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ORIGINAL PAPER

Jinseok Kim · J. N. Reddy

A general third-order theory of functionally graded plates with modified couple stress effect and the von Kármán nonlinearity: theory and finite element analysis

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Abstract Finite element analysis of functionally graded plates based on a general third-order shear deformation plate theory with a modified couple stress effect and the von Kármán nonlinearity is carried out to bring out the effects of couple stress, geometric nonlinearity and power-law variation of the material composition through the plate thickness on the bending deflections of plates. The theory requires no shear correction factors. The principle of virtual displacements is utilized to develop a nonlinear finite element model. The finite element model requires C^1 continuity of all dependent variables. The microstructural effects are captured using a length scale parameter via the modified couple stress theory. The variation of two-constituent material is assumed through the thickness direction according to a power-law distribution. Numerical results are presented for static bending problems of rectangular plates with various boundary conditions to bring out the parametric effects of the power-law index and length scale parameter on the load–deflection characteristics of plates with various boundary conditions.

1 Introduction

The high computational cost associated with the use of three-dimensional finite elements for the analysis of plate structures led to the development of two-dimensional plate finite elements that account for shear deformation and geometric nonlinearity. Among various two-dimensional plate theories, displacement-based plate theories are preferred because they automatically satisfy the compatibility conditions and equilibrium equations [34]. Various higher-order plate theories differ from each other in terms of the number of independent variables in the displacement expansion. The optimal displacement expansion is the one that has the ability to represent a quadratic variation of the transverse shear strains through the plate thickness. A general third-order shear deformation plate theory (GTPT) of functionally graded materials (FGMs) with a power-law distribution and modified couple stress theory [41] was proposed by Reddy and Kim [34]. The GTPT is based on the cubic variation of the in-plane displacements and the quadratic variation of the out-of-plane displacement. This plate theory results in a quadratic variation of transverse shear strains and stresses and does not required a shear correction factor. In the GTPT, three-dimensional constitutive relations and the von Kármán nonlinear strains are used. The FGM used herein is made of two constituents, and the composition varies from one material one surface to another surface according to a power-law. Many micro- and nano-scale devices make use of FGMs. It is well known that the conventional continuum theories do not capture the microstructural material length scale effect in micro- and nano-scale plate-type structures. The modified couple stress theory [41] that accounts for the microstructural effect through a single length scale parameter is used here. A displacement-based weak-form Galerkin finite element model of the GTPT is developed, which requires the use of C^1-continuity of all

J. Kim · J. N. Reddy (✉)
Advanced Computational Mechanics Laboratory, Department of Mechanical Engineering, Texas A&M University, College Station, TX 77843-3123, USA
E-mail: jnreddy@tamu.edu
Tel.: 979 862 2417

generalized displacements. Several numerical examples of static bending of rectangular plates with various boundary conditions are presented to bring out the effects of the length scale parameters and power-law index.

1.1 Functionally graded materials

Functionally graded materials are novel composite materials of which the volume fraction of two or more material constituents varies continuously on the microscopic scale. The common application of FGMs has been in structures involving high-temperature environments, where a high gradient of temperature and/or thermal shock exists [9,15]. Noda [21] reviewed numerus papers on thermo-elastic to thermo-inelastic problems in FGMs. Fukui and Yamanaka [7] and Fukui et al. [8] presented the effect of the variation of constituents on mechanical behavior of thick-walled functionally graded tube under internal pressure and the effect of FGMs on residual stresses under uniform thermal loading. They minimized compressive circumferential stress at the inner surface to determine optimum variation of the FGM tube. Fuchiyama et al. [6] presented transient thermal stresses and stress intensity factors of functionally graded materials with cracks. Jin and Noda [10,11,22] suggested optimum material variation of metal–ceramic functionally gradient material by minimizing the thermal stress intensity factor and presented the steady state and the transient heat conduction problems in FGMs. Obata et al. [24] studied the in-plane thermal stress distributions caused by a temperature distribution through plate thickness. They presented an appropriate law for material gradation and the temperature and stress distribution depended on volume fraction of constituents. Noda and Tsuji [23], Shen [37] and Yang and Shen [42] have studied the thermo-elastic analysis of FGM plates considering temperature-dependent material properties. Praveen and Reddy [26], Reddy [28] and Aliaga and Reddy [1] have included the von Kármán nonlinearity in FGM plates. Reddy and Berry [33] presented axisymmetric bending of functionally graded circular plates based on temperature-dependent material properties and the von Kármán geometric nonlinearity. Saidi et al. [36] presented Levy-type solution for bending–stretching analysis of thick functionally graded rectangular plates using Reddy third-order plate theory [27].

1.2 Modified couple stress theory

In micron-scale structures, the size-dependent behavior which cannot be explained by the classical theories of mechanics has been experimentally observed by various researchers [3,5,19,38]. In classical mechanics, the particles' motion (only translation) is changed due to the applied forces on the material particles, and forces are defined from the kinetic energy of the material particles. In classical couple stress theories, however, the applied forces include a couple to rotate the material particles in addition to a force to change the motion of the material particles [14,20,39]. The classical couple stress theories require two additional material constants in addition to the classical material constants. These additional material constants are related to size-dependent effect, and they are difficult to determine due to their indeterminacy. Eringen [4] proposed a micro-polar theory that the motion of a particle is defined from the location vector and inner product of a rigid vector. In the micro-polar theory, the equilibrium relations are derived from conservational laws of momentum and moment of momentum. Yang et al. [41] proposed a modified couple stress theory based on a higher-order equilibrium condition. Their theory contains only one additional material constant, and only the symmetric part of the displacement gradient and the symmetric part of the rotation gradient contribute to the strain energy. Reddy [30] developed the nonlocal Euler–Bernoulli, Timoshenko, Reddy and Levinson beam models, and Lu et al. [16] and Reddy [32] developed classical and first-order shear deformation plate models using the nonlocal differential constitutive relations of Eringen. Various researchers have developed non-classical beam and plate models using the modified couples stress theory. Park and Gao [25] studied a bending problem of Bernoulli–Euler beam theory, while Ma et al. [17] presented a linear Timoshenko beam model and Asghari et al. [2] presented a nonlinear Timoshenko beam model. Tsiatas [40] developed a classical plate theory for static analysis of isotropic micro-plate, and Jomehzadeh et al. [12] studied free vibration of micro-plate based on classical plate theory. Ma et al. [18] developed a non-classical first-order shear deformation theory and presented an analytical solutions for static bending and free vibration of a rectangular micro-plate. Reddy and Kim [34] developed a nonlinear general third-order shear deformation plate theory and specialized to classical, first- order shear deformation, and the Reddy third-order plate theories that account for the von Kármán functionally graded materials, and Kim and Reddy [13] presented an analytical solutions of a general third-order plate theory to study static bending, free vibration, and buckling problems of micro-plates. Reddy and Berry [33] and Reddy et al. [35] developed nonlinear classical and first-order shear deformation theories based on the von Kármán nonlinearity and functionally graded materials to study axisymmetric bending of circular plates.

2 Displacement and strain fields of GTPT

The displacement field of the GTPT is extended up to third power and second power of thickness direction for in-plane displacements and out-of-plane displacement, respectively [34]:

$$u_1(x, y, z, t) = u(x, y, t) + z\theta_x + z^2\phi_x + z^3\psi_x, \tag{1a}$$

$$u_2(x, y, z, t) = v(x, y, t) + z\theta_y + z^2\phi_y + z^3\psi_y, \tag{1b}$$

$$u_3(x, y, z, t) = w(x, y, t) + z\theta_z + z^2\phi_z, \tag{1c}$$

where $u,\ v,\ w,\ \theta_x,\ \theta_y,\ \theta_z,\ \phi_x,\ \phi_y,\ \phi_z,\ \psi_x$, and ψ_y are unknown generalized displacements. The assumed displacement field allows parabolic variation of transverse shear strains and releases the Kirchhoff hypothesis: The cubic variation of in-plane displacements releases normality and straight condition of transverse normal lines, and the quadratic variation of out-of-plane displacement releases the inextensibility of the transverse normal lines. The von Kármán nonlinear strain–displacement relations associated with the displacement field in Eq. (1) can be obtained by assuming small strains and moderately large rotations and take the form

$$\begin{Bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ 2\varepsilon_{xy} \\ 2\varepsilon_{xz} \\ 2\varepsilon_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial u}{\partial x} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \frac{\partial v}{\partial y} + \frac{1}{2}\left(\frac{\partial w}{\partial x}\right)^2 \\ \theta_z \\ \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} + \frac{\partial v}{\partial x}\frac{\partial u}{\partial y} \\ \theta_x + \frac{\partial w}{\partial x} \\ \theta_y + \frac{\partial w}{\partial y} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \theta_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \\ 2\phi_z \\ \frac{\partial \theta_x}{\partial y} + \frac{\partial \theta_y}{\partial x} \\ 2\phi_x + \frac{\partial \theta_z}{\partial x} \\ 2\phi_y + \frac{\partial \theta_z}{\partial y} \end{Bmatrix} + z^2 \begin{Bmatrix} \frac{\partial \phi_x}{\partial x} \\ \frac{\partial \phi_y}{\partial y} \\ 0 \\ \frac{\partial \phi_x}{\partial y} + \frac{\partial \phi_y}{\partial x} \\ 3\psi_x + \frac{\partial \phi_z}{\partial x} \\ 3\psi_y + \frac{\partial \phi_z}{\partial y} \end{Bmatrix} + z^3 \begin{Bmatrix} \frac{\partial \psi_x}{\partial x} \\ \frac{\partial \theta_y}{\partial y} \\ 0 \\ \frac{\partial \psi_x}{\partial y} + \frac{\partial \psi_y}{\partial x} \\ 0 \\ 0 \end{Bmatrix}, \tag{2}$$

where the subscripts 1, 2 and 3 are replaced with $x,\ y$ and z, respectively. The components of the rotation vector, ω_i, and curvature tensor, χ_{ij}, of the assumed displacement field (1) take the form

$$2\begin{Bmatrix} \omega_x \\ \omega_y \\ \omega_z \end{Bmatrix} = \begin{Bmatrix} \frac{\partial w}{\partial y} - \theta_y \\ \theta_x - \frac{\partial w}{\partial x} \\ \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial \theta_z}{\partial y} - 2\phi_y \\ 2\phi_x - \frac{\partial \theta_z}{\partial x} \\ \frac{\partial \theta_y}{\partial x} - \frac{\partial \theta_x}{\partial y} \end{Bmatrix} + z^2 \begin{Bmatrix} \frac{\partial \phi_z}{\partial y} - 3\psi_y \\ 3\psi_x - \frac{\partial \phi_z}{\partial x} \\ \frac{\partial \phi_y}{\partial x} - \frac{\partial \phi_x}{\partial y} \end{Bmatrix} + z^3 \begin{Bmatrix} 0 \\ 0 \\ \frac{\partial \psi_y}{\partial x} - \frac{\partial \theta_x}{\partial y} \end{Bmatrix}. \tag{3}$$

The curvature tensors ($\chi_{xx},\ \chi_{yy},\ \chi_{zz},\ \chi_{xy},\ \chi_{xz}$, and χ_{yz}) are

$$2\begin{Bmatrix} \chi_{xx} \\ \chi_{yy} \\ \chi_{zz} \\ \chi_{xy} \\ \chi_{xz} \\ \chi_{yz} \end{Bmatrix} = \begin{Bmatrix} \frac{\partial}{\partial x}\left(\frac{\partial w}{\partial y} - \theta_y\right) \\ \frac{\partial}{\partial y}\left(\theta_x - \frac{\partial w}{\partial x}\right) \\ \frac{\partial \theta_y}{\partial x} - \frac{\partial \theta_x}{\partial y} \\ \frac{\partial}{\partial y}\left(\frac{\partial w}{\partial y} - \theta_y\right) + \frac{\partial}{\partial x}\left(\theta_x - \frac{\partial w}{\partial x}\right) \\ \frac{\partial}{\partial x}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + \frac{\partial \theta_z}{\partial y} - 2\phi_y \\ \frac{\partial}{\partial y}\left(\frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}\right) + 2\phi_x - \frac{\partial \theta_z}{\partial x} \end{Bmatrix} + z \begin{Bmatrix} \frac{\partial}{\partial x}\left(\frac{\partial \theta_z}{\partial y} - 2\phi_y\right) \\ \frac{\partial}{\partial y}\left(2\phi_x - \frac{\partial \theta_z}{\partial x}\right) \\ 2\left(\frac{\partial \phi_y}{\partial x} - \frac{\partial \phi_x}{\partial y}\right) \\ \frac{\partial}{\partial y}\left(\frac{\partial \theta_z}{\partial y} - 2\phi_y\right) + \frac{\partial}{\partial x}\left(2\phi_x - \frac{\partial \theta_z}{\partial x}\right) \\ \frac{\partial}{\partial x}\left(\frac{\partial \theta_y}{\partial x} - \frac{\partial \theta_x}{\partial y}\right) + 2\left(\frac{\partial \phi_z}{\partial y} - 3\psi_y\right) \\ \frac{\partial}{\partial y}\left(\frac{\partial \theta_y}{\partial x} - \frac{\partial \theta_x}{\partial y}\right) + 2\left(3\psi_x - \frac{\partial \phi_z}{\partial x}\right) \end{Bmatrix}$$

$$+z^2\begin{Bmatrix} \frac{\partial}{\partial x}\left(\frac{\partial \phi_z}{\partial y}-3\psi_y\right) \\ \frac{\partial}{\partial y}\left(3\psi_x-\frac{\partial \phi_z}{\partial x}\right) \\ 3\left(\frac{\partial \psi_y}{\partial x}-\frac{\partial \psi_x}{\partial y}\right) \\ \frac{\partial}{\partial y}\left(\frac{\partial \phi_z}{\partial y}-3\psi_y\right)+\frac{\partial}{\partial x}\left(3\psi_x-\frac{\partial \phi_z}{\partial x}\right) \\ \frac{\partial}{\partial x}\left(\frac{\partial \phi_y}{\partial x}-\frac{\partial \phi_x}{\partial y}\right) \\ \frac{\partial}{\partial y}\left(\frac{\partial \phi_y}{\partial x}-\frac{\partial \phi_x}{\partial y}\right) \end{Bmatrix} + z^3\begin{Bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \frac{\partial}{\partial x}\left(\frac{\partial \psi_y}{\partial x}-\frac{\partial \psi_x}{\partial y}\right) \\ \frac{\partial}{\partial y}\left(\frac{\partial \psi_y}{\partial x}-\frac{\partial \psi_x}{\partial y}\right) \end{Bmatrix}. \tag{4}$$

3 Constitutive model

We assume isotropic functionally graded plate with the variation of two constituents through thickness. The linear constitutive equations for isotropic plate are given by

$$\sigma_{ij} = 2\mu\varepsilon_{ij} + \lambda\delta_{ij}\varepsilon_{kk}, \tag{5a}$$

$$m_{ij} = 2\mu\ell^2\chi_{ij}, \tag{5b}$$

where μ and λ are the Lamé parameters (see Reddy [31])

$$\lambda = \frac{\nu E}{(1+\nu)(1-2\nu)}, \quad 2\mu = E(1+\nu) \tag{6}$$

with E and ν being Young's modulus and Poisson's ratio, and ℓ being the material length scale parameter. The material length scale parameter is the square root of the ratio of the modulus of curvature to the modulus of shear, and it is a property measuring the effect of the couple stress [34]. For a functionally graded material, Young's modulus, E, varies through thickness direction and Poisson's ratio, ν, is assumed as constant. The variation of Young's modulus varies from one kind of material on one side, $z = -h/2$, to another material on the other side, $z = h/2$ where h is height of plate thickness, and the variation is represented by a power-law distribution

$$E(z) = [E_c - E_m]\, f(z) + E_m, \quad f(z) = \left(\frac{1}{2}+\frac{z}{h}\right)^n, \tag{7}$$

where E_c and E_m are the values of the modulus of the ceramic material and metal, respectively; n denotes the volume fraction exponent, called power-law index. In Eq. (7), we assume fully ceramic on the top surface ($h/2$) and fully metal on bottom surface ($-h/2$) of a plate. When $n = 0$, we obtain the ceramic plate. Figure 1 shows the variation of the volume fraction of ceramic material through the plate thickness for various values of the power-law index, n.

4 Finite element model

The displacement-based weak-form Galerkin finite element model for the GTPT is developed using the principle of virtual displacements (8).

$$\int_0^T (\delta\mathcal{K} - \delta\mathcal{U} - \delta\mathcal{V})\,\mathrm{d}t = 0, \tag{8}$$

where $\delta\mathcal{K}$, $\delta\mathcal{U}$ and $\delta\mathcal{V}$ are the virtual kinetic energy, the virtual strain energy and the virtual work done by external forces, respectively. The virtual kinetic energy $\delta\mathcal{K}$ is

$$\delta\mathcal{K} = \int_\Omega \int_{-\frac{h}{2}}^{\frac{h}{2}} \rho \left(\frac{\partial u_x}{\partial t}\frac{\partial \delta u_x}{\partial t} + \frac{\partial u_y}{\partial t}\frac{\partial \delta u_y}{\partial t} + \frac{\partial u_z}{\partial t}\frac{\partial \delta u_z}{\partial t}\right) \mathrm{d}z\mathrm{d}x\mathrm{d}y. \tag{9}$$

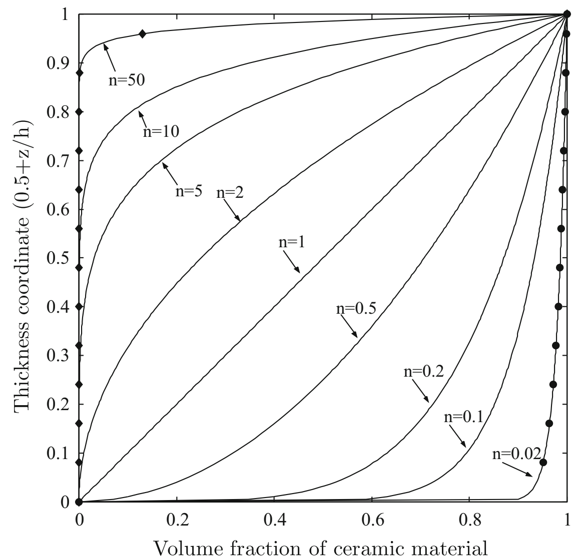

Fig. 1 Volume fraction of ceramic material through the plate thickness for various values of power-law index

The virtual strain energy is given by

$$\delta\mathcal{U} = \int_{\Omega}\int_{-\frac{h}{2}}^{\frac{h}{2}} \left(\sigma_{xx}\delta\varepsilon_{xx} + \sigma_{yy}\delta\varepsilon_{yy} + \sigma_{zz}\delta\varepsilon_{zz} + \sigma_{xy}\delta\gamma_{xy} + \sigma_{xz}\delta\gamma_{xz} + \sigma_{yz}\delta\gamma_{yz}\right) \mathrm{d}z\mathrm{d}x\mathrm{d}y$$
$$+ \int_{\Omega}\int_{-\frac{h}{2}}^{\frac{h}{2}} \left(m_{xx}\delta\chi_{xx} + m_{yy}\delta\chi_{yy} + m_{zz}\delta\chi_{zz} + m_{xy}\delta\chi_{xy} + m_{xz}\delta\chi_{xz} + m_{yz}\delta\chi_{yz}\right) \mathrm{d}z\mathrm{d}x\mathrm{d}y. \quad (10)$$

Then the virtual work done by external forces is

$$\delta\mathcal{V} = -\Bigg[\int_{V} \left(\bar{f}_x\delta u_1 + \bar{f}_y\delta u_2 + \bar{f}_z\delta u_3 + \bar{c}_x\delta\omega_x + \bar{c}_y\delta\omega_y + \bar{c}_z\delta\omega_z\right) \mathrm{d}V$$
$$+ \int_{\Omega^+} \left(q_x^t\delta u_1 + q_y^t\delta u_2 + q_z^t\delta u_3 + p_x^t\delta\omega_x + p_y^t\delta\omega_y + p_z^t\delta\omega_z\right)\mathrm{d}x\mathrm{d}y$$
$$+ \int_{\Omega^-} \left(q_x^b\delta u_1 + q_y^b\delta u_2 + q_z^b\delta u_3 + p_x^b\delta\omega_x + p_y^b\delta\omega_y + p_z^b\delta\omega_z\right)\mathrm{d}x\mathrm{d}y$$
$$+ \int_{S} \left(\bar{t}_x\delta u_1 + \bar{t}_y\delta u_2 + \bar{t}_z\delta u_3 + \bar{s}_x\delta\omega_x + \bar{s}_y\delta\omega_y + \bar{s}_z\delta\omega_z\right) \mathrm{d}S\Bigg], \quad (11)$$

where $\bar{f}_i$ and $\bar{c}_i$ are the body forces and couples, respectively (measured per unit volume), $\bar{t}_i$ and $\bar{s}_i$ are the surface forces and couples (measured per unit area) on the side surfaces, and q_i^α, p_i^α are the distributed forces and couples (measured per unit area) on top ($\alpha = t$) and bottom ($\alpha = b$) surfaces in the coordinate directions, $i = x, y, z$. Reddy and Kim [34] presented the explicit forms of the virtual energies and the virtual work. The principle of virtual displacement (8) contains the second derivative of dependent variables, $(u, v, w, \theta_x, \theta_y, \theta_z, \phi_x, \phi_y, \phi_z, \psi_x, \psi_y)$. Therefore, they can be approximated using the C^1 interpolation functions. The finite element approximation takes the form

$$U^{(I)} = \sum_{j=1}^{n} \bar{\Delta}_j^{(I)}\varphi_j^{(I)}(x, y), \quad (12)$$

where $U^{(I)}$ are the dependent variables, the superscript I indicates Ith variable of a node, e.g., $U^{(1)} = u$, $U^{(2)} = v$, $U^{(3)} = w$, $U^{(4)} = \theta_x$, and so on. $\bar{\Delta}_j^{(I)}$ are the nodal variables and their derivatives of the jth node, and $\varphi_j^{(I)}(x, y)$ are C^1 interpolation functions. In this study, we used a conforming element which has four degrees of freedom$\left(U^{(I)}, \frac{\partial U^{(I)}}{\partial x}, \frac{\partial U^{(I)}}{\partial y}, \text{ and } \frac{\partial^2 U^{(I)}}{\partial x \partial y}\right)$ per node [29]. Since the GTPT has 11 dependent variables, the total degrees of freedom per node become 44. By substituting the Eqs. (1), (2), (3), (4), (5) and (12) into the principle of virtual displacement (8), we obtain the following the finite element model of GTPT in generic matrix form:

$$[M]\{\ddot{U}\} + [K]\{U\} = \{F\}, \tag{13}$$

where $[M]$ and $[K]$ are mass and stiffness matrices, and $\{\ddot{U}\}$ and $\{U\}$ are the acceleration and the displacement vectors, respectively. $\{F\}$ is the force vector. The mass and stiffness matrices have 11 by 11 square submatrices, and the acceleration, the displacement and the force vectors have 11 subvectors whose sizes are 4 times number of node in an element. The simplest rectangular element (4 node element) has 176 degrees of freedom per element. The explicit forms of nonzero mass and stiffness matrices are

$$M_{ij}^{0101} = \int_\Omega m^{(0)}\varphi_i^{(1)}\varphi_j^{(1)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0104} = \int_\Omega m^{(1)}\varphi_i^{(1)}\varphi_j^{(4)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0107} = \int_\Omega m^{(2)}\varphi_i^{(1)}\varphi_j^{(7)}\mathrm{d}x\mathrm{d}y, \tag{14}$$

$$M_{ij}^{0110} = \int_\Omega m^{(3)}\varphi_i^{(1)}\varphi_j^{(10)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0202} = \int_\Omega m^{(0)}\varphi_i^{(2)}\varphi_j^{(2)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0205} = \int_\Omega m^{(1)}\varphi_i^{(2)}\varphi_j^{(5)}\mathrm{d}x\mathrm{d}y, \tag{15}$$

$$M_{ij}^{0208} = \int_\Omega m^{(2)}\varphi_i^{(2)}\varphi_j^{(8)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0211} = \int_\Omega m^{(3)}\varphi_i^{(2)}\varphi_j^{(11)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0303} = \int_\Omega m^{(0)}\varphi_i^{(3)}\varphi_j^{(3)}\mathrm{d}x\mathrm{d}y, \tag{16}$$

$$M_{ij}^{0306} = \int_\Omega m^{(1)}\varphi_i^{(3)}\varphi_j^{(6)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0309} = \int_\Omega m^{(2)}\varphi_i^{(3)}\varphi_j^{(9)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0401} = \int_\Omega m^{(1)}\varphi_i^{(4)}\varphi_j^{(1)}\mathrm{d}x\mathrm{d}y, \tag{17}$$

$$M_{ij}^{0404} = \int_\Omega m^{(2)}\varphi_i^{(4)}\varphi_j^{(4)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0407} = \int_\Omega m^{(3)}\varphi_i^{(4)}\varphi_j^{(7)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0410} = \int_\Omega m^{(4)}\varphi_i^{(4)}\varphi_j^{(10)}\mathrm{d}x\mathrm{d}y, \tag{18}$$

$$M_{ij}^{0502} = \int_\Omega m^{(1)}\varphi_i^{(5)}\varphi_j^{(2)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0505} = \int_\Omega m^{(2)}\varphi_i^{(5)}\varphi_j^{(5)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0508} = \int_\Omega m^{(3)}\varphi_i^{(5)}\varphi_j^{(8)}\mathrm{d}x\mathrm{d}y, \tag{19}$$

$$M_{ij}^{0511} = \int_\Omega m^{(4)}\varphi_i^{(5)}\varphi_j^{(11)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0603} = \int_\Omega m^{(1)}\varphi_i^{(6)}\varphi_j^{(3)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0606} = \int_\Omega m^{(2)}\varphi_i^{(6)}\varphi_j^{(6)}\mathrm{d}x\mathrm{d}y, \tag{20}$$

$$M_{ij}^{0609} = \int_\Omega m^{(3)}\varphi_i^{(6)}\varphi_j^{(9)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0701} = \int_\Omega m^{(2)}\varphi_i^{(7)}\varphi_j^{(1)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0704} = \int_\Omega m^{(3)}\varphi_i^{(7)}\varphi_j^{(4)}\mathrm{d}x\mathrm{d}y, \tag{21}$$

$$M_{ij}^{0707} = \int_\Omega m^{(4)}\varphi_i^{(7)}\varphi_j^{(7)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0710} = \int_\Omega m^{(5)}\varphi_i^{(7)}\varphi_j^{(10)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0802} = \int_\Omega m^{(2)}\varphi_i^{(8)}\varphi_j^{(2)}\mathrm{d}x\mathrm{d}y, \tag{22}$$

$$M_{ij}^{0805} = \int_\Omega m^{(3)}\varphi_i^{(8)}\varphi_j^{(5)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0808} = \int_\Omega m^{(4)}\varphi_i^{(8)}\varphi_j^{(8)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0811} = \int_\Omega m^{(5)}\varphi_i^{(8)}\varphi_j^{(11)}\mathrm{d}x\mathrm{d}y, \tag{23}$$

$$M_{ij}^{0903} = \int_\Omega m^{(2)}\varphi_i^{(9)}\varphi_j^{(3)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0906} = \int_\Omega m^{(3)}\varphi_i^{(9)}\varphi_j^{(6)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{0909} = \int_\Omega m^{(4)}\varphi_i^{(9)}\varphi_j^{(9)}\mathrm{d}x\mathrm{d}y, \tag{24}$$

$$M_{ij}^{1001} = \int_\Omega m^{(3)}\varphi_i^{(10)}\varphi_j^{(1)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{1004} = \int_\Omega m^{(4)}\varphi_i^{(10)}\varphi_j^{(4)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{1007} = \int_\Omega m^{(5)}\varphi_i^{(10)}\varphi_j^{(7)}\mathrm{d}x\mathrm{d}y, \tag{25}$$

$$M_{ij}^{1010} = \int_\Omega m^{(6)}\varphi_i^{(10)}\varphi_j^{(10)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{1102} = \int_\Omega m^{(3)}\varphi_i^{(11)}\varphi_j^{(2)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{1105} = \int_\Omega m^{(4)}\varphi_i^{(11)}\varphi_j^{(5)}\mathrm{d}x\mathrm{d}y, \tag{26}$$

$$M_{ij}^{1108} = \int_\Omega m^{(5)}\varphi_i^{(11)}\varphi_j^{(8)}\mathrm{d}x\mathrm{d}y, \quad M_{ij}^{1111} = \int_\Omega m^{(6)}\varphi_i^{(11)}\varphi_j^{(11)}\mathrm{d}x\mathrm{d}y, \tag{27}$$

$$K_{ij}^{0101} = \int_\Omega A_{11}^{(0)}\frac{\partial\varphi_i^{(1)}}{\partial x}\frac{\partial\varphi_j^{(1)}}{\partial x} + B_{66}^{(0)}\frac{\partial\varphi_i^{(1)}}{\partial y}\frac{\partial\varphi_j^{(1)}}{\partial y} + \frac{1}{4}\left(R_{44}^{(0)}\frac{\partial^2\varphi_i^{(1)}}{\partial y^2}\frac{\partial^2\varphi_j^{(1)}}{\partial y^2} + R_{55}^{(0)}\frac{\partial^2\varphi_i^{(1)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(1)}}{\partial x\partial y}\right)\mathrm{d}x\mathrm{d}y, \tag{28}$$

$$K_{ij}^{0102} = \int_\Omega A_{12}^{(0)}\frac{\partial\varphi_i^{(1)}}{\partial x}\frac{\partial\varphi_j^{(2)}}{\partial y} + B_{66}^{(0)}\frac{\partial\varphi_i^{(1)}}{\partial y}\frac{\partial\varphi_j^{(2)}}{\partial x} - \frac{1}{4}\left(R_{44}^{(0)}\frac{\partial^2\varphi_i^{(1)}}{\partial y^2}\frac{\partial^2\varphi_j^{(2)}}{\partial x\partial y} + R_{55}^{(0)}\frac{\partial^2\varphi_i^{(1)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(2)}}{\partial x^2}\right)\mathrm{d}x\mathrm{d}y, \tag{29}$$

$$K_{ij}^{0103} = \frac{1}{2}\int_\Omega \frac{\partial w}{\partial x}\left(A_{11}^{(0)}\frac{\partial\varphi_i^{(1)}}{\partial x}\frac{\partial\varphi_j^{(3)}}{\partial x} + B_{66}^{(0)}\frac{\partial\varphi_i^{(1)}}{\partial y}\frac{\partial\varphi_j^{(3)}}{\partial y}\right) + \frac{\partial w}{\partial y}\left(A_{12}^{(0)}\frac{\partial\varphi_i^{(1)}}{\partial x}\frac{\partial\varphi_j^{(3)}}{\partial y} + B_{66}^{(0)}\frac{\partial\varphi_i^{(1)}}{\partial y}\frac{\partial\varphi_j^{(3)}}{\partial x}\right)\mathrm{d}x\mathrm{d}y, \tag{30}$$

$$K_{ij}^{0104} = \int_\Omega A_{11}^{(1)}\frac{\partial\varphi_i^{(1)}}{\partial x}\frac{\partial\varphi_j^{(4)}}{\partial x} + B_{66}^{(1)}\frac{\partial\varphi_i^{(1)}}{\partial y}\frac{\partial\varphi_j^{(4)}}{\partial y} + \frac{1}{4}\left(R_{44}^{(1)}\frac{\partial^2\varphi_i^{(1)}}{\partial y^2}\frac{\partial^2\varphi_j^{(4)}}{\partial y^2} + R_{55}^{(1)}\frac{\partial^2\varphi_i^{(1)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(4)}}{\partial x\partial y}\right)\mathrm{d}x\mathrm{d}y, \tag{31}$$

$$K_{ij}^{0105} = \int_\Omega A_{12}^{(1)}\frac{\partial\varphi_i^{(1)}}{\partial x}\frac{\partial\varphi_j^{(5)}}{\partial y} + B_{66}^{(1)}\frac{\partial\varphi_i^{(1)}}{\partial y}\frac{\partial\varphi_j^{(5)}}{\partial x} - \frac{1}{4}\left(R_{44}^{(1)}\frac{\partial^2\varphi_i^{(1)}}{\partial y^2}\frac{\partial^2\varphi_j^{(5)}}{\partial x\partial y} + R_{55}^{(1)}\frac{\partial^2\varphi_i^{(1)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(5)}}{\partial x^2}\right)\mathrm{d}x\mathrm{d}y, \tag{32}$$

$$K_{ij}^{0106} = \int_\Omega A_{13}^{(0)}\frac{\partial\varphi_i^{(1)}}{\partial x}\varphi_j^{(6)} + \frac{1}{4}\left(R_{44}^{(0)}\frac{\partial^2\varphi_i^{(1)}}{\partial y^2}\frac{\partial\varphi_j^{(6)}}{\partial x} - R_{55}^{(0)}\frac{\partial^2\varphi_i^{(1)}}{\partial x\partial y}\frac{\partial\varphi_j^{(6)}}{\partial y}\right)\mathrm{d}x\mathrm{d}y, \tag{33}$$

$$K_{ij}^{0107}=\int_\Omega A_{11}^{(2)}\frac{\partial\varphi_i^{(1)}}{\partial x}\frac{\partial\varphi_j^{(7)}}{\partial x}+B_{66}^{(2)}\frac{\partial\varphi_i^{(1)}}{\partial y}\frac{\partial\varphi_j^{(7)}}{\partial y}+\frac{1}{4}\left(R_{44}^{(2)}\frac{\partial^2\varphi_i^{(1)}}{\partial y^2}\frac{\partial^2\varphi_j^{(7)}}{\partial y^2}+R_{55}^{(2)}\frac{\partial^2\varphi_i^{(1)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(7)}}{\partial x\partial y}\right)-\frac{1}{2}R_{44}^{(0)}\frac{\partial^2\varphi_i^{(1)}}{\partial y^2}\varphi_j^{(7)}\mathrm{d}x\mathrm{d}y, \tag{34}$$

$$K_{ij}^{0108}=\int_\Omega A_{12}^{(2)}\frac{\partial\varphi_i^{(1)}}{\partial x}\frac{\partial\varphi_j^{(8)}}{\partial y}+B_{66}^{(2)}\frac{\partial\varphi_i^{(1)}}{\partial y}\frac{\partial\varphi_j^{(8)}}{\partial x}-\frac{1}{4}\left(R_{44}^{(2)}\frac{\partial^2\varphi_i^{(1)}}{\partial y^2}\frac{\partial^2\varphi_j^{(8)}}{\partial x\partial y}+R_{55}^{(2)}\frac{\partial^2\varphi_i^{(1)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(8)}}{\partial x^2}\right)+\frac{1}{2}R_{55}^{(0)}\frac{\partial^2\varphi_i^{(1)}}{\partial x\partial y}\varphi_j^{(8)}\mathrm{d}x\mathrm{d}y, \tag{35}$$

$$K_{ij}^{0109}=\int_\Omega 2A_{13}^{(1)}\frac{\partial\varphi_i^{(1)}}{\partial x}\varphi_j^{(9)}+\frac{1}{2}\left(R_{44}^{(1)}\frac{\partial^2\varphi_i^{(1)}}{\partial y^2}\frac{\partial\varphi_j^{(9)}}{\partial x}-R_{55}^{(1)}\frac{\partial^2\varphi_i^{(1)}}{\partial x\partial y}\frac{\partial\varphi_j^{(9)}}{\partial y}\right)\mathrm{d}x\mathrm{d}y, \tag{36}$$

$$K_{ij}^{0110}=\int_\Omega A_{11}^{(3)}\frac{\partial\varphi_i^{(1)}}{\partial x}\frac{\partial\varphi_j^{(10)}}{\partial x}+B_{66}^{(3)}\frac{\partial\varphi_i^{(1)}}{\partial y}\frac{\partial\varphi_j^{(10)}}{\partial y}+\frac{1}{4}\left(R_{44}^{(3)}\frac{\partial^2\varphi_i^{(1)}}{\partial y^2}\frac{\partial^2\varphi_j^{(10)}}{\partial y^2}+R_{55}^{(3)}\frac{\partial^2\varphi_i^{(1)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(10)}}{\partial x\partial y}\right)-\frac{3}{2}R_{44}^{(1)}\frac{\partial^2\varphi_i^{(1)}}{\partial y^2}\varphi_j^{(10)}\mathrm{d}x\mathrm{d}y, \tag{37}$$

$$K_{ij}^{0111}=\int_\Omega A_{12}^{(3)}\frac{\partial\varphi_i^{(1)}}{\partial x}\frac{\partial\varphi_j^{(11)}}{\partial y}+B_{66}^{(3)}\frac{\partial\varphi_i^{(1)}}{\partial y}\frac{\partial\varphi_j^{(11)}}{\partial x}-\frac{1}{4}\left(R_{44}^{(3)}\frac{\partial^2\varphi_i^{(1)}}{\partial y^2}\frac{\partial^2\varphi_j^{(11)}}{\partial x\partial y}+R_{55}^{(3)}\frac{\partial^2\varphi_i^{(1)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(11)}}{\partial x^2}\right)+\frac{3}{2}R_{55}^{(1)}\frac{\partial^2\varphi_i^{(1)}}{\partial x\partial y}\varphi_j^{(11)}\mathrm{d}x\mathrm{d}y, \tag{38}$$

$$K_{ij}^{0201}=\int_\Omega A_{21}^{(0)}\frac{\partial\varphi_i^{(2)}}{\partial y}\frac{\partial\varphi_j^{(1)}}{\partial x}+B_{66}^{(0)}\frac{\partial\varphi_i^{(2)}}{\partial x}\frac{\partial\varphi_j^{(1)}}{\partial y}-\frac{1}{4}\left(R_{44}^{(0)}\frac{\partial^2\varphi_i^{(2)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(1)}}{\partial y^2}+R_{55}^{(0)}\frac{\partial^2\varphi_i^{(2)}}{\partial x^2}\frac{\partial^2\varphi_j^{(1)}}{\partial x\partial y}\right)\mathrm{d}x\mathrm{d}y, \tag{39}$$

$$K_{ij}^{0202}=\int_\Omega A_{22}^{(0)}\frac{\partial\varphi_i^{(2)}}{\partial y}\frac{\partial\varphi_j^{(2)}}{\partial y}+B_{66}^{(0)}\frac{\partial\varphi_i^{(2)}}{\partial x}\frac{\partial\varphi_j^{(2)}}{\partial x}+\frac{1}{4}\left(R_{44}^{(0)}\frac{\partial^2\varphi_i^{(2)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(2)}}{\partial x\partial y}+R_{55}^{(0)}\frac{\partial^2\varphi_i^{(2)}}{\partial x^2}\frac{\partial^2\varphi_j^{(2)}}{\partial x^2}\right)\mathrm{d}x\mathrm{d}y, \tag{40}$$

$$K_{ij}^{0203}=\frac{1}{2}\int_\Omega \frac{\partial w}{\partial y}\left(A_{22}^{(0)}\frac{\partial\varphi_i^{(2)}}{\partial y}\frac{\partial\varphi_j^{(3)}}{\partial y}+B_{66}^{(0)}\frac{\partial\varphi_i^{(2)}}{\partial x}\frac{\partial\varphi_j^{(3)}}{\partial x}\right)+\frac{\partial w}{\partial x}\left(A_{21}^{(0)}\frac{\partial\varphi_i^{(2)}}{\partial y}\frac{\partial\varphi_j^{(3)}}{\partial x}+B_{66}^{(0)}\frac{\partial\varphi_i^{(2)}}{\partial x}\frac{\partial\varphi_j^{(3)}}{\partial y}\right)\mathrm{d}x\mathrm{d}y, \tag{41}$$

$$K_{ij}^{0204}=\int_\Omega A_{21}^{(1)}\frac{\partial\varphi_i^{(2)}}{\partial y}\frac{\partial\varphi_j^{(4)}}{\partial x}+B_{66}^{(1)}\frac{\partial\varphi_i^{(2)}}{\partial x}\frac{\partial\varphi_j^{(4)}}{\partial y}-\frac{1}{4}\left(R_{44}^{(1)}\frac{\partial^2\varphi_i^{(2)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(4)}}{\partial y^2}+R_{55}^{(1)}\frac{\partial^2\varphi_i^{(2)}}{\partial x^2}\frac{\partial^2\varphi_j^{(4)}}{\partial x\partial y}\right)\mathrm{d}x\mathrm{d}y, \tag{42}$$

$$K_{ij}^{0205}=\int_\Omega A_{22}^{(1)}\frac{\partial\varphi_i^{(2)}}{\partial y}\frac{\partial\varphi_j^{(5)}}{\partial y}+B_{66}^{(1)}\frac{\partial\varphi_i^{(2)}}{\partial x}\frac{\partial\varphi_j^{(5)}}{\partial x}+\frac{1}{4}\left(R_{44}^{(1)}\frac{\partial^2\varphi_i^{(2)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(5)}}{\partial x\partial y}+R_{55}^{(1)}\frac{\partial^2\varphi_i^{(2)}}{\partial x^2}\frac{\partial^2\varphi_j^{(5)}}{\partial x^2}\right)\mathrm{d}x\mathrm{d}y, \tag{43}$$

$$K_{ij}^{0206}=\int_\Omega A_{23}^{(0)}\frac{\partial\varphi_i^{(2)}}{\partial y}\varphi_j^{(6)}-\frac{1}{4}\left(R_{44}^{(0)}\frac{\partial^2\varphi_i^{(2)}}{\partial x\partial y}\frac{\partial\varphi_j^{(6)}}{\partial x}-R_{55}^{(0)}\frac{\partial^2\varphi_i^{(2)}}{\partial x^2}\frac{\partial\varphi_j^{(6)}}{\partial y}\right)\mathrm{d}x\mathrm{d}y, \tag{44}$$

$$K_{ij}^{0207}=\int_\Omega A_{21}^{(2)}\frac{\partial\varphi_i^{(2)}}{\partial y}\frac{\partial\varphi_j^{(7)}}{\partial x}+B_{66}^{(2)}\frac{\partial\varphi_i^{(2)}}{\partial x}\frac{\partial\varphi_j^{(7)}}{\partial y}-\frac{1}{4}\left(R_{44}^{(2)}\frac{\partial^2\varphi_i^{(2)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(7)}}{\partial y^2}+R_{55}^{(2)}\frac{\partial^2\varphi_i^{(2)}}{\partial x^2}\frac{\partial^2\varphi_j^{(7)}}{\partial x\partial y}\right)+\frac{1}{2}R_{44}^{(0)}\frac{\partial^2\varphi_i^{(2)}}{\partial x\partial y}\varphi_j^{(7)}\mathrm{d}x\mathrm{d}y, \tag{45}$$

$$K_{ij}^{0208}=\int_\Omega A_{22}^{(2)}\frac{\partial\varphi_i^{(2)}}{\partial y}\frac{\partial\varphi_j^{(8)}}{\partial y}+B_{66}^{(2)}\frac{\partial\varphi_i^{(2)}}{\partial x}\frac{\partial\varphi_j^{(8)}}{\partial x}+\frac{1}{4}\left(R_{44}^{(2)}\frac{\partial^2\varphi_i^{(2)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(8)}}{\partial x\partial y}+R_{55}^{(2)}\frac{\partial^2\varphi_i^{(2)}}{\partial x^2}\frac{\partial^2\varphi_j^{(8)}}{\partial x^2}\right)-\frac{1}{2}R_{55}^{(0)}\frac{\partial^2\varphi_i^{(2)}}{\partial x^2}\varphi_j^{(8)}\mathrm{d}x\mathrm{d}y, \tag{46}$$

$$K_{ij}^{0209}=\int_\Omega 2A_{23}^{(1)}\frac{\partial\varphi_i^{(2)}}{\partial y}\varphi_j^{(9)}-\frac{1}{2}\left(R_{44}^{(1)}\frac{\partial^2\varphi_i^{(2)}}{\partial x\partial y}\frac{\partial\varphi_j^{(9)}}{\partial x}-R_{55}^{(1)}\frac{\partial^2\varphi_i^{(2)}}{\partial x^2}\frac{\partial\varphi_j^{(9)}}{\partial y}\right)\mathrm{d}x\mathrm{d}y, \tag{47}$$

$$K_{ij}^{0210} = \int_{\Omega} A_{21}^{(3)} \frac{\partial \varphi_i^{(2)}}{\partial y} \frac{\partial \varphi_j^{(10)}}{\partial x} + B_{66}^{(3)} \frac{\partial \varphi_i^{(2)}}{\partial x} \frac{\partial \varphi_j^{(10)}}{\partial y} - \frac{1}{4}\left(R_{44}^{(3)} \frac{\partial^2 \varphi_i^{(2)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(10)}}{\partial y^2} + R_{55}^{(3)} \frac{\partial^2 \varphi_i^{(2)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(10)}}{\partial x \partial y} \right) + \frac{3}{2} R_{44}^{(1)} \frac{\partial^2 \varphi_i^{(2)}}{\partial x \partial y} \varphi_j^{(10)} \mathrm{d}x\mathrm{d}y, \tag{48}$$

$$K_{ij}^{0211} = \int_{\Omega} A_{22}^{(3)} \frac{\partial \varphi_i^{(2)}}{\partial y} \frac{\partial \varphi_j^{(11)}}{\partial y} + B_{66}^{(3)} \frac{\partial \varphi_i^{(2)}}{\partial x} \frac{\partial \varphi_j^{(11)}}{\partial x} + \frac{1}{4}\left(R_{44}^{(3)} \frac{\partial^2 \varphi_i^{(2)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(11)}}{\partial x \partial y} + R_{55}^{(3)} \frac{\partial^2 \varphi_i^{(2)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(11)}}{\partial x^2} \right) - \frac{3}{2} R_{55}^{(1)} \frac{\partial^2 \varphi_i^{(2)}}{\partial x^2} \varphi_j^{(11)} \mathrm{d}x\mathrm{d}y, \tag{49}$$

$$K_{ij}^{0301} = \int_{\Omega} \frac{\partial w}{\partial x}\left(A_{11}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(1)}}{\partial x} + B_{66}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(1)}}{\partial y} \right) + \frac{\partial w}{\partial y}\left(A_{21}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(1)}}{\partial x} + B_{66}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(1)}}{\partial y} \right) \mathrm{d}x\mathrm{d}y, \tag{50}$$

$$K_{ij}^{0302} = \int_{\Omega} \frac{\partial w}{\partial y}\left(A_{22}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(1)}}{\partial y} + B_{66}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(1)}}{\partial x} \right) + \frac{\partial w}{\partial x}\left(A_{12}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(1)}}{\partial y} + B_{66}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(1)}}{\partial x} \right) \mathrm{d}x\mathrm{d}y, \tag{51}$$

$$\begin{aligned} K_{ij}^{0303} = \int_{\Omega} \frac{1}{2} \Bigg[& \left(\frac{\partial w}{\partial x}\right)^2 \left(A_{11}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(3)}}{\partial x} + B_{66}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(3)}}{\partial y} \right) + 2B_{44}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(3)}}{\partial y} \\ & + \left(\frac{\partial w}{\partial y}\right)^2 \left(A_{22}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(3)}}{\partial y} + B_{66}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(3)}}{\partial x} \right) + 2B_{55}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(3)}}{\partial x} \\ & + \left(\frac{\partial w}{\partial x} \frac{\partial w}{\partial y}\right) \left(A_{12}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(3)}}{\partial y} + A_{21}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(3)}}{\partial x} + B_{66}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(3)}}{\partial y} \right. \\ & \left. + B_{66}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(3)}}{\partial x} \right) \Bigg] + \frac{1}{4} \Bigg[R_{11}^{(0)} \frac{\partial^2 \varphi_i^{(3)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(3)}}{\partial x \partial y} + R_{22}^{(0)} \frac{\partial^2 \varphi_i^{(3)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(3)}}{\partial x \partial y} \\ & + R_{66}^{(0)} \left(\frac{\partial^2 \varphi_i^{(3)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(3)}}{\partial x^2} + \frac{\partial^2 \varphi_i^{(3)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(3)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(3)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(3)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(3)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(3)}}{\partial x^2} \right) \Bigg] \mathrm{d}x\mathrm{d}y, \end{aligned} \tag{52}$$

$$\begin{aligned} K_{ij}^{0304} = \int_{\Omega} & \frac{\partial w}{\partial x}\left(A_{11}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(4)}}{\partial x} + B_{66}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(4)}}{\partial y} \right) \\ & + \frac{\partial w}{\partial y}\left(A_{21}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(4)}}{\partial x} + B_{66}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(4)}}{\partial y} \right) + B_{55}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial x} \varphi_j^{(4)} \\ & + \frac{1}{4}\left[R_{66}^{(0)} \left(\frac{\partial^2 \varphi_i^{(3)}}{\partial y^2} \frac{\partial \varphi_j^{(4)}}{\partial x} - \frac{\partial^2 \varphi_i^{(3)}}{\partial x^2} \frac{\partial \varphi_j^{(4)}}{\partial x} \right) - R_{22}^{(0)} \frac{\partial^2 \varphi_i^{(3)}}{\partial x \partial y} \frac{\partial \varphi_j^{(4)}}{\partial y} \right] \mathrm{d}x\mathrm{d}y, \end{aligned} \tag{53}$$

$$\begin{aligned} K_{ij}^{0305} = \int_{\Omega} & \frac{\partial w}{\partial y}\left(A_{22}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(5)}}{\partial y} + B_{66}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(5)}}{\partial x} \right) \\ & + \frac{\partial w}{\partial x}\left(A_{12}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(5)}}{\partial y} + B_{66}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(5)}}{\partial x} \right) + B_{44}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial y} \varphi_j^{(5)} \\ & + \frac{1}{4}\left[R_{66}^{(0)} \left(\frac{\partial^2 \varphi_i^{(3)}}{\partial x^2} \frac{\partial \varphi_j^{(5)}}{\partial y} - \frac{\partial^2 \varphi_i^{(3)}}{\partial y^2} \frac{\partial \varphi_j^{(5)}}{\partial y} \right) - R_{11}^{(0)} \frac{\partial^2 \varphi_i^{(3)}}{\partial x \partial y} \frac{\partial \varphi_j^{(5)}}{\partial x} \right] \mathrm{d}x\mathrm{d}y, \end{aligned} \tag{54}$$

$$\begin{aligned} K_{ij}^{0306} = \int_{\Omega} & \frac{\partial w}{\partial x} A_{13}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial x} \varphi_j^{(6)} + \frac{\partial w}{\partial y} A_{23}^{(0)} \frac{\partial \varphi_i^{(3)}}{\partial y} \varphi_j^{(6)} + B_{44}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(6)}}{\partial y} + B_{55}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(6)}}{\partial x} \\ & + \frac{1}{4}\Bigg[R_{11}^{(1)} \frac{\partial^2 \varphi_i^{(3)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(6)}}{\partial x \partial y} + R_{22}^{(1)} \frac{\partial^2 \varphi_i^{(3)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(6)}}{\partial x \partial y} + R_{66}^{(1)} \Bigg(\frac{\partial^2 \varphi_i^{(3)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(6)}}{\partial x^2} + \frac{\partial^2 \varphi_i^{(3)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(6)}}{\partial y^2} \\ & - \frac{\partial^2 \varphi_i^{(3)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(6)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(3)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(6)}}{\partial x^2} \Bigg) \Bigg] \mathrm{d}x\mathrm{d}y, \end{aligned} \tag{55}$$

$$K_{ij}^{0307} = \int_{\Omega} \frac{\partial w}{\partial x}\left(A_{11}^{(2)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(7)}}{\partial x} + B_{66}^{(2)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(7)}}{\partial y}\right) + \frac{\partial w}{\partial y}\left(A_{21}^{(2)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(7)}}{\partial x} + B_{66}^{(2)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(7)}}{\partial y}\right) + 2B_{55}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial x} \varphi_j^{(7)} + \frac{1}{2}\left[R_{66}^{(1)}\left(\frac{\partial^2 \varphi_i^{(3)}}{\partial y^2} \frac{\partial \varphi_j^{(7)}}{\partial x} - \frac{\partial^2 \varphi_i^{(3)}}{\partial x^2} \frac{\partial \varphi_j^{(7)}}{\partial x}\right) - R_{22}^{(1)} \frac{\partial^2 \varphi_i^{(3)}}{\partial x \partial y} \frac{\partial \varphi_j^{(7)}}{\partial y}\right] \mathrm{d}x\mathrm{d}y, \quad (56)$$

$$K_{ij}^{0308} = \int_{\Omega} \frac{\partial w}{\partial y}\left(A_{22}^{(2)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(8)}}{\partial y} + B_{66}^{(2)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(8)}}{\partial x}\right) + \frac{\partial w}{\partial x}\left(A_{12}^{(2)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(8)}}{\partial y} + B_{66}^{(2)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(8)}}{\partial x}\right) + 2B_{44}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial y} \varphi_j^{(8)} + \frac{1}{2}\left[R_{66}^{(1)}\left(\frac{\partial^2 \varphi_i^{(3)}}{\partial x^2} \frac{\partial \varphi_j^{(8)}}{\partial y} - \frac{\partial^2 \varphi_i^{(3)}}{\partial y^2} \frac{\partial \varphi_j^{(8)}}{\partial y}\right) - R_{11}^{(1)} \frac{\partial^2 \varphi_i^{(3)}}{\partial x \partial y} \frac{\partial \varphi_j^{(8)}}{\partial x}\right] \mathrm{d}x\mathrm{d}y, \quad (57)$$

$$K_{ij}^{0309} = \int_{\Omega} 2\left(\frac{\partial w}{\partial x} A_{13}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial x} \varphi_j^{(9)} + \frac{\partial w}{\partial y} A_{23}^{(1)} \frac{\partial \varphi_i^{(3)}}{\partial y} \varphi_j^{(9)}\right) + B_{44}^{(2)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(9)}}{\partial y} + B_{55}^{(2)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(9)}}{\partial x} + \frac{1}{4}\left[R_{66}^{(2)}\left(\frac{\partial^2 \varphi_i^{(3)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(9)}}{\partial x^2} + \frac{\partial^2 \varphi_i^{(3)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(9)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(3)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(9)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(3)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(9)}}{\partial x^2}\right) + R_{11}^{(2)} \frac{\partial^2 \varphi_i^{(3)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(9)}}{\partial x \partial y} + R_{22}^{(2)} \frac{\partial^2 \varphi_i^{(3)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(9)}}{\partial x \partial y} +\right] \mathrm{d}x\mathrm{d}y, \quad (58)$$

$$K_{ij}^{0310} = \int_{\Omega} \frac{\partial w}{\partial x}\left(A_{11}^{(3)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(10)}}{\partial x} + B_{66}^{(3)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(10)}}{\partial y}\right) + \frac{\partial w}{\partial y}\left(A_{21}^{(3)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(10)}}{\partial x} + B_{66}^{(3)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(10)}}{\partial y}\right) + 3B_{55}^{(2)} \frac{\partial \varphi_i^{(3)}}{\partial x} \varphi_j^{(10)} + \frac{3}{4}\left[R_{66}^{(2)}\left(\frac{\partial^2 \varphi_i^{(3)}}{\partial y^2} \frac{\partial \varphi_j^{(10)}}{\partial x} - \frac{\partial^2 \varphi_i^{(3)}}{\partial x^2} \frac{\partial \varphi_j^{(10)}}{\partial x}\right) - R_{22}^{(2)} \frac{\partial^2 \varphi_i^{(3)}}{\partial x \partial y} \frac{\partial \varphi_j^{(10)}}{\partial y}\right] \mathrm{d}x\mathrm{d}y, \quad (59)$$

$$K_{ij}^{0311} = \int_{\Omega} \frac{\partial w}{\partial y}\left(A_{22}^{(3)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(11)}}{\partial y} + B_{66}^{(3)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(11)}}{\partial x}\right) + \frac{\partial w}{\partial x}\left(A_{12}^{(3)} \frac{\partial \varphi_i^{(3)}}{\partial x} \frac{\partial \varphi_j^{(11)}}{\partial y} + B_{66}^{(3)} \frac{\partial \varphi_i^{(3)}}{\partial y} \frac{\partial \varphi_j^{(11)}}{\partial x}\right) + 3B_{44}^{(2)} \frac{\partial \varphi_i^{(3)}}{\partial y} \varphi_j^{(11)} + \frac{3}{4}\left[R_{66}^{(2)}\left(\frac{\partial^2 \varphi_i^{(3)}}{\partial x^2} \frac{\partial \varphi_j^{(11)}}{\partial y} - \frac{\partial^2 \varphi_i^{(3)}}{\partial y^2} \frac{\partial \varphi_j^{(11)}}{\partial y}\right) - R_{11}^{(2)} \frac{\partial^2 \varphi_i^{(3)}}{\partial x \partial y} \frac{\partial \varphi_j^{(11)}}{\partial x}\right] \mathrm{d}x\mathrm{d}y, \quad (60)$$

$$K_{ij}^{0401} = \int_{\Omega} A_{11}^{(1)} \frac{\partial \varphi_i^{(4)}}{\partial x} \frac{\partial \varphi_j^{(1)}}{\partial x} + B_{66}^{(1)} \frac{\partial \varphi_i^{(4)}}{\partial y} \frac{\partial \varphi_j^{(1)}}{\partial y} + \frac{1}{4}\left(R_{44}^{(1)} \frac{\partial^2 \varphi_i^{(4)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(1)}}{\partial y^2} + R_{55}^{(1)} \frac{\partial^2 \varphi_i^{(4)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(1)}}{\partial x \partial y}\right) \mathrm{d}x\mathrm{d}y, \quad (61)$$

$$K_{ij}^{0402} = \int_{\Omega} A_{12}^{(1)} \frac{\partial \varphi_i^{(4)}}{\partial x} \frac{\partial \varphi_j^{(2)}}{\partial y} + B_{66}^{(1)} \frac{\partial \varphi_i^{(4)}}{\partial y} \frac{\partial \varphi_j^{(2)}}{\partial x} - \frac{1}{4}\left(R_{44}^{(1)} \frac{\partial^2 \varphi_i^{(4)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(2)}}{\partial x \partial y} + R_{55}^{(1)} \frac{\partial^2 \varphi_i^{(4)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(2)}}{\partial x^2}\right) \mathrm{d}x\mathrm{d}y, \quad (62)$$

$$K_{ij}^{0403} = \frac{1}{2} \int_{\Omega} \frac{\partial w}{\partial x}\left(A_{11}^{(1)} \frac{\partial \varphi_i^{(4)}}{\partial x} \frac{\partial \varphi_j^{(3)}}{\partial x} + B_{66}^{(1)} \frac{\partial \varphi_i^{(4)}}{\partial y} \frac{\partial \varphi_j^{(3)}}{\partial y}\right) + \frac{\partial w}{\partial y}\left(A_{12}^{(1)} \frac{\partial \varphi_i^{(4)}}{\partial x} \frac{\partial \varphi_j^{(3)}}{\partial y} + B_{66}^{(1)} \frac{\partial \varphi_i^{(4)}}{\partial y} \frac{\partial \varphi_j^{(3)}}{\partial x}\right) + 2B_{55}^{(0)} \varphi_i^{(4)} \frac{\partial \varphi_j^{(3)}}{\partial x} + \frac{1}{2}\left[R_{66}^{(0)}\left(\frac{\partial \varphi_i^{(4)}}{\partial x} \frac{\partial^2 \varphi_j^{(3)}}{\partial y^2} - \frac{\partial \varphi_i^{(4)}}{\partial x} \frac{\partial^2 \varphi_j^{(3)}}{\partial x^2}\right) - R_{22}^{(0)} \frac{\partial \varphi_i^{(4)}}{\partial y} \frac{\partial^2 \varphi_j^{(3)}}{\partial x \partial y}\right] \mathrm{d}x\mathrm{d}y, \quad (63)$$

$$K_{ij}^{0404}=\int_{\Omega} A_{11}^{(2)}\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial\varphi_j^{(4)}}{\partial x}+B_{66}^{(2)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(4)}}{\partial y}+B_{55}^{(0)}\varphi_i^{(4)}\varphi_j^{(4)}+\frac{1}{4}\left(R_{22}^{(0)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(4)}}{\partial y}\right.$$
$$\left.+\ R_{33}^{(0)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(4)}}{\partial y}+R_{66}^{(0)}\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial\varphi_j^{(4)}}{\partial x}+R_{44}^{(2)}\frac{\partial^2\varphi_i^{(4)}}{\partial y^2}\frac{\partial^2\varphi_j^{(4)}}{\partial y^2}+R_{55}^{(2)}\frac{\partial^2\varphi_i^{(4)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(4)}}{\partial x\partial y}\right)\mathrm{d}x\mathrm{d}y, \tag{64}$$

$$K_{ij}^{0405}=\int_{\Omega} A_{12}^{(2)}\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial\varphi_j^{(5)}}{\partial y}+B_{66}^{(2)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(5)}}{\partial x}-\frac{1}{4}\left(R_{33}^{(0)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(5)}}{\partial x}+R_{66}^{(0)}\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial\varphi_j^{(5)}}{\partial y}\right.$$
$$\left.+\ R_{44}^{(2)}\frac{\partial^2\varphi_i^{(4)}}{\partial y^2}\frac{\partial^2\varphi_j^{(5)}}{\partial x\partial y}+R_{55}^{(2)}\frac{\partial^2\varphi_i^{(4)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(5)}}{\partial x^2}\right)\mathrm{d}x\mathrm{d}y, \tag{65}$$

$$K_{ij}^{0406}=\int_{\Omega} A_{13}^{(1)}\frac{\partial\varphi_i^{(4)}}{\partial x}\varphi_j^{(6)}+B_{55}^{(1)}\varphi_i^{(4)}\frac{\partial\varphi_j^{(6)}}{\partial x}+\frac{1}{4}\left[R_{66}^{(1)}\left(\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial^2\varphi_j^{(6)}}{\partial y^2}-\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial^2\varphi_j^{(6)}}{\partial x^2}\right)\right.$$
$$\left.-\ R_{22}^{(1)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial^2\varphi_j^{(6)}}{\partial x\partial y}+R_{44}^{(1)}\frac{\partial^2\varphi_i^{(4)}}{\partial y^2}\frac{\partial\varphi_j^{(6)}}{\partial x}-R_{55}^{(1)}\frac{\partial^2\varphi_i^{(4)}}{\partial x\partial y}\frac{\partial\varphi_j^{(6)}}{\partial y}\right]\mathrm{d}x\mathrm{d}y, \tag{66}$$

$$K_{ij}^{0407}=\int_{\Omega} A_{11}^{(3)}\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial\varphi_j^{(7)}}{\partial x}+B_{66}^{(3)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(7)}}{\partial y}+2B_{55}^{(1)}\varphi_i^{(4)}\varphi_j^{(7)}+\frac{1}{2}\left[R_{22}^{(1)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(7)}}{\partial y}\right.$$
$$+\ R_{33}^{(1)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(7)}}{\partial y}-R_{44}^{(1)}\frac{\partial^2\varphi_i^{(4)}}{\partial y^2}\varphi_j^{(7)}+R_{66}^{(1)}\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial\varphi_j^{(7)}}{\partial x}+\frac{1}{2}\left(R_{44}^{(3)}\frac{\partial^2\varphi_i^{(4)}}{\partial y^2}\frac{\partial^2\varphi_j^{(7)}}{\partial y^2}\right.$$
$$\left.\left.+\ R_{55}^{(3)}\frac{\partial^2\varphi_i^{(4)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(7)}}{\partial x\partial y}\right)\right]\mathrm{d}x\mathrm{d}y, \tag{67}$$

$$K_{ij}^{0408}=\int_{\Omega} A_{12}^{(3)}\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial\varphi_j^{(8)}}{\partial y}+B_{66}^{(3)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(8)}}{\partial x}-\frac{1}{2}\left[R_{33}^{(1)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(8)}}{\partial x}-R_{55}^{(1)}\frac{\partial^2\varphi_i^{(4)}}{\partial x\partial y}\varphi_j^{(8)}\right.$$
$$\left.+\ R_{66}^{(1)}\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial\varphi_j^{(8)}}{\partial y}+\frac{1}{2}\left(R_{44}^{(3)}\frac{\partial^2\varphi_i^{(4)}}{\partial y^2}\frac{\partial^2\varphi_j^{(8)}}{\partial x\partial y}+R_{55}^{(3)}\frac{\partial^2\varphi_i^{(4)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(8)}}{\partial x^2}\right)\right]\mathrm{d}x\mathrm{d}y, \tag{68}$$

$$K_{ij}^{0409}=\int_{\Omega} 2A_{13}^{(2)}\frac{\partial\varphi_i^{(4)}}{\partial x}\varphi_j^{(9)}+B_{55}^{(2)}\varphi_i^{(4)}\frac{\partial\varphi_j^{(9)}}{\partial x}+\frac{1}{4}\left[R_{66}^{(2)}\left(\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial^2\varphi_j^{(9)}}{\partial y^2}-\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial^2\varphi_j^{(9)}}{\partial x^2}\right)\right.$$
$$\left.-\ R_{22}^{(2)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial^2\varphi_j^{(9)}}{\partial x\partial y}+2\left(R_{44}^{(2)}\frac{\partial^2\varphi_i^{(4)}}{\partial y^2}\frac{\partial\varphi_j^{(9)}}{\partial x}-R_{55}^{(2)}\frac{\partial^2\varphi_i^{(4)}}{\partial x\partial y}\frac{\partial\varphi_j^{(9)}}{\partial y}\right)\right]\mathrm{d}x\mathrm{d}y, \tag{69}$$

$$K_{ij}^{0410}=\int_{\Omega} A_{11}^{(4)}\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial\varphi_j^{(10)}}{\partial x}+B_{66}^{(4)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(10)}}{\partial y}+3B_{55}^{(2)}\varphi_i^{(4)}\varphi_j^{(10)}+\frac{3}{4}\left[R_{22}^{(2)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(10)}}{\partial y}\right.$$
$$+\ R_{33}^{(2)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(10)}}{\partial y}-2R_{44}^{(2)}\frac{\partial^2\varphi_i^{(4)}}{\partial y^2}\varphi_j^{(10)}+R_{66}^{(2)}\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial\varphi_j^{(10)}}{\partial x}+\frac{1}{3}\left(R_{44}^{(4)}\frac{\partial^2\varphi_i^{(4)}}{\partial y^2}\frac{\partial^2\varphi_j^{(10)}}{\partial y^2}\right.$$
$$\left.\left.+\ R_{55}^{(4)}\frac{\partial^2\varphi_i^{(4)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(10)}}{\partial x\partial y}\right)\right]\mathrm{d}x\mathrm{d}y, \tag{70}$$

$$K_{ij}^{0411}=\int_{\Omega} A_{12}^{(4)}\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial\varphi_j^{(11)}}{\partial y}+B_{66}^{(4)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(11)}}{\partial x}-\frac{3}{4}\left[R_{33}^{(2)}\frac{\partial\varphi_i^{(4)}}{\partial y}\frac{\partial\varphi_j^{(11)}}{\partial x}-2R_{55}^{(2)}\frac{\partial^2\varphi_i^{(4)}}{\partial x\partial y}\varphi_j^{(11)}\right.$$
$$\left.+\ R_{66}^{(2)}\frac{\partial\varphi_i^{(4)}}{\partial x}\frac{\partial\varphi_j^{(11)}}{\partial y}+\frac{1}{3}\left(R_{44}^{(4)}\frac{\partial^2\varphi_i^{(4)}}{\partial y^2}\frac{\partial^2\varphi_j^{(11)}}{\partial x\partial y}+R_{55}^{(4)}\frac{\partial^2\varphi_i^{(4)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(11)}}{\partial x^2}\right)\right]\mathrm{d}x\mathrm{d}y, \tag{71}$$

$$K_{ij}^{0501}=\int_{\Omega} A_{21}^{(1)}\frac{\partial\varphi_i^{(5)}}{\partial y}\frac{\partial\varphi_j^{(1)}}{\partial x}+B_{66}^{(1)}\frac{\partial\varphi_i^{(5)}}{\partial x}\frac{\partial\varphi_j^{(1)}}{\partial y}-\frac{1}{4}\left(R_{44}^{(1)}\frac{\partial^2\varphi_i^{(5)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(1)}}{\partial y^2}+R_{55}^{(1)}\frac{\partial^2\varphi_i^{(5)}}{\partial x^2}\frac{\partial^2\varphi_j^{(1)}}{\partial x\partial y}\right)\mathrm{d}x\mathrm{d}y, \tag{72}$$

$$K_{ij}^{0502}=\int_{\Omega} A_{22}^{(1)}\frac{\partial\varphi_i^{(5)}}{\partial y}\frac{\partial\varphi_j^{(2)}}{\partial y}+B_{66}^{(1)}\frac{\partial\varphi_i^{(5)}}{\partial x}\frac{\partial\varphi_j^{(2)}}{\partial x}+\frac{1}{4}\left(R_{44}^{(1)}\frac{\partial^2\varphi_i^{(5)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(2)}}{\partial x\partial y}+R_{55}^{(1)}\frac{\partial^2\varphi_i^{(5)}}{\partial x^2}\frac{\partial^2\varphi_j^{(2)}}{\partial x^2}\right)\mathrm{d}x\mathrm{d}y, \tag{73}$$

$$K_{ij}^{0503} = \frac{1}{2}\int_{\Omega} \frac{\partial w}{\partial x}\left(A_{21}^{(1)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(3)}}{\partial x} + B_{66}^{(1)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(3)}}{\partial y}\right) + \frac{\partial w}{\partial y}\left(A_{22}^{(1)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(3)}}{\partial y} + B_{66}^{(1)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(3)}}{\partial x}\right) + 2B_{44}^{(0)}\varphi_i^{(5)}\frac{\partial \varphi_j^{(3)}}{\partial y} + \frac{1}{2}\left[R_{66}^{(0)}\left(\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial^2 \varphi_j^{(3)}}{\partial x^2} - \frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial^2 \varphi_j^{(3)}}{\partial y^2}\right) - R_{11}^{(0)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial^2 \varphi_j^{(3)}}{\partial x \partial y}\right]\mathrm{d}x\mathrm{d}y, \tag{74}$$

$$K_{ij}^{0504} = \int_{\Omega} A_{21}^{(2)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(4)}}{\partial x} + B_{66}^{(2)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(4)}}{\partial y} - \frac{1}{4}\left(R_{33}^{(0)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(4)}}{\partial y} + R_{66}^{(0)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(4)}}{\partial x} + R_{44}^{(2)}\frac{\partial \varphi_i^{(5)}}{\partial x \partial y}\frac{\partial \varphi_j^{(4)}}{\partial y^2} + R_{55}^{(2)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x^2}\frac{\partial^2 \varphi_j^{(4)}}{\partial x \partial y}\right)\mathrm{d}x\mathrm{d}y, \tag{75}$$

$$K_{ij}^{0505} = \int_{\Omega} A_{22}^{(2)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(5)}}{\partial y} + B_{66}^{(2)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(5)}}{\partial x} + B_{44}^{(0)}\varphi_i^{(5)}\varphi_j^{(5)} + \frac{1}{4}\left(R_{11}^{(0)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(5)}}{\partial x} + R_{33}^{(0)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(5)}}{\partial x} + R_{66}^{(0)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(5)}}{\partial y} + R_{44}^{(2)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x \partial y}\frac{\partial^2 \varphi_j^{(5)}}{\partial x \partial y} + R_{55}^{(2)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x^2}\frac{\partial^2 \varphi_j^{(5)}}{\partial x^2}\right)\mathrm{d}x\mathrm{d}y, \tag{76}$$

$$K_{ij}^{0506} = \int_{\Omega} A_{23}^{(1)}\frac{\partial \varphi_i^{(5)}}{\partial y}\varphi_j^{(6)} + B_{44}^{(1)}\varphi_i^{(5)}\frac{\partial \varphi_j^{(6)}}{\partial y} + \frac{1}{4}\left[R_{66}^{(1)}\left(\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial^2 \varphi_j^{(6)}}{\partial x^2} - \frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial^2 \varphi_j^{(6)}}{\partial y^2}\right) - R_{11}^{(1)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial^2 \varphi_j^{(6)}}{\partial x \partial y} - R_{44}^{(1)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x \partial y}\frac{\partial \varphi_j^{(6)}}{\partial x} + R_{55}^{(1)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x^2}\frac{\partial \varphi_j^{(6)}}{\partial y}\right]\mathrm{d}x\mathrm{d}y, \tag{77}$$

$$K_{ij}^{0507} = \int_{\Omega} A_{21}^{(3)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(7)}}{\partial x} + B_{66}^{(3)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(7)}}{\partial y} - \frac{1}{2}\left[R_{33}^{(1)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(7)}}{\partial y} - R_{44}^{(1)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x \partial y}\varphi_j^{(7)} + R_{66}^{(1)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(7)}}{\partial x} + \frac{1}{2}\left(R_{44}^{(3)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x \partial y}\frac{\partial^2 \varphi_j^{(7)}}{\partial y^2} + R_{55}^{(3)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x^2}\frac{\partial^2 \varphi_j^{(7)}}{\partial x \partial y}\right)\right]\mathrm{d}x\mathrm{d}y, \tag{78}$$

$$K_{ij}^{0508} = \int_{\Omega} A_{22}^{(3)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(8)}}{\partial y} + B_{66}^{(3)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(8)}}{\partial x} + 2B_{44}^{(1)}\varphi_i^{(5)}\varphi_j^{(8)} + \frac{1}{2}\left[R_{11}^{(1)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(8)}}{\partial x} + R_{33}^{(1)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(8)}}{\partial x} - R_{55}^{(1)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x^2}\varphi_j^{(8)} + R_{66}^{(1)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(8)}}{\partial y} + \frac{1}{2}\left(R_{44}^{(3)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x \partial y}\frac{\partial^2 \varphi_j^{(8)}}{\partial x \partial y} + R_{55}^{(3)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x^2}\frac{\partial^2 \varphi_j^{(8)}}{\partial x^2}\right)\right]\mathrm{d}x\mathrm{d}y, \tag{79}$$

$$K_{ij}^{0509} = \int_{\Omega} 2A_{23}^{(2)}\frac{\partial \varphi_i^{(5)}}{\partial y}\varphi_j^{(9)} + B_{44}^{(2)}\varphi_i^{(5)}\frac{\partial \varphi_j^{(9)}}{\partial y} + \frac{1}{4}\left[R_{66}^{(2)}\left(\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial^2 \varphi_j^{(9)}}{\partial x^2} - \frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial^2 \varphi_j^{(9)}}{\partial y^2}\right) - R_{11}^{(2)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial^2 \varphi_j^{(9)}}{\partial x \partial y} - 2\left(R_{44}^{(2)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x \partial y}\frac{\partial \varphi_j^{(9)}}{\partial x} - R_{55}^{(2)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x^2}\frac{\partial \varphi_j^{(9)}}{\partial y}\right)\right]\mathrm{d}x\mathrm{d}y, \tag{80}$$

$$K_{ij}^{0510} = \int_{\Omega} A_{21}^{(4)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(10)}}{\partial x} + B_{66}^{(4)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(10)}}{\partial y} - \frac{3}{4}\left[R_{33}^{(2)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(10)}}{\partial y} + R_{66}^{(2)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(10)}}{\partial x} - 2R_{44}^{(2)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x \partial y}\varphi_j^{(10)} + \frac{1}{3}\left(R_{44}^{(4)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x \partial y}\frac{\partial^2 \varphi_j^{(10)}}{\partial y^2} + R_{55}^{(4)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x^2}\frac{\partial^2 \varphi_j^{(10)}}{\partial x \partial y}\right)\right]\mathrm{d}x\mathrm{d}y, \tag{81}$$

$$K_{ij}^{0511} = \int_{\Omega} A_{22}^{(4)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(11)}}{\partial y} + B_{66}^{(4)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(11)}}{\partial x} + 3B_{44}^{(2)}\varphi_i^{(5)}\varphi_j^{(11)} + \frac{3}{4}\left[R_{11}^{(2)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(11)}}{\partial x} + R_{33}^{(2)}\frac{\partial \varphi_i^{(5)}}{\partial x}\frac{\partial \varphi_j^{(11)}}{\partial x} - 2R_{55}^{(2)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x^2}\varphi_j^{(11)} + R_{66}^{(2)}\frac{\partial \varphi_i^{(5)}}{\partial y}\frac{\partial \varphi_j^{(11)}}{\partial y} + \frac{1}{3}\left(R_{44}^{(4)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x \partial y}\frac{\partial^2 \varphi_j^{(11)}}{\partial x \partial y} + R_{55}^{(4)}\frac{\partial^2 \varphi_i^{(5)}}{\partial x^2}\frac{\partial^2 \varphi_j^{(11)}}{\partial x^2}\right)\right]\mathrm{d}x\mathrm{d}y, \tag{82}$$

$$K_{ij}^{0601} = \int_\Omega A_{31}^{(0)} \varphi_i^{(6)} \frac{\partial \varphi_j^{(1)}}{\partial x} + \frac{1}{4}\left(R_{44}^{(0)} \frac{\partial \varphi_i^{(6)}}{\partial x} \frac{\partial^2 \varphi_j^{(1)}}{\partial y^2} - R_{55}^{(0)} \frac{\partial \varphi_i^{(6)}}{\partial y} \frac{\partial^2 \varphi_j^{(1)}}{\partial x \partial y} \right) \mathrm{d}x\mathrm{d}y, \tag{83}$$

$$K_{ij}^{0602} = \int_\Omega A_{32}^{(0)} \varphi_i^{(6)} \frac{\partial \varphi_j^{(2)}}{\partial y} - \frac{1}{4}\left(R_{44}^{(0)} \frac{\partial \varphi_i^{(6)}}{\partial x} \frac{\partial^2 \varphi_j^{(2)}}{\partial x \partial y} - R_{55}^{(0)} \frac{\partial \varphi_i^{(6)}}{\partial y} \frac{\partial^2 \varphi_j^{(2)}}{\partial x^2} \right) \mathrm{d}x\mathrm{d}y, \tag{84}$$

$$\begin{aligned} K_{ij}^{0603} = \frac{1}{2}\int_\Omega & \frac{\partial w}{\partial x} A_{31}^{(0)} \varphi_i^{(6)} \frac{\partial \varphi_j^{(3)}}{\partial x} + \frac{\partial w}{\partial y} A_{32}^{(0)} \varphi_i^{(6)} \frac{\partial \varphi_j^{(3)}}{\partial y} + 2\left(B_{44}^{(1)} \frac{\partial \varphi_i^{(6)}}{\partial y} \frac{\partial \varphi_j^{(3)}}{\partial y} + B_{55}^{(1)} \frac{\partial \varphi_i^{(6)}}{\partial x} \frac{\partial \varphi_j^{(3)}}{\partial x} \right) \\ & + \frac{1}{2}\left[R_{11}^{(1)} \frac{\partial^2 \varphi_i^{(6)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(3)}}{\partial x \partial y} + R_{22}^{(1)} \frac{\partial^2 \varphi_i^{(6)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(3)}}{\partial x \partial y} + R_{66}^{(1)} \left(\frac{\partial^2 \varphi_i^{(6)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(3)}}{\partial x^2} + \frac{\partial^2 \varphi_i^{(6)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(3)}}{\partial y^2} \right.\right. \\ & \left.\left. - \frac{\partial^2 \varphi_i^{(6)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(3)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(6)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(3)}}{\partial x^2} \right) \right] \mathrm{d}x\mathrm{d}y, \end{aligned} \tag{85}$$

$$\begin{aligned} K_{ij}^{0604} = \int_\Omega & A_{31}^{(1)} \varphi_i^{(6)} \frac{\partial \varphi_j^{(4)}}{\partial x} + B_{55}^{(1)} \frac{\partial \varphi_i^{(6)}}{\partial x} \varphi_j^{(4)} + \frac{1}{4}\left[R_{66}^{(1)} \left(\frac{\partial^2 \varphi_i^{(6)}}{\partial y^2} \frac{\partial \varphi_j^{(4)}}{\partial x} - \frac{\partial^2 \varphi_i^{(6)}}{\partial x^2} \frac{\partial \varphi_j^{(4)}}{\partial x} \right) \right. \\ & \left. - R_{22}^{(1)} \frac{\partial^2 \varphi_i^{(6)}}{\partial x \partial y} \frac{\partial \varphi_j^{(4)}}{\partial y} + R_{44}^{(1)} \frac{\partial \varphi_i^{(6)}}{\partial x} \frac{\partial^2 \varphi_j^{(4)}}{\partial y^2} - R_{55}^{(1)} \frac{\partial \varphi_i^{(6)}}{\partial y} \frac{\partial^2 \varphi_j^{(4)}}{\partial x \partial y} \right] \mathrm{d}x\mathrm{d}y, \end{aligned} \tag{86}$$

$$\begin{aligned} K_{ij}^{0605} = \int_\Omega & A_{32}^{(1)} \varphi_i^{(6)} \frac{\partial \varphi_j^{(5)}}{\partial y} + B_{44}^{(1)} \frac{\partial \varphi_i^{(6)}}{\partial y} \varphi_j^{(5)} + \frac{1}{4}\left[R_{66}^{(1)} \left(\frac{\partial^2 \varphi_i^{(6)}}{\partial x^2} \frac{\partial \varphi_j^{(5)}}{\partial y} - \frac{\partial^2 \varphi_i^{(6)}}{\partial y^2} \frac{\partial \varphi_j^{(5)}}{\partial y} \right) \right. \\ & \left. - R_{11}^{(1)} \frac{\partial^2 \varphi_i^{(6)}}{\partial x \partial y} \frac{\partial \varphi_j^{(5)}}{\partial x} - R_{44}^{(1)} \frac{\partial \varphi_i^{(6)}}{\partial x} \frac{\partial^2 \varphi_j^{(5)}}{\partial x \partial y} + R_{55}^{(1)} \frac{\partial \varphi_i^{(6)}}{\partial y} \frac{\partial^2 \varphi_j^{(5)}}{\partial x^2} \right] \mathrm{d}x\mathrm{d}y, \end{aligned} \tag{87}$$

$$\begin{aligned} K_{ij}^{0606} = \int_\Omega & A_{33}^{(0)} \varphi_i^{(6)} \varphi_j^{(6)} + B_{44}^{(2)} \frac{\partial \varphi_i^{(6)}}{\partial y} \frac{\partial \varphi_j^{(6)}}{\partial y} + B_{55}^{(2)} \frac{\partial \varphi_i^{(6)}}{\partial x} \frac{\partial \varphi_j^{(6)}}{\partial x} \\ & + \frac{1}{4}\left[R_{11}^{(2)} \frac{\partial^2 \varphi_i^{(6)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(6)}}{\partial x \partial y} + R_{22}^{(2)} \frac{\partial^2 \varphi_i^{(6)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(6)}}{\partial x \partial y} + R_{44}^{(0)} \frac{\partial \varphi_i^{(6)}}{\partial x} \frac{\partial \varphi_j^{(6)}}{\partial x} + R_{55}^{(0)} \frac{\partial \varphi_i^{(6)}}{\partial y} \frac{\partial \varphi_j^{(6)}}{\partial y} \right. \\ & \left. + R_{66}^{(2)} \left(\frac{\partial^2 \varphi_i^{(6)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(6)}}{\partial x^2} + \frac{\partial^2 \varphi_i^{(6)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(6)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(6)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(6)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(6)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(6)}}{\partial x^2} \right) \right] \mathrm{d}x\mathrm{d}y, \end{aligned} \tag{88}$$

$$\begin{aligned} K_{ij}^{0607} = \int_\Omega & A_{31}^{(2)} \varphi_i^{(6)} \frac{\partial \varphi_j^{(7)}}{\partial x} + 2B_{55}^{(2)} \frac{\partial \varphi_i^{(6)}}{\partial x} \varphi_j^{(7)} + \frac{1}{2}\left[R_{66}^{(2)} \left(\frac{\partial^2 \varphi_i^{(6)}}{\partial y^2} \frac{\partial \varphi_j^{(7)}}{\partial x} - \frac{\partial^2 \varphi_i^{(6)}}{\partial x^2} \frac{\partial \varphi_j^{(7)}}{\partial x} \right) \right. \\ & \left. - R_{22}^{(2)} \frac{\partial^2 \varphi_i^{(6)}}{\partial x \partial y} \frac{\partial \varphi_j^{(7)}}{\partial y} - R_{44}^{(0)} \frac{\partial \varphi_i^{(6)}}{\partial x} \varphi_j^{(7)} + \frac{1}{2}\left(R_{44}^{(2)} \frac{\partial \varphi_i^{(6)}}{\partial x} \frac{\partial^2 \varphi_j^{(7)}}{\partial y^2} - R_{55}^{(2)} \frac{\partial \varphi_i^{(6)}}{\partial y} \frac{\partial^2 \varphi_j^{(7)}}{\partial x \partial y} \right) \right] \mathrm{d}x\mathrm{d}y, \end{aligned} \tag{89}$$

$$\begin{aligned} K_{ij}^{0608} = \int_\Omega & A_{32}^{(2)} \varphi_i^{(6)} \frac{\partial \varphi_j^{(8)}}{\partial y} + 2B_{44}^{(2)} \frac{\partial \varphi_i^{(6)}}{\partial y} \varphi_j^{(8)} + \frac{1}{2}\left[R_{66}^{(2)} \left(\frac{\partial^2 \varphi_i^{(6)}}{\partial x^2} \frac{\partial \varphi_j^{(8)}}{\partial y} - \frac{\partial^2 \varphi_i^{(6)}}{\partial y^2} \frac{\partial \varphi_j^{(8)}}{\partial y} \right) \right. \\ & \left. - R_{11}^{(2)} \frac{\partial^2 \varphi_i^{(6)}}{\partial x \partial y} \frac{\partial \varphi_j^{(8)}}{\partial x} - R_{55}^{(0)} \frac{\partial \varphi_i^{(6)}}{\partial y} \varphi_j^{(8)} - \frac{1}{2}\left(R_{44}^{(2)} \frac{\partial \varphi_i^{(6)}}{\partial x} \frac{\partial^2 \varphi_j^{(8)}}{\partial x \partial y} - R_{55}^{(2)} \frac{\partial \varphi_i^{(6)}}{\partial y} \frac{\partial^2 \varphi_j^{(8)}}{\partial x^2} \right) \right] \mathrm{d}x\mathrm{d}y, \end{aligned} \tag{90}$$

$$\begin{aligned} K_{ij}^{0609} = \int_\Omega & 2A_{33}^{(1)} \varphi_i^{(6)} \varphi_j^{(9)} + B_{44}^{(3)} \frac{\partial \varphi_i^{(6)}}{\partial y} \frac{\partial \varphi_j^{(9)}}{\partial y} + B_{55}^{(3)} \frac{\partial \varphi_i^{(6)}}{\partial x} \frac{\partial \varphi_j^{(9)}}{\partial x} \\ & + \frac{1}{4}\left[R_{11}^{(3)} \frac{\partial^2 \varphi_i^{(6)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(9)}}{\partial x \partial y} + R_{22}^{(3)} \frac{\partial^2 \varphi_i^{(6)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(9)}}{\partial x \partial y} + 2\left(R_{44}^{(1)} \frac{\partial \varphi_i^{(6)}}{\partial x} \frac{\partial \varphi_j^{(9)}}{\partial x} + R_{55}^{(1)} \frac{\partial \varphi_i^{(6)}}{\partial y} \frac{\partial \varphi_j^{(9)}}{\partial y} \right) \right. \\ & \left. + R_{66}^{(3)} \left(\frac{\partial^2 \varphi_i^{(6)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(9)}}{\partial x^2} + \frac{\partial^2 \varphi_i^{(6)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(9)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(6)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(9)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(6)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(9)}}{\partial x^2} \right) \right] \mathrm{d}x\mathrm{d}y, \end{aligned} \tag{91}$$

$$K_{ij}^{0610}=\int_{\Omega}A_{31}^{(3)}\varphi_i^{(6)}\frac{\partial\varphi_j^{(10)}}{\partial x}+3B_{55}^{(3)}\frac{\partial\varphi_i^{(6)}}{\partial x}\varphi_j^{(10)}+\frac{3}{4}\left[R_{66}^{(3)}\left(\frac{\partial^2\varphi_i^{(6)}}{\partial y^2}\frac{\partial\varphi_j^{(10)}}{\partial x}-\frac{\partial^2\varphi_i^{(6)}}{\partial x^2}\frac{\partial\varphi_j^{(10)}}{\partial x}\right)\right.$$
$$-R_{22}^{(3)}\frac{\partial^2\varphi_i^{(6)}}{\partial x\partial y}\frac{\partial\varphi_j^{(10)}}{\partial y}+\frac{1}{3}\left(R_{44}^{(3)}\frac{\partial\varphi_i^{(6)}}{\partial x}\frac{\partial^2\varphi_j^{(10)}}{\partial y^2}-R_{55}^{(3)}\frac{\partial\varphi_i^{(6)}}{\partial y}\frac{\partial^2\varphi_j^{(10)}}{\partial x\partial y}\right)$$
$$\left.-2R_{44}^{(1)}\frac{\partial\varphi_i^{(6)}}{\partial x}\varphi_j^{(10)}\right]\mathrm{d}x\mathrm{d}y, \tag{92}$$

$$K_{ij}^{0611}=\int_{\Omega}A_{32}^{(3)}\varphi_i^{(6)}\frac{\partial\varphi_j^{(11)}}{\partial y}+3B_{44}^{(3)}\frac{\partial\varphi_i^{(6)}}{\partial y}\varphi_j^{(11)}+\frac{3}{4}\left[R_{66}^{(3)}\left(\frac{\partial^2\varphi_i^{(6)}}{\partial x^2}\frac{\partial\varphi_j^{(11)}}{\partial y}-\frac{\partial^2\varphi_i^{(6)}}{\partial y^2}\frac{\partial\varphi_j^{(11)}}{\partial y}\right)\right.$$
$$-R_{11}^{(3)}\frac{\partial^2\varphi_i^{(6)}}{\partial x\partial y}\frac{\partial\varphi_j^{(11)}}{\partial x}-\frac{1}{3}\left(R_{44}^{(3)}\frac{\partial\varphi_i^{(6)}}{\partial x}\frac{\partial^2\varphi_j^{(11)}}{\partial x\partial y}-R_{55}^{(3)}\frac{\partial\varphi_i^{(6)}}{\partial y}\frac{\partial^2\varphi_j^{(11)}}{\partial x^2}\right)$$
$$\left.-2R_{55}^{(1)}\frac{\partial\varphi_i^{(6)}}{\partial y}\varphi_j^{(11)}\right]\mathrm{d}x\mathrm{d}y, \tag{93}$$

$$K_{ij}^{0701}=\int_{\Omega}A_{11}^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial\varphi_j^{(1)}}{\partial x}+B_{66}^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial\varphi_j^{(1)}}{\partial y}+\frac{1}{4}\left(R_{44}^{(2)}\frac{\partial^2\varphi_i^{(7)}}{\partial y^2}\frac{\partial^2\varphi_j^{(1)}}{\partial y^2}+R_{55}^{(2)}\frac{\partial^2\varphi_i^{(7)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(1)}}{\partial x\partial y}\right)$$
$$-\frac{1}{2}R_{44}^{(0)}\varphi_i^{(7)}\frac{\partial^2\varphi_j^{(1)}}{\partial y^2}\mathrm{d}x\mathrm{d}y, \tag{94}$$

$$K_{ij}^{0702}=\int_{\Omega}A_{12}^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial\varphi_j^{(2)}}{\partial y}+B_{66}^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial\varphi_j^{(2)}}{\partial x}-\frac{1}{4}\left(R_{44}^{(2)}\frac{\partial^2\varphi_i^{(7)}}{\partial y^2}\frac{\partial^2\varphi_j^{(2)}}{\partial x\partial y}+R_{55}^{(2)}\frac{\partial^2\varphi_i^{(7)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(2)}}{\partial x^2}\right)$$
$$+\frac{1}{2}R_{44}^{(0)}\varphi_i^{(7)}\frac{\partial^2\varphi_j^{(2)}}{\partial x\partial y}\mathrm{d}x\mathrm{d}y, \tag{95}$$

$$K_{ij}^{0703}=\frac{1}{2}\int_{\Omega}\frac{\partial w}{\partial x}\left(A_{11}^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial\varphi_j^{(3)}}{\partial x}+B_{66}^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial\varphi_j^{(3)}}{\partial y}\right)+\frac{\partial w}{\partial y}\left(A_{12}^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial\varphi_j^{(3)}}{\partial y}+B_{66}^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial\varphi_j^{(3)}}{\partial x}\right)+4B_{55}^{(1)}\varphi_i^{(7)}\frac{\partial\varphi_j^{(3)}}{\partial x}$$
$$+R_{66}^{(1)}\left(\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial^2\varphi_j^{(3)}}{\partial y^2}-\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial^2\varphi_j^{(3)}}{\partial x^2}\right)-R_{22}^{(1)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial^2\varphi_j^{(3)}}{\partial x\partial y}\mathrm{d}x\mathrm{d}y, \tag{96}$$

$$K_{ij}^{0704}=\int_{\Omega}A_{11}^{(3)}\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial\varphi_j^{(4)}}{\partial x}+B_{66}^{(3)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial\varphi_j^{(4)}}{\partial y}+2B_{55}^{(1)}\varphi_i^{(7)}\varphi_j^{(4)}+\frac{1}{2}\left[R_{22}^{(1)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial\varphi_j^{(4)}}{\partial y}\right.$$
$$\left.+R_{33}^{(1)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial\varphi_j^{(4)}}{\partial y}-R_{44}^{(1)}\varphi_i^{(7)}\frac{\partial^2\varphi_j^{(4)}}{\partial y^2}+R_{66}^{(1)}\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial\varphi_j^{(4)}}{\partial x}+\frac{1}{2}\left(R_{44}^{(3)}\frac{\partial^2\varphi_i^{(7)}}{\partial y^2}\frac{\partial^2\varphi_j^{(4)}}{\partial y^2}+R_{55}^{(3)}\frac{\partial^2\varphi_i^{(7)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(4)}}{\partial x\partial y}\right)\right]\mathrm{d}x\mathrm{d}y, \tag{97}$$

$$K_{ij}^{0705}=\int_{\Omega}A_{12}^{(3)}\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial\varphi_j^{(5)}}{\partial y}+B_{66}^{(3)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial\varphi_j^{(5)}}{\partial x}-\frac{1}{2}\left[R_{33}^{(1)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial\varphi_j^{(5)}}{\partial x}-R_{44}^{(1)}\varphi_i^{(7)}\frac{\partial^2\varphi_j^{(5)}}{\partial x\partial y}\right.$$
$$\left.+R_{66}^{(1)}\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial\varphi_j^{(5)}}{\partial y}+\frac{1}{2}\left(R_{44}^{(3)}\frac{\partial^2\varphi_i^{(7)}}{\partial y^2}\frac{\partial^2\varphi_j^{(5)}}{\partial x\partial y}+R_{55}^{(3)}\frac{\partial^2\varphi_i^{(7)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(5)}}{\partial x^2}\right)\right]\mathrm{d}x\mathrm{d}y, \tag{98}$$

$$K_{ij}^{0706}=\int_{\Omega}A_{13}^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial x}\varphi_j^{(6)}+2B_{55}^{(2)}\varphi_i^{(7)}\frac{\partial\varphi_j^{(6)}}{\partial x}+\frac{1}{2}\left[R_{66}^{(2)}\left(\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial^2\varphi_j^{(6)}}{\partial y^2}-\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial^2\varphi_j^{(6)}}{\partial x^2}\right)\right.$$
$$\left.-R_{22}^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial^2\varphi_j^{(6)}}{\partial x\partial y}-R_{44}^{(0)}\varphi_i^{(7)}\frac{\partial\varphi_j^{(6)}}{\partial x}+\frac{1}{2}\left(R_{44}^{(2)}\frac{\partial^2\varphi_i^{(7)}}{\partial y^2}\frac{\partial\varphi_j^{(6)}}{\partial x}-R_{55}^{(2)}\frac{\partial^2\varphi_i^{(7)}}{\partial x\partial y}\frac{\partial\varphi_j^{(6)}}{\partial y}\right)\right]\mathrm{d}x\mathrm{d}y, \tag{99}$$

$$K_{ij}^{0707}=\int_{\Omega}A_{11}^{(4)}\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial\varphi_j^{(7)}}{\partial x}+B_{66}^{(4)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial\varphi_j^{(7)}}{\partial y}+4B_{55}^{(2)}\varphi_i^{(7)}\varphi_j^{(7)}$$
$$+R_{22}^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial\varphi_j^{(7)}}{\partial y}+R_{33}^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial y}\frac{\partial\varphi_j^{(7)}}{\partial y}+R_{66}^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial x}\frac{\partial\varphi_j^{(7)}}{\partial x}+R_{44}^{(0)}\varphi_i^{(7)}\varphi_j^{(7)}$$
$$-\frac{1}{2}R_{44}^{(2)}\left(\frac{\partial^2\varphi_i^{(7)}}{\partial y^2}\varphi_j^{(7)}+\varphi_i^{(7)}\frac{\partial^2\varphi_j^{(7)}}{\partial y^2}\right)+\frac{1}{4}\left(R_{44}^{(4)}\frac{\partial^2\varphi_i^{(7)}}{\partial y^2}\frac{\partial^2\varphi_j^{(7)}}{\partial y^2}+R_{55}^{(4)}\frac{\partial^2\varphi_i^{(7)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(7)}}{\partial x\partial y}\right)\mathrm{d}x\mathrm{d}y, \tag{100}$$

$$K_{ij}^{0708} = \int_\Omega A_{12}^{(4)} \frac{\partial \varphi_i^{(7)}}{\partial x} \frac{\partial \varphi_j^{(8)}}{\partial y} + B_{66}^{(4)} \frac{\partial \varphi_i^{(7)}}{\partial y} \frac{\partial \varphi_j^{(8)}}{\partial x} - R_{33}^{(2)} \frac{\partial \varphi_i^{(7)}}{\partial y} \frac{\partial \varphi_j^{(8)}}{\partial x} - R_{66}^{(2)} \frac{\partial \varphi_i^{(7)}}{\partial x} \frac{\partial \varphi_j^{(8)}}{\partial y}$$
$$+ \frac{1}{2} \left[R_{44}^{(2)} \varphi_i^{(7)} \frac{\partial^2 \varphi_j^{(8)}}{\partial x \partial y} + R_{55}^{(2)} \frac{\partial^2 \varphi_i^{(7)}}{\partial x \partial y} \varphi_j^{(8)} - \frac{1}{2} \left(R_{44}^{(4)} \frac{\partial^2 \varphi_i^{(7)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(8)}}{\partial x \partial y} + R_{55}^{(4)} \frac{\partial^2 \varphi_i^{(7)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(8)}}{\partial x^2} \right) \right] \mathrm{d}x\mathrm{d}y, \tag{101}$$

$$K_{ij}^{0709} = \int_\Omega 2A_{13}^{(3)} \frac{\partial \varphi_i^{(7)}}{\partial x} \varphi_j^{(9)} + 2B_{55}^{(3)} \varphi_i^{(7)} \frac{\partial \varphi_j^{(9)}}{\partial x} + \frac{1}{2} \left[R_{66}^{(3)} \left(\frac{\partial \varphi_i^{(7)}}{\partial x} \frac{\partial^2 \varphi_j^{(9)}}{\partial y^2} - \frac{\partial \varphi_i^{(7)}}{\partial x} \frac{\partial^2 \varphi_j^{(9)}}{\partial x^2} \right) \right.$$
$$\left. - R_{22}^{(3)} \frac{\partial \varphi_i^{(7)}}{\partial y} \frac{\partial^2 \varphi_j^{(9)}}{\partial x \partial y} + R_{44}^{(3)} \frac{\partial^2 \varphi_i^{(7)}}{\partial y^2} \frac{\partial \varphi_j^{(9)}}{\partial x} - R_{55}^{(3)} \frac{\partial^2 \varphi_i^{(7)}}{\partial x \partial y} \frac{\partial \varphi_j^{(9)}}{\partial y} - 2R_{44}^{(1)} \varphi_i^{(7)} \frac{\partial \varphi_j^{(9)}}{\partial x} \right] \mathrm{d}x\mathrm{d}y, \tag{102}$$

$$K_{ij}^{0710} = \int_\Omega A_{11}^{(5)} \frac{\partial \varphi_i^{(7)}}{\partial x} \frac{\partial \varphi_j^{(10)}}{\partial x} + B_{66}^{(5)} \frac{\partial \varphi_i^{(7)}}{\partial y} \frac{\partial \varphi_j^{(10)}}{\partial y} + 6B_{55}^{(3)} \varphi_i^{(7)} \varphi_j^{(10)} + 3R_{44}^{(1)} \varphi_i^{(7)} \varphi_j^{(10)}$$
$$+ \frac{3}{2} \left[R_{22}^{(3)} \frac{\partial \varphi_i^{(7)}}{\partial y} \frac{\partial \varphi_j^{(10)}}{\partial y} + R_{33}^{(3)} \frac{\partial \varphi_i^{(7)}}{\partial y} \frac{\partial \varphi_j^{(10)}}{\partial y} - R_{44}^{(3)} \frac{\partial^2 \varphi_i^{(7)}}{\partial y^2} \varphi_j^{(10)} + R_{66}^{(3)} \frac{\partial \varphi_i^{(7)}}{\partial x} \frac{\partial \varphi_j^{(10)}}{\partial x} \right.$$
$$\left. + \frac{1}{6} \left(R_{44}^{(5)} \frac{\partial^2 \varphi_i^{(7)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(10)}}{\partial y^2} + R_{55}^{(5)} \frac{\partial^2 \varphi_i^{(7)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(10)}}{\partial x \partial y} \right) - \frac{1}{3} R_{44}^{(3)} \varphi_i^{(7)} \frac{\partial^2 \varphi_j^{(10)}}{\partial y^2} \right] \mathrm{d}x\mathrm{d}y, \tag{103}$$

$$K_{ij}^{0711} = \int_\Omega A_{12}^{(5)} \frac{\partial \varphi_i^{(7)}}{\partial x} \frac{\partial \varphi_j^{(11)}}{\partial y} + B_{66}^{(5)} \frac{\partial \varphi_i^{(7)}}{\partial y} \frac{\partial \varphi_j^{(11)}}{\partial x} - \frac{3}{2} \left[R_{33}^{(3)} \frac{\partial \varphi_i^{(7)}}{\partial y} \frac{\partial \varphi_j^{(11)}}{\partial x} - R_{55}^{(3)} \frac{\partial^2 \varphi_i^{(7)}}{\partial x \partial y} \varphi_j^{(11)} \right.$$
$$+ R_{66}^{(3)} \frac{\partial \varphi_i^{(7)}}{\partial x} \frac{\partial \varphi_j^{(11)}}{\partial y} - \frac{1}{3} R_{44}^{(3)} \varphi_i^{(7)} \frac{\partial^2 \varphi_j^{(11)}}{\partial x \partial y} + \frac{1}{6} \left(R_{44}^{(5)} \frac{\partial^2 \varphi_i^{(7)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(11)}}{\partial x \partial y} \right.$$
$$\left. \left. + R_{55}^{(5)} \frac{\partial^2 \varphi_i^{(7)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(11)}}{\partial x^2} \right) \right] \mathrm{d}x\mathrm{d}y, \tag{104}$$

$$K_{ij}^{0801} = \int_\Omega A_{21}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(1)}}{\partial x} + B_{66}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(1)}}{\partial y} - \frac{1}{4} \left(R_{44}^{(2)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(1)}}{\partial y^2} + R_{55}^{(2)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(1)}}{\partial x \partial y} \right)$$
$$+ \frac{1}{2} R_{55}^{(0)} \varphi_i^{(8)} \frac{\partial^2 \varphi_j^{(1)}}{\partial x \partial y} \mathrm{d}x\mathrm{d}y, \tag{105}$$

$$K_{ij}^{0802} = \int_\Omega A_{22}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(2)}}{\partial y} + B_{66}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(2)}}{\partial x} + \frac{1}{4} \left(R_{44}^{(2)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(2)}}{\partial x \partial y} + R_{55}^{(2)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(2)}}{\partial x^2} \right)$$
$$- \frac{1}{2} R_{55}^{(0)} \varphi_i^{(8)} \frac{\partial^2 \varphi_j^{(2)}}{\partial x^2} \mathrm{d}x\mathrm{d}y, \tag{106}$$

$$K_{ij}^{0803} = \frac{1}{2} \int_\Omega \frac{\partial w}{\partial x} \left(A_{21}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(3)}}{\partial x} + B_{66}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(3)}}{\partial y} \right)$$
$$+ \frac{\partial w}{\partial y} \left(A_{22}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(3)}}{\partial y} + B_{66}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(3)}}{\partial x} \right) + 4B_{44}^{(1)} \varphi_i^{(8)} \frac{\partial \varphi_j^{(3)}}{\partial y}$$
$$+ R_{66}^{(1)} \left(\frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial^2 \varphi_j^{(3)}}{\partial x^2} - \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial^2 \varphi_j^{(3)}}{\partial y^2} \right) - R_{11}^{(1)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial^2 \varphi_j^{(3)}}{\partial x \partial y} \mathrm{d}x\mathrm{d}y, \tag{107}$$

$$K_{ij}^{0804} = \int_\Omega A_{21}^{(3)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(4)}}{\partial x} + B_{66}^{(3)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(4)}}{\partial y} - \frac{1}{2} \left[R_{33}^{(1)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(4)}}{\partial y} - R_{55}^{(1)} \varphi_i^{(8)} \frac{\partial^2 \varphi_j^{(4)}}{\partial x \partial y} \right.$$
$$\left. + R_{66}^{(1)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(4)}}{\partial x} + \frac{1}{2} \left(R_{44}^{(3)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(4)}}{\partial y^2} + R_{55}^{(3)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(4)}}{\partial x \partial y} \right) \right] \mathrm{d}x\mathrm{d}y, \tag{108}$$

$$K_{ij}^{0805} = \int_{\Omega} A_{22}^{(3)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(5)}}{\partial y} + B_{66}^{(3)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(5)}}{\partial x} + 2B_{44}^{(1)} \varphi_i^{(8)} \varphi_j^{(5)} + \frac{1}{2} \left[R_{11}^{(1)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(5)}}{\partial x} + R_{33}^{(1)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(5)}}{\partial x} - R_{55}^{(1)} \varphi_i^{(8)} \frac{\partial^2 \varphi_j^{(5)}}{\partial x^2} + R_{66}^{(1)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(5)}}{\partial y} + \frac{1}{2} \left(R_{44}^{(3)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(5)}}{\partial x \partial y} + R_{55}^{(3)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(5)}}{\partial x^2} \right) \right] \mathrm{d}x \mathrm{d}y, \quad (109)$$

$$K_{ij}^{0806} = \int_{\Omega} A_{23}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial y} \varphi_j^{(6)} + 2B_{44}^{(2)} \varphi_i^{(8)} \frac{\partial \varphi_j^{(6)}}{\partial y} + \frac{1}{2} \left[R_{66}^{(2)} \left(\frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial^2 \varphi_j^{(6)}}{\partial x^2} - \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial^2 \varphi_j^{(6)}}{\partial y^2} \right) - R_{11}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial^2 \varphi_j^{(6)}}{\partial x \partial y} - R_{55}^{(0)} \varphi_i^{(8)} \frac{\partial \varphi_j^{(6)}}{\partial y} - \frac{1}{2} \left(R_{44}^{(2)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x \partial y} \frac{\partial \varphi_j^{(6)}}{\partial x} - R_{55}^{(2)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x^2} \frac{\partial \varphi_j^{(6)}}{\partial y} \right) \right] \mathrm{d}x \mathrm{d}y, \quad (110)$$

$$K_{ij}^{0807} = \int_{\Omega} A_{21}^{(4)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(7)}}{\partial x} + B_{66}^{(4)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(7)}}{\partial y} - R_{33}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(7)}}{\partial y} - R_{66}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(7)}}{\partial x} + \frac{1}{2} \left[R_{44}^{(2)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x \partial y} \varphi_j^{(7)} + R_{55}^{(2)} \varphi_i^{(8)} \frac{\partial^2 \varphi_j^{(7)}}{\partial x \partial y} - \frac{1}{2} \left(R_{44}^{(4)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(7)}}{\partial y^2} + R_{55}^{(4)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(7)}}{\partial x \partial y} \right) \right] \mathrm{d}x \mathrm{d}y, \quad (111)$$

$$K_{ij}^{0808} = \int_{\Omega} A_{22}^{(4)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(8)}}{\partial y} + B_{66}^{(4)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(8)}}{\partial x} + 4B_{44}^{(2)} \varphi_i^{(8)} \varphi_j^{(8)} + R_{55}^{(0)} \varphi_i^{(8)} \varphi_j^{(8)} + R_{11}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(8)}}{\partial x} + R_{33}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(8)}}{\partial x} + R_{66}^{(2)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(8)}}{\partial y} - \frac{1}{2} R_{55}^{(2)} \left(\varphi_i^{(8)} \frac{\partial^2 \varphi_j^{(8)}}{\partial x^2} + \frac{\partial^2 \varphi_i^{(8)}}{\partial x^2} \varphi_j^{(8)} \right) + \frac{1}{4} \left(R_{44}^{(4)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(8)}}{\partial x \partial y} + R_{55}^{(4)} \frac{\partial^2 \varphi_j^{(8)}}{\partial x^2} \frac{\partial^2 \varphi_i^{(8)}}{\partial x^2} \right) \mathrm{d}x \mathrm{d}y, \quad (112)$$

$$K_{ij}^{0809} = \int_{\Omega} 2A_{23}^{(3)} \frac{\partial \varphi_i^{(8)}}{\partial y} \varphi_j^{(9)} + 2B_{44}^{(3)} \varphi_i^{(8)} \frac{\partial \varphi_j^{(9)}}{\partial y} - R_{55}^{(1)} \varphi_i^{(8)} \frac{\partial \varphi_j^{(9)}}{\partial y} - \frac{1}{2} \left[R_{11}^{(3)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial^2 \varphi_j^{(9)}}{\partial x \partial y} + R_{44}^{(3)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x \partial y} \frac{\partial \varphi_j^{(9)}}{\partial x} - R_{55}^{(3)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x^2} \frac{\partial \varphi_j^{(9)}}{\partial y} + R_{66}^{(3)} \left(\frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial^2 \varphi_j^{(9)}}{\partial y^2} - \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial^2 \varphi_j^{(9)}}{\partial x^2} \right) \right] \mathrm{d}x \mathrm{d}y, \quad (113)$$

$$K_{ij}^{0810} = \int_{\Omega} A_{21}^{(5)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(10)}}{\partial x} + B_{66}^{(5)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(10)}}{\partial y} - \frac{3}{2} \left[R_{33}^{(3)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(10)}}{\partial y} - R_{44}^{(3)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x \partial y} \varphi_j^{(10)} + R_{66}^{(3)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(10)}}{\partial x} - \frac{1}{3} R_{55}^{(3)} \varphi_i^{(8)} \frac{\partial^2 \varphi_j^{(10)}}{\partial x \partial y} + \frac{1}{6} \left(R_{44}^{(5)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(10)}}{\partial y^2} + R_{55}^{(5)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(10)}}{\partial x \partial y} \right) \right] \mathrm{d}x \mathrm{d}y, \quad (114)$$

$$K_{ij}^{0811} = \int_{\Omega} A_{22}^{(5)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(11)}}{\partial y} + B_{66}^{(5)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(11)}}{\partial x} + 6B_{44}^{(3)} \varphi_i^{(8)} \varphi_j^{(11)} + 3R_{55}^{(1)} \varphi_i^{(8)} \varphi_j^{(11)} + \frac{3}{2} \left[R_{11}^{(3)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(11)}}{\partial x} + R_{33}^{(3)} \frac{\partial \varphi_i^{(8)}}{\partial x} \frac{\partial \varphi_j^{(11)}}{\partial x} - R_{55}^{(3)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x^2} \varphi_j^{(11)} + R_{66}^{(3)} \frac{\partial \varphi_i^{(8)}}{\partial y} \frac{\partial \varphi_j^{(11)}}{\partial y} + \frac{1}{6} \left(R_{44}^{(5)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(11)}}{\partial x \partial y} + R_{55}^{(5)} \frac{\partial^2 \varphi_i^{(8)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(11)}}{\partial x^2} \right) - \frac{1}{3} R_{55}^{(3)} \varphi_i^{(8)} \frac{\partial^2 \varphi_j^{(11)}}{\partial x^2} \right] \mathrm{d}x \mathrm{d}y, \quad (115)$$

$$K_{ij}^{0901} = \int_{\Omega} 2A_{31}^{(1)} \varphi_i^{(9)} \frac{\partial \varphi_j^{(1)}}{\partial x} + \frac{1}{2} \left(R_{44}^{(1)} \frac{\partial \varphi_i^{(9)}}{\partial x} \frac{\partial^2 \varphi_j^{(1)}}{\partial y^2} - R_{55}^{(1)} \frac{\partial \varphi_i^{(9)}}{\partial y} \frac{\partial^2 \varphi_j^{(1)}}{\partial x \partial y} \right) \mathrm{d}x \mathrm{d}y, \quad (116)$$

$$K_{ij}^{0902} = \int_{\Omega} 2A_{32}^{(1)} \varphi_i^{(9)} \frac{\partial \varphi_j^{(2)}}{\partial y} - \frac{1}{2} \left(R_{44}^{(1)} \frac{\partial \varphi_i^{(9)}}{\partial x} \frac{\partial^2 \varphi_j^{(2)}}{\partial x \partial y} - R_{55}^{(1)} \frac{\partial \varphi_i^{(9)}}{\partial y} \frac{\partial^2 \varphi_j^{(2)}}{\partial x^2} \right) \mathrm{d}x \mathrm{d}y, \quad (117)$$

$$K_{ij}^{0903} = \int_\Omega \frac{\partial w}{\partial x} A_{31}^{(1)} \varphi_i^{(9)} \frac{\partial \varphi_j^{(3)}}{\partial x} + \frac{\partial w}{\partial y} A_{32}^{(1)} \varphi_i^{(9)} \frac{\partial \varphi_j^{(3)}}{\partial y} + B_{44}^{(2)} \frac{\partial \varphi_i^{(9)}}{\partial y} \frac{\partial \varphi_j^{(3)}}{\partial y} + B_{55}^{(2)} \frac{\partial \varphi_i^{(9)}}{\partial x} \frac{\partial \varphi_j^{(3)}}{\partial x}$$
$$+ \frac{1}{4} \left[R_{66}^{(2)} \left(\frac{\partial^2 \varphi_i^{(9)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(3)}}{\partial x^2} + \frac{\partial^2 \varphi_j^{(3)}}{\partial y^2} \frac{\partial^2 \varphi_i^{(9)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(9)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(3)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(9)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(3)}}{\partial x^2} \right) \right.$$
$$\left. + R_{11}^{(2)} \frac{\partial^2 \varphi_i^{(9)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(3)}}{\partial x \partial y} + R_{22}^{(2)} \frac{\partial^2 \varphi_i^{(9)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(3)}}{\partial x \partial y} \right] \mathrm{d}x\mathrm{d}y, \tag{118}$$

$$K_{ij}^{0904} = \int_\Omega 2A_{31}^{(2)} \varphi_i^{(9)} \frac{\partial \varphi_j^{(4)}}{\partial x} + B_{55}^{(2)} \frac{\partial \varphi_i^{(9)}}{\partial x} \varphi_j^{(4)} + \frac{1}{4} \left[R_{66}^{(2)} \left(\frac{\partial^2 \varphi_i^{(9)}}{\partial y^2} \frac{\partial \varphi_j^{(4)}}{\partial x} - \frac{\partial^2 \varphi_i^{(9)}}{\partial x^2} \frac{\partial \varphi_j^{(4)}}{\partial x} \right) \right.$$
$$\left. - R_{22}^{(2)} \frac{\partial^2 \varphi_i^{(9)}}{\partial x \partial y} \frac{\partial \varphi_j^{(4)}}{\partial y} + 2 \left(R_{44}^{(2)} \frac{\partial \varphi_i^{(9)}}{\partial x} \frac{\partial^2 \varphi_j^{(4)}}{\partial y^2} - R_{55}^{(2)} \frac{\partial \varphi_i^{(9)}}{\partial y} \frac{\partial^2 \varphi_j^{(4)}}{\partial x \partial y} \right) \right] \mathrm{d}x\mathrm{d}y, \tag{119}$$

$$K_{ij}^{0905} = \int_\Omega 2A_{32}^{(2)} \varphi_i^{(9)} \frac{\partial \varphi_j^{(5)}}{\partial y} + B_{44}^{(2)} \frac{\partial \varphi_i^{(9)}}{\partial y} \varphi_j^{(5)} + \frac{1}{4} \left[R_{66}^{(2)} \left(\frac{\partial^2 \varphi_i^{(9)}}{\partial x^2} \frac{\partial \varphi_j^{(5)}}{\partial y} - \frac{\partial^2 \varphi_i^{(9)}}{\partial y^2} \frac{\partial \varphi_j^{(5)}}{\partial y} \right) \right.$$
$$\left. - R_{11}^{(2)} \frac{\partial^2 \varphi_i^{(9)}}{\partial x \partial y} \frac{\partial \varphi_j^{(5)}}{\partial x} - 2 \left(R_{44}^{(2)} \frac{\partial \varphi_i^{(9)}}{\partial x} \frac{\partial^2 \varphi_j^{(5)}}{\partial x \partial y} - R_{55}^{(2)} \frac{\partial \varphi_i^{(9)}}{\partial y} \frac{\partial^2 \varphi_j^{(5)}}{\partial x^2} \right) \right] \mathrm{d}x\mathrm{d}y, \tag{120}$$

$$K_{ij}^{0906} = \int_\Omega 2A_{33}^{(1)} \varphi_i^{(9)} \varphi_j^{(6)} + B_{44}^{(3)} \frac{\partial \varphi_i^{(9)}}{\partial y} \frac{\partial \varphi_j^{(6)}}{\partial y} + B_{55}^{(3)} \frac{\partial \varphi_i^{(9)}}{\partial x} \frac{\partial \varphi_j^{(6)}}{\partial x}$$
$$+ \frac{1}{4} \left[R_{11}^{(3)} \frac{\partial^2 \varphi_i^{(9)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(6)}}{\partial x \partial y} + R_{22}^{(3)} \frac{\partial^2 \varphi_i^{(9)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(6)}}{\partial x \partial y} + 2 \left(R_{44}^{(1)} \frac{\partial \varphi_i^{(9)}}{\partial x} \frac{\partial \varphi_j^{(6)}}{\partial x} + R_{55}^{(1)} \frac{\partial \varphi_i^{(9)}}{\partial y} \frac{\partial \varphi_j^{(6)}}{\partial y} \right) \right.$$
$$\left. + R_{66}^{(3)} \left(\frac{\partial^2 \varphi_i^{(9)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(6)}}{\partial x^2} + \frac{\partial^2 \varphi_i^{(9)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(6)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(9)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(6)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(9)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(6)}}{\partial x^2} \right) \right] \mathrm{d}x\mathrm{d}y, \tag{121}$$

$$K_{ij}^{0907} = \int_\Omega 2A_{31}^{(3)} \varphi_i^{(9)} \frac{\partial \varphi_j^{(7)}}{\partial x} + 2B_{55}^{(3)} \frac{\partial \varphi_i^{(9)}}{\partial x} \varphi_j^{(7)} + \frac{1}{2} \left[R_{66}^{(3)} \left(\frac{\partial^2 \varphi_i^{(9)}}{\partial y^2} \frac{\partial \varphi_j^{(7)}}{\partial x} - \frac{\partial^2 \varphi_i^{(9)}}{\partial x^2} \frac{\partial \varphi_j^{(7)}}{\partial x} \right) \right.$$
$$\left. - R_{22}^{(3)} \frac{\partial^2 \varphi_i^{(9)}}{\partial x \partial y} \frac{\partial \varphi_j^{(7)}}{\partial y} + R_{44}^{(3)} \frac{\partial \varphi_i^{(9)}}{\partial x} \frac{\partial^2 \varphi_j^{(7)}}{\partial y^2} - R_{55}^{(3)} \frac{\partial \varphi_i^{(9)}}{\partial y} \frac{\partial^2 \varphi_j^{(7)}}{\partial x \partial y} - 2R_{44}^{(1)} \frac{\partial \varphi_i^{(9)}}{\partial x} \varphi_j^{(7)} \right] \mathrm{d}x\mathrm{d}y, \tag{122}$$

$$K_{ij}^{0908} = \int_\Omega 2A_{32}^{(3)} \varphi_i^{(9)} \frac{\partial \varphi_j^{(8)}}{\partial y} + 2B_{44}^{(3)} \frac{\partial \varphi_i^{(9)}}{\partial y} \varphi_j^{(8)} + \frac{1}{2} \left[R_{66}^{(3)} \left(\frac{\partial^2 \varphi_i^{(9)}}{\partial x^2} \frac{\partial \varphi_j^{(8)}}{\partial y} - \frac{\partial^2 \varphi_i^{(9)}}{\partial y^2} \frac{\partial \varphi_j^{(8)}}{\partial y} \right) \right.$$
$$\left. - R_{11}^{(3)} \frac{\partial^2 \varphi_i^{(9)}}{\partial x \partial y} \frac{\partial \varphi_j^{(8)}}{\partial x} - R_{44}^{(3)} \frac{\partial \varphi_i^{(9)}}{\partial x} \frac{\partial^2 \varphi_j^{(8)}}{\partial x \partial y} + R_{55}^{(3)} \frac{\partial \varphi_i^{(9)}}{\partial y} \frac{\partial^2 \varphi_j^{(8)}}{\partial x^2} - 2R_{55}^{(1)} \frac{\partial \varphi_i^{(9)}}{\partial y} \varphi_j^{(8)} \right] \mathrm{d}x\mathrm{d}y, \tag{123}$$

$$K_{ij}^{0909} = \int_\Omega 4A_{33}^{(2)} \varphi_i^{(9)} \varphi_j^{(9)} + B_{44}^{(4)} \frac{\partial \varphi_i^{(9)}}{\partial y} \frac{\partial \varphi_j^{(9)}}{\partial y} + B_{55}^{(4)} \frac{\partial \varphi_i^{(9)}}{\partial x} \frac{\partial \varphi_j^{(9)}}{\partial x} + R_{44}^{(2)} \frac{\partial \varphi_i^{(9)}}{\partial x} \frac{\partial \varphi_j^{(9)}}{\partial x}$$
$$+ R_{55}^{(2)} \frac{\partial \varphi_i^{(9)}}{\partial y} \frac{\partial \varphi_j^{(9)}}{\partial y} + \frac{1}{4} \left[R_{11}^{(4)} \frac{\partial^2 \varphi_i^{(9)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(9)}}{\partial x \partial y} + R_{22}^{(4)} \frac{\partial^2 \varphi_i^{(9)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(9)}}{\partial x \partial y} \right.$$
$$\left. + R_{66}^{(4)} \left(\frac{\partial^2 \varphi_i^{(9)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(9)}}{\partial x^2} + \frac{\partial^2 \varphi_i^{(9)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(9)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(9)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(9)}}{\partial y^2} - \frac{\partial^2 \varphi_i^{(9)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(9)}}{\partial x^2} \right) \right] \mathrm{d}x\mathrm{d}y, \tag{124}$$

$$K_{ij}^{0910} = \int_\Omega 2A_{31}^{(4)} \varphi_i^{(9)} \frac{\partial \varphi_j^{(10)}}{\partial x} - 3 \left(R_{44}^{(2)} \frac{\partial \varphi_i^{(9)}}{\partial x} \varphi_j^{(10)} - B_{55}^{(4)} \frac{\partial \varphi_i^{(9)}}{\partial x} \varphi_j^{(10)} \right)$$
$$- \frac{3}{4} \left[R_{22}^{(4)} \frac{\partial^2 \varphi_i^{(9)}}{\partial x \partial y} \frac{\partial \varphi_j^{(10)}}{\partial y} + R_{66}^{(4)} \left(\frac{\partial^2 \varphi_i^{(9)}}{\partial x^2} \frac{\partial \varphi_j^{(10)}}{\partial x} - \frac{\partial^2 \varphi_i^{(9)}}{\partial y^2} \frac{\partial \varphi_j^{(10)}}{\partial x} \right) \right.$$
$$\left. - \frac{2}{3} \left(R_{44}^{(4)} \frac{\partial \varphi_i^{(9)}}{\partial x} \frac{\partial^2 \varphi_j^{(10)}}{\partial y^2} - R_{55}^{(4)} \frac{\partial \varphi_i^{(9)}}{\partial y} \frac{\partial^2 \varphi_j^{(10)}}{\partial x \partial y} \right) \right] \mathrm{d}x\mathrm{d}y, \tag{125}$$

$$K_{ij}^{0911} = \int_\Omega 2A_{32}^{(4)}\varphi_i^{(9)}\frac{\partial\varphi_j^{(11)}}{\partial y} + 3\left(B_{44}^{(4)}\frac{\partial\varphi_i^{(9)}}{\partial y}\varphi_j^{(11)} - R_{55}^{(2)}\frac{\partial\varphi_i^{(9)}}{\partial y}\varphi_j^{(11)}\right)$$
$$- \frac{3}{4}\left[R_{11}^{(4)}\frac{\partial^2\varphi_i^{(9)}}{\partial x\partial y}\frac{\partial\varphi_j^{(11)}}{\partial x} + R_{66}^{(4)}\left(\frac{\partial^2\varphi_i^{(9)}}{\partial y^2}\frac{\partial\varphi_j^{(11)}}{\partial y} - \frac{\partial^2\varphi_i^{(9)}}{\partial x^2}\frac{\partial\varphi_j^{(11)}}{\partial y}\right)\right.$$
$$\left. + \frac{2}{3}\left(R_{44}^{(4)}\frac{\partial\varphi_i^{(9)}}{\partial x}\frac{\partial^2\varphi_j^{(11)}}{\partial x\partial y} - R_{55}^{(4)}\frac{\partial\varphi_i^{(9)}}{\partial y}\frac{\partial^2\varphi_j^{(11)}}{\partial x^2}\right)\right]\mathrm{d}x\mathrm{d}y, \quad (126)$$

$$K_{ij}^{1001} = \int_\Omega A_{11}^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial\varphi_j^{(1)}}{\partial x} + B_{66}^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial\varphi_j^{(1)}}{\partial y} + \frac{1}{4}\left(R_{44}^{(3)}\frac{\partial^2\varphi_i^{(10)}}{\partial y^2}\frac{\partial^2\varphi_j^{(1)}}{\partial y^2} + R_{55}^{(3)}\frac{\partial^2\varphi_i^{(10)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(1)}}{\partial x\partial y}\right)$$
$$- \frac{3}{2}R_{44}^{(1)}\varphi_i^{(10)}\frac{\partial^2\varphi_j^{(1)}}{\partial y^2}\mathrm{d}x\mathrm{d}y, \quad (127)$$

$$K_{ij}^{1002} = \int_\Omega A_{12}^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial\varphi_j^{(2)}}{\partial y} + B_{66}^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial\varphi_j^{(2)}}{\partial x} - \frac{1}{4}\left(R_{44}^{(3)}\frac{\partial^2\varphi_i^{(10)}}{\partial y^2}\frac{\partial^2\varphi_j^{(2)}}{\partial x\partial y} + R_{55}^{(3)}\frac{\partial^2\varphi_i^{(10)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(2)}}{\partial x^2}\right)$$
$$+ \frac{3}{2}R_{44}^{(1)}\varphi_i^{(10)}\frac{\partial^2\varphi_j^{(2)}}{\partial x\partial y}\mathrm{d}x\mathrm{d}y, \quad (128)$$

$$K_{ij}^{1003} = \frac{1}{2}\int_\Omega \frac{\partial w}{\partial x}\left(A_{11}^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial\varphi_j^{(3)}}{\partial x} + B_{66}^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial\varphi_j^{(3)}}{\partial y}\right)$$
$$+ \frac{\partial w}{\partial y}\left(A_{12}^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial\varphi_j^{(3)}}{\partial y} + B_{66}^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial\varphi_j^{(3)}}{\partial x}\right) + 6B_{55}^{(2)}\varphi_i^{(10)}\frac{\partial\varphi_j^{(3)}}{\partial x}$$
$$+ \frac{3}{2}\left[R_{66}^{(2)}\left(\frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial^2\varphi_j^{(3)}}{\partial y^2} - \frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial^2\varphi_j^{(3)}}{\partial x^2}\right) - R_{22}^{(2)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial^2\varphi_j^{(3)}}{\partial x\partial y}\right]\mathrm{d}x\mathrm{d}y, \quad (129)$$

$$K_{ij}^{1004} = \int_\Omega A_{11}^{(4)}\frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial\varphi_j^{(4)}}{\partial x} + B_{66}^{(4)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial\varphi_j^{(4)}}{\partial y} + 3B_{55}^{(2)}\varphi_i^{(10)}\varphi_j^{(4)} + \frac{3}{4}\left[R_{22}^{(2)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial\varphi_j^{(4)}}{\partial y}\right.$$
$$+ R_{33}^{(2)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial\varphi_j^{(4)}}{\partial y} - 2R_{44}^{(2)}\varphi_i^{(10)}\frac{\partial^2\varphi_j^{(4)}}{\partial y^2} + R_{66}^{(2)}\frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial\varphi_j^{(4)}}{\partial x} + \frac{1}{3}\left(R_{44}^{(4)}\frac{\partial^2\varphi_i^{(10)}}{\partial y^2}\frac{\partial^2\varphi_j^{(4)}}{\partial y^2}\right.$$
$$\left.\left. + R_{55}^{(4)}\frac{\partial^2\varphi_i^{(10)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(4)}}{\partial x\partial y}\right)\right]\mathrm{d}x\mathrm{d}y, \quad (130)$$

$$K_{ij}^{1005} = \int_\Omega A_{12}^{(4)}\frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial\varphi_j^{(5)}}{\partial y} + B_{66}^{(4)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial\varphi_j^{(5)}}{\partial x} - \frac{3}{4}\left[R_{33}^{(2)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial\varphi_j^{(5)}}{\partial x} + R_{66}^{(2)}\frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial\varphi_j^{(5)}}{\partial y}\right.$$
$$\left. - 2R_{44}^{(2)}\varphi_i^{(10)}\frac{\partial^2\varphi_j^{(5)}}{\partial x\partial y} + \frac{1}{3}\left(R_{44}^{(4)}\frac{\partial^2\varphi_i^{(10)}}{\partial y^2}\frac{\partial^2\varphi_j^{(5)}}{\partial x\partial y} + R_{55}^{(4)}\frac{\partial^2\varphi_i^{(10)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(5)}}{\partial x^2}\right)\right]\mathrm{d}x\mathrm{d}y, \quad (131)$$

$$K_{ij}^{1006} = \int_\Omega A_{13}^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial x}\varphi_j^{(6)} + 3B_{55}^{(3)}\varphi_i^{(10)}\frac{\partial\varphi_j^{(6)}}{\partial x} + \frac{3}{4}\left[R_{66}^{(3)}\left(\frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial^2\varphi_j^{(6)}}{\partial y^2} - \frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial^2\varphi_j^{(6)}}{\partial x^2}\right)\right.$$
$$- R_{22}^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial^2\varphi_j^{(6)}}{\partial x\partial y} + \frac{1}{3}\left(R_{44}^{(3)}\frac{\partial^2\varphi_i^{(10)}}{\partial y^2}\frac{\partial\varphi_j^{(6)}}{\partial x} - R_{55}^{(3)}\frac{\partial^2\varphi_i^{(10)}}{\partial x\partial y}\frac{\partial\varphi_j^{(6)}}{\partial y}\right)$$
$$\left. - 2R_{44}^{(1)}\varphi_i^{(10)}\frac{\partial\varphi_j^{(6)}}{\partial x}\right]\mathrm{d}x\mathrm{d}y, \quad (132)$$

$$K_{ij}^{1007} = \int_\Omega A_{11}^{(5)}\frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial\varphi_j^{(7)}}{\partial x} + B_{66}^{(5)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial\varphi_j^{(7)}}{\partial y} + 6B_{55}^{(3)}\varphi_i^{(10)}\varphi_j^{(7)} + 3R_{44}^{(1)}\varphi_i^{(10)}\varphi_j^{(7)}$$
$$+ \frac{3}{2}\left[R_{22}^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial\varphi_j^{(7)}}{\partial y} + R_{33}^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial y}\frac{\partial\varphi_j^{(7)}}{\partial y} - R_{44}^{(3)}\varphi_i^{(10)}\frac{\partial^2\varphi_j^{(7)}}{\partial y^2} + R_{66}^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial x}\frac{\partial\varphi_j^{(7)}}{\partial x}\right.$$
$$\left. + \frac{1}{6}\left(R_{44}^{(5)}\frac{\partial^2\varphi_i^{(10)}}{\partial y^2}\frac{\partial^2\varphi_j^{(7)}}{\partial y^2} + R_{55}^{(5)}\frac{\partial^2\varphi_i^{(10)}}{\partial x\partial y}\frac{\partial^2\varphi_j^{(7)}}{\partial x\partial y}\right) - \frac{1}{3}R_{44}^{(3)}\frac{\partial^2\varphi_i^{(10)}}{\partial y^2}\varphi_j^{(7)}\right]\mathrm{d}x\mathrm{d}y, \quad (133)$$

$$K_{ij}^{1008} = \int_\Omega A_{12}^{(5)} \frac{\partial \varphi_i^{(10)}}{\partial x} \frac{\partial \varphi_j^{(8)}}{\partial y} + B_{66}^{(5)} \frac{\partial \varphi_i^{(10)}}{\partial y} \frac{\partial \varphi_j^{(8)}}{\partial x} - \frac{3}{2} \left[R_{33}^{(3)} \frac{\partial \varphi_i^{(10)}}{\partial y} \frac{\partial \varphi_j^{(8)}}{\partial x} - R_{44}^{(3)} \varphi_i^{(10)} \frac{\partial^2 \varphi_j^{(8)}}{\partial x \partial y} \right.$$
$$\left. + R_{66}^{(3)} \frac{\partial \varphi_i^{(10)}}{\partial x} \frac{\partial \varphi_j^{(8)}}{\partial y} - \frac{1}{3} R_{55}^{(3)} \frac{\partial^2 \varphi_i^{(10)}}{\partial x \partial y} \varphi_j^{(8)} + \frac{1}{6} \left(R_{44}^{(5)} \frac{\partial^2 \varphi_i^{(10)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(8)}}{\partial x \partial y} + R_{55}^{(5)} \frac{\partial^2 \varphi_i^{(10)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(8)}}{\partial x^2} \right) \right] \mathrm{d}x \mathrm{d}y, \tag{134}$$

$$K_{ij}^{1009} = \int_\Omega 2A_{13}^{(4)} \frac{\partial \varphi_i^{(10)}}{\partial x} \varphi_j^{(9)} - 3 \left(R_{44}^{(2)} \varphi_i^{(10)} \frac{\partial \varphi_j^{(9)}}{\partial x} - B_{55}^{(4)} \varphi_i^{(10)} \frac{\partial \varphi_j^{(9)}}{\partial x} \right)$$
$$- \frac{3}{4} \left[R_{22}^{(4)} \frac{\partial \varphi_i^{(10)}}{\partial y} \frac{\partial^2 \varphi_j^{(9)}}{\partial x \partial y} + R_{66}^{(4)} \left(\frac{\partial \varphi_i^{(10)}}{\partial x} \frac{\partial^2 \varphi_j^{(9)}}{\partial x^2} - \frac{\partial \varphi_i^{(10)}}{\partial x} \frac{\partial^2 \varphi_j^{(9)}}{\partial y^2} \right) \right.$$
$$\left. - \frac{2}{3} \left(R_{44}^{(4)} \frac{\partial^2 \varphi_i^{(10)}}{\partial y^2} \frac{\partial \varphi_j^{(9)}}{\partial x} - R_{55}^{(4)} \frac{\partial^2 \varphi_i^{(10)}}{\partial x \partial y} \frac{\partial \varphi_j^{(9)}}{\partial y} \right) \right] \mathrm{d}x \mathrm{d}y, \tag{135}$$

$$K_{ij}^{1010} = \int_\Omega A_{11}^{(6)} \frac{\partial \varphi_i^{(10)}}{\partial x} \frac{\partial \varphi_j^{(10)}}{\partial x} + B_{66}^{(6)} \frac{\partial \varphi_i^{(10)}}{\partial y} \frac{\partial \varphi_j^{(10)}}{\partial y} + 9 \left(B_{55}^{(4)} \varphi_i^{(10)} \varphi_j^{(10)} + R_{44}^{(2)} \varphi_i^{(10)} \varphi_j^{(10)} \right)$$
$$- \frac{3}{2} \left[R_{44}^{(4)} \left(\varphi_i^{(10)} \frac{\partial^2 \varphi_j^{(10)}}{\partial y^2} + \frac{\partial^2 \varphi_i^{(10)}}{\partial y^2} \varphi_j^{(10)} \right) - \frac{3}{2} \left(R_{22}^{(4)} \frac{\partial \varphi_i^{(10)}}{\partial y} \frac{\partial \varphi_j^{(10)}}{\partial y} + R_{33}^{(4)} \frac{\partial \varphi_i^{(10)}}{\partial y} \frac{\partial \varphi_j^{(10)}}{\partial y} \right. \right.$$
$$\left. \left. + R_{66}^{(4)} \frac{\partial \varphi_i^{(10)}}{\partial x} \frac{\partial \varphi_j^{(10)}}{\partial x} \right) - \frac{1}{6} \left(R_{44}^{(6)} \frac{\partial^2 \varphi_i^{(10)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(10)}}{\partial y^2} + R_{55}^{(6)} \frac{\partial^2 \varphi_i^{(10)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(10)}}{\partial x \partial y} \right) \right] \mathrm{d}x \mathrm{d}y, \tag{136}$$

$$K_{ij}^{1011} = \int_\Omega A_{12}^{(6)} \frac{\partial \varphi_i^{(10)}}{\partial x} \frac{\partial \varphi_j^{(11)}}{\partial y} + B_{66}^{(6)} \frac{\partial \varphi_i^{(10)}}{\partial y} \frac{\partial \varphi_j^{(11)}}{\partial x} + \frac{3}{2} \left[\left(R_{44}^{(4)} \varphi_i^{(10)} \frac{\partial^2 \varphi_j^{(11)}}{\partial x \partial y} + R_{55}^{(4)} \frac{\partial^2 \varphi_i^{(10)}}{\partial x \partial y} \varphi_j^{(11)} \right) \right.$$
$$\left. - \frac{3}{2} \left(R_{33}^{(4)} \frac{\partial \varphi_i^{(10)}}{\partial y} \frac{\partial \varphi_j^{(11)}}{\partial x} + R_{66}^{(4)} \frac{\partial \varphi_i^{(10)}}{\partial x} \frac{\partial \varphi_j^{(11)}}{\partial y} \right) - \frac{1}{6} \left(R_{44}^{(6)} \frac{\partial^2 \varphi_i^{(10)}}{\partial y^2} \frac{\partial^2 \varphi_j^{(11)}}{\partial x \partial y} + R_{55}^{(6)} \frac{\partial^2 \varphi_i^{(10)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(11)}}{\partial x^2} \right) \right] \mathrm{d}x \mathrm{d}y, \tag{137}$$

$$K_{ij}^{1101} = \int_\Omega A_{21}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(1)}}{\partial x} + B_{66}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(1)}}{\partial y} - \frac{1}{4} \left(R_{44}^{(3)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(1)}}{\partial y^2} + R_{55}^{(3)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(1)}}{\partial x \partial y} \right)$$
$$+ \frac{3}{2} R_{55}^{(1)} \varphi_i^{(11)} \frac{\partial^2 \varphi_j^{(1)}}{\partial x \partial y} \mathrm{d}x \mathrm{d}y, \tag{138}$$

$$K_{ij}^{1102} = \int_\Omega A_{22}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(2)}}{\partial y} + B_{66}^{(3)} \frac{\partial \varphi_j^{(2)}}{\partial x} \frac{\partial \varphi_i^{(11)}}{\partial x} + \frac{1}{4} \left(R_{44}^{(3)} \frac{\partial^2 \varphi_j^{(2)}}{\partial x \partial y} \frac{\partial^2 \varphi_i^{(11)}}{\partial x \partial y} + R_{55}^{(3)} \frac{\partial^2 \varphi_j^{(2)}}{\partial x^2} \frac{\partial^2 \varphi_i^{(11)}}{\partial x^2} \right)$$
$$- \frac{3}{2} R_{55}^{(1)} \varphi_i^{(11)} \frac{\partial^2 \varphi_j^{(2)}}{\partial x^2} \mathrm{d}x \mathrm{d}y, \tag{139}$$

$$K_{ij}^{1103} = \frac{1}{2} \int_\Omega \frac{\partial w}{\partial x} \left(A_{21}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(3)}}{\partial x} + B_{66}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(3)}}{\partial y} \right)$$
$$+ \frac{\partial w}{\partial y} \left(A_{22}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(3)}}{\partial y} + B_{66}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(3)}}{\partial x} \right) + 6 B_{44}^{(2)} \varphi_i^{(11)} \frac{\partial \varphi_j^{(3)}}{\partial y}$$
$$+ \frac{3}{2} \left[R_{66}^{(2)} \left(\frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial^2 \varphi_j^{(3)}}{\partial x^2} - \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial^2 \varphi_j^{(3)}}{\partial y^2} \right) - R_{11}^{(2)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial^2 \varphi_j^{(3)}}{\partial x \partial y} \right] \mathrm{d}x \mathrm{d}y, \tag{140}$$

$$K_{ij}^{1104} = \int_\Omega A_{21}^{(4)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(4)}}{\partial x} + B_{66}^{(4)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(4)}}{\partial y} - \frac{3}{4} \left[R_{33}^{(2)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(4)}}{\partial y} - 2 R_{55}^{(2)} \varphi_i^{(11)} \frac{\partial^2 \varphi_j^{(4)}}{\partial x \partial y} \right.$$
$$\left. + R_{66}^{(2)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(4)}}{\partial x} + \frac{1}{3} \left(R_{44}^{(4)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(4)}}{\partial y^2} + R_{55}^{(4)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(4)}}{\partial x \partial y} \right) \right] \mathrm{d}x \mathrm{d}y, \tag{141}$$

$$K_{ij}^{1105} = \int_{\Omega} A_{22}^{(4)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(5)}}{\partial y} + B_{66}^{(4)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(5)}}{\partial x} + 3B_{44}^{(2)} \varphi_i^{(11)} \varphi_j^{(5)} + \frac{3}{4} \left[R_{11}^{(2)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(5)}}{\partial x} + R_{33}^{(2)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(5)}}{\partial x} - 2R_{55}^{(2)} \varphi_i^{(11)} \frac{\partial^2 \varphi_j^{(5)}}{\partial x^2} + R_{66}^{(2)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(5)}}{\partial y} + \frac{1}{3} \left(R_{44}^{(4)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(5)}}{\partial x \partial y} + R_{55}^{(4)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(5)}}{\partial x^2} \right) \right] \mathrm{d}x\mathrm{d}y, \tag{142}$$

$$K_{ij}^{1106} = \int_{\Omega} A_{23}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial y} \varphi_j^{(6)} + 3B_{44}^{(3)} \varphi_i^{(11)} \frac{\partial \varphi_j^{(6)}}{\partial y} + \frac{3}{4} \left[R_{66}^{(3)} \left(\frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial^2 \varphi_j^{(6)}}{\partial x^2} - \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial^2 \varphi_j^{(6)}}{\partial y^2} \right) - R_{11}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial^2 \varphi_j^{(6)}}{\partial x \partial y} - \frac{1}{3} \left(R_{44}^{(3)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x \partial y} \frac{\partial \varphi_j^{(6)}}{\partial x} - R_{55}^{(3)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x^2} \frac{\partial \varphi_j^{(6)}}{\partial y} \right) - 2R_{55}^{(1)} \varphi_i^{(11)} \frac{\partial \varphi_j^{(6)}}{\partial y} \right] \mathrm{d}x\mathrm{d}y, \tag{143}$$

$$K_{ij}^{1107} = \int_{\Omega} A_{21}^{(5)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(7)}}{\partial x} + B_{66}^{(5)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(7)}}{\partial y} - \frac{3}{2} \left[R_{33}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(7)}}{\partial y} - R_{55}^{(3)} \varphi_i^{(11)} \frac{\partial^2 \varphi_j^{(7)}}{\partial x \partial y} + R_{66}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(7)}}{\partial x} - \frac{1}{3} R_{44}^{(3)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x \partial y} \varphi_j^{(7)} + \frac{1}{6} \left(R_{44}^{(5)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(7)}}{\partial y^2} + R_{55}^{(5)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(7)}}{\partial x \partial y} \right) \right] \mathrm{d}x\mathrm{d}y, \tag{144}$$

$$K_{ij}^{1108} = \int_{\Omega} A_{22}^{(5)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(8)}}{\partial y} + B_{66}^{(5)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(8)}}{\partial x} + 6B_{44}^{(3)} \varphi_i^{(11)} \varphi_j^{(8)} + 3R_{55}^{(1)} \varphi_i^{(11)} \varphi_j^{(8)} + \frac{3}{2} \left[R_{11}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(8)}}{\partial x} + R_{33}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(8)}}{\partial x} - R_{55}^{(3)} \varphi_i^{(11)} \frac{\partial^2 \varphi_j^{(8)}}{\partial x^2} + R_{66}^{(3)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(8)}}{\partial y} + \frac{1}{6} \left(R_{44}^{(5)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(8)}}{\partial x \partial y} + R_{55}^{(5)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(8)}}{\partial x^2} \right) - \frac{1}{3} R_{55}^{(3)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x^2} \varphi_j^{(8)} \right] \mathrm{d}x\mathrm{d}y, \tag{145}$$

$$K_{ij}^{1109} = \int_{\Omega} 2A_{23}^{(4)} \frac{\partial \varphi_i^{(11)}}{\partial y} \varphi_j^{(9)} + 3\left(B_{44}^{(4)} \varphi_i^{(11)} \frac{\partial \varphi_j^{(9)}}{\partial y} - R_{55}^{(2)} \varphi_i^{(11)} \frac{\partial \varphi_j^{(9)}}{\partial y} \right) - \frac{3}{4} \left[R_{11}^{(4)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial^2 \varphi_j^{(9)}}{\partial x \partial y} + R_{66}^{(4)} \left(\frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial^2 \varphi_j^{(9)}}{\partial y^2} - \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial^2 \varphi_j^{(9)}}{\partial x^2} \right) + \frac{2}{3} \left(R_{44}^{(4)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x \partial y} \frac{\partial \varphi_j^{(9)}}{\partial x} - R_{55}^{(4)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x^2} \frac{\partial \varphi_j^{(9)}}{\partial y} \right) \right] \mathrm{d}x\mathrm{d}y, \tag{146}$$

$$K_{ij}^{1110} = \int_{\Omega} A_{21}^{(6)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(10)}}{\partial x} + B_{66}^{(6)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(10)}}{\partial y} + \frac{3}{2} \left[\left(R_{44}^{(4)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x \partial y} \varphi_j^{(10)} + R_{55}^{(4)} \varphi_i^{(11)} \frac{\partial^2 \varphi_j^{(10)}}{\partial x \partial y} \right) - \frac{3}{2} \left(R_{33}^{(4)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(10)}}{\partial y} + R_{66}^{(4)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(10)}}{\partial x} \right) - \frac{1}{6} \left(R_{44}^{(6)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(10)}}{\partial y^2} + R_{55}^{(6)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(10)}}{\partial x \partial y} \right) \right] \mathrm{d}x\mathrm{d}y, \tag{147}$$

$$K_{ij}^{1111} = \int_{\Omega} A_{22}^{(6)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(11)}}{\partial y} + B_{66}^{(6)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(11)}}{\partial x} + 9\left(B_{44}^{(4)} \varphi_i^{(11)} \varphi_j^{(11)} + R_{55}^{(2)} \varphi_i^{(11)} \varphi_j^{(11)} \right) - \frac{3}{2} \left[R_{55}^{(4)} \left(\varphi_i^{(11)} \frac{\partial^2 \varphi_j^{(11)}}{\partial x^2} + \frac{\partial^2 \varphi_i^{(11)}}{\partial x^2} \varphi_j^{(11)} \right) - \frac{3}{2} \left(R_{11}^{(4)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(11)}}{\partial x} + R_{33}^{(4)} \frac{\partial \varphi_i^{(11)}}{\partial x} \frac{\partial \varphi_j^{(11)}}{\partial x} + R_{66}^{(4)} \frac{\partial \varphi_i^{(11)}}{\partial y} \frac{\partial \varphi_j^{(11)}}{\partial y} \right) - \frac{1}{6} \left(R_{44}^{(6)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x \partial y} \frac{\partial^2 \varphi_j^{(11)}}{\partial x \partial y} + R_{55}^{(6)} \frac{\partial^2 \varphi_i^{(11)}}{\partial x^2} \frac{\partial^2 \varphi_j^{(11)}}{\partial x^2} \right) \right] \mathrm{d}x\mathrm{d}y, \tag{148}$$

where the resultants of mass $\left(m^{(k)}\right)$ are

$$m^{(k)} = \int_{-\frac{h}{2}}^{\frac{h}{2}} (z)^k \, \rho(z) \, \mathrm{d}z, \tag{149}$$

and the plate stiffnesses ($A_{ij}^{(k)}$, $B_{mm}^{(k)}$, and $R_{nn}^{(k)}$) are

$$A_{ij}^{(k)} = \frac{1-\nu}{(1+\nu)(1-2\nu)} \int_{-\frac{h}{2}}^{\frac{h}{2}} (z)^k\, E(z)\, \mathrm{d}z, \qquad i = j, \tag{150}$$

$$A_{ij}^{(k)} = \frac{\nu}{(1+\nu)(1-2\nu)} \int_{-\frac{h}{2}}^{\frac{h}{2}} (z)^k\, E(z)\, \mathrm{d}z, \qquad i \neq j, \tag{151}$$

$$B_{mm}^{(k)} = \frac{1}{2(1+\nu)} \int_{-\frac{h}{2}}^{\frac{h}{2}} (z)^k\, E(z)\, \mathrm{d}z, \tag{152}$$

$$R_{ii}^{(k)} = \frac{l^2}{(1+\nu)} \int_{-\frac{h}{2}}^{\frac{h}{2}} (z)^k\, E(z)\, \mathrm{d}z, \tag{153}$$

$$R_{mm}^{(k)} = \frac{l^2}{2(1+\nu)} \int_{-\frac{h}{2}}^{\frac{h}{2}} (z)^k\, E(z)\, \mathrm{d}z \tag{154}$$

for $i = 1, 2, 3,\ j = 1, 2, 3,\ m = 4, 5, 6$ and $k = 1, 2, \ldots, 6$. The superscripts of mass and stiffness matrices vary 01 to 11 to distinguish $I = 1$ and $J = 11$ from $I = 11$ and $J = 1$. The explicit form of the force vector, $\{F\}$, is

$$F_i^1 = \int_\Omega f_x^{(0)}\varphi_i^{(1)} - \frac{1}{2}c_z^{(0)}\frac{\partial\varphi_i^{(1)}}{\partial y}\mathrm{d}x\mathrm{d}y + \int_\Gamma t_x^{(0)}\varphi_i^{(1)} - \frac{1}{2}s_z^{(0)}\frac{\partial\varphi_i^{(1)}}{\partial y}\mathrm{d}s, \tag{155}$$

$$F_i^2 = \int_\Omega f_y^{(0)}\varphi_i^{(2)} + \frac{1}{2}c_z^{(0)}\frac{\partial\varphi_i^{(2)}}{\partial x}\mathrm{d}x\mathrm{d}y + \int_\Gamma t_y^{(0)}\varphi_i^{(2)} + \frac{1}{2}s_z^{(0)}\frac{\partial\varphi_i^{(2)}}{\partial x}\mathrm{d}s, \tag{156}$$

$$F_i^3 = \int_\Omega f_z^{(0)}\varphi_i^{(3)} + \frac{1}{2}\left(c_x^{(0)}\frac{\partial\varphi_i^{(3)}}{\partial y} - c_y^{(0)}\frac{\partial\varphi_i^{(3)}}{\partial x}\right)\mathrm{d}x\mathrm{d}y + \int_\Gamma t_z^{(0)}\varphi_i^{(3)} + \frac{1}{2}\left(s_x^{(0)}\frac{\partial\varphi_i^{(3)}}{\partial y} - s_y^{(0)}\frac{\partial\varphi_i^{(3)}}{\partial x}\right)\mathrm{d}s, \tag{157}$$

$$F_i^4 = \int_\Omega f_x^{(1)}\varphi_i^{(4)} + \frac{1}{2}\left(c_y^{(0)}\varphi_i^{(4)} - c_z^{(1)}\frac{\partial\varphi_i^{(4)}}{\partial y}\right)\mathrm{d}x\mathrm{d}y + \int_\Gamma t_x^{(1)}\varphi_i^{(4)} + \frac{1}{2}\left(s_y^{(0)}\varphi_i^{(4)} - s_z^{(1)}\frac{\partial\varphi_i^{(4)}}{\partial y}\right)\mathrm{d}s, \tag{158}$$

$$F_i^5 = \int_\Omega f_y^{(1)}\varphi_i^{(5)} - \frac{1}{2}\left(c_x^{(0)}\varphi_i^{(5)} - c_z^{(1)}\frac{\partial\varphi_i^{(5)}}{\partial x}\right)\mathrm{d}x\mathrm{d}y + \int_\Gamma t_y^{(1)}\varphi_i^{(5)} - \frac{1}{2}\left(s_x^{(0)}\varphi_i^{(5)} - s_z^{(1)}\frac{\partial\varphi_i^{(5)}}{\partial x}\right)\mathrm{d}s, \tag{159}$$

$$F_i^6 = \int_\Omega f_z^{(1)}\varphi_i^{(6)} + \frac{1}{2}\left(c_x^{(1)}\frac{\partial\varphi_i^{(6)}}{\partial y} - c_y^{(1)}\frac{\partial\varphi_i^{(6)}}{\partial x}\right)\mathrm{d}x\mathrm{d}y + \int_\Gamma t_z^{(1)}\varphi_i^{(6)} + \frac{1}{2}\left(s_x^{(1)}\frac{\partial\varphi_i^{(6)}}{\partial y} - s_y^{(1)}\frac{\partial\varphi_i^{(6)}}{\partial x}\right)\mathrm{d}s, \tag{160}$$

$$F_i^7 = \int_\Omega f_x^{(2)}\varphi_i^{(7)} + \frac{1}{2}\left(2c_y^{(1)}\varphi_i^{(7)} - c_z^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial y}\right)\mathrm{d}x\mathrm{d}y + \int_\Gamma t_x^{(2)}\varphi_i^{(7)} + \frac{1}{2}\left(2s_y^{(1)}\varphi_i^{(7)} - s_z^{(2)}\frac{\partial\varphi_i^{(7)}}{\partial y}\right)\mathrm{d}s, \tag{161}$$

$$F_i^8 = \int_\Omega f_y^{(2)}\varphi_i^{(8)} - \frac{1}{2}\left(2c_x^{(1)}\varphi_i^{(8)} - c_z^{(2)}\frac{\partial\varphi_i^{(8)}}{\partial x}\right)\mathrm{d}x\mathrm{d}y + \int_\Gamma t_y^{(2)}\varphi_i^{(8)} - \frac{1}{2}\left(2s_x^{(1)}\varphi_i^{(8)} - s_z^{(2)}\frac{\partial\varphi_i^{(8)}}{\partial x}\right)\mathrm{d}s, \tag{162}$$

$$F_i^9 = \int_\Omega f_z^{(2)}\varphi_i^{(9)} + \frac{1}{2}\left(c_x^{(2)}\frac{\partial\varphi_i^{(9)}}{\partial y} - c_y^{(2)}\frac{\partial\varphi_i^{(9)}}{\partial x}\right)\mathrm{d}x\mathrm{d}y + \int_\Gamma t_z^{(2)}\varphi_i^{(9)} + \frac{1}{2}\left(s_x^{(2)}\frac{\partial\varphi_i^{(9)}}{\partial y} - s_y^{(2)}\frac{\partial\varphi_i^{(9)}}{\partial x}\right)\mathrm{d}s, \tag{163}$$

$$F_i^{10} = \int_\Omega f_x^{(3)}\varphi_i^{(10)} + \frac{1}{2}\left(3c_y^{(2)}\varphi_i^{(10)} - c_z^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial y}\right)\mathrm{d}x\mathrm{d}y + \int_\Gamma t_x^{(3)}\varphi_i^{(10)} + \frac{1}{2}\left(3s_y^{(2)}\varphi_i^{(10)} - s_z^{(3)}\frac{\partial\varphi_i^{(10)}}{\partial y}\right)\mathrm{d}s, \tag{164}$$

$$F_i^{11} = \int_\Omega f_y^{(3)}\varphi_i^{(11)} - \frac{1}{2}\left(3c_x^{(2)}\varphi_i^{(11)} - c_z^{(3)}\frac{\partial\varphi_i^{(11)}}{\partial x}\right)\mathrm{d}x\mathrm{d}y + \int_\Gamma t_y^{(3)}\varphi_i^{(11)} - \frac{1}{2}\left(3s_x^{(2)}\varphi_i^{(11)} - s_z^{(3)}\frac{\partial\varphi_i^{(11)}}{\partial x}\right)\mathrm{d}s, \tag{165}$$

where

$$f_\xi^{(i)} = \int_{-\frac{h}{2}}^{\frac{h}{2}} (z)^i\ \bar{f}_\xi\, \mathrm{d}z + \left(\frac{h}{2}\right)^i \left[q_\xi^t + (-1)^i\, q_\xi^b\right], \quad t_\xi^{(i)} = \int_{-\frac{h}{2}}^{\frac{h}{2}} (z)^i\ \bar{t}_\xi\, \mathrm{d}z, \tag{166}$$

$$c_\xi^{(i)} = \int_{-\frac{h}{2}}^{\frac{h}{2}} (z)^i\ \bar{c}_\xi\, \mathrm{d}z + \left(\frac{h}{2}\right)^i \left[p_\xi^t + (-1)^i\, p_\xi^b\right], \quad s_\xi^{(i)} = \int_{-\frac{h}{2}}^{\frac{h}{2}} (z)^i\ \bar{s}_\xi\, \mathrm{d}z \tag{167}$$

for $i = 0, 1, 2, 3$ and $\xi = x, y, z$.

5 Numerical examples

Here we present the results of a static bending analysis using the developed finite element model. For the purpose of illustration, we take following material properties. The Young's moduli of top and bottom surfaces are $E_t = 14.4 \times 10^9$ N/m^2 and $E_b = 1.44 \times 10^9$ N/m^2, respectively. The Poisson's ratio, ν, is assumed as 0.38 for both materials. The plate thickness, h, is assumed to be 17.6×10^{-6} m, and the length of a square plate, a, is assumed to be $20h$. The full plate is used as computational domain shown in Fig. 2. The 16 by 16 mesh is used to analyse the micro-plate.

Simply supported and clamped boundary conditions are applied to $x = \pm\frac{a}{2}$ and $y = \pm\frac{b}{2}$. In the case of the simply supported boundary conditions, SS1 and SS3 types [29] are considered. The SS1 type boundary condition is that the in-plane displacement (u_1 or u_2) whose direction is parallel to the normal direction on a side of plates is free to move. The SS3 type boundary condition is that all bending displacements with respect to in-plane coordinates are free to move on all sides of plates. The clamped boundary condition is that all in-plane displacements are constrained on all sides of plates. For all types of boundary conditions, we constrain the transverse defection on middle surface, w, on all sides of plates. Since the developed finite element model is based on a modified couple stress theory, we need to define additional higher-order boundary conditions (derivatives of displacements) in addition to boundary conditions of classical plate models. We keep the meaning of each boundary condition and define additional boundary conditions with respect to the rotation vector (3). For the SS1 type, the rigid body rotation, ω_i, whose direction is parallel to the tangential direction of the side of plates, is free to move, and other rotations are constrained. For example, only ω_y is not constrained at $x = \pm\frac{a}{2}$. Note that the rigid body rotation, ω_y, and the bending rotations (θ_x , ϕ_x and ψ_x) of u_1 are in the same direction. For the SS3 type, the rigid body rotations with respect to in-plane coordinates are free to move, and all rigid body rotations are constrained for the clamped boundary condition. We assume that the rigid body rotation with respect to transverse direction, z, is constrained for all boundary cases. To compare with a classical model (the first-order shear deformable plate model), the SS1-0 type boundary condition which does not include any higher-order terms is considered. The constrained degree of freedoms is shown in Table 1. Note that u_i and ω_j are defined in Eqs. (1) and (3) in terms of generalized displacements and their derivatives, $i = 1, 2, 3$ and $j = x, y, z$.

To clearly see nonlinear behavior of a micro-plate, $q_z^t = 5.4 \times 10^6$ N/m^2 is incrementally applied through 20 load steps. The Newton iteration scheme is used to solve the nonlinear equations [29]. Figures 3 and 4 show comparisons of FSDT (8 by 8 quadratic element) and GTPT (16 by 16 cubic element) in the case of homogenous material ($n = 0$). The displacement, $\bar{u}_3 = \frac{u_3}{h}$, through thickness direction at the center of the plate and the middle plane deflection, $\bar{w} = \frac{w}{h}$, versus the load parameter, $\bar{q} = \frac{q_z^t a^4}{E_b h^4}$, are shown in Fig. 3. Since the GTPT considers an extensible plate thickness, the deflection through thickness shows a quadratic variation (shown in Fig. 3a). Since the SS1 boundary condition constrains more degree of freedoms (derivatives of dependent variables), the system becomes slightly stiffer than with the SS1-0 boundary condition. Figure 4 shows the comparison of bending and shear stresses of FSDT and GTPT at load parameter $\bar{q} = 50$.

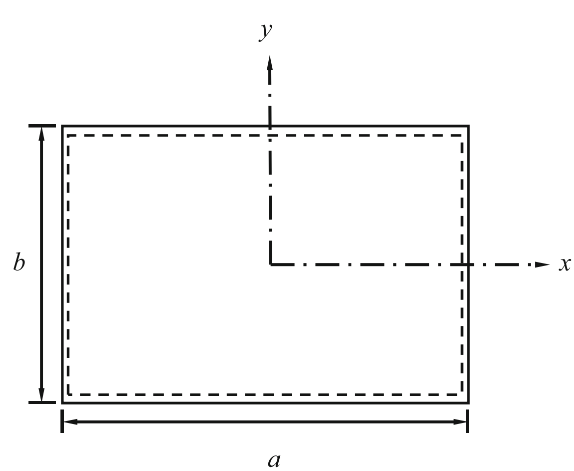

Fig. 2 Computational domain of a full plate

Table 1 Constrained degree of freedoms of each boundary condition

SS1-0						
$x = \pm a/2$	u_2,	w				
$y = \pm b/2$	u_1,	w				
SS1						
$x = \pm a/2$	u_2,	w,	ω_x,	ω_z		
$y = \pm b/2$	u_1,	w,	ω_y,	ω_z		
SS3						
$x = \pm a/2$	u,	v,	w,	ω_z		
$y = \pm b/2$	u,	v,	w,	ω_z		
Clamped						
$x = \pm a/2$, $y = \pm b/2$	u_1,	u_2,	w,	ω_x,	ω_y,	ω_z

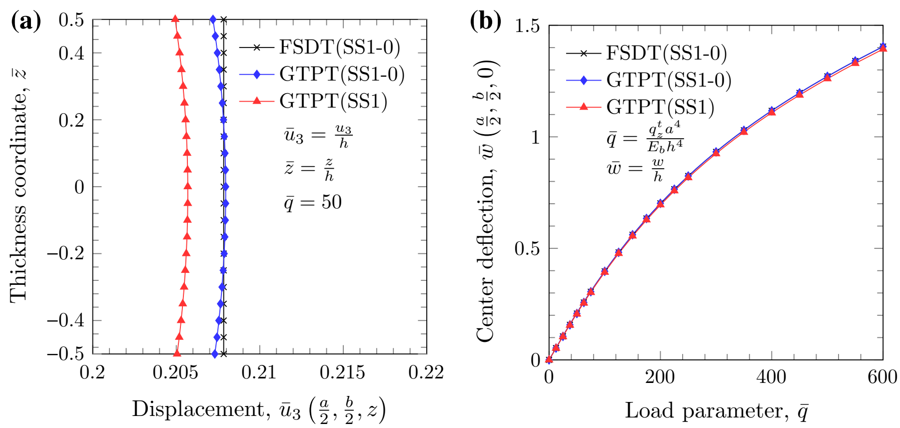

Fig. 3 Comparison of deflections of FSDT (8 by 8 quad.) and GTPT (16 by 16 cubic). **a** Center deflection through thickness, **b** middle plane deflection versus load parameter

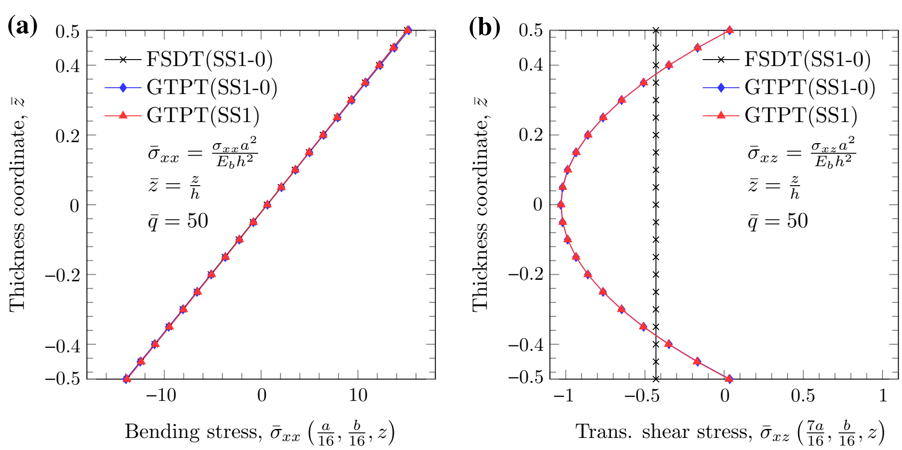

Fig. 4 Comparison of stresses of FSDT (8 by 8 quad.) and GTPT (16 by 16 cubic). **a** Bending stress through thickness, **b** transverse shear stress through thickness

The stresses of the GTPT are computed at nodal points instead of integration points since the displacement gradients are computed at nodes. To compare with the FSDT (8 by 8 quadratic element), the stresses of the FSDT are computed using one-point Gaussian quadrature rule. The bending stress, σ_{xx}, and the transverse shear stress, σ_{xz}, are computed at $\left(\frac{a}{16}, \frac{b}{16}, z\right)$ and at $\left(\frac{7a}{16}, \frac{b}{16}, z\right)$, respectively. The bending stresses of the GTPT do not much differ from the bending stresses of the FSDT (shown in Fig. 4a). Since the variation of transverse

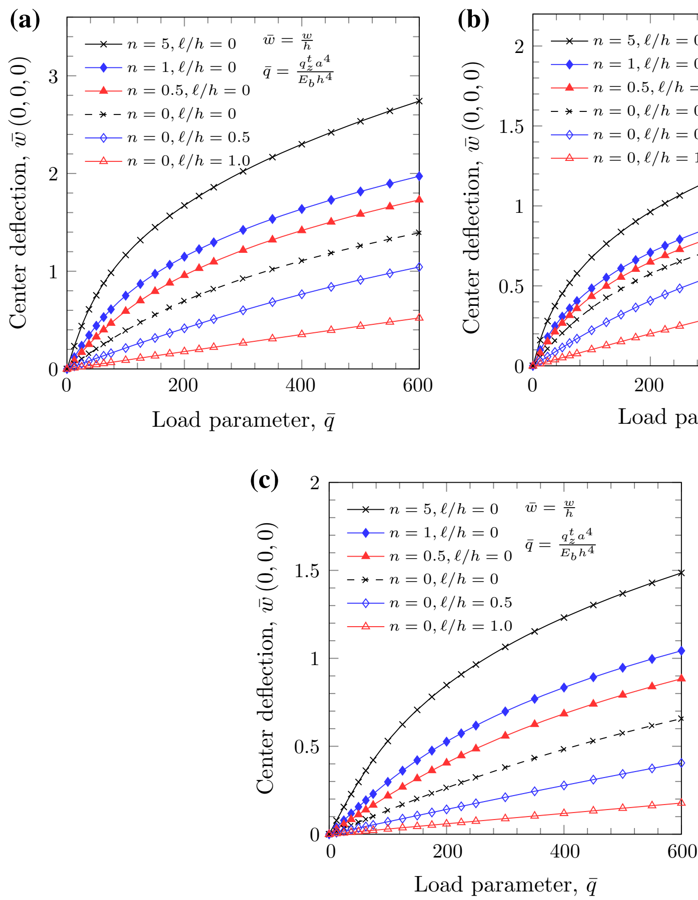

Fig. 5 Middle plane deflections versus load parameter. **a** SS1, **b** SS3, **c** clamped

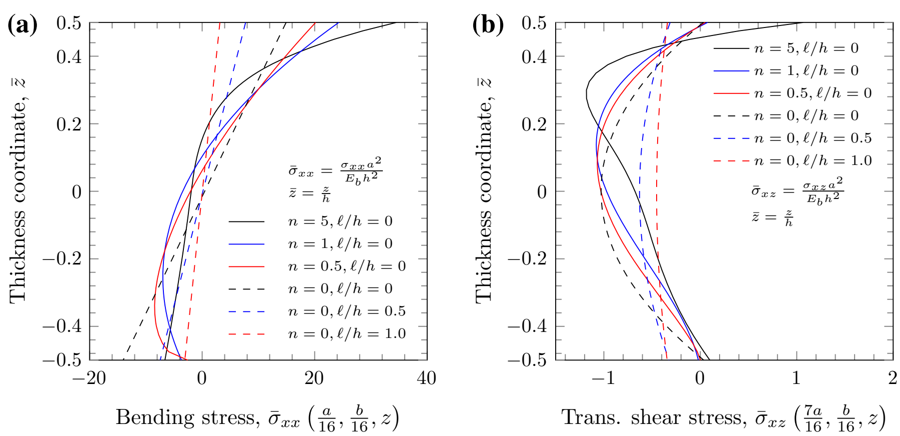

Fig. 6 Bending and shear stresses through thickness (SS1). **a** Bending stress through thickness, **b** transverse shear stress through thickness

shear strains of the GTPT has a form of quadratic variation through thickness, the transverse shear stress in the case of homogenous material shows a quadratic variation (shown in Fig. 4b). Note that we do not force the transverse shear strain to be zero on top and bottom surfaces, and the transverse shear stresses on top and bottom surfaces are not exactly zero. Figure 5 shows center deflection of middle plane, $w\left(\frac{a}{2}, b2, 0\right)$, versus load parameter, $\bar{q}$, with SS1, SS3 and clamped boundary conditions. When the power-law index is larger,

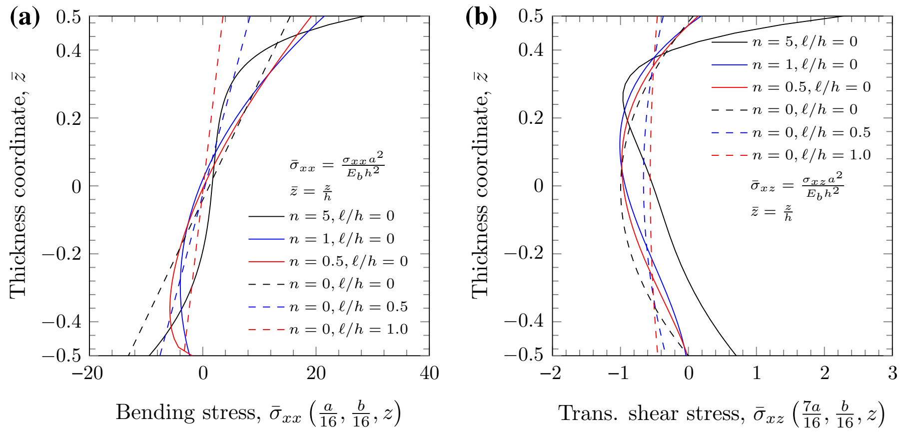

Fig. 7 Bending and shear stresses through thickness (SS3). **a** Bending stress through thickness, **b** transverse shear stress through thickness

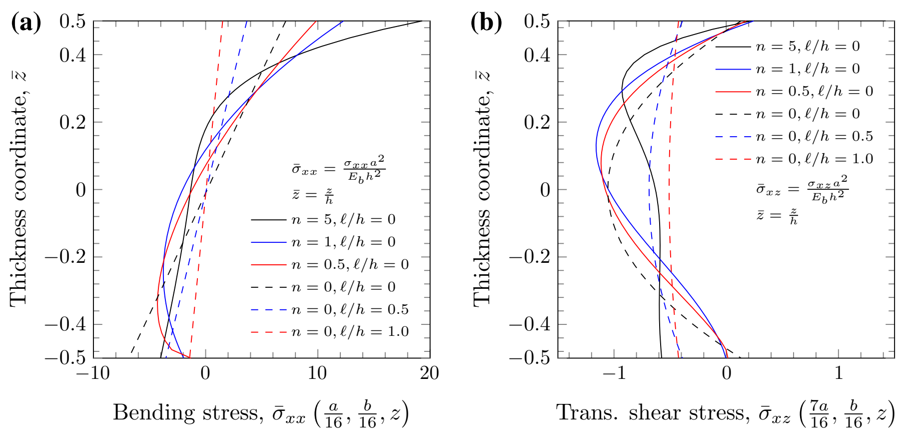

Fig. 8 Bending and shear stresses through thickness (clamped). **a** Bending stress through thickness, **b** transverse shear stress through thickness

the volume fraction of top surface material which is stiffer material decreases, and the plate stiffnesses in Eqs. (150)–(154) become softer, and therefore, larger displacements are obtained for larger power-law index. In the case of considering the microstructural size effect, the stiffness matrices become stiffer due to the effect of the couple stress-related terms, $R_{nn}^{(k)}, n = 1, 2, \ldots, 6$ in Eqs. (153) and (154), and smaller deflections are presented. Figures 6, 7 and 8 show the bending and shear stresses through thickness direction at $\bar{q} = 50$.

The bending stresses and shear stresses are computed at $\left(\frac{a}{16}, \frac{b}{16}, z\right)$ and at $\left(\frac{7a}{16}, \frac{b}{16}, z\right)$, respectively. Unlike for the homogenous plate, the variation of stresses of the FGM plate depends on not only the variation of strains but also on the variation of the material properties in a body. For nonzero power-law index, the odd order plate stiffnesses, $(A_{ij}^{(k)}, B_{mm}^{(k)}$, and $R_{nn}^{(k)}), k = 1, 3, 5$ are nonzero and contribute to system Eq. (13). When the odd order plate stiffnesses are nonzero, the contribution of the higher-order generalized displacements to overall displacements increases. The effect of these phenomena makes larger nonzero transverse shear stresses on top and bottom surfaces.

6 Conclusions

In this paper, a C^1-continuous displacement finite element model of a general third-order plate theory that accounts for a modified couple stress effect and two-constituent power-law variation of functionally graded material through the thickness, and the von Kármán nonlinearity is developed. A microstructural size effect is captured by a length scale parameter through a modified couple stress theory, and the variation of the two-constituent material is considered using a power-law distribution. The finite element model developed requires C^1 continuity for all dependent variables. The two-dimensional Hermite interpolation functions are used to represent the generalized dependent variables. The Newton iteration scheme is used to solve the resulting nonlinear finite element equations. Numerical results for rectangular plates with various boundary conditions are presented to study the parametric effects of the power-law index and the length scale parameter the bending response. The numerical results obtained clearly show that the length scale parameter has the effect of stiffening the plate. Since the plate theory does not explicitly include the vanishing of the transverse shear stresses, they are not exactly zero on the bounding surfaces; however, a quadratic variation of the transverse shear stresses is accounted for, and no shear correction factors are used.

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