss bridge joints using Artificial Neural Network" authored by Mehrjoo et al. [30]. This truss presented in Fig. 3 is a simplistic representation of the Louisville Bridge in the USA. Assumed geometric and section details are listed in Table 2. Due to the symmetry of the truss, only data corresponding to half of the truss are given in Table 2. The FE model of the truss is subjected to a zero-mean white noise excitation at all the top nodes and responses are collected with a sampling frequency of 1000 Hz for a time span of 5 s and subsequently contaminated with 2 % white noise. Progressive damage is simulated in the member B5B6 through a reduction in the cross-sectional area following the same degradation rate used for the beam experiment with $t_s = -3$ s, $t_e = 5$ s and $p = 50\,\%$. Thus, in this problem damage is initiated prior to sampling and gradually increased up to the fifth second of sampling between which the cross-sectional area of B5B6 is reduced by 50 %. Again two case studies are undertaken for different numbers of channels in the recorded signal. Details of the number of accelerometers used and their locations for each case are given in Table 3. Since it has been observed that with all 14 free nodes of the truss instrumented with accelerometers capable of record; vertical acceleration response, the parameter estimation is quite easy, the case studies presented in this article considered a less extensive instrumentation (7 and 5 channels, respectively). In addition, while the former considers collocated measurements, the latter extends it to non-collocated measurement conditions. The contaminated signal is used for the DEKF identification algorithm from which information about time-varying properties of all 29 members is obtained.

In this numerical experiment, the ratio of current area to initial area of cross section, termed here as area ratio (AR), is considered as the control parameter which is identified using the proposed algorithm. Figure 4 shows the identification result.

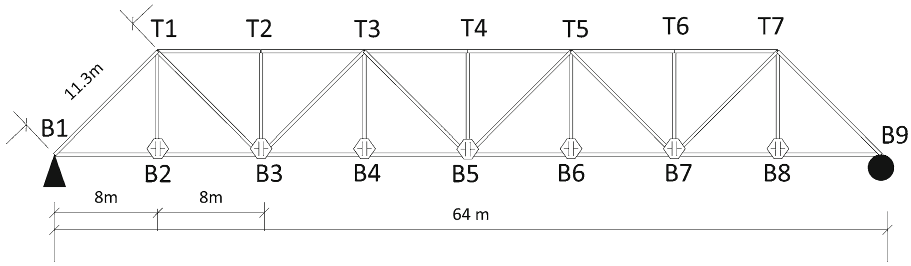

Fig. 3 Schematic diagram of the truss

Table 2 Geometry and section details of the truss

Elements	Length (m)	Cross-sectional area (cm^2)	Section details
B1B2, B2B3	8.0	181.0	IPB360
B3B4, B4B5	8.0	373.0	IPBV300 + 2PL350 × 10
B1T1	11.3	463.0	IPBV300 + 2PL400 × 20
B2T1, B4T3	8.0	72.7	IPE360
B3T2, B5T4	8.0	143.0	IPBL360
B3T1, B3T3, B5T3	11.3	181.0	IPB360
T1T2, T2T3	8.0	373.0	IPBV300 + 2PL350 × 10
T3T4	8.0	463.0	IPBV300 + 2PL400 × 20

Table 3 Details of accelerometer locations for three case studies

Case studies	Accelerometer location
Case 1 (7 nos)	B2, B3, B4, B5, B6, B7 and B8
Case 2 (5 nos)	B2, B4, T4, B6 and B8

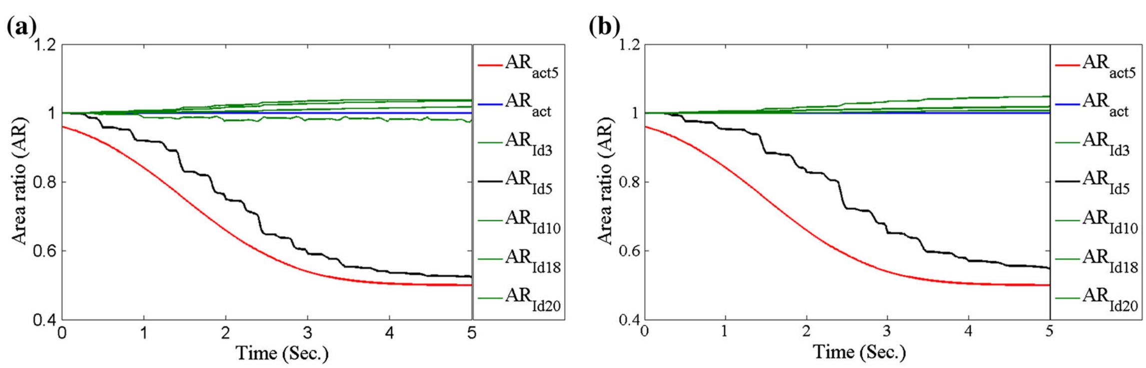

Fig. 4 Progressive damage identification of truss with different numbers of response signal channels. AR = area ratio and subscript act or Id signifies actual or identified quantities, respectively, and the associated number signifies corresponding element numbers. **a** Estimation result for 7 channel measured signal, **b** estimation result for 5 channel measured signal

In Fig. 4 results are presented for only a set of selected members, i.e., the 3rd, 5th, 10th, 18th and 20th member corresponding to B3B4, B5B6, T2T3, T3B5 and T5B7. Time variance of area ratio (AR) for these selected members shows that the algorithm successfully captured the exact location of the damage along with the rate at which the AR is degrading while using a limited amount of signal data. It can also be observed from the result that as we decrease the number of available signal channels, the performance of the algorithm degrades. However, even with a 5 channel non-collocated measured signal, the parameter filter still perfectly localizes the damage while capturing the rate of degradation with sufficient accuracy. Ultimately, considering the required precision level and associated cost for instrumenting the structure, a cost-effective decision should be made regarding the number of required signal channels.

7 Conclusion

In this article we employed dual extended Kalman filtering (DEKF) algorithm for health monitoring of time-varying structural systems using a reduced-order model and thus by avoiding unnecessary estimation of unmeasured states and subsequent ill influence of improper estimates on the parameter filter.

The key feature of the algorithm is the simultaneous but decoupled estimation strategy for states and parameters which are conditioned upon each other. The approach when tested for simulated numerical problems is found to be robust against noisy measurements, while the parameter estimate is observed to be stable with respect to sudden variation of parameters.

While the consideration of location-based parameters as system parameters lessens the dimensionality of the parameter states, the system reduction approach reduces the dimensionality of the response state vector to be estimated and thus by reducing the estimation complexity as a whole. However, the reduced dimensionality is achieved at the expense of enhanced computation due to additional FE modelling and the system reduction step.

Damage localization is achieved beyond sensor resolution which is beneficial for limited instrumented structures. Additionally, the selection of physical structural properties as parameters is more straightforward in the context of SHM purposes. Future work will involve an economic study involving the trade-off between the economical benefit due to reduced instrumentation and the consequential sacrifice in accuracy in order to select the optimal number of sensors.

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