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Study on SH-SAW in imperfectly bonded piezoelectric structures loaded with viscous liquid

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Abstract We analytically investigate shear horizontal surface acoustic wave (SH-SAW) propagation in layered piezoelectric structures loaded with viscous liquid, which involves a thin piezoelectric layer imperfectly bonded to an unbounded elastic substrate. The coupling wave equations are obtained based on the linear piezoelectric theory. The governing equations are solved by means of the analytical method with consideration of electrically open and shorted cases, respectively. The dispersive relations are obtained, and the effects of the imperfect constant on the properties of waves are presented and discussed. From the numerical results, we can find that the phase velocity decreases with the increase of the interface parameter n, and for a specified viscosity, the attenuation increases with the interface parameter. The results show that the effects of the imperfect constant on the properties of SH-SAW are remarkable.

1 Introduction

Surface acoustic waves (SAWs) are widely used in resonators, actuators and sensors since the interdigital transducers (IDTs) were invented in 1965 [1]. The acoustic energy concentrates in the region under the surface within a few wavelengths, which results in some important advantages for the microacoustic devices, such as high sensitivities, fast responses and low cost. Shear horizontal surface acoustic wave (SH-SAW) sensors are normally used in gas detection, chemical analysis, medical analysis, environmental monitoring and so on. Studying the effects of the viscous liquid on the propagation of SH-SAW is of great significance in biosensing or chemical sensing applications. Guo and Sun [2] analyzed the propagation of Bleustein–Gulyaev wave (B-G wave) in 6-mm piezoelectric materials loaded with viscous liquid. Zhang et al. [3] studied B-G wave for liquid sensing applications. Zaitsev et al. [4] investigated the acoustic waves in piezoelectric plates bordered with viscous and conductive liquid. Wu and Wu [5] investigated surface waves in a coated anisotropic medium loaded with viscous liquid. The propagation of Love waves in prestressed layered piezoelectric structures loaded with viscous liquid [6] and the SH-SAW propagation in layered functionally graded piezoelectric material structures loaded with viscous liquid [7] were discussed by Du et al. Chen et al. [8] considered the viscous effects on shear horizontal surface acoustic waves in semi-infinite superlattices.

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In the above-mentioned researches, the combination of piezoelectric material and substrate is assumed to be perfectly bonded, which implies the continuity of stresses and displacements across the interface. In practice, due to various reasons such as damage, aging of adhesive layers, microcracks and diffusion impurity, the piezoelectric layer is not perfectly coated on the substrate. Thus, it is of practical importance to take the possible interface damages into consideration in the designs and applications of piezoelectric sensors. The imperfection has been taken into account some simplified interfacial models, such as spring-layer mode [9], multi-layered homogeneous model [10] and spring-mass mode [11]. In these models, the spring–layer relationship is widely accepted to characterize the constitutive behaviors of imperfect interfaces, which assumes the stresses are continuous and the displacements are jumped across the interface. Previously, weak bonded structures with elastic waves have been widely studied [12–18]. Fan et al. [12] studied the piezoelectric waves near an imperfectly bonded interface between two half-spaces. Li and Lee [13] investigated the effect of the imperfect interface on the SH wave propagating in a cylindrical piezoelectric sensor. Liu et al. [14] studied shear horizontal surface waves in a layered piezoelectric half-space with imperfect interface. Huang and Li [15] analyzed shear waves in two magnetoelectric materials bonded imperfectly. Chen et al. [16] studied shear horizontal waves in rotated Y-cut quartz with imperfect interface.

In this paper, we focus on the propagation of SH-SAW in imperfectly bonded piezoelectric structures loaded with viscous liquid. The linear spring model is used to simulate the imperfection of bonding behavior at the interface. The effects of the imperfect interface on wave propagation are analyzed and discussed in detail. The method and the results are useful for the design of the acoustic wave sensors in liquid-phase application.

2 Formulation of the problem

A layered piezoelectric structure loaded with viscous liquid involving a thin piezoelectric layer imperfectly bonded to an elastic substrate is illustrated in Fig. 1. The piezoelectric material and the liquid occupy the half-space $x < 0$ and the substrate covers the half-space $x > 0$. The piezoelectric material and the substrate are polarized along the z-direction. We here only consider the so-called anti-plane motion, and the coupled wave equations and the constitutive equations can be given as [7]

$$\begin{aligned} &c_{44}\nabla^2 w + e_{15}\nabla^2\phi = \rho\frac{\partial^2 w}{\partial t^2}, \\ &e_{15}\nabla^2 w = \varepsilon_{11}\nabla^2\phi, \end{aligned} \tag{1}$$

$$\begin{aligned} T_{xz} &= c_{44}w_{,x} + e_{15}\phi_{,x}, \\ T_{zy} &= c_{44}w_{,y} + e_{15}\phi_{,y}, \\ D_x &= e_{15}w_{,x} - \varepsilon_{11}\phi_{,x}, \\ D_y &= e_{15}w_{,y} - \varepsilon_{11}\phi_{,y}, \end{aligned} \tag{2}$$

where $\nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}$ is the two-dimensional Laplace operator in Cartesian coordinates, w is the displacement component in z-direction, ϕ is electric potential and ρ is the mass density. T_{ij} and D_i are the stress and electric

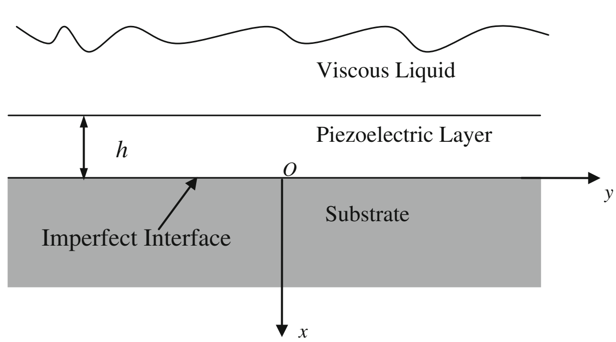

Fig. 1 A half-space layered piezoelectric structure loaded with viscous liquid

displacements, respectively. c_{pq} and ε_{ik} are the elastic constants and dielectric permeability coefficients, respectively; e_{iq} is the piezoelectric coefficient.

We can define

$$\psi = \phi - \frac{e_{15}}{\varepsilon_{11}} w. \tag{3}$$

Then, Eq. (1) can be reduced as follows:

$$c_{44}^{*} \nabla^2 w = \rho \frac{\partial^2 w}{\partial t^2}, \quad \nabla^2 \psi = 0, \tag{4}$$

where c_{44}^{*} is given by

$$c_{44}^{*} = c_{44} + \frac{e_{15}^2}{\varepsilon_{11}}. \tag{5}$$

The displacement and electric potential in the substrate tend to zero when the coordinate x approaches infinity along the positive x-axis, namely

$$x \to +\infty, \quad w^m = 0, \quad \phi^m = 0, \tag{6}$$

where the superscript m indicates the quantities in the substrate. The continuity conditions and electrically open conditions for the interface between liquid and the piezoelectric layer can be given by

$$\begin{aligned} &\dot{w}\,(-h, y, t) = v^L\,(-h, y, t), \\ &T_{xz}\,(-h, y, t) = T_{xz}^L\,(-h, y, t), \\ &D_x\,(-h, y, t) = 0, \end{aligned} \tag{7}$$

where v^L is the liquid particle velocity in the z-direction. The superscript L indicates the quantities in the liquid. The continuity conditions and electrically shorted conditions for the interface between liquid and the piezoelectric layer can be given by

$$\begin{aligned} &\dot{w}\,(-h, y, t) = v^L\,(-h, y, t), \\ &T_{xz}\,(-h, y, t) = T_{xz}^L\,(-h, y, t), \\ &\phi\,(-h, y, t) = 0. \end{aligned} \tag{8}$$

For the linear spring model [9], the bond between the piezoelectric layer and the elastic substrate is not perfect, and it is a "mechanical spring" pattern, i.e., the stress is continuous and the displacements are jumped, so we can assume the conditions as follows:

$$\begin{aligned} &T_{xz}\,(0, y, t) = T_{xz}^m\,(0, y, t) = \alpha \left[w^m (0, y, t) - w\,(0, y, t) \right], \\ &\phi\,(0, y, t) = \phi^m\,(0, y, t)\,, D_x\,(0, y, t) = D_x^m\,(0, y, t), \end{aligned} \tag{9}$$

where α is the bond coefficient, which indicates the intensity of the bond between the piezoelectric layer and the elastic substrate. For $\alpha \to \infty$, the bond is perfect, and for $\alpha \to 0$, there is no mechanics bond between the piezoelectric layer and the elastic substrate, i.e., it is a sliding interface.

In addition to the aforementioned boundary and interface, the displacement and electric potential in the substrate tend to zero when coordinate x approaches infinity along the negative x-axis, namely

$$x \to -\infty, \quad w^L, \phi^L \to 0. \tag{10}$$

3 Solution to the problem

3.1 Solutions in the piezoelectric layer

We consider the following solution forms of (4):

$$\begin{aligned} w &= w(x)\exp\left[\mathrm{i}\xi\,(y-ct)\right], \\ \psi &= \psi(x)\exp\left[\mathrm{i}\xi\,(y-ct)\right], \end{aligned} \tag{11}$$

where ξ is the complex wave number, and $\xi = \frac{\omega}{c}(1+\gamma i) = k(1+\gamma i)$. c and k are the phase velocity and the real part of the wave number in the y-direction, respectively. γ is the attenuation coefficient, and ω is the angular frequency. Substituting (11) into (4), we can obtain

$$w''(x) - \xi^2 b^2 w(x) = 0,\ \ \psi''(x) - \xi^2 \psi(x) = 0, \tag{12}$$

where $b^2 = \frac{\rho c^2}{c_{44}^*} - 1$. The solutions to (12) are assumed as

$$\begin{aligned} w(x) &= C_1\cos(\xi bx) + C_2\sin(\xi bx), \\ \psi(x) &= C_3 e^{\xi x} + C_4 e^{-\xi x}, \end{aligned} \tag{13}$$

where C_1, C_2, C_3 and C_4 are unknown constants to be determined. Then, the displacement and electric potential are given by

$$\begin{aligned} w(x,y,t) &= \left[C_1\cos(\xi bx) + C_2\sin(\xi bx)\right]\exp\left[\mathrm{i}\xi\,(y-ct)\right], \\ \phi(x,y,t) &= \left[C_3 e^{\xi x} + C_4 e^{-\xi x} + \frac{e_{15}}{\varepsilon_{11}}\left(C_1\cos(\xi bx) + C_2\sin(\xi bx)\right)\right]\exp\left[\mathrm{i}\xi\,(y-ct)\right]. \end{aligned} \tag{14}$$

3.2 Solutions in the viscous liquid

The liquid is assumed to be viscous and nonconductive. Suppose the motion of liquid is induced only by the wave propagation in the piezoelectric material and also propagates in the form of a harmonic wave. The embroil inertial term in the Navier–Stokes equation can be omitted for this problem. Moreover, the pressure gradient also can be ignored since only shear deformation occurs during the wave propagation. Therefore, the governing equation for the liquid is reduced to be

$$\mu^L \nabla^2 v^L = \rho^L \dot{v}^L, \tag{15}$$

where ρ^L is the mass density of liquid and μ^L is the dynamic viscous coefficient of the liquid. The solution for Eq. (15) is assumed as:

$$v^L = v^L(x)\exp\left[\mathrm{i}\xi\,(y-ct)\right]. \tag{16}$$

Substituting (16) into (15) and considering the radiation conditions in the liquid far from the interface, we can obtain

$$v^L(x) = D_1 e^{\lambda x}, \tag{17}$$

where $\lambda^2 = \xi^2 - i\omega\frac{\rho^L}{\mu^L}$, $R_e(\lambda) > 0$ and D_1 are unknown constants to be determined. Then, we can obtain

$$v^L = D_1 e^{\lambda x}\exp\left[\mathrm{i}\xi\,(y-ct)\right]. \tag{18}$$

The shear stress can be obtained by the Newtonian liquid law, namely

$$T_{xz}^L = \mu^L \frac{\partial v^L}{\partial x}.$$

3.3 Solutions in the elastic substrate

The solutions to the displacement and the electric potential of waves in the substrate are given by

$$\begin{aligned} w^m &= B_1^m e^{-\xi b^m x} \exp[\mathrm{i}\xi (y - ct)], \\ \phi^m &= B_2^m e^{-\xi x} \exp[\mathrm{i}\xi (y - ct)], \end{aligned} \tag{19}$$

where $b^m = \sqrt{1 - \rho^m c^2 / c_{44}^m}$, B_1^m, B_2^m are unknown constants.

4 The phase velocity equation

From the conditions (9), we can obtain the following:

$$\begin{aligned} & C_1\alpha + C_2\left(c_{44} + \frac{e_{15}^2}{\varepsilon_{11}}\right)\xi b + C_3 e_{15}\xi - C_4 e_{15}\xi - \alpha B_1^m = 0, \\ & - C_1\alpha + B_1^m \left(c_{44}^m \xi b^m + \alpha\right) + B_2^m e_{15}^m \xi = 0, \\ & C_1 \frac{e_{15}}{\varepsilon_{11}} + C_3 + C_4 - B_2^m = 0, \\ & - C_3\varepsilon_{11} + C_4\varepsilon_{11} + B_1^m e_{15}^m b^m - B_2^m \varepsilon_{11}^m = 0. \end{aligned} \tag{20}$$

4.1 Solutions to the electrically open conditions at the interface between the liquid and the piezoelectric layer

From the electrically open conditions at the interface between the liquid and the piezoelectric layer (7), we can obtain the following:

$$\begin{aligned} & \mathrm{i}\omega \cos(bh\xi) C_1 - \mathrm{i}\omega \sin(bh\xi) C_2 + D_1 \exp(-h\lambda) = 0, \\ & C_1 b\xi (c_{44} + e_{15}^2/\varepsilon_{11}) \sin(b\xi h) + C_2 b\xi (c_{44} + e_{15}^2/\varepsilon_{11}) \cos(b\xi h) \\ & \quad + C_3 e_{15}\xi \exp(-\xi h) - C_4 e_{15}\xi \exp(\xi h) - D_1 \mu^{\mathrm{L}} \lambda \exp(-h\lambda) = 0, \\ & \quad - C_3 \exp(-h\xi) + C_4 \exp(h\xi) = 0. \end{aligned} \tag{21}$$

(20) and (21) are the linear algebraic equations about constants C_1, C_2, C_3, C_4, D_1, B_1^{m}, B_2^{m}. In order to obtain the nontrivial solutions to the above-mentioned unknown constants, the determinant of the coefficient matrix of these linear algebraic equations needs to be zero. Then, the dispersive relation for the electrically open conditions can be obtained.

4.2 Solutions of the electrically shorted conditions at the interface between the liquid and the piezoelectric layer

From the electrically shorted conditions at the interface between the liquid and the piezoelectric layer (8), we can obtain the following:

$$\begin{aligned} & \mathrm{i}\omega \cos(bh\xi) C_1 \text{-} \mathrm{i}\omega \sin(bh\xi) C_2 + D_1 \exp(-h\lambda) = 0, \\ & C_1 b\xi (c_{44} + e_{15}^2/\varepsilon_{11}) \sin(b\xi h) + C_2 b\xi (c_{44} + e_{15}^2/\varepsilon_{11}) \cos(b\xi h) \\ & \quad + C_3 e_{15}\xi \exp(-\xi h) - C_4 e_{15}\xi \exp(\xi h) - D_1 \mu^{\mathrm{L}} \lambda \exp(-h\lambda) = 0, \\ & C_1 \frac{\mathrm{e}_{15}}{\varepsilon_{11}} \cos(bh\xi) - C_2 \frac{\mathrm{e}_{15}}{\varepsilon_{11}} \sin(bh\xi) + C_3 \exp(-h\xi) + C_4 \exp(h\xi) = 0. \end{aligned} \tag{22}$$

Similarly, (20) and (22) are the homogeneous algebraic equations about constants C_1, C_2, C_3, C_4, D_1, B_1^{m}, B_2^{m}. In order to obtain the nontrivial solutions of the above-mentioned unknown constants, the determinant of the coefficient matrix of these linear algebraic equations needs to be zero. The dispersive relation for the electrically shorted conditions can be obtained.

5 Numerical results and discussion

We can obtain the dispersive relations from (20) and (21) for the electrically open case and from (20) and (22) for the electrically shorted case. The material constants of piezoelectric layer and elastic substrate are given in Tables 1 and 2, respectively. Unless specifically mentioned, the thickness of the piezoelectric layer is assumed to be $h = 0.1$ mm, and the mass density of the liquid is $\rho^L = 1 \times 10^3$ kg/m^3 in the following discussion. For simplicity, we bring in a dimensionless interface parameter n to measure the interfacial imperfection, namely

$$n = c_{44}^*/\alpha h, c_{44}^* = c_{44} + e_{15}^2/\varepsilon_{11}.$$

Figures 2 and 3 show the phase velocity of the first mode for electrically open and shorted cases with viscosity $\mu^L = 0.5$ N s/m^2, respectively. From the results, we can find that the phase velocity decreases with the interface parameter. Furthermore, it can be seen that for smaller interface parameters, such as $n = 0$, the phase velocity curves are monotonous, and the phase velocity decreases with the increase in the nondimensional wave number. For the larger interface parameters, i.e., for the weaker interfaces, the curves are not monotonous. The phase velocity decreases with the increase in the wave number at first, and then, it increases until approaching a horizontal limit value.

As we know, the SH-SAW will attenuate during propagating because of the viscosity of the liquid. Figures 4 and 5 illustrate the electrically open and shorted cases, respectively, in order to show the effect of the imperfect interface on the relationship between the attenuation ($k\gamma$) and the frequency. For the perfect bond, we can find that the attenuation increases with the frequency, and the relationship between the attenuation and the frequency is nonlinear. If the bond is getting to be weaker, it can be seen that the attenuation increases with the interface parameter n.

The relationships between the phase velocity and the viscosity of the liquid for electrically open and shorted cases are illustrated in Figs. 6 and 7, respectively. It can be seen that the phase velocity decreases with the increase in liquid viscosity. No matter the bond is perfect or imperfect, the tendencies of the curves are similar. In addition, we can see clearly that the phase velocity decreases with the increase of the interface parameter once more.

Table 1 Material coefficients of the piezoelectric $BaTiO_3$

c_{44} (10^9 N/m^2)	e_{15} (C/m^2)	ε_{11} (10^{-9} C^2/Nm^2)	ρ (10^3 kg/m^3)
43	11.6	11.2	5.8

Table 2 Material coefficients of the SiO_2

c_{44}^m (10^9 N/m^2)	ε_{11}^m (10^{-9} C^2/Nm^2)	ρ^m (10^3 kg/m^3)
31.2	3.36	2.2

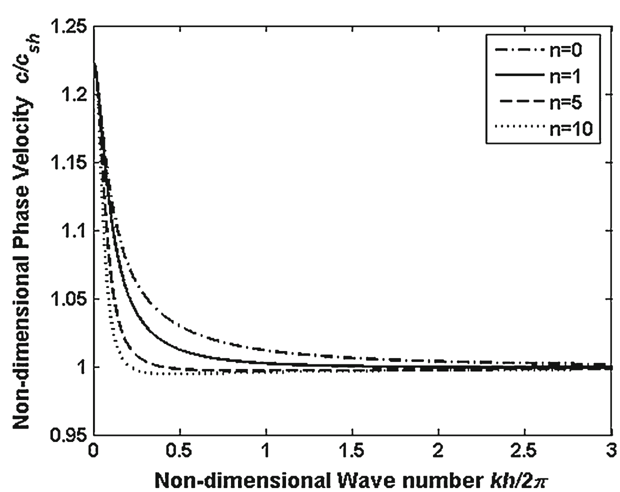

Fig. 2 Dispersive relationship for electrically open case

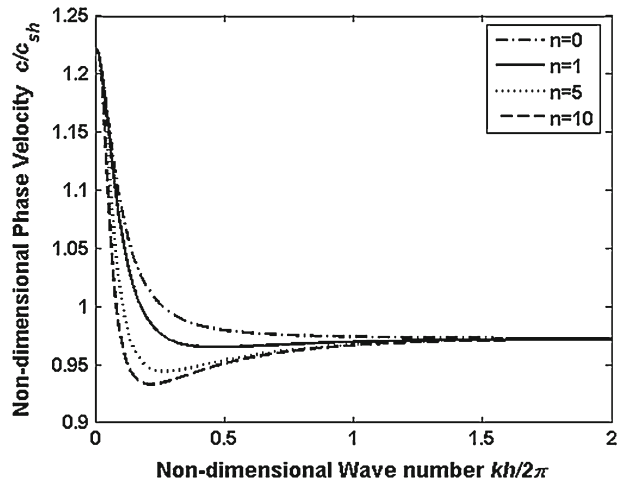

Fig. 3 Dispersive relationship for electrically shorted case

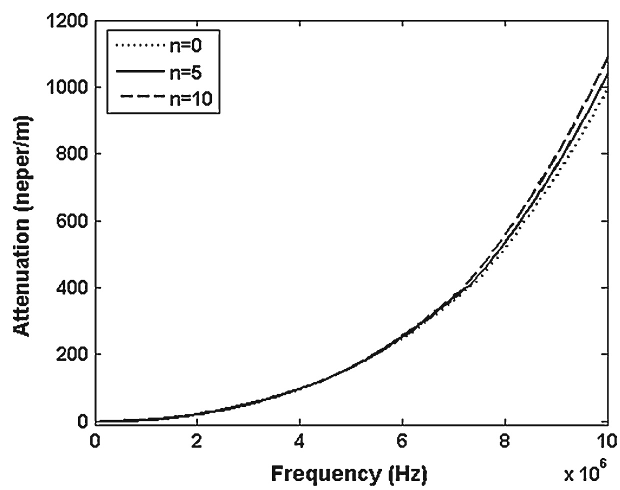

Fig. 4 Attenuation versus frequency for electrically open case

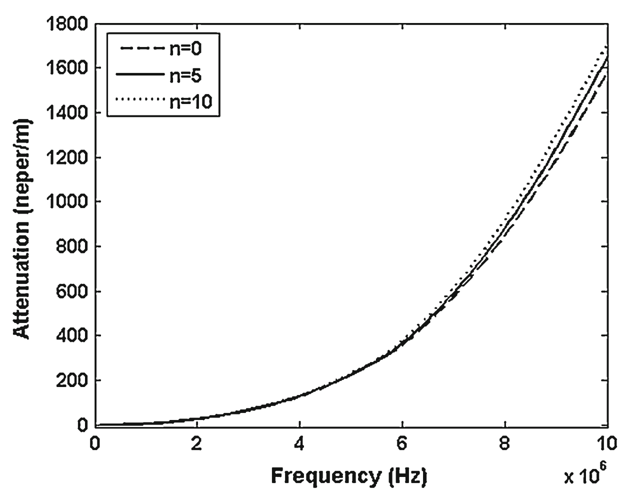

Fig. 5 Attenuation versus frequency for electrically shorted case

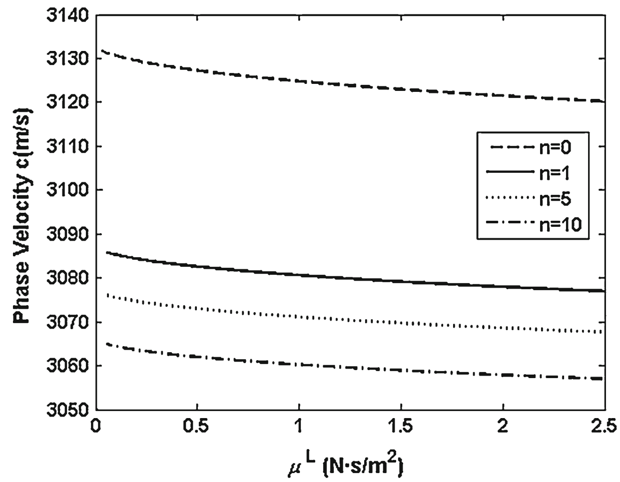

Fig. 6 Phase velocity versus viscosity for electrically open case ($kh = 10$)

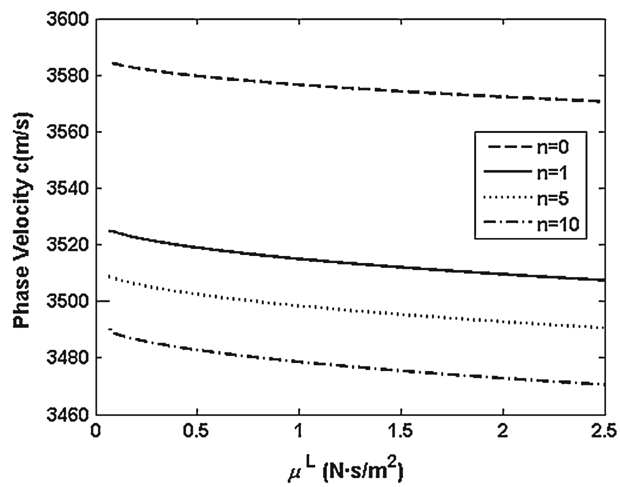

Fig. 7 Phase velocity versus viscosity for electrically shorted case ($kh = 10$)

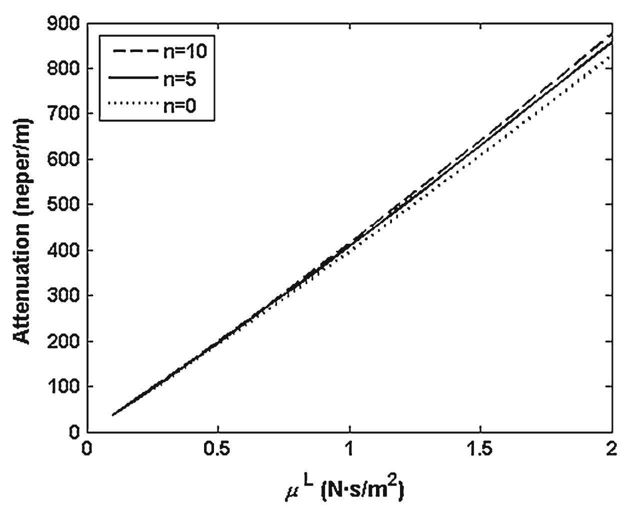

Fig. 8 Attenuation versus viscosity for electrically open case

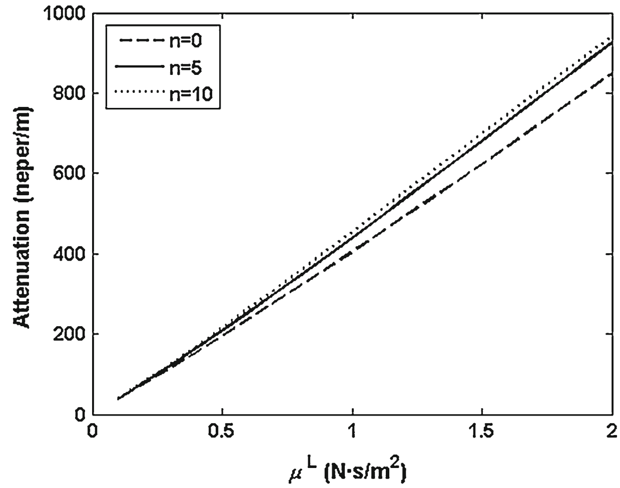

Fig. 9 Attenuation versus viscosity for electrically shorted case

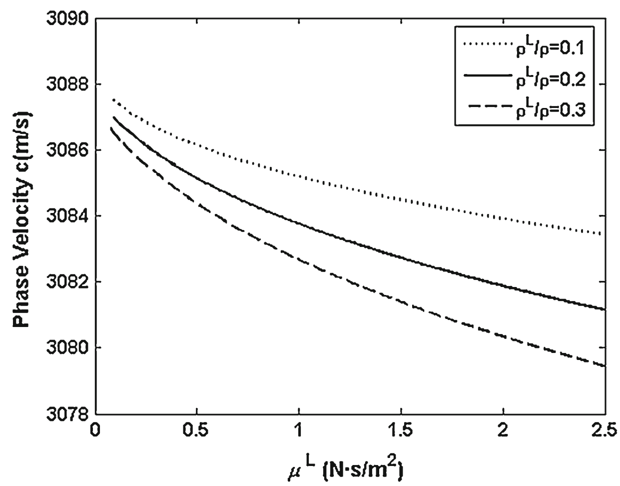

Fig. 10 Phase velocity versus viscosity with different mass densities of liquid for electrically open case ($f = 40$ MHz, $n = 1$)

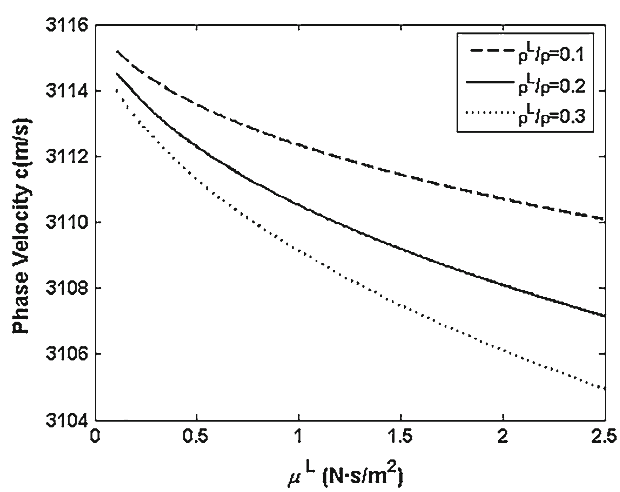

Fig. 11 Phase velocity versus viscosity with different mass densities of liquid for electrically open case ($f = 40$ MHz, $n = 0$)

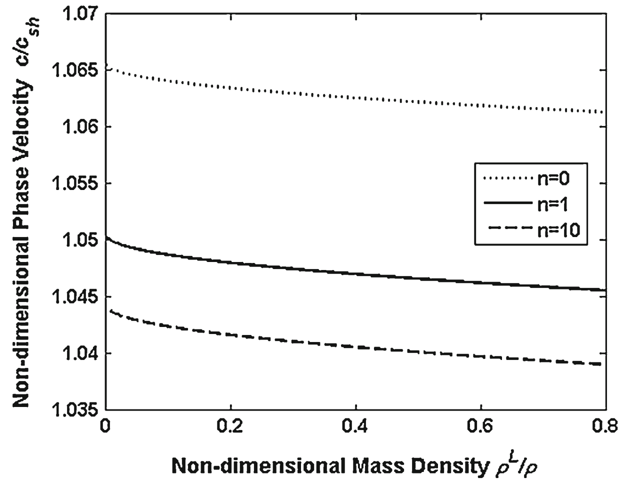

Fig. 12 Phase velocity versus density ratio ρ^L/ρ with different interface parameters for electrically open case ($kh = 10$)

Figures 8 and 9 show that the attenuation increases with the viscous coefficient. In particular, for a specified viscosity, the attenuation increases with the increase in interface parameter. Whether the bond is perfect or not, the shapes of different curves are similar.

Figures 10 and 11 show the phase velocity with different mass densities of liquid ρ^L for interface parameter $n = 1$ and $n = 0$, respectively. From the results, we can see that the phase velocity decreases with the mass density of the liquid, which can be seen more clearly in Fig. 12.

6 Conclusions

The properties of SH-SAW propagation in layered piezoelectric material structures loaded with viscous liquid are investigated, which involves a thin piezoelectric layer imperfectly bonded to an unbounded elastic substrate. A generalized linear spring-layer model is used to characterize the interfacial bonding conditions. The governing equations are solved by means of the analytical method with consideration of electrically open and shorted cases, respectively. The effects of the imperfect interface on wave propagation are analyzed and discussed in detail. From numerical results, we can find that the phase velocity decreases with the increase in the interface parameter, and for a specified viscosity, the attenuation increases with the increase in the interface parameter. The method and the results are useful for the design of the acoustic wave sensors in liquid-phase application.

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