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A numerical study of a plate with a hole for a new class of elastic bodies

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Abstract It has been shown recently that the class of elastic bodies is much larger than the classical Cauchy and Green elastic bodies, if by an elastic body one means a body incapable of dissipation (converting working into heat). In this paper, we study the boundary value problem of a hole in a finite nonlinear elastic plate that belongs to a subset of this class of the generalization of elastic bodies, subject to a uniaxial state of traction at the boundary (see Fig. 1). We consider several different specific models, including one that exhibits limiting strain. As the plate is finite, we have to solve the problem numerically, and we use the finite element method to solve the problem. In marked contrast to the results for the classical linearized elastic body, we find that the strains grow far slower than the stress.

1 Introduction

The stress concentration due to the presence of defects, such as holes or inclusions, is an important factor in determining the design and development of most load-bearing structural elements and has thus been studied with assiduity within the context of several constitutive theories including classical linearized elasticity. One of the classical problems in linearized elasticity is the stress concentration due to the presence of a circular hole in an infinite plate subject to traction on the boundary (see Bickley [1] and Love [2]). The generalization of this problem has been carried out for plates with a variety of holes of different shapes (see Murakami [3]) and for circular and elliptic holes for different nonlinear elastic bodies subject to a variety of loads. In this paper, we study the problem within the context of a new class of elastic bodies, which is a subset of a recent generalization of classical Cauchy and Green elastic bodies. While some simple problems have been studied within this class of new elastic bodies, the boundary value problem of a plate with a hole and problems such as inclusions, etc., wherein stress concentration occurs, has not been studied within the context of such bodies.

In Cauchy elastic bodies (see Truesdell and Noll [4]), the Cauchy stress is given in terms of the deformation gradient, and in Green elastic bodies (see [4,5]), the stored energy is given in terms of the deformation gradient with the stress being derivable from the stored energy. The linearization of the constitutive model for a general nonlinear elastic body under the assumption that the displacement gradient is sufficiently small leads to the constitutive theory for a linearized elastic body. While the response of a linearized elastic body can be expressed by either prescribing the stress in terms of the linearized strain or the linearized strain in terms of the stress (a similar situation presents itself in the case of a linear(ized) viscoelastic body), this is not true in

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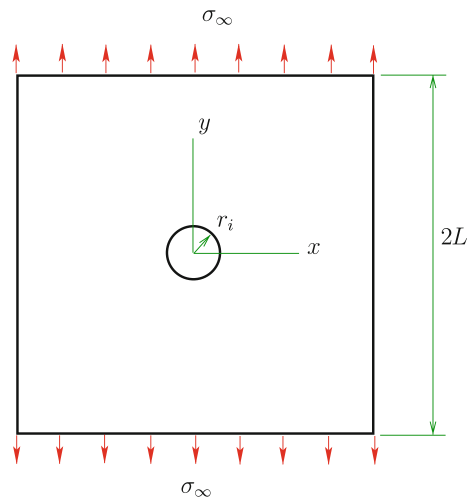

Fig. 1 Plate with an hole under uniaxial boundary traction

the case of general nonlinear elastic bodies. In general, the nonlinear constitutive relation for the stress is not invertible.[1] Linearization of the new class of elastic bodies, wherein implicit relationships exist between the deformation gradient and the stress or those in which the deformation gradient is expressed as a function of the stress, under the usual assumption that the Frobenius norm of the displacement gradient is small, in marked departure from the linearized case, leads to models wherein the linearized strain can be given as a nonlinear function of the stress.

As mentioned earlier, it has recently been shown that the class of elastic bodies is much more general than previously thought (see Rajagopal [7,8] and Rajagopal and Srinivasa [9,10]). The generalization that has been put into place allows for an elastic body to be defined through implicit constitutive relations between the nonlinear Cauchy–Green stretch and the Cauchy stress by relations of the form:

$$\boldsymbol{f}(\mathbf{B}, \mathbf{T}, \rho) = \mathbf{0}, \tag{1}$$

where $\mathbf{B}$ is the Cauchy–Green stretch tensor,[2] $\mathbf{T}$ is the Cauchy stress tensor, and ρ is the mass density. In virtue of the balance of mass, we can recast the dependence on the density with the dependence on the determinant of $\mathbf{B}$ and so express the relationship between $\mathbf{B}$ and $\mathbf{T}$.

A special subclass of the above class of implicit models is the following class of explicit models that provide an expression for $\mathbf{B}$ in terms of $\mathbf{T}$, namely (see Rajagopal [7])

$$\mathbf{B} = \boldsymbol{g}(\mathbf{T}, \rho). \tag{2}$$

For isotropic bodies, we have

$$\mathbf{B} = \alpha_0 \mathbf{I} + \alpha_1 \mathbf{T} + \alpha_2 \mathbf{T}^2, \tag{3}$$

where α_i, $i = 0, 1, 2$, depend on the principal invariants of $\mathbf{T}$ and the density. Model (3) essentially reverses the role of the Cauchy–Green stretch tensor and the stress tensor from the classical model, for the response of isotropic homogeneous compressible elastic solids. While the classical model can be inverted when invertible, to obtain a model that belongs to the class defined by (2), not all models of the class defined by (2) can be obtained by such an inversion.

The classical procedure of linearizing the relationship (1) under the assumption that the displacement gradient be sufficiently small leads to a relationship between the linearized strain $\boldsymbol{\varepsilon}$ and the stress $\mathbf{T}$ of the form [8,11]

$$\boldsymbol{\varepsilon} = \boldsymbol{h}(\mathbf{T}). \tag{4}$$

[1] Truesdell and Moon [6] have determined conditions under which the relation is invertible, but their interest lay in determining the invertibility of isotropic functions.

[2] Though it is referred to as the stretch tensor, to be more precise, it is the square of the stretch tensor $\mathbf{V}$.

For isotropic bodies, we then have

$$\boldsymbol{\varepsilon} = \beta_0 \mathbf{I} + \beta_1 \mathbf{T} + \beta_2 \mathbf{T}^2, \tag{5}$$

where β_i, $i = 0, 1, 2$, depend on the principal invariants of $\mathbf{T}$. A nonlinear relationship such as (5) between the strain and the stress is impossible to obtain by linearizing classical Cauchy or Green elasticity (see [11,12] for a detailed discussion of the same).

Constitutive theories wherein the linearized strain is a nonlinear function of the stress allows one to describe the response of elastic bodies that were hitherto not possible, for instance the problem of fracture in brittle materials in which the body fractures within the realm of small strains. As is well known, within the context of the linearized theory of elasticity, both the strain and the stress grow proportional to the square root of the inverse of r, where r is the radial distance from the tip of a crack, within the context of the theory of linearized elasticity. But such a growth is self-contradictory, as the linearized theory is only valid if the displacement gradient and hence the strain is very small. It turns out that the generalization (5) that leads to models in which the linearized strain is a nonlinear function of the stress, allows one to obtain bounded strains that can be fixed to be as small as we wish a priori, while the stress is allowed to grow and even become unbounded. Rajagopal and Walton [13] have shown that in the case of anti-plane strain involving a crack in an infinite body, for a large class of the generalized elastic bodies, the strain remains bounded even at the crack tip. The ability to predict bounded strains for the crack problem is insufficient to advocate the use of models belonging to this new class of elastic bodies. It is necessary to further study whether models belonging to this new class predict meaningful physical results for a variety of boundary value problems, before they can be adopted for further use. It is with such a view in mind that several simple boundary value problems have been studied within the context of these new constitutive models. However, even more studies, in which the results are in agreement with observations and experimental results, are necessary to provide some confidence with regard to the usefulness and efficacy of such models.

As mentioned above, recently, several boundary value problems have been studied within the context of this new class of elastic bodies. The problems of uniaxial extension, shear, circumferential shear, and torsion for different subclasses of bodies belonging to this new class of elastic bodies have been studied by Rajagopal [12]. Bustamante and Rajagopal [11,14] studied plane stress and plane strain problems involving members of this new class of elastic bodies. Interestingly, even within the context of the simple shearing of members of this new class of elastic bodies, Bustamante and Rajagopal [15] find the possibility of multiple solutions, as well as solutions wherein one finds the presence of pronounced stress boundary layers in that there are narrow regions adjacent to the boundary wherein the gradients of the stress are very large, while in the region outside the narrow region, the stresses are essentially constant. Rajagopal and Saravanan [16] have studied the inflation of a compressible spherical annulus of members of this new class of elastic materials and found the development of pronounced stress boundary layers in the case of a spherical inclusion Rajagopal and Saravanan [17] have also studied the extension, inflation, and the circumferential shearing of a cylindrical annulus of such compressible elastic solids and once again found the development of stress boundary layers. Most recently, Bustamante and Rajagopal [15] have studied the simple shearing of a class of incompressible isotropic elastic solids belonging to this new class.

In particular, we are concerned with the response of a planar slab of such a material with a hole when subject to tensile loading (see Fig. 1). In the case of the classical linearized elastic body, the stress concentration factor is 3 and occurs at the location $(r_i, 0)$ in a polar coordinate system (see, for example, [18]), where r_i is the radius of the hole. In the classical linearized elastic body, the stress and the strain grow in the same manner as they are related linearly. In the case of the model that we study, in view of the fact that there is a limiting strain as the stress grows, we should expect the growth in the strain to be far more moderate than the growth in the stress. We find this to indeed be the case. We find that the strain grows very much slower than the stress as we approach the hole. While the stress concentration for the material being studied is higher than that for a linearized elastic body, the strains that are engendered are much smaller than that in the elastic body obtained by linearizing the nonlinear relationship.

The organization of the paper is as follows. In Sect. 2, we introduce the basic kinematics, the constitutive theory and record the boundary value problem. As the problem considered leads to a nonlinear system of partial differential equations in a finite and reasonably complex geometry, it is necessary to resort to a numerical solution of the problems; in Sect. 3, we provide the details concerning the finite element method as well as the documentation of the results of the numerical procedure, and in Sect. 4, we discuss the implications of the results.

2 Basic equations

2.1 Kinematics

Let $\mathbf{X} \in \kappa_R(\mathcal{B})$ denote a particle belonging to a body $\mathcal{B}$ in the reference configuration $\kappa_R(\mathcal{B})$, and let $\mathbf{x} \in \kappa_t(\mathcal{B})$ denote the position of the same particle in the current configuration $\kappa_t(\mathcal{B})$, at time t. We assume the mapping $\boldsymbol{\chi}$ that assigns to each particle $\mathbf{X} \in \kappa_R(\mathcal{B})$ the position $\mathbf{x}$ at time t, that is, $\mathbf{x} = \boldsymbol{\chi}(\mathbf{X}, t)$, is sufficiently smooth to make all the derivatives that are taken to be meaningful. The displacement $\mathbf{u}$ and the deformation gradient $\mathbf{F}$ are defined through

$$\mathbf{u} = \mathbf{x} - \mathbf{X}, \quad \mathbf{F} = \frac{\partial \boldsymbol{\chi}}{\partial \mathbf{X}}, \tag{6}$$

respectively. The Cauchy–Green stretch tensors $\mathbf{B}$ and $\mathbf{C}$ are defined through $\mathbf{B} = \mathbf{F}\mathbf{F}^{\mathrm{T}}$, $\mathbf{C} = \mathbf{F}^{\mathrm{T}}\mathbf{F}$, and the Green–St.Venant strain $\mathbf{E}$ and the linearized strain $\boldsymbol{\varepsilon}$ are defined through

$$\mathbf{E} = \frac{1}{2}(\mathbf{C} - \mathbf{I}), \tag{7}$$

$$\boldsymbol{\varepsilon} = \frac{1}{2}\left[\left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}}\right) + \left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}}\right)^{\mathrm{T}}\right], \tag{8}$$

respectively.

2.2 Constitutive equations for a new class of elastic bodies

In this paper, we consider only ‘isotropic’ bodies, and it follows from (5) that

$$\boldsymbol{\varepsilon} = \beta_0\mathbf{I} + \beta_1\mathbf{T} + \beta_2\mathbf{T}^2,$$

where β_0, β_1, and β_2 are scalar functions that depend on three mutually independent invariants. We consider the set

$$I_T = \mathrm{tr}\,\mathbf{T}, \quad II_T = \frac{1}{2}\mathrm{tr}\,(\mathbf{T}^2), \quad III_T = \frac{1}{3}\mathrm{tr}\,(\mathbf{T}^3). \tag{9}$$

A special subclass of the class of materials defined through (5) was proposed by Bustamante [19], who proved the existence of a scalar function $W = W(\mathbf{T})$ s.t.

$$\boldsymbol{\varepsilon} = \frac{\partial W}{\partial \mathbf{T}}. \tag{10}$$

In the case of an isotropic material, we have $W(\mathbf{T}) = W(I_T, II_T, III_T)$, and it follows from (9) and (10) that

$$\boldsymbol{\varepsilon} = W_1\mathbf{I} + W_2\mathbf{T} + W_3\mathbf{T}^2, \tag{11}$$

where we have used the notation $W_1 = \frac{\partial W}{\partial I_T}$, $W_2 = \frac{\partial W}{\partial II_T}$ and $W_3 = \frac{\partial W}{\partial III_T}$.

2.3 Boundary value problem

In the absence of body forces, under the assumption that the body is static, the stress tensor $\mathbf{T}$ has to satisfy the equilibrium equation

$$\mathrm{div}\,\mathbf{T} = \mathbf{0}. \tag{12}$$

The compatibility equations for the components of the linearized strain tensor are [2]

$$\varepsilon_{kn,lm} + \varepsilon_{lm,kn} - \varepsilon_{km,ln} - \varepsilon_{ln,km} = 0, \tag{13}$$

where only six of the above equations are independent.

In the linearized theory of elasticity, there are two methods that are used to solve boundary value problems; to work with the displacement field as the main unknown variable which leads to the Navier equations, or to use the stress potential such that for 2D problems one arrives at the biharmonic equation.

In the case of the new class of materials we are interested in, if we follow the procedures described above, we obtain highly nonlinear equations for the following reasons:

- If we consider the first alternative of working with the displacement field $\mathbf{u}$ as the main unknown variable, we obtain $\boldsymbol{\varepsilon}$ by appealing to (8) and as a consequence (13) is satisfied automatically.
  For this new class of materials from (5), we would need to calculate the components of $\mathbf{T}$. In general, (5) may not be invertible (in some interesting problems, we may obtain more than one solution); therefore, we may need numerical methods to determine $\mathbf{T}$.
  Once we obtain $\mathbf{T}$, we need to solve (12) for $\mathbf{u}$.
- If we consider the second method of working with a stress potential, such that (12) can be satisfied automatically, then for the two-dimensional problem, we can use the solution in terms of the Airy's stress potential Φ

$$T^{11} = \Phi_{,22}, \quad T^{22} = \Phi_{,11}, \quad T^{12} = -\Phi_{,12}, \tag{14}$$

  whereas for three-dimensional problems, we can use the stress (symmetric) tensor potential $\mathbf{a}$, where [20]

$$T^{km} = e^{krp} e^{msq} a_{rs,pq}, \tag{15}$$

  where e^{ijk} is the permutation symbol.
  With the above representations, (12) is satisfied automatically. The next step for this new class of materials is to obtain the components of $\boldsymbol{\varepsilon}$ from (5) and to replace these components in (13). In the two-dimensional case, we would obtain a highly nonlinear fourth-order partial differential equation for Φ (see, for example, [11]), and in the three-dimensional case, we would obtain a system of six highly nonlinear fourth-order partial differential equations for the six independent components of $\mathbf{a}$.

The equation to be solved for Φ for the two-dimensional case working with Cartesian coordinates is documented in [11]. To date, there has been little success in finding exact solutions for such problems, and even the numerical resolution of the problems seems quite daunting; therefore, we choose to work with $\mathbf{u}$ as the main variable, and in this case, we would need to solve the boundary value problem defined through:

$$\operatorname{div}\mathbf{T} = \mathbf{0} \quad \mathbf{x} \in \boldsymbol{\kappa}_t(\mathcal{B}), \quad \mathbf{Tn} = \hat{\mathbf{t}} \quad \mathbf{x} \in \partial\boldsymbol{\kappa}_t^t(\mathcal{B}), \quad \mathbf{u} = \hat{\mathbf{u}} \quad \mathbf{x} \in \partial\boldsymbol{\kappa}_t^u(\mathcal{B}), \tag{16}$$

where $\hat{\mathbf{t}}$ and $\hat{\mathbf{u}}$ are the prescribed traction and displacement on the boundaries $\partial\boldsymbol{\kappa}_t^t(\mathcal{B})$, $\partial\boldsymbol{\kappa}_t^u(\mathcal{B})$, respectively, where $\partial\boldsymbol{\kappa}_t^t(\mathcal{B}) \cup \partial\boldsymbol{\kappa}_t^u(\mathcal{B}) = \partial\boldsymbol{\kappa}_t(\mathcal{B})$ and $\partial\boldsymbol{\kappa}_t^t(\mathcal{B}) \cap \partial\boldsymbol{\kappa}_t^u(\mathcal{B}) = \emptyset$. The components of $\mathbf{T}$ are related to $\mathbf{u}$ through (using (8) and (11))

$$\boldsymbol{\varepsilon} = W_1\mathbf{I} + W_2\mathbf{T} + W_3\mathbf{T}^2, \quad \boldsymbol{\varepsilon} = \frac{1}{2}\left[\left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}}\right) + \left(\frac{\partial \mathbf{u}}{\partial \mathbf{x}}\right)^{\mathrm{T}}\right], \tag{17}$$

where W depends (in general, nonlinearly) on $\mathbf{T}$.

3 Numerical solution of the boundary value problem using the finite element method

3.1 Finite element approximation

We are interested in studying the behavior of a thin plane square plate with a circular hole, which is under the effect of a uniform traction σ_∞ applied on two of the edges of the plate (see Fig. 1). We can consider the problem as a two-dimensional problem, and we shall denote by x and y the coordinates of a point in the plate and the origin of the coordinate system being located at the center of the hole. Let r_i and L denote the radius of the hole and half of the length of the plate, respectively (see Fig. 1). Far away, we apply a uniform tension σ_∞. We will assume that $r_i \ll L$.

In Fig. 2, we have a depiction of the mesh for a quarter of the whole plate. Due to the symmetries of this problem, only a quarter of this plate is needed for our analysis. We notice that near the hole, the density of the

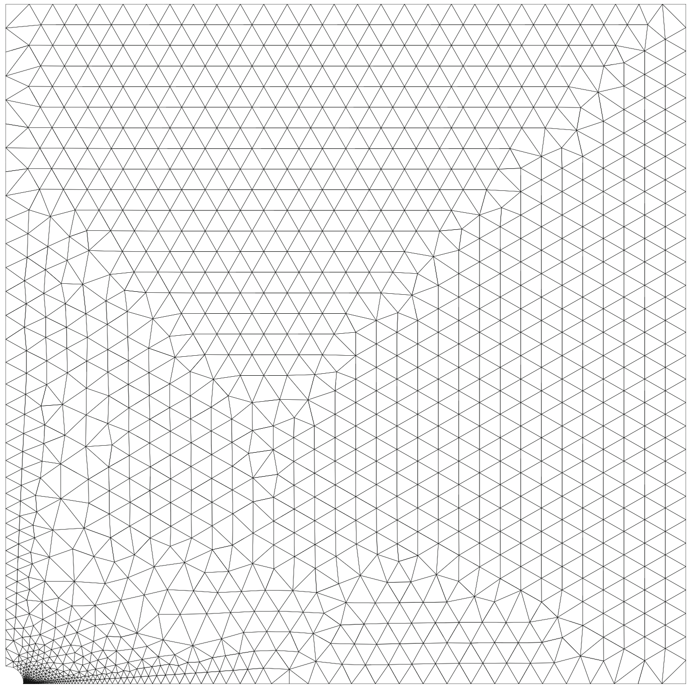

Fig. 2 Mesh for a quarter of the whole plate

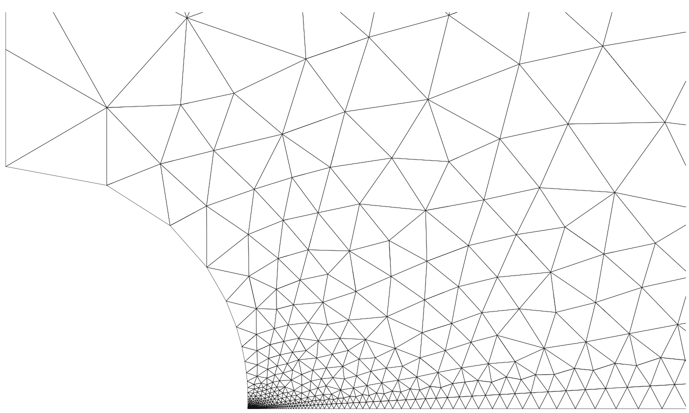

Fig. 3 Mesh near the surface of the hole

meshing is far greater than away from the hole, as we expect a more rapid change in the distributions of the stresses and strains near the hole.

In Fig. 3, we provide a closer view of the meshing near the hole.

The main purpose of the study is to determine the growth of the stress and strain in a body whose constitutive relation belongs to the subclass of elastic bodies (10). We shall pick a specific form for the function W as given below:

$$W(I_T, II_T) = -\alpha \left[I_T - \frac{1}{\beta} \ln(1 + \beta I_T) \right] + \frac{\alpha\gamma}{\iota} \sqrt{1 + 2\iota II_T}, \tag{18}$$

where I_T and II_T are as defined in $(9)_{1,2}$, and where α, β, γ, and ι are constants. The values for these constants are assumed to be (using the international system)

$$\alpha = 10^{-3}, \quad \beta = 10^{-5} \frac{1}{\text{Pa}}, \quad \gamma = 10^{-1}, \quad \iota = 1 \frac{1}{\text{Pa}^2}. \tag{19}$$

Remark We need to point out that the model (18) and the values (19) have not been obtained by corroboration against actual experimental data. The constitutive theory (5), (10) has been presented recently (see [11,12,14, 15,19]), and it is necessary to explore the implications and consequences of these new class of constitutive relations by solving some boundary value problems. The particular form for W used in this paper has been considered in previous works (see [14]), and it follows from (11) that the explicit expression for the linearized strain that corresponds to the special choice for the function W is

$$\boldsymbol{\varepsilon} = -\alpha\left[1 - \frac{1}{(1+\beta I_T)}\right]\mathbf{I} + \frac{\alpha\gamma}{\sqrt{1+2\iota II_T}}\mathbf{T}. \tag{20}$$

It follows that in a simple uniaxial extension problem, the constitutive relation (20) leads to a limiting strain. The values for the constants shown in (19) are similar in magnitude to the values used in [14].

To solve the boundary value problem (16), we used the finite element method (nonlinear analysis), developing our own code written in Matlab. Figure 2 shows one of the meshes used. We also provide some additional details with regard to the computations:

- Statistics for the mesh: 1,989 nodes and 3,659 elements.
- Type of element: 3-nodes linear triangle.
- Method used to solve the nonlinear equations: Quasi-Newton method with relaxation (line search minimization).
- Number of increments: 200.

The external load and dimensions for the geometry are as follows:

$$\sigma_\infty = 1\,\text{Pa}, \quad L = 1\,\text{m}, \quad r_i = 0.025\,\text{m}. \tag{21}$$

3.2 Results

We need to study the influence of the mesh density on our results, and in order to do so, we would need to study the error, with regard to the numerical results as a function of the mesh density (for example, in relation to the number of nodes). If we know the exact solution for this boundary value problem, which can be denoted as $\mathbf{u}^e$ for the displacement field, and if $\mathbf{u}^i$ denotes the displacement field for the different mesh densities considered, then we could define the error through

$$e^i = \frac{\int_{\kappa_t(\mathcal{B})} \|\mathbf{u}^e - \mathbf{u}^i\|\,\mathrm{d}v}{\int_{\kappa_t(\mathcal{B})} \|\mathbf{u}^e\|\,\mathrm{d}v}. \tag{22}$$

However, we do not have an exact solution for this problem; therefore, we determine (approximately) the rate of convergence in the following way. Let us define $\mathbf{u}^f$ as the displacement field obtained using the finest mesh; then, we can calculate

$$f^i = \frac{\int_{\kappa_t(\mathcal{B})} \|\mathbf{u}^f - \mathbf{u}^i\|\,\mathrm{d}v}{\int_{\kappa_t(\mathcal{B})} \|\mathbf{u}^f\|\,\mathrm{d}v}. \tag{23}$$

Thereafter, we calculate the approximated 'rate of convergence' R^i as

$$R^i = \frac{f^i - f^{i-1}}{n_i - n_{i-1}}, \tag{24}$$

where n_k would be the number of nodes for the meshes considered.

For simplicity, we do not calculate the above integrals for the whole body, but only for a part of the line $(x, 0)$ (see Fig. 1), where we expect to obtain maximum stress concentration. Also, for that line, we replace the norms $\|\mathbf{u}^f - \mathbf{u}^i\|$, $\|\mathbf{u}^f\|$ by $|u_1^f(x,0) - u_1^i(x,0)|$, and $|u_1^f(x,0)|$, respectively. Therefore, we have

$$f^i = \frac{\int_r^L |u_1^f(x,0) - u_1^i(x,0)|\,\mathrm{d}x}{\int_r^L |u_1^f(x,0)|\,\mathrm{d}x}. \tag{25}$$

Figure 4 displays the behavior of R^i as a function of the number of nodes for different meshes.

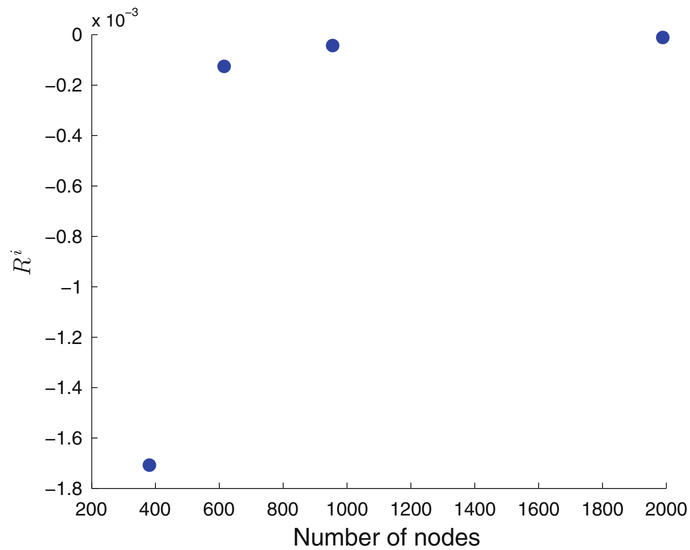

Fig. 4 Convergence error

Fig. 5 Contour plot for the strain ε_{22}

In Figs. 5, 6, we have depicted ε_{22} and T_{22} as functions of the radial distance in a neighborhood adjacent to the hole.

If T_{22}^{∞} and $\varepsilon_{22}^{\infty}$ are the stress and strain components evaluated far away from the surface of the hole[3], we can define

$$\bar{T}_{22}(x) = \frac{T_{22}(x, 0)}{T_{22}^{\infty}}, \quad \bar{\varepsilon}_{22}(x) = \frac{\varepsilon_{22}(x, 0)}{\varepsilon_{22}^{\infty}}. \tag{26}$$

Figure 7 portrays the behavior of $\bar{T}_{22}$ and $\bar{\varepsilon}_{22}$. In that figure, we have also plotted the 'linear solution,' which is the result for $\bar{T}_{22}$ and $\bar{\varepsilon}_{22}$; in the case, we consider the linear constitutive equation

$$\boldsymbol{\varepsilon} = -\alpha\beta I_T \mathbf{I} + \alpha\gamma\mathbf{T}, \tag{27}$$

which can be obtained from (20) when $\iota = 0$ and $\beta I_T \ll 1$. In this case, the curves for $\bar{T}_{22}$ and $\bar{\varepsilon}_{22}$ are the same as expected for the linearized solution.

We notice in Figs. 5 and 6, as is to be expected, that the highest strain and stress occur near the point $(ri, 0)$. *We also see that the maximum strain is of the order of ten to the power of minus four. The fact that the*

[3] In the practice, we obtain T_{22}^{∞} and $\varepsilon_{22}^{\infty}$ evaluating $T_{22}(x, 0)$ and $\varepsilon_{22}(x, 0)$ for x large enough, such that the stress and strain distributions are almost uniform and unaffected by the presence of the hole. In that case, $T_{22}^{\infty} \approx \sigma_{\infty}$.

Fig. 6 Contour plot for the stress T_{22}. Stress in [Pa]

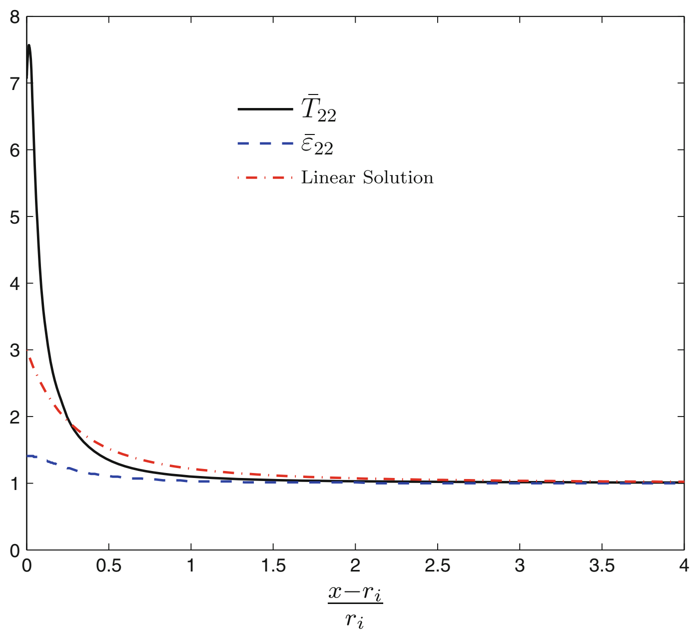

Fig. 7 Variation of the normalized stress and strain near the hole

linearized strain remains small, and more importantly, the growth of the linearized strain being much slower than the stresses, as one approaches the hole, cannot be overemphasized. It is also worth noting that the stress concentration factor is close to 9, this is, however, not unexpected, as the constitutive relation for bodies with such a limiting stress, allows for the body to withstand a much higher stress concentration, as the strains are yet limited and within the small strain approximation.

Figures 4, 5, 6, 7 were obtained assuming $\sigma_\infty = 1\,\text{Pa}$.

In Fig. 8, we display the manner in which $\bar{T}_{22}$ and $\bar{\varepsilon}_{22}$ vary with x, for different values for the external load σ_∞.

4 Final remarks

The aim of this study was to determine the stresses and strains that manifest themselves in a classical problem in solid mechanics, namely the problem of a plate with a hole subject to uniform loading at infinity, within the context of a new class of constitutive relations that shows much promise with regard to the resolution of problems, which usually lead to a singularity in the linearized strain, thereby contradicting the basic assumption

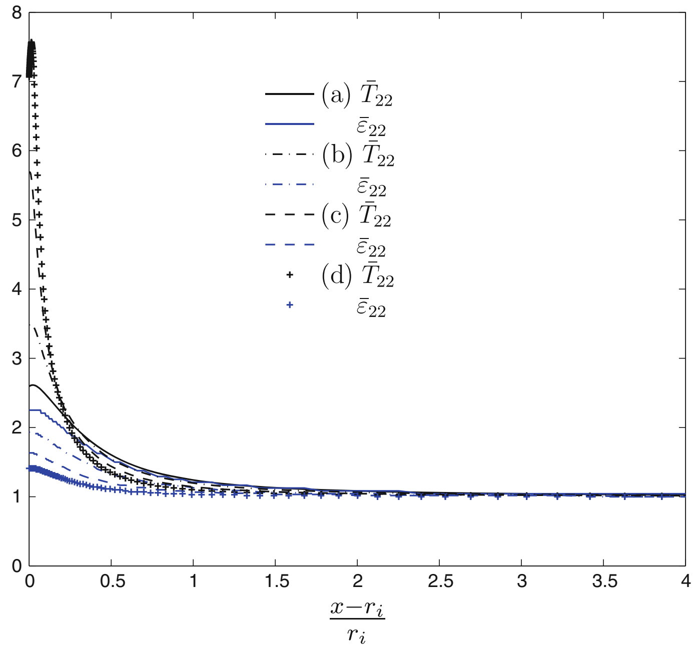

Fig. 8 Behavior of the normalized stress and strain near the hole for different values of the external load σ_∞ Pa. (*a*) 0.25, (*b*) 0.5, (*c*) 0.75, and (*d*) 1

within which the classical linearized elastic model is derived, namely that the strain is very small. As expected, the growth of the strain as one approaches the hole is much slower than the growth of the stress. This result is in keeping with the result obtained by Rajagopal and Walton [13], who found that the linearized strain, even at the tip of a crack (the body being subject to a state of anti-plane strain), is bounded. As mentioned earlier, it is necessary to evaluate the usefulness of the new class of models by solving several other specific boundary value problems.

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