Acta Mech 226, 4017–4029 (2015)
DOI 10.1007/s00707-015-1443-y

ORIGINAL PAPER

Anshu Mli Gaur · Dinesh Singh Rana

Dispersion relations for SH waves propagation in a porous piezoelectric (PZT–PVDF) composite structure

Received: 4 January 2015 / Revised: 13 July 2015 / Published online: 4 September 2015

Abstract The propagation of shear horizontal waves in a porous piezoelectric composite structure is investigated analytically in this paper. A two-layer structure comprised of perfectly bonded piezoelectric layers is considered. The solution of mechanical displacement and electrical potential is found by solving the constitutive equation for a porous piezoelectric composite structure. Then the dispersion relation is derived for the wave propagation along the direction normal to layering and in the direction of layering. The results obtained from the investigation show the dependence of the wave number on the thickness of the alternative layer in piezoelectric material. Also the phase velocity has been significantly influenced by the variation of elastic constant and porosity. The results provide a theoretical framework for designing and development of underwater acoustic devices for sensing and nondestructive testing.

1 Introduction

Piezoelectric materials are a class of active materials that are used these days invariably in numerous applications. Most thrust application area includes damageable detection sensor, sonar, microphones, energy harvesting devices, transducers [1,2]. In recent years, a new configuration of these materials has opened a broad field ranging from undersea applications up to biomedical application areas. Two or more piezoelectric materials can be utilized to form a new type of piezoelectric composite materials having multiple layers of materials bonded alternatively [3]. But these composite materials suffer from drawbacks that include failure of the device under electrical or mechanical loading. The other limitation is higher acoustic impedance that restricts the use of the device in undersea application up to a certain depth. The acoustic impedance is an important prerequisite in design of sonar and other underwater object detection devices. The piezoelectric response of piezoelectric material depends upon numerous factors such as elastic constant, porosity, dielectric constant, viscosity, density [4,5]. The acoustic impedance can be controlled by the variation in porosity. By the variation in porosity of the piezoelectric material, the mismatching of impedance and failure rate of the piezoelectric composite device can be overcome altogether.

In the last decade, numerous works have been reported on the propagation of shear waves (SH) in a piezoelectric composite structure. Singh and Gaur et al. [6,7] found the dispersion relation for the propagation of shear waves in a piezoelectric medium. Piliospian et al. [8] discussed the shear wave propagation in a periodic phonic/photonic piezoelectric structure and considered the effect of piezoelectricity on band gap width. SH waves in piezoelectric composites were extensively studied theoretically and experimentally by Kieleyznki et al. [9]. Son et al. [10] and Du et al. [11] discuss the effect of initial stress on propagation of an SH wave in piezoelectric layers. Liu et al. [12] and Qian et al. [13] study the Love wave propagation in a piezoelectric structure in the presence of initial stress and found that the phase velocity depends on the value of stress at the

A. M. Gaur (✉) · D. S. Rana
Department of Instrumentation (I.I.E), Kurukshetra University, Kurukshetra, Haryana, 136119 India
E-mail: gauranshu10@gmail.com

interface. A three-layer model was presented by Li et al. [14] and investigated the effect of an intermediate soft layer on the propagation characteristics of Love waves. The propagation behavior of Lamb waves propagating in a functionally graded piezoelectric–piezomagnetic (PE–PM) composite was studied by Cao et al. [15]. The Love wave propagating in a piezoelectric composite with imperfect interface was discussed by Liu et al. [16] and Wang et al. [17]. Daros et al. [18] have obtained a mathematical model for the propagation of Buleteen–Gulagaev waves in non-homogeneous piezoelectric media. Soh et al. [19], Singh et al. [20] and Du et al. [21] considered the parameters affecting the propagation of PE–PM layered structure and investigated the effect of gradient variation on the frequency of the SH wave. A new approach was presented by Qian et al. [22] and derived the dispersion relation for a piezoelectric composite with hard metal as an intermediate layer. Liu et al. [23] and Pang et al. [24] found the dispersion relation of an SH wave propagating in PE–PM layers. The scattering of shear waves in an inhomogeneous structure was studied by Du et al. [25]. Yang et al. [26] and Chen et al. [27] investigated the shear horizontal vibration of piezoelectric plates. But only few authors have focused on the effect of porosity on SH wave propagation in a piezoelectric multilayered structure. Vashisth et al. [28] have analyzed the vibration of porous piezoelectric ceramic plates. Manna et al. considered the propagation of Love waves in piezoelectric media with non-homogeneous space. Recently Sharma [29] has analytically shown the effect of porosity on the velocity of a shear wave propagating in a poroelastic medium. The objectives of the present study mainly deal with the following objectives:

(a) To provide a mathematical model and constitutive equations for a porous piezoelectric composite structure.
(b) To determine the effect of porosity on the phase velocity and dispersion equation.
(c) To evaluate the effect of variation in elastic constant, dielectric constant, density, and volume fraction on the phase velocity.

The work in this paper is arranged as follows: Sect. 1 briefs the work that has been done on shear wave propagation in a piezoelectric composite; Sect. 2 focuses on the problem formulation; the solution SH wave propagating along and in the direction normal to the layering is found in Sect. 3; Sect. 4 outlines the determination of the dispersion relation; Sect. 5 is devoted to numerical simulation and results. Finally, the last Section is devoted to the main findings of this study.

2 Formulation of problem and boundary constraints

Consider a composite structure having an alternative layer of piezoelectric material perfectly bonded to each other. Let h_1 and h_2 be the thickness of the alternative layers of the piezoelectric composite of two different piezoelectric materials. The piezoelectric material is considered to be polarized in z axis direction. The propagation of the SH wave is taken along the direction normal to layering and along the direction of layering. Figure 1 shows the geometrical layout of the composite under consideration.

The constitutive equation of the piezoelectric medium can be expressed as [13,30]

$$\begin{aligned} \sigma_{ij} &= c_{ijkl}\varepsilon_{kl} - e_{kij}E_k, \\ D_j &= e_{jk}\varepsilon_{kl} + \epsilon_{jk}E_k \end{aligned} \tag{1}$$

where σ_{ij}, ε_{kl} is stress and strain tensor respectively. D_j, E_k is electric displacement and electric field intensity, c_{ijkl}, e_{jkl} are elastic and piezoelectric constant of the piezoelectric media.

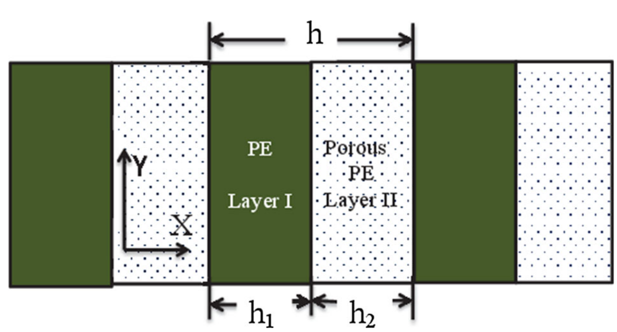

Fig. 1 Schematic of porous PE–PE composite layered structure

The motion differential equation for piezoelectric media can be described as [32]

$$\begin{aligned} \sigma_{ij,j} &= \rho \ddot{u}_i, \\ D_{i,j} &= 0 \end{aligned} \tag{2}$$

where u_i is mechanical displacement component, ρ is mass density of the medium. The dot represents the time derivative, and comma followed by subscript indicates the space derivative.

The strain tensor ε_{ij} and electric field intensity E_k in piezoelectric media can be written as

$$\begin{aligned} \varepsilon_{ij} &= \tfrac{1}{2}\left(u_{j,i} + u_{i,j}\right), \\ E_i &= -\tfrac{\partial \varphi}{\partial x_i}. \end{aligned} \tag{3}$$

The constitutive equation for a porous piezoelectric medium can be represented as [31]

$$\begin{aligned} \sigma_{ij} &= (\rho_{11})_{ij}\,\ddot{u}'_j + (\rho_{12})_{ij}\,\ddot{u}'^{*}_j, \\ \sigma^*_i &= (\rho_{12})_{ij}\,\ddot{u}'_j + (\rho_{22})_{ij}\,\ddot{u}'^{*}_j, \\ D_{i,i} &= 0, \\ D^*_{i,i} &= 0 \end{aligned} \tag{4}$$

where $(\rho_{11})_{ij}$, $(\rho_{12})_{ij}$ and $(\rho_{22})_{ij}$ are dynamical mass coefficients. U^* is the mechanical displacement component for porous media. Φ^* represents the electric potential function for porous media.

The motion differential equation for a porous piezoelectric material is given by

$$\begin{aligned} \varepsilon'_{ij} &= \tfrac{1}{2}\left(u'_{j,i} + u'_{i,j}\right), \\ E'_i &= -\tfrac{\partial \varphi'}{\partial x_i}, \quad E'^{*}_i = -\tfrac{\partial \varphi'^{*}}{\partial x_i}. \end{aligned} \tag{5}$$

From Eq. (1), the constitutive equation for piezoelectric media 1 is obtained as

$$\begin{pmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{zy} \\ \tau_{zx} \\ \tau_{xy} \end{pmatrix} = \begin{pmatrix} c_{11} & c_{12} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{12} & 0 & 0 & 0 \\ c_{12} & c_{12} & c_{11} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{44} \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ 2\varepsilon_{zy} \\ 2\varepsilon_{zx} \\ 2\varepsilon_{xy} \end{pmatrix} - \begin{pmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}. \tag{6}$$

Expanding Eq. (6) leads to the following equations:

$$\begin{aligned} \sigma_x &= c_{11}\varepsilon_x + c_{12}\varepsilon_y + c_{12}\varepsilon_z - e_{31}E_z, \\ \sigma_y &= c_{12}\varepsilon_x + c_{11}\varepsilon_y + c_{12}\varepsilon_z - e_{31}E_z, \\ \sigma_z &= c_{12}\varepsilon_x + c_{12}\varepsilon_y + c_{11}\varepsilon_z - e_{33}E_z, \\ \tau_{zy} &= 2c_{44}\varepsilon_{zy} - e_{15}E_y, \\ \tau_{zx} &= 2c_{44}\varepsilon_{zx} - e_{15}E_x, \\ \tau_{xy} &= 2c_{44}\varepsilon_{xy}. \end{aligned} \tag{7}$$

Similarly, from Eq. (1), we have

$$\begin{pmatrix} D_x \\ D_y \\ D_z \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 & e_{15} & 0 \\ 0 & 0 & 0 & e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_z \\ 2\varepsilon_{zy} \\ 2\varepsilon_{zx} \\ 2\varepsilon_{xy} \end{pmatrix} + \begin{pmatrix} \epsilon_{11} & 0 & 0 \\ 0 & \epsilon_{11} & 0 \\ 0 & 0 & \epsilon_{33} \end{pmatrix} \begin{pmatrix} E_x \\ E_y \\ E_z \end{pmatrix}. \tag{8}$$

Now expanding the set of equations (8) results in

$$
\begin{aligned}
D_x &= 2e_{15}\varepsilon_x + \epsilon_{11}E_x,\\
D_y &= 2e_{15}\varepsilon_y + \epsilon_{11}E_y,\\
D_z &= e_{31}\varepsilon_x + e_{31}\varepsilon_y + e_{33}\varepsilon_z + \epsilon_{33}E_z.
\end{aligned}
\tag{9}
$$

The constitutive equation for porous piezoelectric media obtained from Eq. (4) is

$$
\begin{pmatrix}\sigma'_x\\ \sigma'_y\\ \sigma'_z\\ \tau'_{zy}\\ \tau'_{zx}\\ \tau'_{xy}\end{pmatrix}
=
\begin{pmatrix}
c'_{11} & c'_{12} & c'_{13} & 0 & 0 & 0\\
c'_{12} & c'_{11} & c'_{13} & 0 & 0 & 0\\
c'_{13} & c'_{13} & c'_{33} & 0 & 0 & 0\\
0 & 0 & 0 & c'_{44} & 0 & 0\\
0 & 0 & 0 & 0 & c'_{44} & 0\\
0 & 0 & 0 & 0 & 0 & c'_{14}
\end{pmatrix}
\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ 2\varepsilon_{zy}\\ 2\varepsilon_{zx}\\ 2\varepsilon_{xy}\end{pmatrix}
-
\begin{pmatrix}
0 & 0 & e'_{31}\\
0 & 0 & e'_{31}\\
0 & 0 & e'_{33}\\
0 & e'_{15} & 0\\
e'_{15} & 0 & 0\\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix}
+
\begin{pmatrix}
m_{11} & 0 & 0\\
0 & m_{11} & 0\\
0 & 0 & m_{33}\\
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}\varepsilon^*\\ \varepsilon^*\\ \varepsilon^*\end{pmatrix}.
\tag{10}
$$

Expanding the set of equations (10), the following equations are derived:

$$
\begin{aligned}
\sigma'_x &= c'_{11}\varepsilon_x + c'_{12}\varepsilon_y + c_{13}\varepsilon_z - e'_{31}E_z + m_{11}\varepsilon^*,\\
\sigma'_y &= c'_{12}\varepsilon_x + c'_{11}\varepsilon_y + c'_{13}\varepsilon_z - e'_{31}E_z + m_{11}\varepsilon^*,\\
\sigma'_z &= c'_{13}\varepsilon_x + c'_{13}\varepsilon_y + c'_{33}\varepsilon_z - e'_{33}E_z + m_{33}\varepsilon^*,\\
\tau'_{zy} &= 2c'_{44}\varepsilon_{zy} - e'_{15}E_y,\\
\tau'_{zx} &= 2c'_{14}\varepsilon_{zx} - e'_{15}E_x,\\
\tau'_{xy} &= 2c'_{44}\varepsilon_{xy}.
\end{aligned}
\tag{11}
$$

From Eq. (4), we have

$$
\begin{pmatrix}D''_x\\ D''_y\\ D''_z\end{pmatrix}
=
\begin{pmatrix}
0 & 0 & 0 & 0 & e'_{15} & 0\\
0 & 0 & 0 & e'_{15} & 0 & 0\\
e'_{31} & e'_{31} & e'_{33} & 0 & 0 & 0
\end{pmatrix}
\begin{pmatrix}\varepsilon_x\\ \varepsilon_y\\ \varepsilon_z\\ 2\varepsilon_{zy}\\ 2\varepsilon_{zx}\\ 2\varepsilon_{xy}\end{pmatrix}
+
\begin{pmatrix}
\epsilon'_{11} & 0 & 0\\
0 & \epsilon'_{11} & 0\\
0 & 0 & \epsilon'_{33}
\end{pmatrix}
\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix}
+
\begin{pmatrix}
A_{11} & 0 & 0\\
0 & A_{11} & 0\\
0 & 0 & A_{33}
\end{pmatrix}
\begin{pmatrix}E^*_x\\ E^*_y\\ E^*_z\end{pmatrix},
\tag{12}
$$

$$
\begin{pmatrix}D'^*_x\\ D'^*_y\\ D'^*_z\end{pmatrix}
=
\begin{pmatrix}
A_{11} & 0 & 0\\
0 & A_{11} & 0\\
0 & 0 & A_{33}
\end{pmatrix}
\begin{pmatrix}E_x\\ E_y\\ E_z\end{pmatrix}
+
\begin{pmatrix}
\epsilon'_{11} & 0 & 0\\
0 & \epsilon'_{11} & 0\\
0 & 0 & \epsilon'_{33}
\end{pmatrix}
\begin{pmatrix}E^*_x\\ E^*_y\\ E^*_z\end{pmatrix}
+
\begin{pmatrix}
0 & 0 & 0\\
0 & 0 & 0\\
0 & 0 & e^*_3
\end{pmatrix}
\begin{pmatrix}\varepsilon^*\\ \varepsilon^*\\ \varepsilon^*\end{pmatrix}.
\tag{13}
$$

Adding Eqs. (12) and (13), we have

$$
\begin{aligned}
D'_x &= D''_x + D'^*_x,\\
D'_y &= D''_y + D'^*_y,\\
D'_z &= D''_z + D'^*_z.
\end{aligned}
\tag{14}
$$

An SH wave propagates either in positive direction of x axis or in the positive direction of y axis. So the displacement and electric potential component can be represented as

$$u = 0, \quad \upsilon = 0, \quad w = w(x, y, t), \quad \varphi = \varphi(x, y, t), \quad \varphi^* = \varphi^*(x, y, t). \tag{15}$$

From (7) and (9), the following equations are obtained:

$$\begin{aligned} c_{44}\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right) + e_{15}\left(\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{\partial^2 \varphi_1}{\partial y^2}\right) = \rho\frac{\partial^2 w_1}{\partial t^2}, \\ e_{15}\left(\frac{\partial^2 w_1}{\partial x^2} + \frac{\partial^2 w_1}{\partial y^2}\right) - \epsilon_{11}\left(\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{\partial^2 \varphi_1}{\partial y^2}\right) = 0. \end{aligned} \tag{16}$$

Using Eqs. (11) and (14), the governing equations for a porous piezoelectric material can be written as

$$\begin{aligned} c'_{44}\left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right) + e'_{15}\left(\frac{\partial^2 \varphi_2}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial y^2}\right) = \rho^{\mathcal{P}}\frac{\partial^2 w_2}{\partial^2}, \\ e'_{15}\left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}\right) - \epsilon_{11}^{\prime\mathcal{P}}\left(\frac{\partial^2 \varphi_2}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial y^2}\right) = 0 \end{aligned} \tag{17}$$

where

$$\begin{aligned} c_{44} &= \frac{(c_{11} - c_{12})}{2}, \quad c'_{14} = \frac{(c'_{11} - c'_{12})}{2}, \\ \epsilon_{11}^{\prime\mathcal{P}} &= \epsilon_{11} - \frac{A_{11}^2}{\epsilon_{11}^{\prime *}}, \quad \nabla^2\phi^* = -\frac{A_1 1}{\epsilon_{11}^{\prime *}}\nabla^2\phi, \\ \rho^{\mathcal{P}} &= (\rho_{11})_{33} - \frac{(\rho_{12})_{33}^2}{(\rho_{22})_{33}}. \end{aligned}$$

The propagation of an SH wave in this developed mathematical model should satisfy the following boundary conditions:

$$\begin{aligned} &w_1(0, y) = w_2(0, y), \\ &\varphi_1(0, y) = \varphi_2(0, y), \\ &\tau_{zx1}(0, y) = \tau_{zx2}(0, y), \\ &D_{x1}(0, y) = D_{x2}(0, y) + D_{x2}^*(0, y); \end{aligned} \tag{18}$$

$$\begin{aligned} &w_1(h_1, y) = w_2(-h_2, y), \\ &\varphi_1(h_1, y) = \varphi_2(-h_2, y), \\ &\tau_{zx1}(h_1, y) = \tau_{zx2}(-h_2, y), \\ &D_{x1}(h_1, y) = D_{x2}(-h_2, y) + D_{x2}^*(-h_2, y) \end{aligned} \tag{19}$$

3 Solution of the problem

3.1 Case I: Solution of an SH wave propagating along the direction normal to the layering

Consider the SH wave propagating along the positive direction of x axis. The solution of Eq. (16) can be assumed as

$$\begin{aligned} w_1(x, t) = W_1(x)\,\mathrm{e}^{ik(x-ct)}, \\ \varphi_1(x, t) = \Phi_1(x)\,\mathrm{e}^{ik(x-ct)}. \end{aligned} \tag{20}$$

Similarly, the solution of Eq. (17) can be assumed as

$$\begin{aligned} w_2(x, t) = W_2(x)\,\mathrm{e}^{ik(x-ct)}, \\ \varphi_2(x, t) = \Phi_2(x)\,\mathrm{e}^{ik(x-ct)} \end{aligned} \tag{21}$$

where k is the wave number $(2\pi/\lambda)$, λ is the wavelength, $i = \sqrt{-1}$, and c is the phase velocity. Let $W_1(x)\, W_2(x)$,
$\Phi_1(x)$ and $\Phi_2(x)$ be the unknown functions.

Now equating Eq. (20) in (16), we have

$$\begin{aligned} &c_{44}\left(W_1'' + 2ikW_1' - k^2W_1\right) + e_{15}\left(\Phi_1'' + 2ik\Phi_1' - k^2\Phi_1\right) = -\rho k^2c^2W_1,\\ &e_{15}\left(W_1'' + 2ikW_1' - k^2W_1\right) - \epsilon_{11}\left(\Phi_1'' + 2ik\Phi_1' - k^2\Phi_1\right) = 0. \end{aligned} \tag{22}$$

Similarly substituting Eq. (21) into (17) yields

$$\begin{aligned} &c_{44}'\left(W_2'' + 2ikW_2' - k^2W_2\right) + e_{15}'\left(\Phi_2'' + 2ik\Phi_2' - k^2\Phi_2\right) = -\rho^{\mathcal{P}}k^2c^2W_2,\\ &e_{15}'\left(W_2'' + 2ikW_2' - k^2W_2\right) - \epsilon_{11}^{'\mathcal{P}}\left(\Phi_2'' + 2ik\Phi_2' - k^2\Phi_2\right) = 0. \end{aligned} \tag{23}$$

From Eq. (22), we have

$$\begin{aligned} &W_1 = M_1\mathrm{e}^{(-1+c/c_{\mathrm{sh}})ikx} + N_1\mathrm{e}^{(-1-c/c_{\mathrm{sh}})ikx},\\ &\Phi_1 = \left(M_1' + N_1'x\right)\mathrm{e}^{-ikx} + \tfrac{e_{15}}{\epsilon_{11}}\left(M_1\mathrm{e}^{(-1+c/c_{\mathrm{sh}})ikx} + N_1\mathrm{e}^{(-1-c/c_{\mathrm{sh}})ikx}\right). \end{aligned} \tag{24}$$

Equation (23) yields the following:

$$\begin{aligned} &W_2 = M_2\mathrm{e}^{(-1+c/c_{\mathrm{sh}}')ikx} + N_2\mathrm{e}^{(-1-c/c_{\mathrm{sh}}')ikx},\\ &\Phi_2 = \left(M_2' + N_2'x\right)\mathrm{e}^{-ikx} + \tfrac{e_{15}'}{\epsilon_{11}^{'\mathcal{P}}}\left(M_2\mathrm{e}^{(-1+c/c_{\mathrm{sh}}')ikx} + N_2\mathrm{e}^{(-1-c/c_{\mathrm{sh}}')ikx}\right) \end{aligned} \tag{25}$$

where c_{sh} and c_{sh}' represent the bulk shear velocity in piezoelectric and porous piezoelectric media, respectively,

$$c_{\mathrm{sh}} = \sqrt{\left(\epsilon_{11}c_{44} + e_{15}^2\right)/\rho\epsilon_{11}},\ c_{\mathrm{sh}}' = \sqrt{\left(\epsilon_{11}^{'\mathcal{P}}c_{44}' + e_{15}^{'2}\right)/\rho^{\mathcal{P}}\epsilon_{11}^{'\mathcal{P}}}.$$

Substituting Eq. (24) into (20) results in

$$\begin{aligned} &w_1(x,t) = \left[M_1\mathrm{e}^{(-1+c/c_{\mathrm{sh}})ikx} + N_1\mathrm{e}^{(-1-c/c_{\mathrm{sh}})ikx}\right]\mathrm{e}^{ik(x-ct)},\\ &\varphi_1(x,t) = \left[\left(M_1' + N_1'x\right)\mathrm{e}^{-ikx} + \tfrac{e_{15}}{\epsilon_{11}}\left(M_1\mathrm{e}^{(-1+c/c_{\mathrm{sh}})ikx} + N_1\mathrm{e}^{(-1-c/c_{\mathrm{sh}})ikx}\right)\right].\\ &\quad \mathrm{e}^{ik(x-ct)} \end{aligned} \tag{26}$$

Substituting Eq. (25) into (21), we have

$$\begin{aligned} &w_2(x,t) = \left[M_2\mathrm{e}^{(-1+c/c_{\mathrm{sh}}')ikx} + N_2\mathrm{e}^{(-1-c/c_{\mathrm{sh}}')ikx}\right]\mathrm{e}^{ik(x-ct)},\\ &\varphi_2(x,t) = \left[\left(M_2' + N_2'x\right)\mathrm{e}^{-ikx} + \tfrac{e_{15}'}{\epsilon_{11}^{'\mathcal{P}}}\left(M_2\mathrm{e}^{(-1+c/c_{\mathrm{sh}}')ikx} + N_2\mathrm{e}^{(-1-c/c_{\mathrm{sh}}')ikx}\right)\right].\\ &\quad \mathrm{e}^{ik(x-ct)} \end{aligned} \tag{27}$$

3.2 Case II: Solution of SH wave propagation along the direction of the layering

In this case, we consider the SH wave propagating in the positive direction of y axis. Mechanical displacement and electrical potential component can be represented as

$$u = 0, \quad v = 0, \quad w = w(x,y,t), \quad \varphi = \varphi(x,y,t).$$

The solution of Eq. (16) is expressed as

$$\begin{aligned} &w_1(x,y,t) = W_1(x)\,\mathrm{e}^{ik(y-ct)},\\ &\varphi_1(x,y,t) = \Phi_1(x)\,\mathrm{e}^{ik(y-ct)}. \end{aligned} \tag{28}$$

The solution of Eq. (17) is written as

$$\begin{aligned} &w_2(x,y,t) = W_2(x)\,\mathrm{e}^{ik(y-ct)},\\ &\varphi_2(x,y,t) = \Phi_2(x)\,\mathrm{e}^{ik(y-ct)}. \end{aligned} \tag{29}$$

The earlier procedure is followed to obtain the following set of equations as

$$\begin{aligned} w_1(x,y,t) &= \left[M_1 e^{-ib_1x} + N_1 e^{ib_1x}\right] e^{ik(y-ct)}, \\ \varphi_1(x,y,t) &= \left[M_1' e^{-kx} + N_1' e^{kx} + \tfrac{e_{15}}{\epsilon_{11}}\left(M_1 e^{-ib_1x} + N_1 e^{ib_1x}\right)\right] e^{ik(y-ct)}; \end{aligned} \tag{30}$$

$$\begin{aligned} w_2(x,y,t) &= \left[M_2 e^{-b_2x} + N_2 e^{b_2x}\right] e^{ik(y-ct)}, \\ \varphi_2(x,y,t) &= \left[\left(M_2' e^{-kx} + N_2' e^{kx}\right) + \tfrac{e_{15}'}{\epsilon_{11}'^{\mathcal{P}}}\left(M_2 e^{-b_2x} + N_2 e^{b_2x}\right)\right] e^{ik(y-ct)} \end{aligned} \tag{31}$$

where

$$b_1 = k\sqrt{\tfrac{c^2}{c_{\text{sh}}^2} - 1}, \quad b_2 = k\sqrt{1 - \tfrac{c^2}{c_{\text{sh}}'^2}}.$$

4 Dispersion equations

4.1 Case I: SH wave propagation along the direction normal to the layering

Substituting Eqs. (26) and (27) into (7), (9), (11), and (14), we obtain the stress and electrical displacement components as

$$\begin{aligned} \tau_{zx1} &= \left[e_{15}N_1' e^{-ikx} + \tfrac{ikcP}{c_{\text{sh}}}\left(M_1 e^{(-1+c/c_{\text{sh}})ikx} - N_1 e^{(-1-c/c_{\text{sh}})ikx}\right)\right] e^{ik(x-ct)}, \\ D_{x1} &= \left[-\epsilon_{11}N_1' e^{-ikx}\right] e^{ik(x-ct)}; \end{aligned} \tag{32}$$

$$\begin{aligned} \tau_{zx2} &= \left[e_{15}'N_2' e^{-ikx} + \tfrac{ikcP_1}{c_{\text{sh}}'}\left(M_2 e^{(-1+c/c_{\text{sh}}')ikx} - N_2 e^{(-1-c/c_{\text{sh}}')ikx}\right)\right] e^{ik(x-ct)}, \\ D_{x2} &= \left[\tfrac{ikc}{c_{\text{sh}}'}e_{15}'\left(1 - \tfrac{l_{11}}{\epsilon_{11}'^{\mathcal{P}}}\right)\left(M_2 e^{(-1+c/c_{\text{sh}}')ikx} - N_2 e^{(-1-c/c_{\text{sh}}')ikx}\right) - N_2' l_{11} e^{-ikx}\right] e^{ik(x-ct)}, \\ D_{x2}^* &= \left[-\tfrac{ikc}{c_{\text{sh}}'}\tfrac{l_{12}e_{15}'}{\epsilon_{11}'^{\mathcal{P}}}\left(M_2 e^{(-1+c/c_{\text{sh}}')ikx} - N_2 e^{(-1-c/c_{\text{sh}}')ikx}\right) - N_2' l_{12} e^{-ikx}\right] e^{ik(x-ct)} \end{aligned} \tag{33}$$

where

$$P = \frac{\epsilon_{11}c_{44} + e_{15}^2}{\epsilon_{11}}, \quad P_1 = \frac{\epsilon_{11}'^{\mathcal{P}}c_{44}' + e_{15}'^2}{\epsilon_{11}'^{\mathcal{P}}}.$$

Substituting the boundary conditions (18) into (26), (27), (32) and (33), we obtain the following set of homogeneous linear equations:

$$\begin{aligned} &M_1 + N_1 - M_2 - N_2 = 0, \\ &M_1' + \tfrac{e_{15}}{\epsilon_{11}}M_1 + \tfrac{e_{15}}{\epsilon_{11}}N_1 - \tfrac{e_{15}'}{\epsilon_{11}'^{\mathcal{P}}}M_2 - \tfrac{e_{15}'}{\epsilon_{11}'^{\mathcal{P}}}N_2 - M_2' = 0, \\ &M_1 - N_1 + \tfrac{e_{15}c_{\text{sh}}}{ikcP}N_1' - \tfrac{e'_{15}c_{\text{sh}}}{ikcP}N_2' - QM_2 + QN_2 = 0, \\ &-\epsilon_{11}N_1' - \tfrac{ikcl_{13}}{c_{\text{sh}}'}M_2 + \tfrac{ikcl_{13}}{c_{\text{sh}}'}N_2 + lN_2' = 0, \\ &e^{i\alpha}M_1 + e^{-i\alpha}N_1 - e^{i(kh-\beta)}M_2 - e^{i(kh+\beta)}N_2 = 0, \\ &M_1' + h_1N_1' + \tfrac{e_{15}}{\epsilon_{11}}e^{i\alpha}M_1 + \tfrac{e_{15}}{\epsilon_{11}}e^{-i\alpha}N_1 - e^{ikh}M_2' + h_2e^{ikh}N_2' - \tfrac{e_{15}'}{\epsilon_{11}'^{\mathcal{P}}}e^{i(kh-\beta)}M_2 - \tfrac{e_{15}'}{\epsilon_{11}'^{\mathcal{P}}}e^{i(kh+\beta)}N_2 = 0, \\ &\tfrac{e_{15}c_{\text{sh}}}{ikcP}N_1' + e^{i\alpha}M_1 - e^{-i\alpha}N_1 - \tfrac{e_{15}'c_{\text{sh}}}{ikcP}e^{ikh}N_2' - Qe^{i(kh-\beta)}M_2 + Qe^{i(kh+\beta)}N_2 = 0, \\ &-\epsilon_{11}N_1' - \tfrac{ikcl_{13}}{c_{\text{sh}}'}M_2e^{i(kh-\beta)} + \tfrac{ikcl_{13}}{c_{\text{sh}}'}N_2e^{i(kh+\beta)} + le^{ikh}N_2' = 0 \end{aligned} \tag{34}$$

where

$$l_{13} = 1 - \frac{l_{11}}{\epsilon_{11}'^{\mathcal{P}}} - \frac{l_{12}}{\epsilon_{11}'^{\mathcal{P}}}, \quad l = l_{11} + l_{12}, \quad l_{11} = \epsilon_{11}' - \frac{A_{11}^2}{\epsilon_{11}'^{*}} C_v, \quad l_{12} = A_{11}(1 - C_v),$$
$$C_v = \frac{C_{\phi^*}^2}{C_\phi^2}, \quad l_{15} = \frac{1}{l_{14}}, \quad l_{14} = \frac{l P C_{\text{sh}}'}{l_{13} e_{15} C_{\text{sh}}}, \quad l_{19} = \frac{e_{15} l}{e_{15}' e_{11}}.$$
$$\alpha = \frac{ckh_1}{c_{\text{sh}}}, \quad \beta = \frac{ckh_2}{c_{\text{sh}}'}, \quad Q = \frac{P_1 c_{\text{sh}}}{P c_{\text{sh}}'}, \quad w = kc.$$

For obtaining the non-trivial solution, the determinant of equations (34) must be equated to zero, and then we obtain the dispersion equation as

$$\cos(hk) = \cos\alpha\cos\beta - \frac{(1+Q^2)}{2Q}\sin\alpha\sin\beta - \frac{l_{15}}{Q}\cos(hk) + \frac{l_{15}}{2Q}(l_{19}-1)(\cos\alpha + \cos\beta) + \frac{l_{15}}{2Q}(1+l_{19})(\cos\alpha+\cos\beta) + \frac{l_{15}}{2}\left(1 + l_{19} + \frac{l_{19} l_{15}}{Q}\right)\sin\alpha\sin\beta - \frac{l_{15}}{2Q}(l_{19}-1). \quad (35)$$

The above equation is reduced by equating $l_{15} = 0$, and now the dispersion equation expressed as

$$\sin(\alpha)\sin(\beta) + 2Q\cos(hk) - 2Q\cos(\alpha)\cos(\beta) + Q^2\sin(\alpha)\sin(\beta) = 0. \quad (36)$$

Rearranging Eq. (36), the equation can be written as [13,32]

$$\cos(hk) - \cos\left(\frac{ckh_1}{c_{\text{sh}}}\right)\cos\left(\frac{ckh_2}{c_{\text{sh}}'}\right) + \frac{(1+Q^2)}{2Q}\sin\left(\frac{ckh_1}{c_{\text{sh}}}\right)\sin\left(\frac{ckh_2}{c_{\text{sh}}'}\right) = 0 \quad (37)$$

where

$$h = (h_1 + h_2).$$

4.2 Case II: SH wave propagation along the direction of the layering

Substituting Eqs. (30) and (31) into (11) and (14), we obtain the following equations:

$$\tau_{zx1} = \left[ib_1 P\left(-M_1 e^{-ib_1 x} + N_1 e^{ib_1 x}\right) + e_{15} k\left(-M_1' e^{-kx} + N_1' e^{kx}\right)\right] e^{ik(y-ct)},$$
$$D_{x1} = \left[-\epsilon_{11} k\left(-M'_1 e^{-kx} + N'_1 e^{kx}\right)\right] e^{ik(y-ct)}; \quad (38)$$

$$\tau_{zx2} = \left[b_2 P_1\left(-M_2 e^{-b_2 x} + N_2 e^{b_2 x}\right) + e_{15}' k\left(-M_2' e^{-kx} + N_2' e^{kx}\right)\right] e^{ik(y-ct)},$$
$$D_{x2} = \left[\left(b_2 e_{15}' - \frac{b_2 e_{15}' l_{11}}{\epsilon_{11}'^{\mathcal{P}}}\right)\left(-M_2 e^{-b_2 x} + N_2 e^{b_2 x}\right) - l_{11} k\left(-M_2' e^{-kx} + N_2' e^{kx}\right)\right] e^{ik(y-ct)}, \quad (39)$$
$$D^{*x2} = \left[-l_{12} k\left(-M_2' e^{-kx} + N_2' e^{kx}\right) - \frac{b_2 e_{15}' l_{12}}{\epsilon'^{\mathcal{P}}{}_{11}}\left(-M_2 e^{-b_2 x} + N_2 e^{b_2 x}\right)\right] e^{ik(y-ct)}.$$

Substituting the boundary conditions (19) into (30), (31), (38) and (39) results in a set of linear algebraic homogeneous equations,

$$\begin{aligned}
&M_1 + N_1 - M_2 - N_2 = 0,\\
&\tfrac{e_{15}}{\varepsilon_{11}} M_1 + \tfrac{e_{15}}{\epsilon_{11}} N_1 + M_1' + N_1' - M_2' - N_2' - \tfrac{e'_{15}}{\epsilon'^{\mathcal{P}}_{11}} M_2 - \tfrac{e'_{15}}{\epsilon'^{\mathcal{P}}_{11}} N_2 = 0,\\
&-ib_1 P M_1 + ib_1 P N_1 - e_{15} k M_1' + e_{15} k N_1' + b_2 P_1 M_2 - b_2 P_1 N_2 + e'_{15} k M_2' - e'_{15} k N_2' = 0,\\
&\epsilon_{11} k M_1' - \epsilon_{11} k N_1' - lk M_2' + lk N_2' + l_{13} M_2 - l_{13} N_2 = 0,\\
&\mathrm{e}^{-ib_1h_1} M_1 + \mathrm{e}^{ib_1h_1} N_1 - \mathrm{e}^{b_2h_2} M_2 - \mathrm{e}^{-b_2h_2} N_2 = 0,\\
&\tfrac{e_{15}}{\epsilon_{11}} \mathrm{e}^{-ib_1h_1} M_1 + \tfrac{e_15}{\epsilon_{11}} \mathrm{e}^{ib_1h_1} N_1 + \mathrm{e}^{-kh_1} M_1' + \mathrm{e}^{kh_1} N_1' - \mathrm{e}^{kh_2} M_2' - \mathrm{e}^{-kh_2} N_2'\\
&\quad - \tfrac{e'_{15}}{\epsilon'^{\mathcal{P}}_{11}} \mathrm{e}^{b_2h_2} M_2 - \tfrac{e'_{15}}{\epsilon'^{\mathcal{P}}_{11}} \mathrm{e}^{-b_2h_2} N_2 = 0,\\
&-ib_1 P \mathrm{e}^{-ib_1h_1} M_1 + ib_1 P \mathrm{e}^{ib_1h_1} N_1 - e_{15} k \mathrm{e}^{-kh_1} M_1' + e_{15} k \mathrm{e}^{kh_1} N_1' + b_2 P_1 \mathrm{e}^{b_2h_2} M_2\\
&\quad - b_2 P_1 \mathrm{e}^{-b_2h_2} N_2 + e'_{15} k \mathrm{e}^{kh_2} M_2' - e'_{15} k \mathrm{e}^{-kh_2} N_2' = 0,\\
&\epsilon_{11} k \mathrm{e}^{-kh_1} M_1' - \epsilon_{11} k \mathrm{e}^{kh_1} N_1' + b_2 l_{13} \mathrm{e}^{b_2h_2} M_2 - b_2 l_{13} \mathrm{e}^{-b_2h_2} N_2 - kl \mathrm{e}^{kh_2} M_2' + kl \mathrm{e}^{-kh_2} N_2' = 0.
\end{aligned} \tag{40}$$

For obtaining the non-trivial solution, the determinant of Eq. (40) must be equated to zero, i.e.,

$$\begin{aligned}
&(k^2 \exp(b_1h_1(-i) - b_2h_2 - h_1k - h_2k)^*(e_{15}{}^4\epsilon'_{11} k^2 l^2 - \epsilon_{11}{}^3 e_{15} e'^{3}_{15} k^2 + P^2 b_1^2 \epsilon_{11}{}^4 \epsilon'_{11} + P_1^2 b_2^2 \epsilon_{11}{}^4 \epsilon'_{11}\\
&+ P b_1 \epsilon_{11}{}^4 e'^{2}_{15} k(-i) - P_1 b_2 \epsilon_{11}{}^4 e'^2_{15} k - P^2 b_1^2 \epsilon_{11}{}^4 \epsilon'_{11} \exp(b_1h_1(2i) + 2b_2h_2)\\
&+ P_1^2 b_2^2 \epsilon_{11}{}^4 \epsilon'_{11} \exp(b_1h_1 * (2*i) + 2b_2h_2) + P^2 b_1^2 \epsilon_{11}{}^4 \epsilon'_{11} \exp(2b_2h_2 + 2h_1k)\\
&+ P_1^2 b_2^2 \epsilon_{11}{}^4 \epsilon'_{11} \exp(2b_2h_2 + 2h_1k) - P^2 b_1^2 \epsilon_{11}{}^4 \epsilon'_{11} \exp(b_1h_1(2i)) + \cdots = 0.
\end{aligned} \tag{41}$$

5 Numerical simulation and results

We have obtained the analytical solution in the previous Section. For numerical simulation, PZT-5H and polyvinylidene fluoride (PVDF) are considered as two piezoelectric materials as alternative layers of the composite. The material coefficients (piezoelectric constant, density, elastic constant, porosity) for both materials are given in Tables 1 and 2 [31].

For numerical computation, we consider a relationship $\omega = kc$, where ω is circular frequency, k is non-dimensional wave number and c is Phase velocity. Firstly, we have considered for given wave velocity the variation in circular frequency with respect to the thickness of both layers. Next, we studied the variation in wave number with respect to the variation in the thickness of layers. Figure 2a, b shows the variation in circular frequency ω in response to the variation in the thickness of alternative layers h_1 and h_2. Figure 3 illustrates the variation in circular frequency ω with total thickness h. For an SH wave propagating in the direction normal to layering, the circular frequency ω tends to decrease as the thickness of successive layers increases. The results obtained were similar to those of earlier investigations [6,28].

Figure 4a, b shows the variation in wave number k with respect to the variation in the thickness of layers h_2 and h_1. Figure 5 shows the variation in wave number when the thickness of both the layers becomes equal. It is observed from the curves that as the thickness of the successive layer increases, the wave number k decreases [4,32].

In order to show the wave filter effect, a new factor volume fraction is introduced as $\zeta = h_1/(h_1 + h_2)$. In dispersion Eq. (38), two variables have been defined as $\omega h_2/c'_{\mathrm{sh}}$ and kh_2. By taking into consideration these variables, the dispersion relation for the direction normal to layering is changed to a dimensionless equation. The wave filter effect is shown in Fig. 6a–d for different values of volume fraction. It is observed from the curves that as the wave number kh_2 increases, the non-dimensional frequency also increases [13]. For the increase in the value of volume fraction, the number of stop band increases.

The variation in phase velocity c versus volume fraction is shown in Fig. 7 for different values of ω ranging from 500–2000 Hz. It is clearly indicated from the curves that circular frequency has a significant effect on the phase velocity c (m/s). As the volume fraction increases, a sharp fall in the value of phase velocity was observed for the different circular frequencies ω.

Table 1 Material coefficients of PVDF

Piezoelectric constant (C/m^2)	Dielectric constant (E-10 F/m)	Elastic constant (E10 N/m^2)	Mass density (E3 kg/m^3)
$e_{15} = -0.16$	$\epsilon_{11} = 1.062$	$c_{44} = 0.91$	$\rho = 1.78$

Table 2 Material coefficients of PZT-5H

Piezoelectric constant (C/m^2)	Dielectric constant (E-10 F/m)	Elastic constant (E10 N/m^2)	Mass density (E3 kg/m^3)
$e'_{15} = 17$	$\epsilon'_{11} = 277$	$c'_{44} = 2.30$	$(\rho_{11})_{33} = 4950$
	$\epsilon'^{*}_{11} = 299$		$(\rho_{12})_{33} = -1125$
	$A_{11} = 112$		$(\rho_{22})_{33} = 4800$

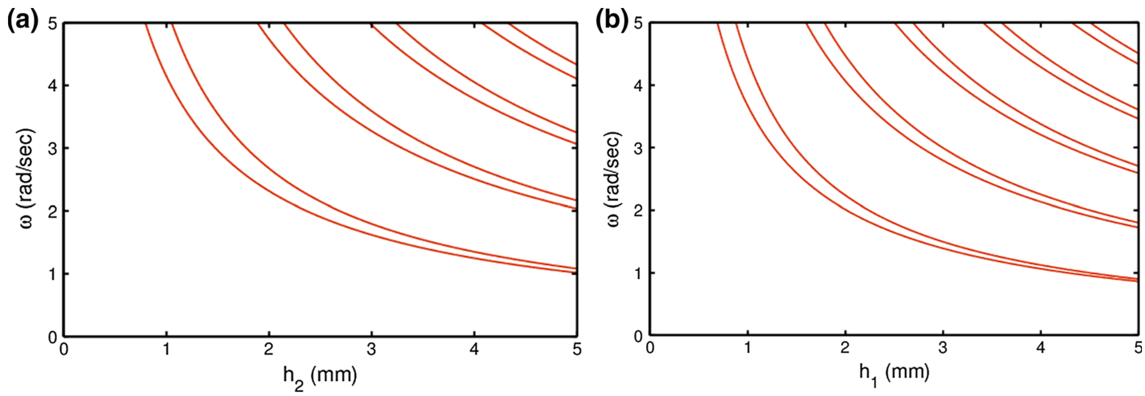

Fig. 2 Variation in ω versus individual layer thickness for **a** $h_1 = 0.1$ and **b** $h_2 = 0.1$

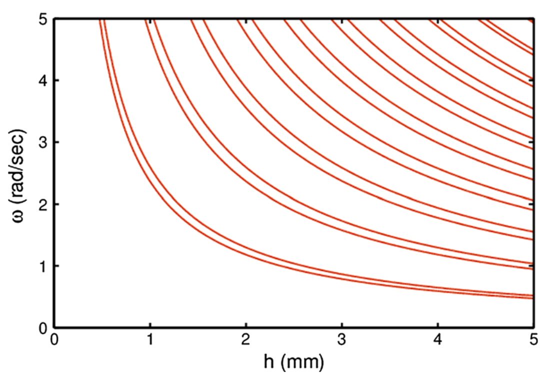

Fig. 3 Variation in ω versus layer thickness for $h_1 = h_2 = h$

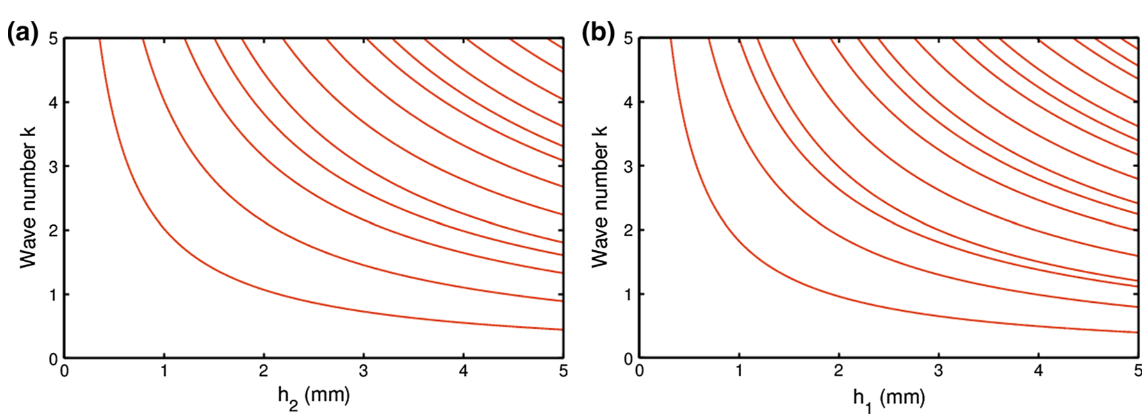

Fig. 4 Variation in wave number versus individual layer thickness for **a** $h_1 = 0.1$ and **b** $h_2 = 0.1$

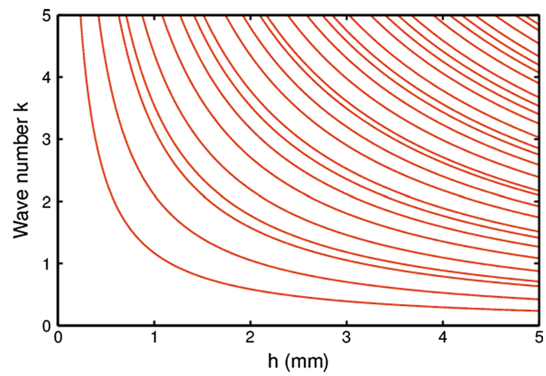

Fig. 5 Variation in wave number versus layer thickness for $h_1 = h_2 = h$

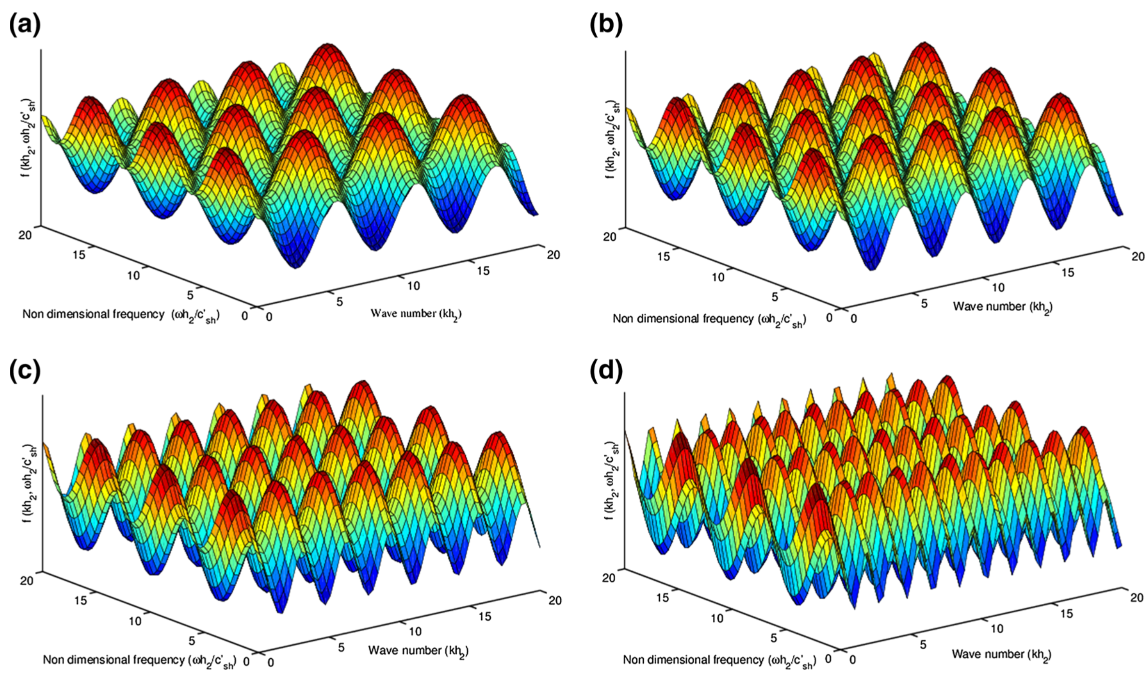

Fig. 6 Stop band effect for different values of volume fraction ζ. **a** 1, **b** 3, **c** 5 and **d** 7

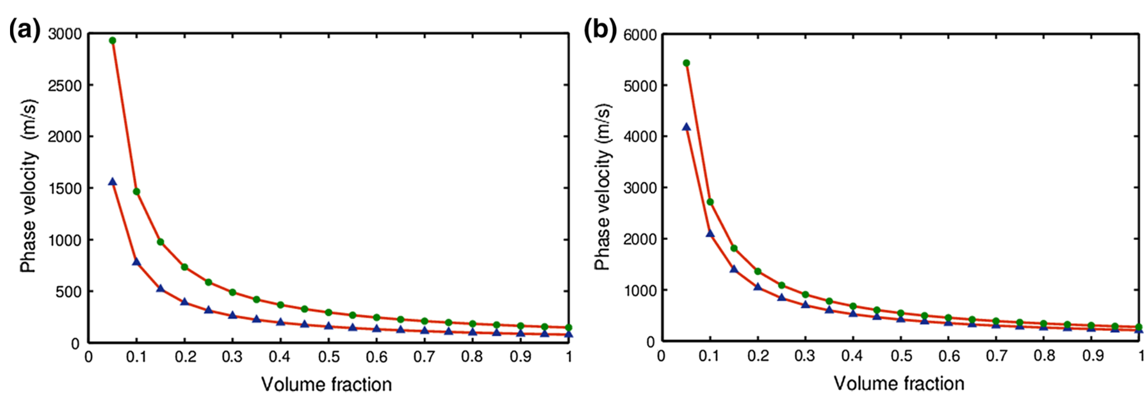

Fig. 7 Variation in phase velocity c (m/s) versus volume fraction for circular frequency. **a** 500–1000 Hz and **b** 1500–2000 Hz

6 Concluding remarks

The propagation of an SH wave is considered in a porous piezoelectric–piezoelectric composite structure. The propagation behavior of an SH wave is discussed along the direction normal to the layering and in the direction of the layering. Dispersion equation and numerical simulation are carried out for two different piezoelectric materials PVDF and PZT. The effect of material constant such as density, elastic constant, porosity, dielectric constant on SH wave propagation is investigated also. It can be concluded from this study that the wave number is influenced by the thickness of the individual layer in the composite. As the value of α increases, the number of stop band also increases followed by the successive decrease in the width of individual stop band. The phase velocity has been significantly influenced by the variation in elastic constant and porosity. Numerically, simulation shows the dependence of phase velocity on volume fraction and indicates the increase in the value of volume fraction with decreases in phase velocity. The work presented in this paper outlines the theoretical and numerical computation for design and development of underwater acoustic sensors and transducers.

References

1. Li, P., Jin, F.: Excitation and propagation of shear horizontal waves in a piezoelectric layer imperfectly bonded to a metal or elastic substrate. Acta Mech. **226**, 267–284 (2014)
2. Allard, J.F., Atalla, N.: Propagation of Sound in Porous Media: Modelling Sound Absorbing Materials. Wiley, Chichester (2009)
3. Christensen, R.M.: Mechanics of Composite Materials. Wiley, New York (1979)
4. Son, M.S., Kang, Y.J.: Propagation behavior of SH waves in layered piezoelectric plates. J. Mech. Sci. Technol. **25**, 613–619 (2011)
5. Huang, M., Fang, X., Zhao, X., Liu, J., Feng, W.: Effective shear modulus of piezoelectric film embedded with square nano-fibers under anti-plane shear waves. J. Appl. Phys. **115**, 154315 (2014)
6. Singh, B.: Propagation of shear waves in a piezoelectric medium. Mech. Adv. Mater. Struct. **20**, 434–440 (2013)
7. Gaur, A.M., Rana, D.S.: Shear wave propagation in piezoelectric–piezoelectric composite layered structure. Lat. Am. J. Solids Struct. **11**, 2483–2496 (2014)
8. Piliposian, G.T., Avetisyan, A.S., Ghazaryanb, K.B.: Shear wave propagation in periodic phononic/photonic piezoelectric medium. Wave Motion **49**, 125–134 (2012)
9. Kielczyfiski, P., Pajewski, W., Szalewski, M.: Shear horizontal surface waves on piezoelectric ceramic with layered structure. Appl. Phys. A **50**, 301–304 (1990)
10. Son, M.S., Kang, Y.J.: The effect of initial stress on the propagation behavior of SH waves in piezoelectric coupled plates. Ultrasonics **51**, 489–495 (2011)
11. Du, J., Jin, X., Wang, J., Zhou, Y.: SH wave propagation in a cylindrically layered piezoelectric structure with initial stress. Acta Mech. **191**, 59–74 (2007)
12. Liu, H., Wang, Z.K., Wang, T.J.: Effect of initial stress on the propagation behavior of Love waves in a layered piezoelectric structure. Int. J. Solids. Struct. **38**, 37–51 (2001)
13. Qian, Z., Jina, F., Kishimoto, K., Wang, Z.: Effect of initial stress on the propagation behavior of SH-waves in multilayered piezoelectric composite structures. Sensor Actuat. A-Phys. **112**, 368–375 (2004)
14. Li, O., Jin, F., Lu, T.J.: A three-layer structure model for the effect of a soft middle layer on Love waves propagating in layered piezoelectric systems. Acta Mech Sin. **28**, 1087–1097 (2012)
15. Cao, X., Shi, J., Jin, F.: Lamb wave propagation in the functionally graded piezoelectric–piezomagnetic material plate. Acta Mech. **223**, 1081–1091 (2012)
16. Liu, J., Cao, X.S., Wang, Z.K.: Love waves in a smart functionally graded piezoelectric composite structure. Acta Mech. **208**, 63–80 (2009)
17. Wang, H.M., Zhao, Z.C.: Love waves in a two-layered piezoelectric/elastic composite plate with an imperfect interface. Arch. Appl. Mech. **83**, 43–51 (2013)
18. Daros, C.H.: On modelling Bleustein–Gulyaev waves in non-homogeneous, transversely isotropic, piezoelectric media via stress equations of motion. Acta Mech. **163**, 121–126 (2003)
19. Soh, A.K., Liu, J.X.: Interfacial shear horizontal waves in a piezoelectric–piezomagnetic biomaterial. Philos. Mag. Lett. **86**, 31–35 (2006)
20. Singh, B.M., Rokne, J.: Propagation of SH waves in layered functionally gradient piezoelectric–piezomagnetic structures. Philos. Mag. **93**, 1690–1700 (2013)
21. Du, J., Xian, K., Wang, J.: SH surface acoustic wave propagation in a cylindrically layered piezomagnetic/piezoelectric structure. Ultrasonics **49**, 131–138 (2009)
22. Qian, Z.H., Jin, F., Hirose, S.: Dispersion characteristics of transverse surface waves in piezoelectric coupled solid media with hard metal interlayer. Ultrasonics **51**, 853–856 (2011)
23. Liu, J., Wei, W., Fang, D.: Propagation behaviors of shear horizontal waves in piezoelectric–piezomagnetic periodically layered structures. Acta Mech. Solid. Sin. **23**, 77–84 (2010)
24. Pang, Y., Liu, J., Wang, Y., Fang, D.: Wave propagation in piezoelectric/piezomagnetic layered periodic composites. Acta Mech. Solid. Sin. **21**, 483–490 (2008)
25. Du, J.K., Shen, Y.P., Ye, D.Y., Yue, F.R.: Scattering of anti-plane shear waves by a partially debonded magneto–electro-elastic circular cylindrical inhomogeneity. Int. J. Eng. Sci **42**, 887–913 (2004)
26. Yang, J.S.: Shear horizontal vibrations of a piezoelectric/ferroelectric wedge. Acta Mech. **173**, 13–17 (2004)

27. Chen, W.Q., Ding, H.J.: On free vibration of a functionally graded piezoelectric rectangular plate. Acta Mech. **153**, 207–216 (2002)
28. Vashishth, A.K., Gupta, V.: Vibrations of porous piezoelectric ceramic plates. J. Sound Vib. **325**, 781–797 (2009)
29. Sharma, M. D.: Piezoelectric effect on the velocities of waves in an anisotropic piezo–poroelastic medium. Proc. R. Soc. **466**, 1977–1992 (2010)
30. Manna, S., Kundu, S., Gupta, V.: Love wave propagation in a piezoelectric layer overlying in an inhomogeneous elastic half-space. J. Vib. Control (2013). doi:10.1177/1077546313513626
31. Vashishth, A.K., Dahiya, A.: Shear waves in a piezoceramic layered structure. Acta Mech. **224**, 727–744 (2013)
32. Qian, Z., Jin, F., Wang, Z., Kishimoto, K.: Dispersion relations for SH-wave propagation in periodic piezoelectric composite layered structures. Int. J. Eng. Sci. **42**, 673–689 (2004)