Acta Mech 223, 1307–1321 (2012)
DOI 10.1007/s00707-012-0651-y

B. J. Yang · B. R. Kim · H. K. Lee

Micromechanics-based viscoelastic damage model for particle-reinforced polymeric composites

Received: 20 June 2011 / Revised: 8 December 2011 / Published online: 31 March 2012

Abstract The objective of this study is to develop a micromechanics-based viscoelastic damage model that can predict the overall viscoelastic behavior of particle-reinforced polymeric composites undergoing damage. The emphasis here is that the present model successfully combines a rate-dependent viscoelastic constitutive model and a damage model. The Laplace transform based on the Boltzmann superposition principle and the ensemble-volume averaged method suggested by Ju and Chen (Acta Mech 103:103–121, 1994a; Acta Mech 103:123–144, 1994b) are extended toward effective viscoelastic properties. Further, the probability of the distribution function of Weibull (J Appl Mech 18:293–297, 1951) is adopted to describe a damage model that is dependent on damage parameters. A series of numerical simulations including parametric studies, and experimental comparisons are carried out to give insight into the potential capacity of the present micromechanics-based viscoelastic damage framework.

1 Introduction

Particle-reinforced polymeric composites have been widely used in many engineering applications due to their potential to provide desirable mechanical properties such as high modulus and high strength or toughness [1,2]. The particle-reinforced polymeric composites consist of randomly dispersed elastic particles embedded in an inelastic matrix that is known to degrade with time and under mechanical, thermal, and environmental loads [3]. These types of composites generally exhibit nonlinear constitutive response due to various factors. In particular, the rate-dependent behavior of these composites and the mechanical degradation due to the existence of damage in the composites can play a significant role in the nonlinear constitutive response [4–8].

It is known that the rate-dependent nature of particle-reinforced polymeric composites can be mainly explained by the viscoelastic theory [9,10]. In the viscoelastic theory, the viscoelastic phenomenon can be described as several simple physical models that include various configurations of the spring element, which stands for the elastic characteristic, and the dashpot element, which depicts the viscous characteristic [11,12]. The typical models among the simple physical models are the Maxwell and Voigt models [11,12]. The Maxwell model consists of the spring and the dashpot in series, and the Voigt model consists of them in parallel [11–13]. Each model has a limitation as the Maxwell model cannot predict creep accurately, while the Voigt model is much less accurate with regard to relaxation [13]. In particular, polymeric composites have strong

B. J. Yang · H. K. Lee (✉)
Department of Civil and Environmental Engineering, Korea Advanced Institute of Science and Technology,
Guseong-dong, Yuseong-gu, Daejeon 305-701, South Korea
E-mail: leeh@kaist.ac.kr
Tel.: +82-42-8693623

B. R. Kim
Civil and Architecture Engineering Department, KEPCO Engineering & Construction Company Inc.,
2354 Yonggudaero, Giheung-gu, Yongin-si, Gyeonggi-do 446-713, South Korea

viscoelastic characteristics compared to conventional material [14]. In the case of these polymeric composites, the Maxwell model is widely used because it predicts that the stress decays exponentially with time, which is accurate for most polymers [13].

Micromechanical methods can estimate the overall performance of composites from individual unique properties of the material. In addition, the methods have an advantage in that an a priori assumption is not needed for the establishment of the composite response [15]. The micromechanical methods have been extensively developed for predicting the mechanical behavior of particle-reinforced composites, and in particular, micromechanics-based viscoelastic modeling can also be found in several studies. Li and Weng [16] extended the Mori-Tanaka method toward viscoelastic material to find its properties. They focused on the influence of inclusion shape and examined cases with an elastic inclusion and viscoelastic matrix under constant strain rate and cyclic uniform strain. Lévesque et al. [17] also used the Mori-Tanaka method to predict a linearly homogenized scheme of particulate composites. It was assumed in Lévesque et al. [17] that the particle is a linear elastic one and only resin has a viscoelastic property. In particular, in Lévesque et al. [17], Schapery's nonlinear viscoelastic model [18] was incorporated into the matrix system for modeling the effective viscoelastic behavior of particulate composites. Muliana and Kim [19] proposed a micromechanical model using a unit-cell micromodel. In Muliana and Kim [19], the micromechanical method to find stiffness and compliances of a unit-cell was defined by following Haj-Ali and Pecknold's model [20], while stress–strain relations of each subcell were expressed in terms of phase average concentration factors, which were developed by Hill [21]. In addition, the time-integration for considering the viscoelastic property in a matrix was performed for nonlinear analyses in Muliana and Kim [19]. Kurnatowski and Matzenmiller [22] introduced a micromechanical model named as generalized method of cells (GMC) to provide the homogenized constitutive tensor of composites. The macroscale of the problem was solved by the finite element method, while the microscale behavior was analyzed by the GMC to predict viscoelastic matrix/elastic particle composites in Kurnatowski and Matzenmiller [22].

The objective of this study is to develop a micromechanics-based viscoelastic damage model that can predict the overall viscoelastic behavior of particle-reinforced polymeric composites undergoing damage. Damage in materials is due to many reasons and leads to a decrease in stiffness and a reduction in strength. Hence, the progressive evolution of damage in materials causes overall nonlinear behavior so that it must be considered to predict realistic response in applications. The emphasis here is that the present model successfully combines a rate-dependent viscoelastic constitutive model and a damage model. The Laplace transform based on the Boltzmann superposition principle and the ensemble-volume averaged method suggested by Ju and Chen [23,24] are extended toward effective viscoelastic properties. Further, the probability of the distribution function of Weibull [25] is adopted to describe a damage model that is dependent on damage parameters. Numerical simulations including parametric studies and experimental comparisons of the developed model are carried out to give insight into the potential capacity of the present micromechanics-based viscoelastic damage framework.

2 Effective viscoelastic moduli of particle-reinforced polymeric composites considering damage

2.1 Recapitulation of the Boltzmann superposition principle

The Boltzmann superposition principle is well accepted as the simplest and more powerful approach among a variety of possible approaches for expressing viscoelastic behavior [26–28]. In the Boltzmann superposition principle, the strain history is described as a function of the rate of the loading history, and the total deformation is the sum of each step of independent contribution [28]. According to the Boltzmann superposition principle, the stress–strain relationship for a viscoelastic material can be expressed as [11,29]

$$\sigma_{ij}(t) = \int_{-\infty}^{t} C^{*}_{ijkl}(t-\tau)\frac{\mathrm{d}\epsilon_{kl}(\tau)}{\mathrm{d}\tau}\mathrm{d}\tau \tag{1}$$

where t and τ denote the arbitrary time and the time under instantaneous application of $\sigma_{ij}(t)$, respectively. In addition, the fourth-rank tensor C^{*}_{ijkl} denotes the effective relaxation modulus which becomes zero assuming that the material is unstressed and unreformed in the range $-\infty \le t \le 0$ [29]. In order to make use of the Boltzmann superposition principle, the Laplace transform of a function $f(t)$ is used in the present study as

$$\mathcal{L}[f(t)] = \int_0^t f(t)e^{-st}\mathrm{d}t \tag{2}$$

where $\mathcal{L}[f(t)]$ is the Laplace transform of function $f(t)$ and s is the Laplace parameter. Taking the Laplace transform of Eq. (1) yields [11]

$$\tilde{\sigma}_{ij}(s) = s\tilde{C}^{*}_{ijkl}(s)\tilde{\epsilon}_{kl}(s) \tag{3}$$

in which a tilde ($\sim$) signifies the transform domain (TD). The Laplace-transformed Eq. (3) is now an analogous form of stress–strain relations. This is the basis of the correspondence principle of estimating the behavior of a material, and viscoelastic problems can be solved by combining the respective solution of the elastic problem and the integral Laplace transform domain [30,31]. Details of the Boltzmann superposition principle and the Laplace transform can be found in Gibson [11], Lakes [12], and McCrum et al. [13].

2.2 Micromechanics-based constitutive equations for the viscoelastic behavior of particle-reinforced polymeric composites

We consider here particle-reinforced polymeric composites made up with a viscoelastic matrix (phase 0) and randomly dispersed spherical elastic particles (phase 1) embedded in the matrix. It is assumed that all of the particles are perfectly bonded in the initial state, but some of the particles are damaged by increasing loading or deformation on the composites, and those particles could be then separately regarded as damaged particles (completely debonded particles, phase 2) that may lose their load-carrying capacity. The effective elastic constitutive equation of the particle-reinforced composites has been studied by many researchers [32–38]. In particular, following the ensemble-averaged volume method proposed by Ju and Chen [23,24], the effective elastic tensor $\mathbf{C}^*$ of the three-phase composites can be given by

$$\begin{aligned}\mathbf{C}^* &= \mathbf{C}_0\left[\mathbf{I} + \sum_{r=1}^{2}\left\{\phi_r\left[(\mathbf{C}_r - \mathbf{C}_0)^{-1}\cdot\mathbf{C}_0 + \mathbf{S}\right]^{-1}\cdot\left[\mathbf{I} - \phi_r\mathbf{S}\cdot\left\{(\mathbf{C}_r - \mathbf{C}_0)^{-1}\cdot\mathbf{C}_0 + \mathbf{S}\right\}^{-1}\right]^{-1}\right\}\right] \\ &= \lambda_*\delta_{ij}\delta_{kl} + \mu_*(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk})\end{aligned} \tag{4}$$

where ϕ_r denotes the volume fraction of the r-phase, $\mathbf{C}_r$ is the stiffness tensor for the r-phase, $\mathbf{S}$ signifies the Eshelby's tensor for a spherical inclusion [39], and δ is Kronecker delta. In addition, λ_* and μ_* are the effective Lamé constants and can be expressed as [35–38]

$$\lambda_* = \kappa_0\left\{1 + \sum_{r=1}^{2}\frac{30(1-\nu_0)\phi_r}{3\alpha_r + 2\beta_r - 10(1+\nu_0)\phi_r}\right\} - \frac{2}{3}\mu_0\left\{1 + \sum_{r=1}^{2}\frac{15(1-\nu_0)\phi_r}{\beta_r - 2(4-5\nu_0)\phi_r}\right\}, \tag{5}$$

$$\mu_* = \mu_0\left\{1 + \sum_{r=1}^{2}\frac{15(1-\nu_0)\phi_r}{\beta_r - 2(4-5\nu_0)\phi_r}\right\}, \tag{6}$$

with

$$\alpha_1 = 2(5\nu_0 - 1) + 10(1-\nu_0)\left(\frac{\kappa_0}{\kappa_1 - \kappa_0} - \frac{\mu_0}{\mu_1 - \mu_0}\right), \qquad \alpha_2 = 2(5\nu_0 - 1), \tag{7}$$

$$\beta_1 = 2(4 - 5\nu_0) + 15(1-\nu_0)\left(\frac{\mu_0}{\mu_1 - \mu_0}\right), \qquad \beta_2 = -7 + 5\nu_0 \tag{8}$$

in which κ_0, μ_0, and ν_0 are bulk modulus, shear modulus, and Poisson's ratio of the matrix, respectively. Details of the effective elastic constitutive equations can be found in Ju and Chen [23,24], Ju and Lee [40,41], and Liu et al. [42].

In order to derive the effective viscoelastic constitutive equations of particle-reinforced polymeric composites, two assumptions are made in the present study as [16,43]: (a) the particle-reinforced polymeric composites consist of elastic particles and a viscoelastic matrix, and (b) the matrix is viscoelastic in shear and dilatation. According to Hashin [43], the effective viscoelastic modulus can be derived by replacing the elastic phase by a time-domain phase. That is, the effective viscoelastic constitutive equations in the Laplace-transformed domain can be obtained as

$$\lambda_*^{TD} = \kappa_0^{TD}\left\{1+\sum_{r=1}^{2}\frac{30\left(1-\nu_0^{TD}\right)\phi_r}{3\alpha_r^{TD}+2\beta_r^{TD}-10\left(1+\nu_0^{TD}\right)\phi_r}\right\} - \frac{2}{3}\mu_0^{TD}\left\{1+\sum_{r=1}^{2}\frac{15\left(1-\nu_0^{TD}\right)\phi_r}{\beta_r^{TD}-2\left(4-5\nu_0^{TD}\right)\phi_r}\right\}, \quad (9)$$

$$\mu_*^{TD} = \mu_0^{TD}\left\{1+\sum_{r=1}^{2}\frac{15\left(1-\nu_0^{TD}\right)\phi_r}{\beta_r^{TD}-2\left(4-5\nu_0^{TD}\right)\phi_r}\right\} \quad (10)$$

with

$$\alpha_r^{TD} = 2\left(5\nu_0^{TD}-1\right)+10\left(1-\nu_0^{TD}\right)\left(\frac{\kappa_0^{TD}}{\kappa_r-\kappa_0^{TD}}-\frac{\mu_0^{TD}}{\mu_r-\mu_0^{TD}}\right), \quad (11)$$

$$\beta_r^{TD} = 2\left(4-5\nu_0^{TD}\right)+15\left(1-\nu_0^{TD}\right)\left(\frac{\mu_0^{TD}}{\mu_r-\mu_0^{TD}}\right), \quad (12)$$

where the Laplace-transformed bulk modulus κ_0^{TD} and shear modulus μ_0^{TD} of the viscoelastic matrix are defined based on the Maxwell model [43–45] as

$$\kappa_0^{TD} = \frac{\eta_0\kappa_0 s}{\mu_0+s\eta_0}, \qquad \mu_0^{TD} = \frac{\eta_0\mu_0 s}{\mu_0+s\eta_0}, \quad (13)$$

in which η_0 is the shear viscosity of the matrix. The Laplace-transformed Poisson's ratio ν_0^{TD} can be obtained through the following relation:

$$\nu_0^{TD} = \frac{3\kappa_0^{TD}-2\mu_0^{TD}}{6\kappa_0^{TD}+2\mu_0^{TD}}. \quad (14)$$

The effective viscoelasticity tensor of the particle-reinforced polymeric composites can be described by taking the inverse Laplace transform as

$$\mathbf{C}^*(t) = \lambda_*(t)\delta_{ij}\delta_{kl}+\mu_*(t)(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk}) \quad (15)$$

with

$$\lambda_*(t) = \frac{\eta_0\kappa_0}{t\mu_0}\left\{\frac{\Gamma_7(\nu_0-1)^2\phi_1}{\Gamma_4}+\frac{\Gamma_5}{\Gamma_1\Gamma_3}+\frac{\exp\left(\frac{-\mu_0 t}{\eta_0}\right)\Gamma_6}{\Gamma_2\Gamma_3}\right\} - \frac{2}{3}\frac{\eta_0}{2t}\left[\frac{\xi_4(\nu_0-1)^2\phi_1}{\xi_1\xi_3}+\frac{\xi_5+\xi_8}{\xi_2\xi_3}+\frac{2\xi_7\exp\left(\frac{-\mu_0 t}{\eta_0}\right)}{\xi_1\xi_2}\right], \quad (16)$$

$$\mu_*(t) = \frac{\eta_0}{2t}\left[\frac{\xi_4(\nu_0-1)^2\phi_1}{\xi_1\xi_3}+\frac{\xi_5+\xi_8}{\xi_2\xi_3}+\frac{2\xi_7\exp\left(\frac{-\mu_0 t}{\eta_0}\right)}{\xi_1\xi_2}\right] \quad (17)$$

where the parameters Γ_i $(i = 1, \ldots, 7)$, and ξ_i $(i = 1, \ldots, 8)$ can be expressed as

$$\Gamma_1 = \nu_0(1+\phi_1) - \phi_1 + 1, \qquad \Gamma_2 = \nu_0(4+\phi_1) - \phi_1 - 2, \tag{18}$$

$$\Gamma_3 = \nu_0(4+\phi_2) - \phi_2 - 2, \qquad \Gamma_4 = \{\phi_1(\nu_0-1) + \nu_0 + 1\}\{\nu_0(4+\phi_1) - \phi_1 - 2\}, \tag{19}$$

$$\Gamma_5 = -\nu_0^2\{\phi_1(5\phi_2+8) + 2(\phi_2-2)\} + 2\nu_0\{\phi_1(5\phi_2+6)+1\} - \phi_1(5\phi_2+4) + 2(\phi_2-1), \tag{20}$$

$$\Gamma_6 = \phi_2\{\nu_0(5\phi_1+8) - 5\phi_1 - 4\}(\nu_0-1) + 4\{\nu_0(\phi_1-2) - \phi_1 + 1\}(2\nu_0-1), \tag{21}$$

$$\Gamma_7 = 9\exp\left(\frac{t\kappa_1\mu_0\Gamma_1}{\eta_0(\kappa_0\Gamma_2 - \kappa_1\Gamma_1)}\right), \tag{22}$$

$$\xi_1 = 5\nu_0(1+2\phi_1) - 7 - 8\phi_1, \qquad \xi_2 = 5\nu_0(1+2\phi_2) - 7 - 8\phi_2, \tag{23}$$

$$\xi_3 = (5\nu_0-4)(\phi_1-1), \qquad \xi_4 = 225\exp\left\{\frac{2t\mu_0\mu_1\xi_3}{\eta_0(-2\mu_1\xi_3 + \mu_0\xi_1)}\right\}, \tag{24}$$

$$\xi_5 = 56(\phi_2-1) - \phi_1(49+176\phi_2) - 25\nu_0^2\{\phi_1 + 2\phi_2(4\phi_1-1) + 2\}, \tag{25}$$

$$\xi_6 = 5\nu_0\left[(5\nu_0-14)(\phi_1-1) + \phi_2\{5\nu_0(8\phi_1+1) - 2(38\phi_1+7)\}\right], \tag{26}$$

$$\xi_7 = \phi_1(176\phi_2+49) + 49(\phi_2-1) + \xi_6, \qquad \xi_8 = 10\nu_0\{7\phi_1 + \phi_2(38\phi_1-11) + 11\}. \tag{27}$$

2.3 Damage model

The probability of damage in particle-reinforced polymeric composites is modeled as a two-parameter Weibull process [40,46–49]. The damage model can be described as a change of the volume fraction of damaged particle ϕ_2 at a level of uniaxial tensile loading as [50–55]

$$\phi_2 = \phi P_d\,(\bar{\sigma}_{11})_1 = \phi\left\{1 - \exp\left[-\left(\frac{(\bar{\sigma}_{11})_1}{S_0}\right)^M\right]\right\} \tag{28}$$

where ϕ is the original particle volume fraction, and S_0 and M are the Weibull damage parameters. The internal stress $\bar{\boldsymbol{\sigma}}_1$ of particles can be obtained as [40]

$$\bar{\boldsymbol{\sigma}}_1 = \mathbf{C}_1 : \left[\mathbf{I} - \mathbf{S}\cdot(\mathbf{A}_1+\mathbf{S})^{-1}\right]\cdot\left[\mathbf{I} - \sum_{r=1}^{2}\phi_r\mathbf{S}\cdot(\mathbf{A}_r+\mathbf{S})^{-1}\right]^{-1} : \bar{\boldsymbol{\epsilon}} \equiv \mathbf{U} : \bar{\boldsymbol{\epsilon}} \tag{29}$$

where $\mathbf{A}_r = (\mathbf{C}_r - \mathbf{C}_0)^{-1}\cdot\mathbf{C}_0$, and details of the internal stress $\bar{\boldsymbol{\sigma}}_1$ of particles can be found in Eqs. (69)–(71) of [40].

The component of the positive-definite fourth-rank tensor $\mathbf{U}$ is given by

$$U_{ijkl} = U_1\delta_{ij}\delta_{kl} + U_2(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{30}$$

where the coefficients U_1 and U_2 are given by [40]

$$U_1 = \frac{3(3\chi_1+2\chi_2)\varphi_2\kappa_1 - 2(3\varphi_1+2\varphi_2)\chi_2\mu_1}{3\varphi_2(3\chi_1+\varphi_2)}, \qquad U_2 = \frac{\chi_2\mu_1}{\varphi_2} \tag{31}$$

with

$$\chi_1 = -2\left[\frac{-(4\alpha_1+\beta_1) + 5\nu_0(\alpha_1+\beta_1)}{\beta_1(3\alpha_1+2\beta_1)}\right], \qquad \chi_2 = -2\left[\frac{4-5\nu_0}{2\beta_1} - \frac{1}{4}\right], \tag{32}$$

$$\varphi_1 = \sum_{r=1}^{2}\left[-2\phi_r\left\{\frac{-(4\alpha_r+\beta_r) + 5\nu_0(\alpha_r+\beta_r)}{\beta_r(3\alpha_r+2\beta_r)}\right\}\right], \tag{33}$$

$$\varphi_2 = \sum_{r=1}^{2}\left[-2\phi_r\left\{\frac{4-5\nu_0}{2\beta_r} - \frac{1}{8\phi_r}\right\}\right]. \tag{34}$$

Details of the damage model can be found in Lee and Pyo [56,57], Kim and Lee [37], and Lee et al. [58].

The Boltzmann superposition principle is also applied to the damage model. The elastic phase of the fourth-rank tensor **U** is, therefore, replaced with the Laplace-transformed domain as

$$U_1^{\mathrm{TD}} = \frac{3\left(3\chi_1^{\mathrm{TD}} + 2\chi_2^{\mathrm{TD}}\right)\varphi_2^{\mathrm{TD}}\kappa_1 - 2\left(3\varphi_1^{\mathrm{TD}} + 2\varphi_2^{\mathrm{TD}}\right)\chi_2^{\mathrm{TD}}\mu_1}{3\varphi_2^{\mathrm{TD}}\left(3\chi_1^{\mathrm{TD}} + \varphi_2^{\mathrm{TD}}\right)}, \qquad U_2^{\mathrm{TD}} = \frac{\chi_2^{\mathrm{TD}}\mu_1}{\varphi_2^{\mathrm{TD}}} \tag{35}$$

with

$$\chi_1^{\mathrm{TD}} = -2\left[\frac{-\left(4\alpha_1^{\mathrm{TD}} + \beta_1^{\mathrm{TD}}\right) + 5\nu_0^{\mathrm{TD}}\left(\alpha_1^{\mathrm{TD}} + \beta_1^{\mathrm{TD}}\right)}{\beta_1^{\mathrm{TD}}\left(3\alpha_1^{\mathrm{TD}} + 2\beta_1^{\mathrm{TD}}\right)}\right], \qquad \chi_2^{\mathrm{TD}} = -2\left[\frac{4 - 5\nu_0^{\mathrm{TD}}}{2\beta_1^{\mathrm{TD}}} - \frac{1}{4}\right], \tag{36}$$

$$\varphi_1^{\mathrm{TD}} = \sum_{r=1}^{2}\left[-2\phi_r\left\{\frac{-\left(4\alpha_r^{\mathrm{TD}} + \beta_r^{\mathrm{TD}}\right) + 5\nu_0^{\mathrm{TD}}\left(\alpha_r^{\mathrm{TD}} + \beta_r^{\mathrm{TD}}\right)}{\beta_r^{\mathrm{TD}}\left(3\alpha_r^{\mathrm{TD}} + 2\beta_r^{\mathrm{TD}}\right)}\right\}\right], \tag{37}$$

$$\varphi_2^{\mathrm{TD}} = \sum_{r=1}^{2}\left[-2\phi_r\left\{\frac{4 - 5\nu_0^{\mathrm{TD}}}{2\beta_r^{\mathrm{TD}}} - \frac{1}{8\phi_r}\right\}\right]. \tag{38}$$

Following the aforementioned procedure, the inverse Laplace transform reads

$$\mathbf{U}(t) = U_1(t)\delta_{ij}\delta_{kl} + U_2(t)(\delta_{ik}\delta_{jl} + \delta_{il}\delta_{jk}) \tag{39}$$

with

$$U_1(t) = \frac{(-1+\nu_0)}{4t}\left[\frac{\eta_0 J_3 J_5}{\mu_0^3\{7 + \nu_0(-19 + 10\nu_0)\}^2} - \frac{\eta_0 J_2 \kappa_0 \Psi_3}{\mu_0(1-2\nu_0)^2} - J_4 + J_6\right], \tag{40}$$

$$U_2(t) = \frac{3\mu_0^2(-1+\nu_0)}{2t\Psi_2^2\Psi_5^2}\left[-10\eta_0 J_5 \Psi_1 \Psi_2^2 + J_1\left\{\eta_0\Psi_1\Psi_2^2 - 30t\mu_0\mu_1\phi_1\Psi_5(4+\Psi_6)\right\}\right], \tag{41}$$

in which

$$\Psi_1 = \frac{3}{2}\left[5\nu_0\left\{2(\phi_1+\phi_2)+1\right\} - 8(\phi_1+\phi_2) - 7\right], \tag{42}$$

$$\Psi_2 = \frac{3}{2}\left[\mu_0(5\nu_0 - 7) + 2\mu_1(5\nu_0 - 4)\right], \tag{43}$$

$$\Psi_3 = \nu_0(\phi_1 + \phi_2 - 4) + \phi_1 + \phi_2 + 2, \qquad \Psi_4 = \kappa_0(2 - 4\nu_0) + \kappa_1(1 + \nu_0), \tag{44}$$

$$\Psi_5 = \mu_0(7 - 5\nu_0), \qquad \Psi_6 = \nu_0(5\nu_0 - 9), \tag{45}$$

$$J_1 = 10\exp\left\{\frac{2t\mu_0\mu_1(4 - 5\nu_0)}{\eta_0\Psi_2}\right\}, \qquad J_2 = 3\exp\left\{\frac{-t\kappa_1\mu_0(1+\nu_0)}{\eta_0\Psi_4}\right\}, \tag{46}$$

$$J_3 = 40\mu_0^3\Psi_1(1-2\nu_0)^2 + 3\kappa_0\Psi_3\Psi_5^2, \qquad J_4 = \frac{12tJ_1\mu_0^3\mu_1(4+\Psi_6)\phi_1}{\Psi_2^2\Psi_5}, \tag{47}$$

$$J_5 = \exp\left(\frac{-t\mu_0}{\eta_0}\right), \qquad J_6 = \frac{6tJ_2\kappa_0^2\kappa_1(-1+\nu_0^2)\phi_1}{(-1+2\nu_0)\Psi_4^2} - \frac{4\eta_0 J_1\mu_0^2\Psi_1}{\Psi_5^2}. \tag{48}$$

3 Numerical simulations

A series of parametric studies is conducted to examine the influence of the strain rate $\dot{\epsilon}$ and damage parameters S_0 and M on the viscoelastic behavior of particle-reinforced polymeric composites. Numerical uniaxial tensile tests on vinyl-ester matrix/glass bead particle composites are carried out for the parametric studies. The material properties used in these tests are $E_0 = 3.5\,\mathrm{GPa}$, $\nu_0 = 0.35$, and $\eta_0 = 180\,\mathrm{GPa}\cdot\mathrm{s}$ for the matrix and $E_1 = 70\,\mathrm{GPa}$ and $\nu_1 = 0.25$ for the particles, respectively [59]. Various strain rates are considered to show the strain rate sensitivity to the viscoelastic stress–strain response of particle-reinforced polymeric composites.

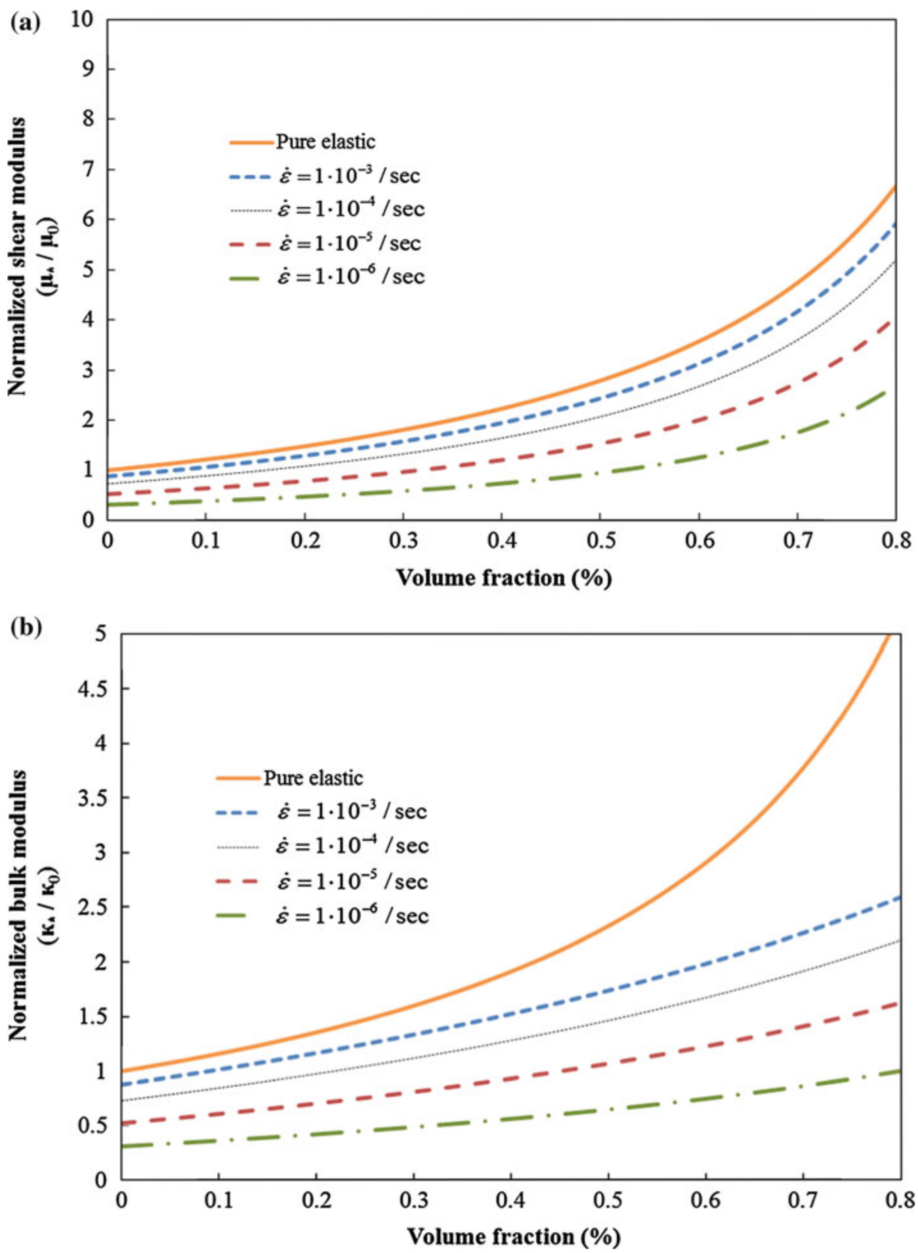

Fig. 1 The predicted normalized shear (**a**) and bulk (**b**) modulus of particle-reinforced polymeric composites with various strain rates and volume fractions of particles

Figure 1 shows the predictions of normalized shear and bulk modulus of particle-reinforced polymeric composites with various strain rates and volume fractions of particles. It is noted that as the strain rate $\dot{\epsilon}$ continues to increase, the predicted normalized shear and bulk moduli tend to increase, eventually reaching the purely elastic state.

The strain rate effect is clearly seen, and normalized shear and bulk moduli are closer to the elastic case with the increase in strain rate in the figure. In particular, the difference between the elastic and viscoelastic cases is getting more pronounced as the strain rate decreased, indicating that the strain rate significantly affects the viscoelastic properties of particle-reinforced polymeric composites. The viscoelastic stress–strain responses of the particle-reinforced polymeric composites with the volume fraction $\phi_1 = 5\,\%$ corresponding to Fig. 1 are shown in Fig. 2. It is observed from the figure that the stress–strain curves become stiffer as the strain rate continues to increase.

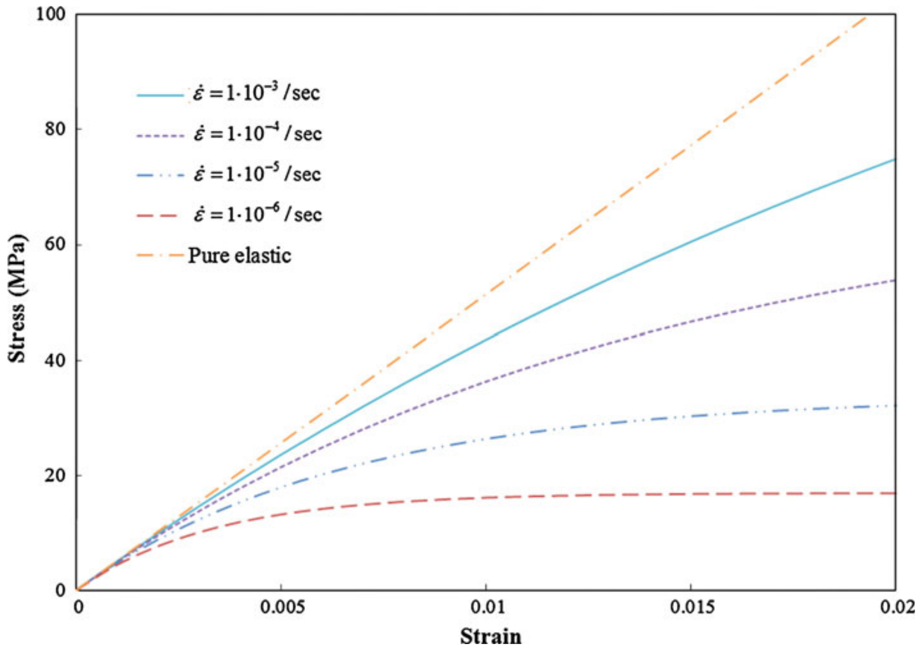

Fig. 2 The viscoelastic stress–strain responses of the particle-reinforced polymeric composites with the volume fraction $\phi = 5\,\%$

An additional parametric study is carried out to investigate the influence of the damage parameters S_0 and M on the overall viscoelastic damage behavior of particle-reinforced polymeric composites. The predicted stress–strain curves with various damage parameters (S_0, M) and the corresponding damage evolutions are shown in Figs. 3 and 4. It is seen from the figures that the damage parameter S_0 has a significant influence on the overall damage evolution compared to the damage parameter M. It is also seen that particles evolve more rapidly to damaged particles as the M value increases and the S_0 value decreases.

In addition, biaxial and hydrostatic numerical tensile tests to examine the viscoelastic stress–strain responses of the composites under different loading conditions are performed. Figures 5 and 6 show the present predicted stress–strain responses of particle-reinforced polymeric composites under biaxial and hydrostatic loadings, respectively. Similar to the aforementioned uniaxial tensile test, a lower S_0 leads to a faster evolution of damage in particle-reinforced polymeric composites.

4 Experimental comparisons

Comparisons between the present predictions and available experimental data are made to assess the predictive capability of the proposed micromechanics-based viscoelastic damage model. Firstly, a comparison between the proposed micromechanics-based viscoelastic damage model and experimental data reported by Meddad and Fisa [60] is made for validation of the proposed model. Meddad and Fisa [60] conducted experiments on glass bead particulate polystyrene composites with different volume fractions ($\phi_1 = 10, 20, 40\,\%$). We adopt the same material properties as those in Meddad and Fisa [60]: $E_0 = 3.3\,\text{GPa}$, $\nu_0 = 0.34$, and $\eta_0 = 70\,\text{GPa} \cdot \text{s}$ for the matrix; $E_1 = 70\,\text{GPa}$ and $\nu_1 = 0.25$ for particles, respectively. In addition, the damage parameters are chosen to be: $S_0 = 50\,\text{MPa}$ and $M = 2$. The strain rate is assumed to be $\dot{\epsilon} = 0.00006/\text{s}$.

Note that the damage parameters and strain rate are fitted at $\phi_1 = 20\,\%$, and then the same values are applied to different volume fraction cases ($\phi_1 = 10, 40\,\%$). Figure 7 shows the comparison of predicted stress–strain curves between the proposed micromechanics-based viscoelastic damage model and the experimental data [60] of glass bead particulate composites with various particle volume fractions. The predicted stress–strain curves obtained from the proposed micromechanics-based viscoelastic damage model are shown to have a good correlation with those of the experimental results [60]. Good agreements between the present predictions and results from the experimental data [60] show the predictive capability of the proposed micromechanics-based viscoelastic damage model.

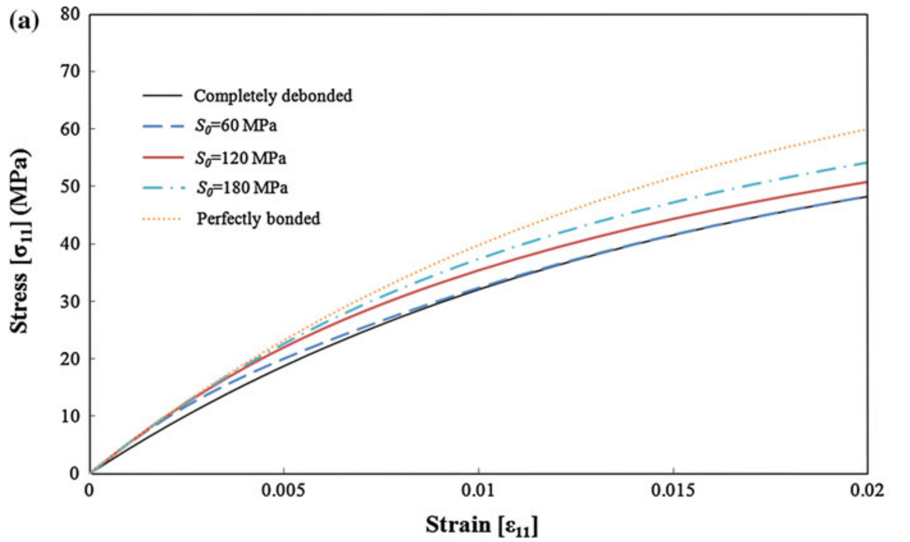

Fig. 3 The predicted stress–strain curves of particle-reinforced polymeric composites with various S_0 values (**a**) and M values (**b**)

Another comparison between the present predictions and available experimental data is made to assess the predictive capability of the proposed micromechanics-based viscoelastic damage model. Cho et al. [59] conducted experiments to characterize the viscoelastic behaviors of alumina (Al_2O_3)-reinforced vinyl-ester matrix composites and glass bead reinforced vinyl-ester matrix composites. In the present study, the experimental data reported by Cho et al. [59] are compared with the present predictions.

Following Cho et al. [59], the material properties of alumina-reinforced vinyl-ester matrix composites are assumed to be $E_0 = 3.5\,\text{GPa}$, $\nu_0 = 0.35$, and $\eta_0 = 180\,\text{GPa}\cdot\text{s}$ for the vinyl-ester matrix; and $E_1 = 145\,\text{GPa}$, $\nu_1 = 0.22$, and $\phi_1 = 3\,\%$ for the alumina particles, respectively. Based on the parametric studies in Section 3, the strain rate and the damage parameters are selected as: $\dot{\epsilon} = 0.00008/\text{s}$, $S_0 = 105\,\text{MPa}$, and $M = 1$ in this simulation. In addition, the material properties of glass bead reinforced vinyl-ester matrix composites are given as [59]: $E_0 = 3.5\,\text{GPa}$, $\nu_0 = 0.35$, and $\eta_0 = 180\,\text{GPa}\cdot\text{s}$ for the vinyl-ester matrix; and $E_1 = 70\,\text{GPa}$, $\nu_1 = 0.25$, and $\phi_1 = 5\,\%$ for the glass bead particles, respectively. The strain rate and damage parameters are assumed to be $\dot{\epsilon} = 0.00009/\text{s}$, $S_0 = 50\,\text{MPa}$, and $M = 1$.

Figures 8 and 9 show the comparisons between the present predictions with and without the damage mechanism and experimental data [59] for alumina-reinforced vinyl-ester matrix composites and glass bead reinforced vinyl-ester matrix composites, respectively. It is observed from the figures that the predictions

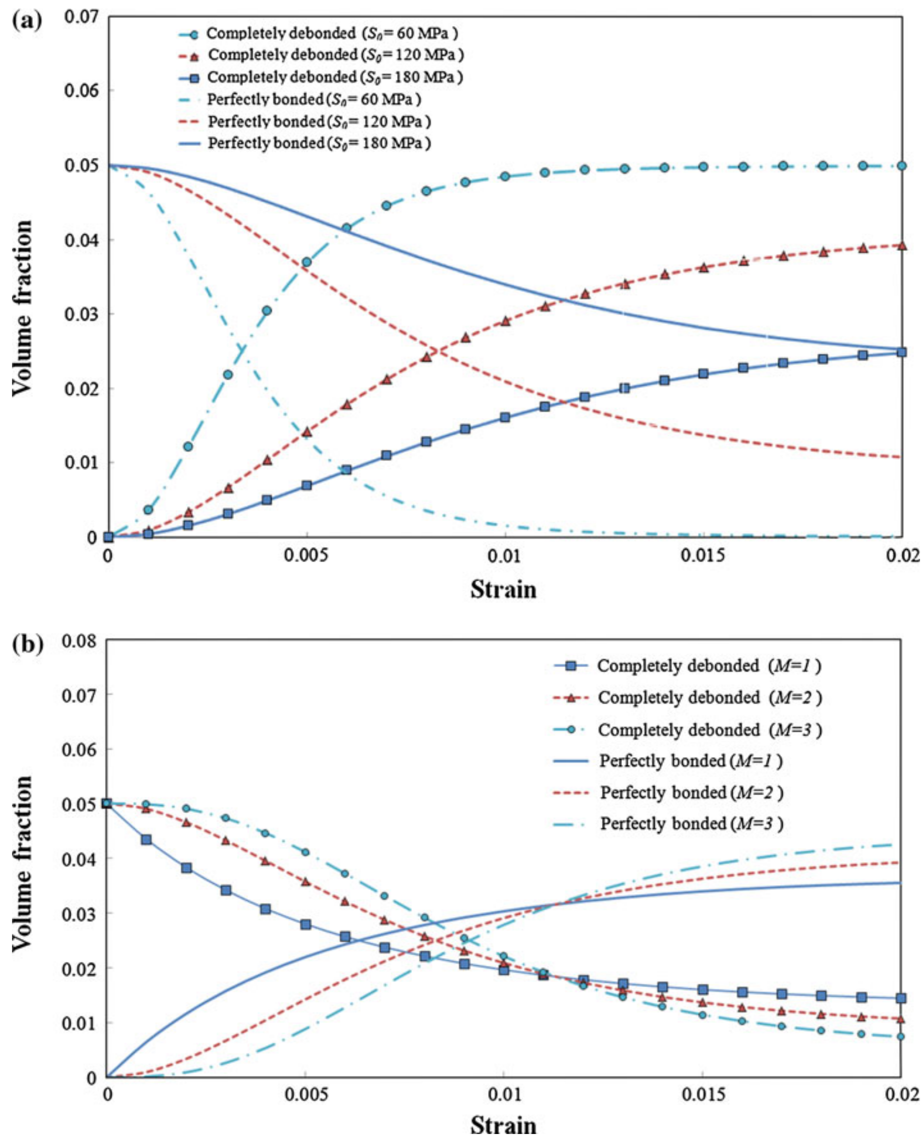

Fig. 4 The damage evolution curves corresponding to the predicted stress–strain curve

considering the damage mechanism are lower than those without considering the damage mechanism. In addition, a higher particle volume fraction leads to a larger difference between the predictions with and without the damage mechanism. The predicted viscoelastic stress–strain responses of the particle-reinforced polymeric composites are shown to be in good agreement with the experimental data. The predicted damage evolution curves corresponding to Figs. 8 and 9 are depicted in Fig. 10.

5 Concluding remarks

A micromechanics-based viscoelastic damage model for particle-reinforced polymeric composites has been presented to predict the overall stress–strain response and damage evolution in the material. The Laplace-transformed Boltzmann superposition principle is applied to a micromechanical framework derived by the ensemble-averaged method [23,24]. A damage model is considered in accordance with Weibull's probabilistic function to characterize the varying probability of particle debonding. A series of numerical simulations including parametric studies, and experimental comparisons are carried out to give insight into the potential

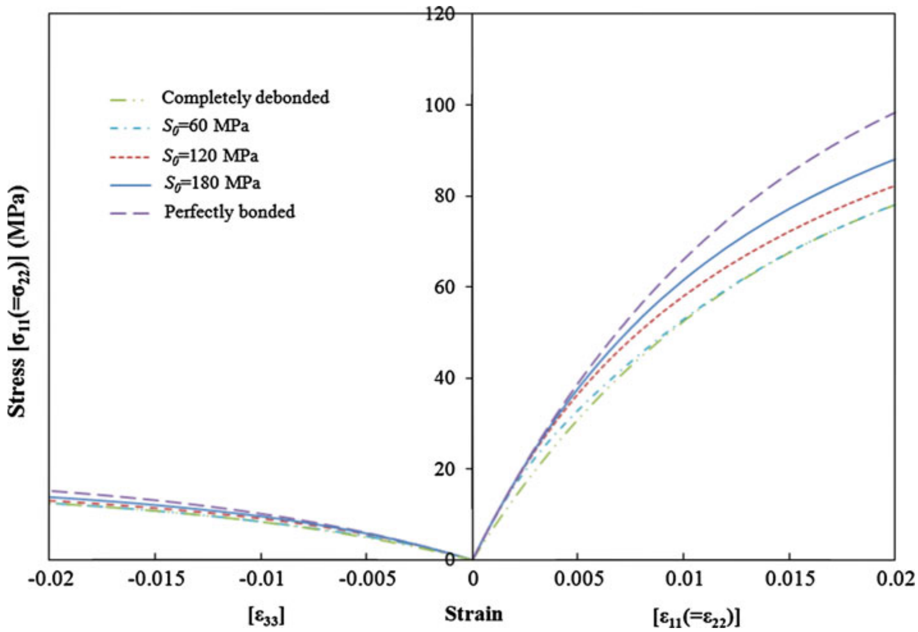

Fig. 5 The present predicted stress–strain response of particle-reinforced polymeric composites under biaxial loading with various S_0 values

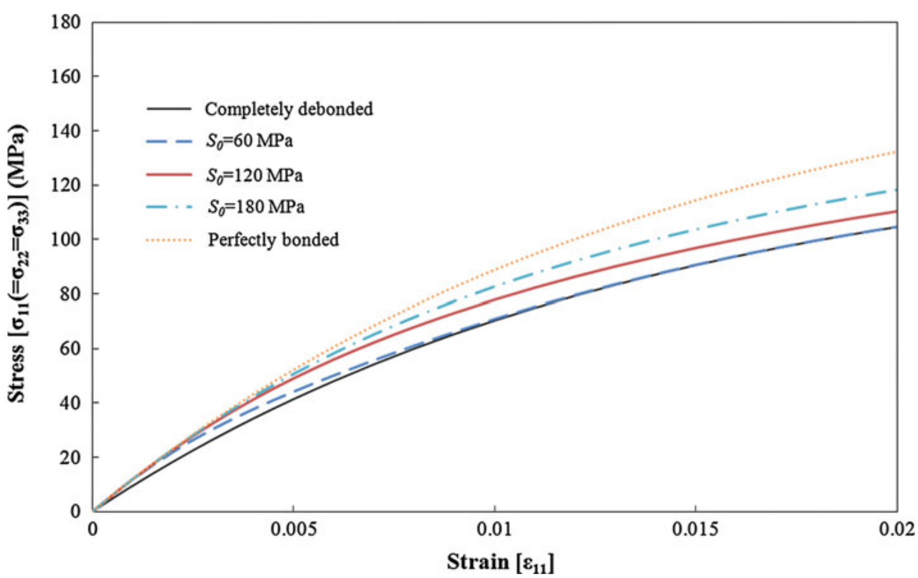

Fig. 6 The present predicted stress–strain response of particle-reinforced polymeric composites under hydrostatic loading with various S_0 values

capacity of the present micromechanics-based viscoelastic damage framework. The findings of the present study can be summarized as follows.

(i) The viscous property in the composites is more pronounced with the decrease in strain rate and is clearly shown in uniaxial, biaxial, and hydrostatic loading conditions.
(ii) The effect of the strain rate on the viscoelastic behavior is shown to be influential, and the influences of the damage parameters are quite remarkable.

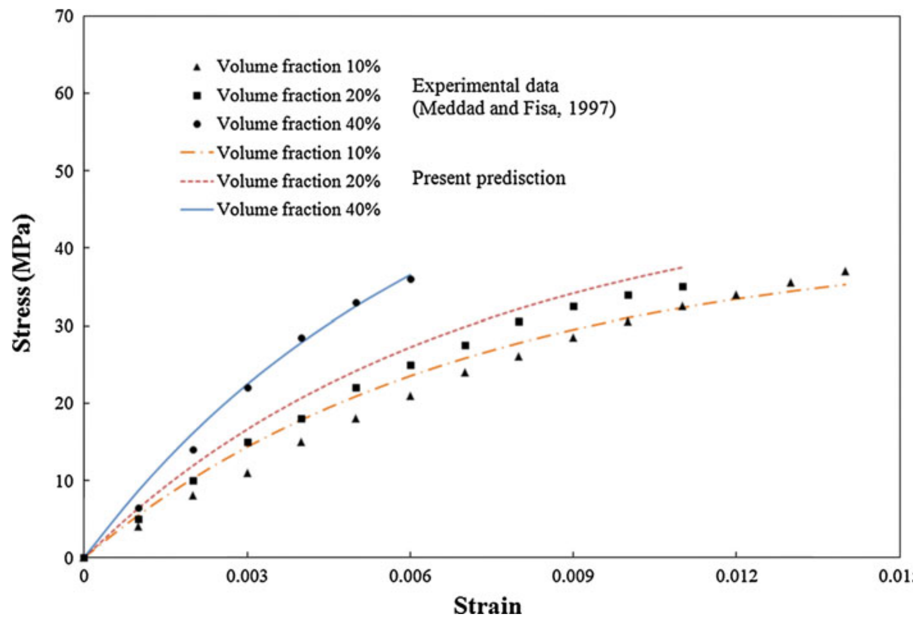

Fig. 7 The comparison of predicted stress–strain curves of glass bead particulate polystyrene composites with various particle volume fractions ($\phi = 10, 20, 40\,\%$) between the proposed micromechanics-based viscoelastic model and experimental data [60]

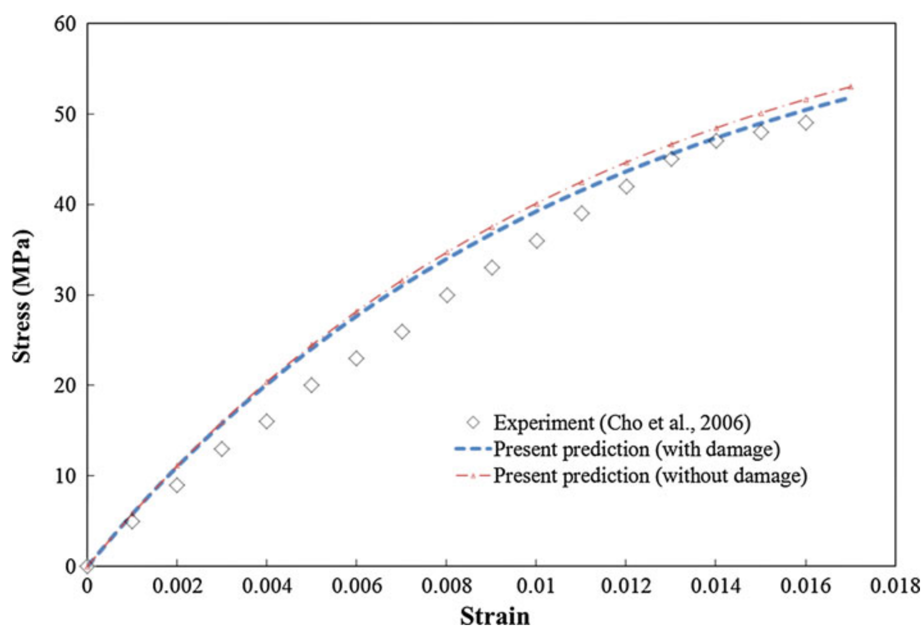

Fig. 8 The comparison between the present prediction and experimental data [59] for uniaxial tensile responses of an alumina (Al_2O_3) particle-reinforced vinyl-ester matrix composite

(iii) Good agreements between the present predictions and experimental data show the predictive capability of the proposed model.

This study has demonstrated the capability of the proposed micromechanical framework for predicting the viscoelastic behavior of particle-reinforced polymeric composites. However, a unified experimental and numerical study needs to be carried out for the calibration of the model parameters of the proposed model.

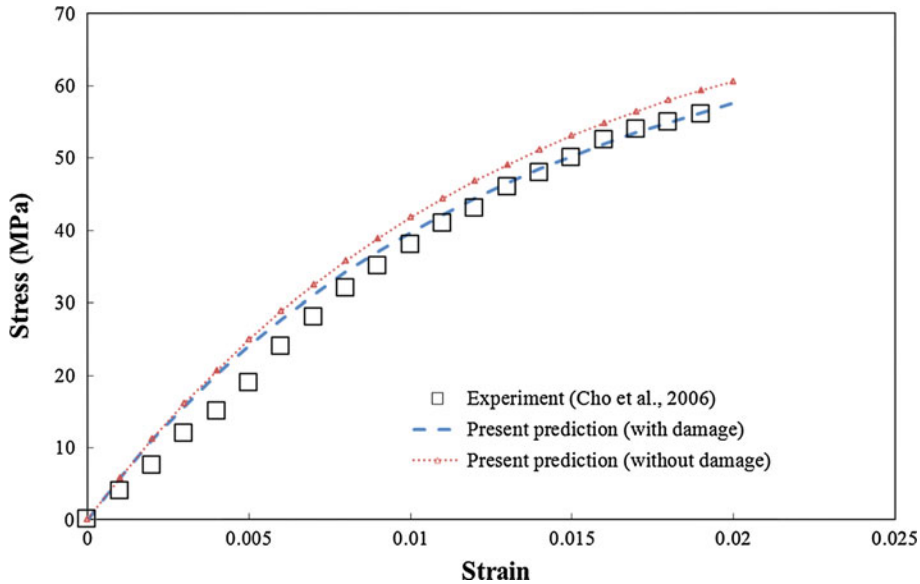

Fig. 9 The comparison between the present prediction and experimental data [59] for uniaxial tensile responses of glass bead particle-reinforced vinyl-ester matrix composites

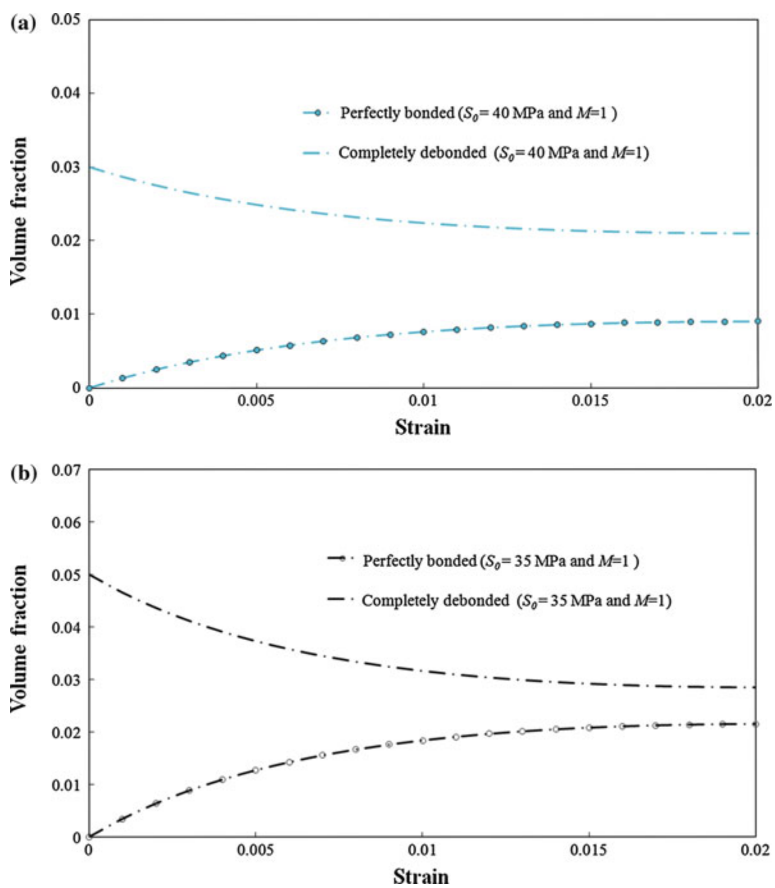

Fig. 10 The predicted damage evolution as a function of strain of particle-reinforced polymeric composite: corresponding to Figs. 8 and 9: **a** alumina particle-reinforced vinyl-ester matrix composite and **b** glass bead particle-reinforced vinyl-ester matrix composite

Acknowledgments This research was supported by grants from the Construction Technology Innovation Program (CTIP) and the U-City Master and Doctor Course Grant Program funded by the Ministry of Land, Transportation and Maritime (MLTM) Affairs of the Korean government.

References

1. Ravichandran, G., Liu, C.T.: Modeling constitutive behavior of particulate composites undergoing damage. Int. J. Solids Struct. **32**, 979–990 (1995)
2. Dai, L.H., Ling, Z., Bai, Y.L.: Size-dependent inelastic behavior of particle-reinforced metal-matrix composites. Compos. Sci. Technol. **61**, 1057–1063 (2001)
3. Kim, Y.R., Allen, D.H., Seidel, G.D.: Damage-induced modeling of elastic-viscoelastic randomly oriented particulate composites. J. Eng. Mater. Technol. ASME. **128**, 18–27 (2006)
4. Shen, W., Peng, L., Yue, Y.: Damage-dependent viscoelastic constitutive relations for glass-fiber woven/polyester composite plate. Eng. Fract. Mech. **47**, 867–872 (1994)
5. Schapery, R.A.: Nonlinear viscoelastic solids. Int. J. Solids Struct. **37**, 359–366 (2000)
6. Raghavan, P., Moorthy, S., Ghosh, S., Pagano, N.J.: Revisiting the composite laminate problem with an adaptive multi-level computational model. Compos. Sci. Technol. **61**, 1017–1040 (2001)
7. Kim, S.J., Lee, C.S., Yeo, H.J.: Direct numerical simulation of composite structures. J. Compos. Mater. **36**, 2765–2785 (2002)
8. Zhang, Y., Xia, Z., Ellyin, F.: Nonlinear viscoelastic micromechanical analysis of fibre-reinforced polymer laminates with damage evolution. Int. J. Solids Struct. **42**, 591–604 (2005)
9. Ward, I.M., Pinnock, P.R.: The mechanical properties of solid polymers. Br. J. Appl. Phys. **17**, 3–32 (1966)
10. Matzenmiller, A., Gerlach, S.: Micromechanical modeling of viscoelastic composites with compliant fiber-matrix bonding. Comput. Mater. Sci. **29**, 283–300 (2004)
11. Gibson, R.F.: Principles of Composites Material Mechanics. McGraw-Hill Inc, New York (1994)
12. Lakes, R.: Viscoelastic Materials. Cambridge University Press, Cambridge (2009)
13. McCrum, N.G., Buckley, C.P., Bucknall, C.B.: Principles of Polymer Engineering. Oxford University Press, Oxford (1997)
14. Roylance, D.: Engineering Viscoelasticity. MIT Press, Cambridge (2001)
15. Schjodt-Thomsen, J., Pyrz, R.: Non-linear creep modeling of single-fibre model composites. Compos. Sci. Technol. **60**, 1791–1800 (2000)
16. Li, J., Weng, G.J.: Strain-rate sensitivity, relaxation behavior, and complex moduli of a class of isotropic viscoelastic composites. J. Eng. Mater. Technol. ASME. **116**, 495–504 (1994)
17. Lévesque, M., Derrien, K., Mishnaevski, L.: A micromechanical model for nonlinear viscoelastic particle reinforced polymeric composite materials-undamaged state. Compos. Part. A Appl. Sci. **35**, 905–913 (2004)
18. Schapery, R.A.: On the characterisation of nonlinear viscoelastic materials. Polym. Eng. Sci. **9**, 295–310 (1969)
19. Muliana, A.H., Kim, J.S.: A concurrent micromechanical model for predicting nonlinear viscoelastic responses of composites reinforced with solid spherical particles. Int. J. Solids Struct. **44**, 6891–6913 (2007)
20. Haj-Ali, R.M., Pecknold, D.A.: Hierarchical material models with microstructure for nonlinear analysis of progressive damage in laminated composites structure. Structural Research Series No. 611, Department of Civil Engineering, University of Illionos at Urbana-Champaign, UILU-ENG-96-2007 (1996)
21. Hill, R.: A self consistent mechanics of composites materials. J. Mech. Phys. Solids. **13**, 213–222 (1965)
22. Kurnatowski, B., Matzenmiller, A.: Finite element analysis of viscoelastic composite structures based on a micromechanical material model. Comp. Mater. Sci. **43**, 957–973 (2008)
23. Ju, J.W., Chen, T.M.: Micromechanics and effective moduli of elastic composites containing randomly dispersed ellipsoidal inhomogeneities. Acta Mech. **103**, 103–121 (1994a)
24. Ju, J.W., Chen, T.M.: Effective elastic moduli of two-phase composites containing randomly dispersed spherical inhomogeneities. Acta Mech. **103**, 123–144 (1994b)
25. Weibull, W.: A statistical distribution function of wide applicability. J. Appl. Mech. **18**, 293–297 (1951)
26. Leaderman, H.: Elastic and creep properties of filamentous materials. Massachusetts Institute of Technology, Dept. of Mechanical Engineering (1941)
27. Aklonis, J., MacKnight, W.: Introduction to Polymer Viscoelasticity. Wiley, New York (1983)
28. Ferry, J.D.: Viscoelastic Properties of Polymers. Wiley, New York (1980)
29. Hashin, Z.: Viscoelastic behavior of heterogeneous media. J. Appl. Mech. **32**, 630–636 (1965)
30. Megnis, M., Varna, J.: Micromechanics based modeling of nonlinear viscoplastic response of unidirectional composite. Compos. Sci. Technol. **63**, 19–31 (2003)
31. Pichler, C., Lackner, R.: Upscaling of viscoelastic properties of highly-filled composites: Investigation of matrix-inclusion-type morphologies with power-law viscoelastic material response. Compos. Sci. Technol. **69**, 2410–2420 (2009)
32. Ju, J.W., Sun, L.Z.: Effective elastoplastic behavior of metal matrix composites containing randomly located aligned spheroidal inhomogeneities. Part I: Micromechanics-based formulation. Int. J. Solids Struct. **38**, 183–201 (2001)
33. Liu, H.T., Sun, L.Z., Ju, J.W.: An interfacial debonding model for particle-reinforced composites. Int. J. Damage Mech. **13**, 163–185 (2004)
34. Liu, H.T., Sun, L.Z., Ju, J.W.: Elastoplastic modeling of progressive interfacial debonding for particle-reinforced metal-matrix composites. Acta Mech. **181**, 1–17 (2006)
35. Lee, H.K., Pyo, S.H.: Micromechanics-based elastic damage modeling of particulate composites with weakened interfaces. Int. J. Solids Struct. **42**, 8390–8406 (2007)
36. Lee, H.K., Pyo, S.H.: Multi-level modeling of effective elastic behavior and progressive weakened interface in particulate composites. Compos. Sci. Technol. **468**, 387–397 (2008)

37. Kim, B.R., Lee, H.K.: Modeling of particle debonding and void evolution in particulate ductile matrix composites. CMES-Comput. Model. Eng. **47**, 253–281 (2009)
38. Kim, B.R., Lee, H.K.: Elastic-damage modeling for particulate composites considering cumulative damage. Int. J. Solids Struct. **20**, 131–158 (2011)
39. Eshelby, J.D.: The determination of the field of an ellipsoidal inclusion and related problems. Proc. R. Soc. A. **241**, 376–396 (1957)
40. Ju, J.W., Lee, H.K.: A micromechanical damage model for effective elastoplastic behavior of ductile matrix composites considering evolutionary complete particle debonding. Comput. Method Appl. M. Mech. Eng. **183**, 201–222 (2000)
41. Ju, J.W., Lee, H.K.: Micromechanical damage model for effective elastoplastic behavior of partially debonded ductile matrix composites. Int. J. Solids Struct. **38**, 6307–6332 (2001)
42. Liu, H.T., Sun, L.Z., Wu, H.C.: Monte Carlo simulation of particle-cracking damage evolution in metal matrix composites. J. Eng. Mater. Technol. ASME. **127**, 318–328 (2005)
43. Hashin, Z.: Complex moduli of viscoelastic composites-I. General theory and application to particulate composites. Int. J. Solids Struct. **6**, 539–552 (1970)
44. Li, J., Weng, G.J.: Effect of a viscoelastic interphase on the creep and stress/strain behavior of fiber-reinforced polymer matrix composites. Compos. Part. B-Eng. **27**, 589–598 (1996)
45. Li, J., Weng, G.J.: A unified approach from elasticity to viscoelasticity to viscoplasticity of particle-reinforced solids. Int. J. Plast. **14**, 193–208 (1998)
46. Tohgo, K., Weng, G.J.: A progress damage mechanics in particle-reinforced metal-matrix composites under high triaxial tension. J. Eng. Mater. Technol. ASME. **116**, 414–420 (1994)
47. Zhao, Y.H., Weng, G.J.: A theory of inclusion debonding and its influence on the stress-stain relations of a ductile matrix. Int. J. Damage Mech. **4**, 196–211 (1995)
48. Zhao, Y.H., Weng, G.J.: Plasticity of a two-phase composite with partially debonded inclusions. Int. J. Plast. **12**, 781–804 (1996)
49. Zhao, Y.H., Weng, G.J.: Transversely isotropic moduli of two partially debonded composites. Int. J. Solids Struct. **34**, 493–507 (1997)
50. Lee, H.K., Simunovic, S.: Modeling of progressive damage in aligned and randomly oriented discontinuous fiber polymer matrix composites. Compos. Part. B-Eng. **31**, 77–86 (2000)
51. Lee, H.K.: Computational approach to the investigation of impact damage evolution in discontinuously reinforced fiber composites. Comput. Mech. **27**, 504–512 (2001)
52. Lee, H.K., Pyo, S.H.: An elastoplastic multi-level damage model for ductile matrix composites considering evolutionary weakened interface. Int. J. Solids Struct. **45**, 1614–1631 (2008)
53. Liang, Z., Lee, H.K., Suaris, W.: Micromechanics-based constitutive modeling for unidirectional laminated composites. Int. J. Solids Struct. **43**, 5674–5689 (2006)
54. Lee, H.K., Simunovic, S., Shin, D.K.: A computational approach for prediction of the damage evolution and crushing behavior of chopped random fiber composites. Comput Mater. Sci. **29**, 459–474 (2004)
55. Lee, H.K., Liang, Z.: Computational modeling of the response and damage behavior of fiber-reinforced cellular concrete. Comput. Struct. **82**, 581–592 (2004)
56. Lee, H.K., Pyo, S.H.: 3D-damage model for fiber-reinforced brittle composites with microcracks and imperfect interfaces. J. Eng. Mech. **135**, 1108–1118 (2009)
57. Lee, H.K., Pyo, S.H.: An elastoplastic damage model for metal matrix composites considering progressive imperfect interface under transverse loading. Int. J. Plast. **26**, 25–41 (2010)
58. Lee, H.K., Simunovic, S., Shin, D.K.: A damage mechanics model of crack-weakened, chopped fiber composites under impact loading. Compos. Part. B-Eng. **33**, 25–34 (2002)
59. Cho, J., Joshi, M.S., Sun, C.T.: Effect of inclusion size on mechanical properties of polymeric composites with micro and nano particles. Compos. Sci. Technol. **66**, 1941–1952 (2006)
60. Meddad, A., Fisa, B.: Filler-matrix debonding in glass bead-filled polystyrene. J. Mater. Sci. **32**, 1177–1185 (1997)