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ORIGINAL PAPER

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A study on fiber orientation influence on the mechanical response of a short fiber composite structure

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Abstract This paper deals with the structural analysis of composite materials with non-homogenous orientation of the reinforcement. During this research, a short fiber-reinforced polymer matrix composite is studied. In this case, inhomogeneity of the reinforcement orientation caused by injection molding manufacturing process is analyzed. The main objective of the paper is the investigation of an influence of process-induced orientation of the reinforcement on mechanical properties of the material in comparison with unidirectional and random reinforcement orientation. In particular, natural frequencies and transient response of an exemplary composite component are investigated. To specify effective properties of the composite, Mori–Tanaka's micromechanical model is assumed. Orientation distribution of the reinforcement is determined by injection molding simulation. To determine elastic material properties dependent on non-homogenous orientation of the reinforcement, an orientation averaging procedure is taken into account. Therefore, during this study, effectiveness of the orientation averaging procedure and different closure approximations influence on the results are studied. Orientation averaging results are compared with numerical results obtained by finite element-based homogenization of composites with prescribed second-order orientation tensor. Finally, the obtained material parameters were applied into a macroscale finite element model, and numerical simulation with different boundary conditions was conducted.

1 Introduction

Short fiber composites find wide industrial application due to their versatile properties, low cost and easy processing. The orientation of fibers depends on the processing conditions and may vary from random to approximately unidirectional. One of the most common processes used in the manufacturing of short fiber composite parts is injection molding. Parts manufactured by injection molding have advantages of economy, vast quantity and no post-molding finishing operations [10,13]. Therefore, during this study, an analysis of molded parts is taken into consideration. From the practical point of view, a very important issue is the capability of the prediction of mechanical properties of composite parts at an early design step. In this case, an influence of composite constituents properties and manufacturing process parameters has to be taken into account. Mentioned assumptions make the analysis of short glass fiber composite structures complex; however, a combination of existing software and numerical procedures can lead to solve this problem.

One of the main aims of this study is the investigation of the influence of fiber arrangement on the dynamic response of a particular composite structure. During this research, a polymer matrix composite reinforced with

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glass fibers, which is the most popular material among the other short fiber-reinforced composites, is considered [10]. Effective properties of the composite can be evaluated on microlevel by using homogenization procedures and taking into account the properties of the constituents. One of the most popular homogenization schemes is based on finite element analysis of a representative volume element (RVE). Results of such an analysis related to short fiber-reinforced composites can be found in [12,14,25]. Other widely used homogenization schemes are mean field methods [5]. Determination of the effective properties of short glass fiber composites by using a mean field approach can be found in [11,17,21,23,26]. Usage of mean field methods can lead to obtain accurate results with low computational cost. Orientation distribution of fibers in an exemplary component in early-stage design process can be determined by using injection molding numerical simulation [1]. There exists commercial software dedicated to the analysis of injection molding, for example Moldflow or Moldex. Analyses that compare the results of numerical simulation of injection molding process and experimental data are presented, for example, in [9,16]. The influence of the molding process parameters (mold temperature, injection time or pressure) was not taken into account. Their influence on fiber orientation and finally on components properties is presented in [24]. The main aim of the presented approach is a general investigation of the influence of fiber arrangement on the mechanical response of a particular composite structure and presentation of the methodology where different methods are implemented.

The approach that was considered during this research is based on the coupling of injection molding simulation, mean field homogenization scheme and finite element structural analysis of a composite part in macroscale. To couple fiber orientation data obtained from injection-molded analysis and homogenized material parameters, the orientation averaging approach proposed by Advani and Tucker is considered [2]. There are several works that are connected with the usage of a similar approach [11,17,25]; however, this study focuses on the estimation of natural frequencies, transient response of exemplary composite part and numerical validation of orientation averaging procedure. To create the finite element model, LS-Dyna software was adopted. In the previous work of the authors connected with multi-scale modeling [22], also LS-Dyna software was used which is a useful tool for solving the presented problems. Effective material parameters were determined by homogenization at microlevel. Finally, to each finite element, different material properties were assigned and finite element computations were conducted. This study is also focused on testing an effectiveness of orientation averaging procedure and different closure approximations influence on the material stiffness prediction. Therefore, orientation averaging results are compared with numerical results obtained by finite element-based homogenization of composites with prescribed second-order orientation tensor.

2 Properties prediction of a composite with misaligned fibers

2.1 Orientation averaging

The orientation averaging procedure assumes that properties of the composite are taken as an average of unidirectional fiber composite properties over all directions weighted by the orientation distribution function $\Psi(p)$ [2]:

$$C_{ijkl} = \oint \Psi(p) C^{U}_{ijkl} \mathrm{d}p \tag{1}$$

where C_{ijkl} is the stiffness tensor of the composite with misaligned fiber orientation, C^{U}_{ijkl} is the stiffness tensor of unidirectional composite, and p is a vector defining an orientation of a single fiber.

In case of estimating fiber orientation induced by the manufacturing process, the orientation distribution function is usually not known. In addition, from the computational point of view using an orientation distribution function, description is cumbersome. Instead, prediction of stiffness tensor of the composite can be realized by assuming second- and fourth-order orientation tensors a_{ij}, a_{ijkl} and scalar constants $B_1 - B_5$ related to the components of the stiffness tensor of a unidirectional composite [2]:

$$\begin{aligned} C_{ijkl} = {} & B_1(a_{ijkl}) + B_2(a_{ij}\delta_{kl} + a_{kl}\delta_{ij}) + B_3(a_{ik}\delta_{jl} + a_{il}\delta_{jk} + a_{jl}\delta_{ik} + a_{jk}\delta_{il}) \\ & + B_4(\delta_{ij}\delta_{kl}) + B_5(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \end{aligned} \tag{2}$$

where

$$a_{ij} = \oint p_i p_j \psi(p) \mathrm{d}p, \tag{3}$$

$$a_{ijkl} = \oint p_i p_j p_k p_l \psi(p) \mathrm{d}p. \tag{4}$$

The drawback of this approach is that the fourth-order tensor a_{ijkl} is required to calculate the effective stiffness. In practice, very frequently, only the second-order orientation tensor is known. For example, it can be evaluated by using software dedicated to the simulation of an injection molding process. During this study, Moldflow software was used. The second-order orientation tensor has the following properties:

$$a_{ii} = 1, \tag{5}$$

$$a_{ij} = a_{ji}. \tag{6}$$

Therefore, the second-order orientation tensor has five independent components. To estimate the fourth-order orientation tensor, commonly closure approximations are considered where fourth-order tensor components are calculated from components of the second-order tensor. There are several works that deal with the analysis of different closure approximation methods [3,7,15]. One of the most popular is hybrid closure approximation [2]:

$$a_{ijkl} = (1-f)a_{ijkl}^{\mathrm{LIN}} + f a_{ijkl}^{\mathrm{QUA}}, \tag{7}$$

$$a_{ijkl}^{\mathrm{LIN}} = \frac{1}{7}\left(a_{ij}\delta_{kl} + a_{ik}\delta_{jl} + a_{il}\delta_{jk} + a_{kl}\delta_{ij} + a_{jl}\delta_{ik} + a_{jk}\delta_{il}\right) - \frac{1}{35}\left(\delta_{ij}\delta_{kl} + \delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}\right), \tag{8}$$

$$a_{ijkl}^{\mathrm{QUA}} = a_{ij}a_{kl}, \tag{9}$$

$$f = 1 - 27\det(a_{ij}). \tag{10}$$

Despite drawbacks of this method discussed in [3], it is still in use due to simplicity and computational efficiency. Another family of closure approximations proposed by Cintra and Tucker are orthotropic closures [6]. Orthotropic closures are based on the assumption that principal axes of the approximated fourth-order tensors are in alignment with the principal axes of the second-order tensor. Definition of orthotropic closure approximation reduces to choosing three scalar functions f, such that $a_{iiii} = f_i(a_1, a_2)$, Cintra and Tucker in their work used second-degree polynomials [6,7]:

$$a_{iiii} = T_{i1}^{1} + T_{i2}^{2}a_{(1)} + T_{i3}^{3}a_{(1)}^{2} + T_{i4}^{4}a_{(2)} + T_{i5}^{5}a_{(2)}^{2} + T_{i6}^{6}a_{(1)}a_{(2)} \tag{11}$$

where there is no sum on i, $a_{(1)}$ and $a_{(2)}$ are the first and the second eigenvalues of the second-order orientation tensor, T_m^i is a matrix of coefficients which can be determined by using specific orientation distributions (smooth version) or by fitting to distributions evaluated for a set of flow fields (fitted version). In opposite to hybrid closure, it requires additional transformation between the global coordinate and the principal coordinate. The orthotropic closure typically provides the most accurate results in comparison with experimentally measured fourth-order tensors [3,7].

2.2 Determination of unidirectional composite properties

To determine the effective properties of the composite from Eq. (2) besides second- and fourth-order orientation tensors also, scalar constants $B_1 - B_5$ have to be calculated. These constants are related to the components of the stiffness tensor of a unidirectional composite as follows [2]:

$$\begin{aligned}
B_1 &= C_{1111}^{U} + C_{2222}^{U} - 2C_{1122}^{U} - 4C_{1212}^{U},\\
B_2 &= C_{1122}^{U} - C_{2233}^{U},\\
B_3 &= C_{1212}^{U} + \frac{1}{2}\left(C_{2233}^{U} - C_{2222}^{U}\right),\\
B_4 &= C_{2233}^{U},\\
B_5 &= \frac{1}{2}\left(C_{2222}^{U} - C_{2233}^{U}\right).
\end{aligned} \tag{12}$$

Evaluation of effective elasticity tensors of heterogeneous materials is connected with the usage of homogenization procedures. The homogenization procedure involves replacing the heterogeneous material with an equivalent homogeneous material. In order to estimate a stiffness of the heterogeneous material, classical mean field methods can be used. This group of methods is based on the well-known equivalent inclusion approach of Eshelby [8]. An advantage of this approach is computational efficiency. Tucker and Liang in their work [26] showed that among the other mean field homogenization methods, Mori–Tanaka's method gives the most accurate stiffness predictions of short fiber composites. Mori–Tanaka's strain concentration tensor can be written as [19]:

$$A^{\mathrm{MT}} = S[(1 - v_{\mathrm{f}})I + v_{\mathrm{f}} S]^{-1}, \tag{13}$$

and the effective stiffness of the unidirectional composite C^U can be written in closed form as follows [4]:

$$C^U = C_{\mathrm{m}} + f_{\mathrm{f}} \left[\left(C_{\mathrm{f}} - C_{\mathrm{m}} \right) A^{\mathrm{MT}} \left(f_{\mathrm{m}} I + f_{\mathrm{f}} A^{\mathrm{MT}} \right) \right]^{-1} \tag{14}$$

where S is an Eshelby tensor [8,20], I is the identity tensor, C_{m} and C_{f} are stiffness tensors of matrix and fiber material, respectively, f_{m} and f_{f} are volume fractions of matrix and fiber phases.

Another widely used homogenization method is based on the volume averaging of stress and strain values in a representative volume element (RVE) by solving the boundary value problem by the finite element method. Calculation of the equivalent material properties requires to solve six RVE boundary value problems (BVPs) in three-dimensional case [18]. For each BVP, a prescribed strain is applied in accordance with Eq. (1) (a superscript indicates the number of analysis),

$$\varepsilon^{(1)} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \varepsilon^{(2)} = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \varepsilon^{(3)} = \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \quad \varepsilon^{(4)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \quad \varepsilon^{(5)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \quad \varepsilon^{(6)} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{bmatrix}. \tag{15}$$

In addition, periodic boundary conditions are introduced. Strains and periodic boundary conditions are prescribed via multi-point constraints (MPC). After solving six BVPs, both the stresses (16) and strains (17) are averaged in the post-processing stage of the analysis,

$$\langle \sigma_{ij} \rangle = \frac{1}{V_{\mathrm{RVE}}} \int_{V_{\mathrm{RVE}}} \sigma_{ij} \mathrm{d}V_{\mathrm{RVE}}, \tag{16}$$

$$\langle \varepsilon_{ij} \rangle = \frac{1}{V_{\mathrm{RVE}}} \int_{V_{\mathrm{RVE}}} \varepsilon_{ij} \mathrm{d}V_{\mathrm{RVE}} \tag{17}$$

where $\langle \sigma_{ij} \rangle$ is average stress, $\langle \varepsilon_{ij} \rangle$ is average strain, σ_{ij} is stress in the RVE, ε_{ij} is strain in the RVE, and V_{RVE} is volume of the RVE. The elasticity matrix C_{RAW} that binds average stress and average strains is expressed by:

$$\langle \sigma_{ij} \rangle = C_{\mathrm{RAW}} \langle \varepsilon_{ij} \rangle. \tag{18}$$

2.3 Methodology of multi-scale modeling

The flowchart of the proposed methodology is presented in Fig. 1. At first, material parameters of fiber and matrix phases are determined, and the next step is the evaluation of effective properties of a composite with perfectly aligned fibers by using Mori–Tanaka's method. Second-order orientation tensors are calculated for each finite element used in mesh prepared for injection molding simulation performed in Moldflow software. To provide good accuracy in structural finite element analysis, different types of finite elements than those used in injection molding simulation software are preferred. The problem of dissimilar meshes is also underlined in [11]. During this study, second-order orientation tensors evaluated for tetrahedral finite elements are mapped on hexagonal finite elements. Then, a fourth-order orientation tensor is computed by using closure approximation. At this stage, the effective stiffness tensor for misaligned fiber composite can be evaluated by using Eq. (2). For each finite element, a different stiffness tensor is computed. Finally, anisotropic material parameters are prescribed to each finite element by creating an appropriate input file to LS-DYNA software which is used as finite element solver.

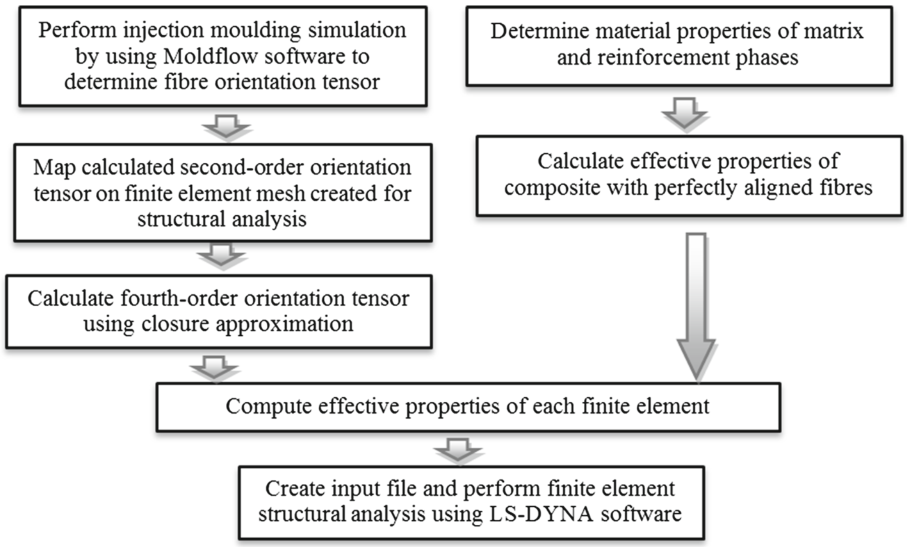


Fig. 1 Flowchart of proposed methodology

3 Numerical simulations results

3.1 Numerical validation of orientation averaging procedure

In order to investigate the effectiveness of the orientation averaging procedure and influence of different closure approximations on the results, analyses of the composite with prescribed second-order orientation tensors were conducted. Orientation averaging results were compared with that obtained by finite element-based homogenization of misaligned composites. To apply the orientation averaging procedure, at first properties of the unidirectional composite have to be determined. In order to compare orientation averaging results with numerical ones in a proper way, effective properties of the composite with perfectly aligned fibers are evaluated. This is done using the homogenization procedure based on volume averaging of stress and strain values in RVE solving the boundary value problem by the finite element method. Figure 2 shows the geometry of the created RVE that represents the material with perfectly aligned fibers (the geometry is created using Digimat-FE software). In comparative analysis, a composite with 0.1 volume fraction of fibers and aspect ratio 10 is considered. The RVE is discretized by voxels of uniform size creating a discrete system of 3 million degrees of freedom.

The obtained stiffness tensor of a composite with perfectly aligned fibers can be presented in the form of tensor:

$$C^U = \begin{bmatrix} 8861.00 & 3591.28 & 3589.65 & 0 & 0 & 0 \\ 3591.28 & 5979.03 & 3655.79 & 0 & 0 & 0 \\ 3589.65 & 3655.79 & 5968.90 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1130.50 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1167.98 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1171.12 \end{bmatrix}. \tag{19}$$

Representative volume elements that contain fibers distributed in accordance with the particular orientation tensor are presented in Figs. 3, 4 and 5. The components of the stiffness tensor of misaligned composites were computed by the finite element-based homogenization procedure in this same manner as in case of a unidirectional composite. Finally, the results of finite element-based homogenization and orientation averaging procedure with considered different closure approximations variants (hybrid, orthotropic fitted, orthotropic smooth) were compared. The results are collected in Tables 1, 2 and 3 that show percent relative error between stiffness tensor components evaluated by finite element-based homogenization C_{ij}^{FE} and orientation averaging procedure C_{ij}^{OA}:

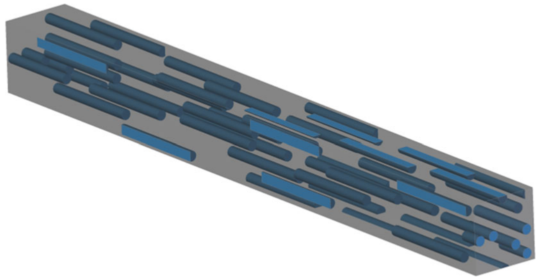
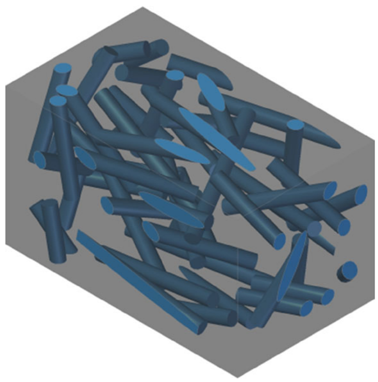
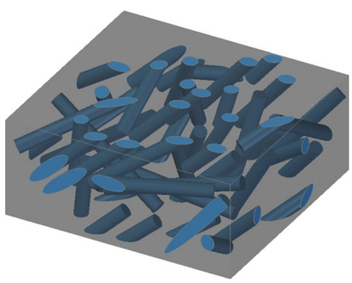


Fig. 2 Geometry of the RVE of the composite with perfectly aligned fibers



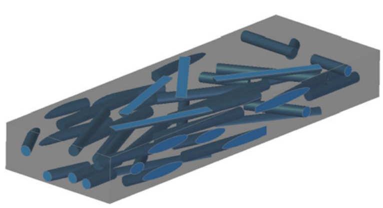
$$a = \begin{bmatrix} 0.580 & 0.019 & -0.015 \\ 0.019 & 0.170 & -0.012 \\ -0.015 & -0.012 & 0.250 \end{bmatrix}$$

Fig. 3 Geometry of the first RVE with corresponding orientation tensor



$$a = \begin{bmatrix} 0.400 & 0.069 & 0.260 \\ 0.069 & 0.170 & -0.001 \\ 0.260 & -0.001 & 0.430 \end{bmatrix}$$

Fig. 4 Geometry of the second RVE with corresponding orientation tensor



$$a = \begin{bmatrix} 0.190 & 0.028 & 0.000 \\ 0.028 & 0.810 & 0.000 \\ 0.000 & 0.000 & 0.000 \end{bmatrix}$$

Fig. 5 Geometry of the third RVE with corresponding orientation tensor

$$\chi C_{ij} = \frac{\left| C_{ij}^{\mathrm{FE}} - C_{ij}^{\mathrm{OA}} \right|}{C_{ij}^{\mathrm{FE}}} \times 100\,\%. \tag{20}$$

In each analyzed case, application of the fitted version of orthotropic closure approximation led to the achievement of the lowest average percent relative error.

Table 1 Percent relative error for the material stiffness components for the first RVE

Applied closure approximation	χC_{11}	χC_{22}	χC_{33}	χC_{12}	χC_{13}	χC_{23}	χC_{44}	χC_{55}	χC_{66}	Average error
Hybrid	2.83	1.56	2.41	1.77	3.48	0.22	1.29	0.42	1.65	1.74
Orthotropic (F)	0.47	0.02	0.88	0.46	1.17	0.08	2.28	0.37	3.70	1.05
Orthotropic (S)	1.44	0.97	0.18	2.20	0.76	0.05	2.00	5.20	1.21	1.56

Table 2 Percent relative error for the material stiffness components for the second RVE

Applied closure approximation	χC_{11}	χC_{22}	χC_{33}	χC_{12}	χC_{13}	χC_{23}	χC_{44}	χC_{55}	χC_{66}	Average error
Hybrid	1.98	2.45	2.55	1.65	1.17	1.23	4.14	7.92	1.21	2.70
Orthotropic (F)	0.79	1.79	2.00	1.92	3.34	2.55	2.62	4.43	0.63	2.23
Orthotropic (S)	1.61	1.00	2.60	1.08	3.92	2.08	2.08	8.38	1.03	2.64

Table 3 Percent relative error for the material stiffness components for the third RVE

Applied closure approximation	χC_{11}	χC_{22}	χC_{33}	χC_{12}	χC_{13}	χC_{23}	χC_{44}	χC_{55}	χC_{66}	Average error
Hybrid	3.27	1.39	0.26	5.81	0.13	0.05	0.25	2.01	16.19	3.26
Orthotropic (F)	0.79	1.83	0.61	0.14	1.08	1.50	1.19	3.35	0.60	1.23
Orthotropic (S)	0.83	3.24	0.26	2.37	1.23	1.06	0.23	2.96	6.33	2.06

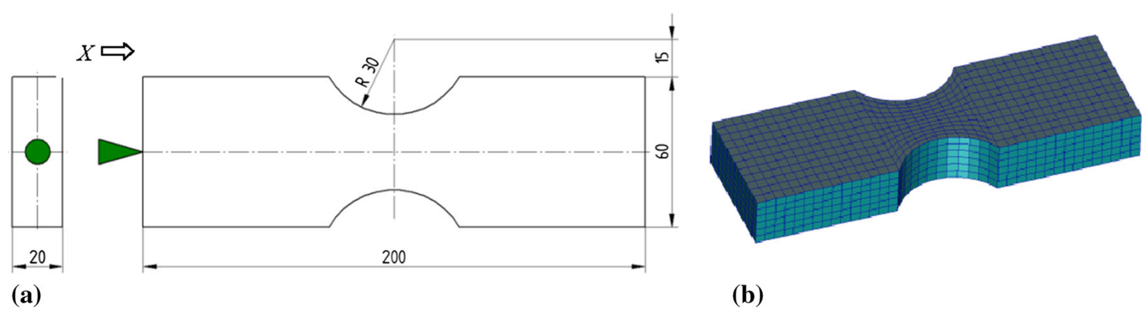


Fig. 6 Analyzed sample **a** dimensions and marked injection point; **b** finite element mesh

3.2 Composite behavior under various loads

To investigate fibers arrangement influence on the mechanical response of the composite, the specimen presented in Fig. 6a was tested (dimensions are expressed in mm). Figure 6b shows the finite discretization of the specimen for the structural analysis. Table 4 contains the material properties of the composite constituents that were considered.

Three different fibers configurations were taken into account:

- perfectly aligned, parallel to X axis,
- random 3D,
- in accordance with second-order orientation tensor computed in injection process simulation software, the injection point setting is marked in Fig. 3.

Table 4 Properties of composite constituents

Fiber properties		Matrix properties:	
Young's modulus, MPa	72000	Young's modulus, MPa	2600
Poisson's ratio	0.21	Poisson's ratio	0.39
Density, g/cm3	2.55	Density, g/cm3	1.18
Aspect ratio	10		
Volume fraction	0.1		

Table 5 Effective properties of perfectly aligned composite

	M-T	FE
Axial Young's modulus, MPa	12960.0	12710.5
In-plane Young's modulus, MPa	4038.2	4367.1
Poisson's ratio	0.359	0.354
Transverse shear modulus, MPa	1372.4	1397.9
In-plane shear modulus, MPa	1307.0	1450.9

Table 6 Effective properties of composites with random 3D fibers arrangement

	M-T	FE
Young's modulus, MPa	5300.9	5468.4
Poisson's ratio	0.349	0.345

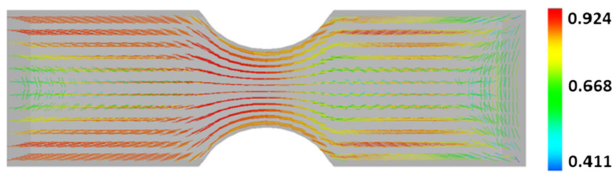


Fig. 7 Non-homogenous distribution of orientation tensor: *Colored scale* indicates the first eigenvalue of the second-order orientation tensor, *lines* direction indicates the first eigenvector of the second-order orientation tensor (color figure online)

Table 7 First six natural frequencies of the specimen with different fibers arrangement

Fiber orientation	Natural frequencies, Hz					
	1	2	3	4	5	6
Perfectly aligned	137.0	345.3	678.6	909.8	1466.0	2210.4
Random 3D	156.0	358.0	808.6	918.8	1617.7	2261.7
Orientation tensor	170.6	379.9	749.5	942.9	1646.2	2325.6

Results of material effective properties predictions of a perfectly aligned composite and composite with random fibers distribution are collected in Tables 5 and 6, respectively. Tables 5 and 6 show effective properties evaluated by Mori–Tanaka's method (M–T) and finite element-based homogenization (FE). During multi-scale computations, Mori–Tanaka's method is used.

For a composite with fibers orientations obtained from injection molding simulation, 21 stiffness tensor components were calculated separately for each finite element in accordance with the methodology presented in chapter 2. Due to high accuracy, fitted orthotropic closure approximation was taken into account. Figure 7 shows graphically the first eigenvectors and corresponding eigenvalues of the calculated second-order orientation tensor.

One of the aims of this study is the estimation of natural frequencies of the analyzed specimen. Modal analysis was carried out by the finite element method considering the specimen constrained from one side. The obtained first six natural frequencies of the specimen with different fibers arrangements are presented in Table 7.

Apart from modal analysis, the transient response of the specimen was investigated. Three different load cases were taken into account. Figure 8a shows prescribed loads that were analyzed separately. Each of the loads is time dependent in accordance with Fig. 8b, and load magnitude is different for each load.

The results of carried out simulations are presented in Figs. 9, 10 and 11 in the form of graphs that show displacement magnitude and maximum principal stress for the finite element connected with the highest stress values as functions of time.

4 Discussion and conclusions

The main objective of the paper is the investigation of an influence of process-induced orientation of the reinforcement on mechanical properties of the short fiber-reinforced composite material. The proposed method

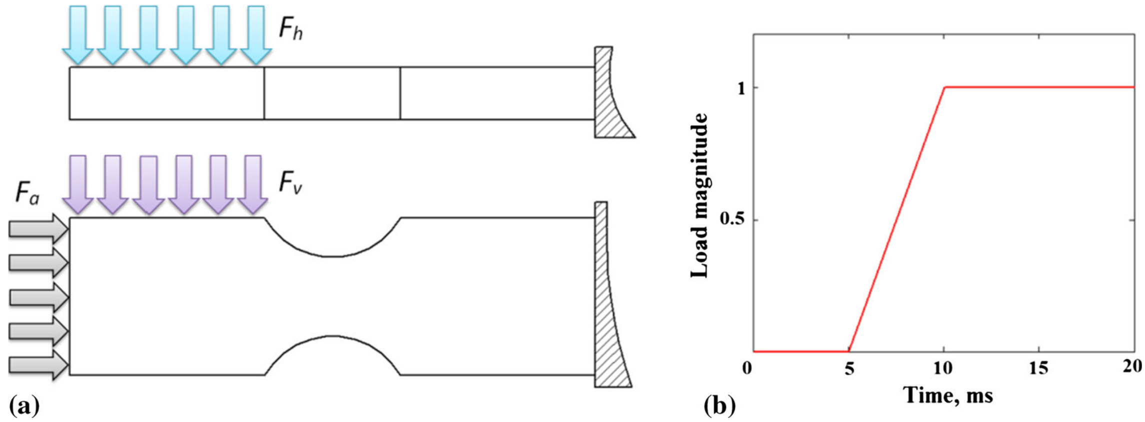


Fig. 8 Boundary conditions for finite element transient response analysis: **a** loads: F_a—axial load, F_v—vertical load, F_h—horizontal load; **b** load–time dependence

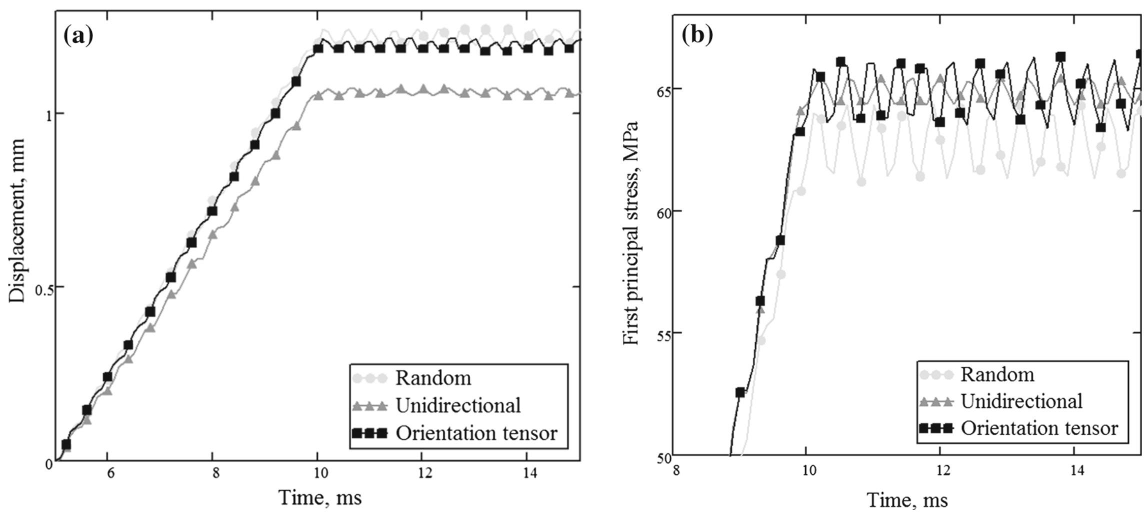


Fig. 9 Structure response on axial load: **a** displacement in time; **b** maximum principal stress in time

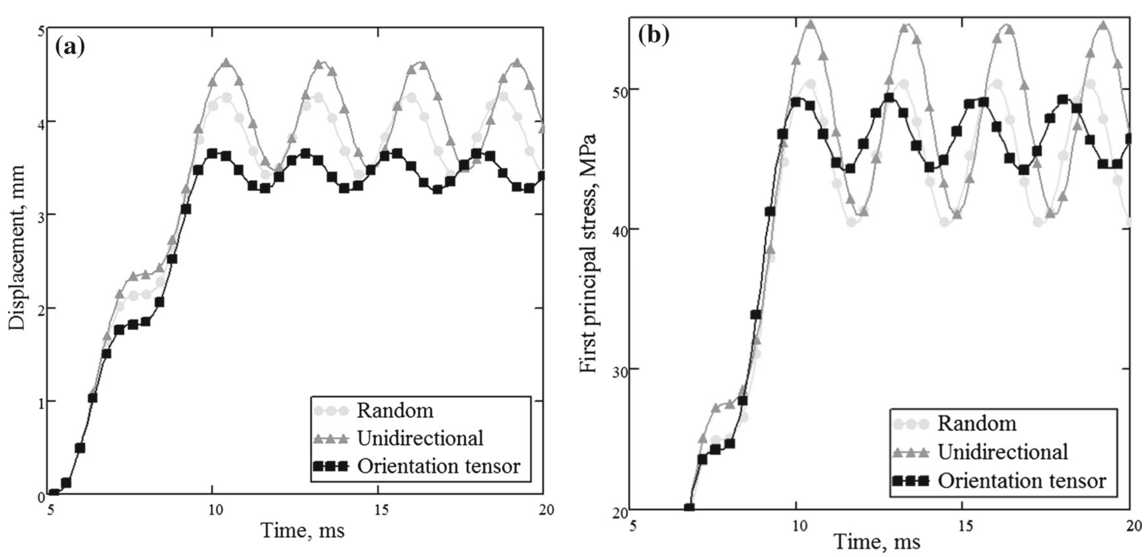


Fig. 10 Structure response on vertical load: **a** displacement in time; **b** maximum principal stress in time

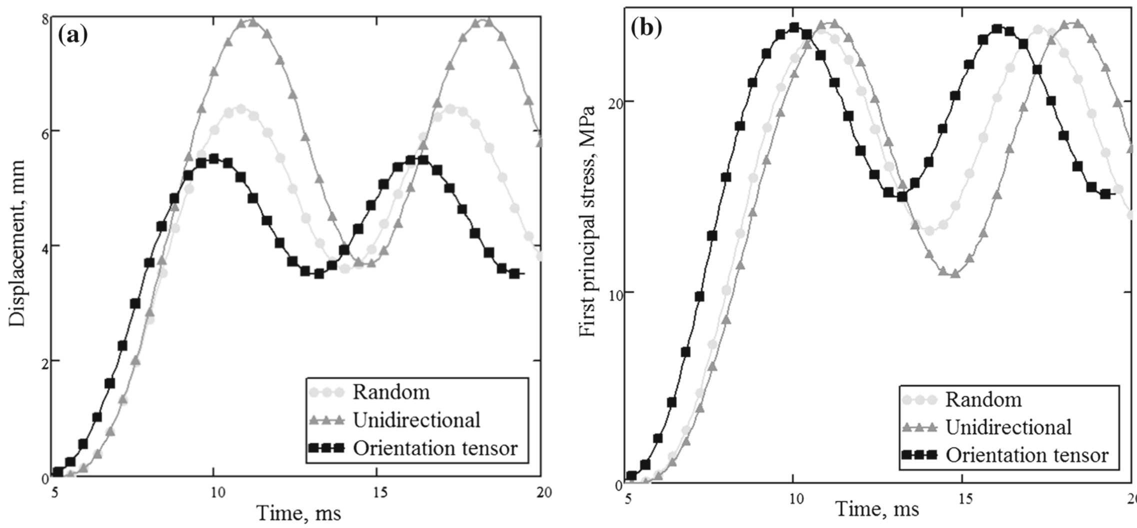


Fig. 11 Structure response on horizontal load: **a** displacement in time; **b** maximum principal stress in time

based on the coupling of injection molding simulation, material modeling at microscale and finite element structural analysis of the composite part in macroscale can be an efficient tool in designing the fiber composite structures. Before conducting multi-scale computations, orientation averaging procedure was validated numerically by comparing orientation averaging results with finite element-based homogenization of composites with prescribed second-order orientation tensor. Three different RVEs containing complex fiber distributions have been created. In addition, three different closure approximations were tested. The fitted orthotropic closure approximation leads to the lowest error related to the results of the finite element analysis of the RVE accounting a complex geometry. This conclusion agrees with observations of other researchers [3,7]. The conducted comparative analysis showed an effectiveness of the orientation averaging procedure. Accounting the most accurate closure approximation for analyzed three RVEs, average percentage errors in the estimation of material stiffness are 1.05, 2.23, 1.23 %, respectively.

Carried out numerical multi-scale simulations based on the presented approach show that fiber orientation distribution in short fiber-reinforced composite parts has got a significant influence on their mechanical response. A comparative analysis of the composite with different fiber orientations shows that simplification of the material model of injection-molded parts and considering it as isotropic corresponding to the random orientation of the fibers or orthotropic corresponding to unidirectional orientation can lead to unacceptable errors in determining the natural frequencies, displacement and stresses. For example, in case of estimation of natural frequencies for the sixth mode, the difference between the natural frequency of the composite with random fibers orientation and the composite with process-induced fiber orientation reached almost 70 Hz. Further work will be devoted to analyzing an influence of injection molding process parameters on the mechanical properties of manufactured parts, experimental validation of obtained numerical results and considering nonlinear material behavior, in particular strain rate sensitivity.

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