Acta Mech 223, 811–828 (2012)
DOI 10.1007/s00707-011-0591-y

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Magneto-thermo-elastic response in a perfectly conducting medium with three-phase-lag effect

Received: 25 July 2011 / Published online: 30 December 2011

Abstract This paper deals with the problem of magneto-thermo-elastic interactions in an unbounded, perfectly conducting elastic medium due to the presence of periodically varying heat sources in the context of linear theory of generalized thermo-elasticity with energy dissipation (TEWED or GN-III model), without energy dissipation (TEWOED or GN-II model) and three-phase-lag model (3P model). The governing equations of generalized thermo-elasticity of the above models under the influence of a magnetic field are established. The Laplace-Fourier double transform technique has been used to get the solution. The inversion of the Fourier transform has been done by using residual calculus, where poles of the integrand are obtained numerically in a complex domain by using Laguerre's method, and the inversion of the Laplace transformation is done numerically using a method based on Fourier series expansion technique. Displacement, temperature, stress and strain distributions have been computed numerically and presented graphically in numbers of figures. A comparison of the results for different theories (GN-II, GN-III and 3P model) and the effect of magnetic field and damping coefficient on the physical quantities has been discussed.

List of symbols

$\mathbf{u}$	Displacement vector
λ, μ	Lamé's constants
ρ	Constant mass density of the medium
γ	Thermal modulus
α_t	Coefficient of linear thermal expansion
T_0	Uniform reference temperature
T	Small temperature increase above the reference temperature T_0
$\mathbf{J}$	Electric current density vector
$\mathbf{B}$	Magnetic induction vector
c_v	Specific heat of the medium at constant strain
K^*	A material constant characteristic for the GN theory
$\mathbf{H}$	Total magnetic field vector at any time

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$\mathbf{E}$	Electric field vector
μ_e	Magnetic permeability of the medium
σ	Electric conductivity of the medium
c_{T}	Non-dimensional finite thermal wave speed of GN theory of thermo-elasticity II
ϵ_{T}	Thermo-elastic coupling constant
K	Thermal conductivity
κ	Thermal diffusivity

1 Introduction

The classical theories of thermo-elasticity, involving infinite speed of propagation of thermal signals, contradict physical facts. During the last three decades, non-classical theories involving a finite speed of heat transport in elastic solids have been developed to remove this paradox. In contrast to the conventional coupled thermo-elasticity theory, which involves a parabolic-type heat transport equation, these generalized theories involving a hyperbolic-type heat transport equation are supported by experiments exhibiting the actual occurrence of wave-type heat transport in solids, called the second-sound effect. The extended thermo-elasticity theory (ETE) proposed by Lord and Shulman [1] incorporates a flux-rate term into Fourier's law of heat conduction and formulates a generalized form that involves a hyperbolic-type heat transport equation with a finite speed of the thermal signal. The temperature-rate-dependent generalized thermo-elasticity (TEDTE) theory proposed by Green and Lindsay [2] involving two relaxation times do not violate the classical Fourier law of heat conduction, and this theory also predicts a finite speed for heat propagation. Because of the experimental evidence in support of the finiteness of the speed of propagation of a heat wave, generalized thermo-elasticity theories are more realistic than conventional thermo-elasticity theories in dealing with practical problems involving very short time intervals and high heat fluxes like those occurring in laser units, energy channels, nuclear reactors.

The phenomenon of coupling between the thermo-mechanical behavior of materials and the electro-magnetic behavior of materials has been studied since the nineteenth century. By the middle of the twentieth century, piezoelectric materials were finding their first applications in hydrophones. In the last two decades, the concept of electro-magnetic composite materials has arisen. Such composites can exhibit field coupling that is not present in any of the monolithic constituent materials. These so called "Smart" materials, and composites have applications in ultrasonic imaging devices, sensors, actuators, transducers and many other emerging components. Magneto-electro-elastic materials are used in various applications. Due to the ability of converting energy from one kind to another (among mechanical, electric and magnetic energies), these materials have been used in high-tech areas such as lasers, supersonic devices, microwave, infrared applications. Furthermore, magneto-electro-elastic materials exhibit coupling behavior among mechanical, electric, and magnetic fields and are inherently anisotropic. Problems related to the wave propagation in thermo-elastic or magneto-thermo-elastic solids using these generalized theories have been studied by several authors. Among them, Paria [3] has presented some ideas about magneto-thermo-elastic plane waves. Nayfeh and Nemat-Nasser [4,5] have studied thermo-elastic waves and electro-magneto-elastic waves in solids with a thermal relaxation time. Roychoudhuri and Chatterjee [6] have introduced a coupled magneto-thermo-elastic problem in a perfectly conducting elastic half-space with thermal relaxation. Hsieh [7] has considered modeling of new electro-magnetic materials. Ezzat [8] has studied the state space approaches to generalized magneto-thermo-elasticity with two relaxation times in a perfectly conducting medium. Ezzat et al. [9] have studied electro-magneto-thermo-elastic plane waves, with thermal relaxation in a medium of perfect conductivity. Problems related to magneto-thermo-elasticity with thermal relaxation have been investigated by Sherief and Yossef [10], Baksi and Bera [11] and by Ezzat and Karamany [12].

Hetnarski and Ignaczak [13,14] developed low-temperature generalized thermo-elasticity called H-I theory. Green and Naghdi [15–17] provided sufficient basic modifications in the constitutive equations that permit the treatment of a much wider class of heat flow problems labeled as GN-I, GN-II and GN-III. GN models include a term called 'thermal displacement gradient' among the independent constitutive variables. When the three theories are linearized, the heat transport equation of GN-I is the same as the classical equation, whereas both GN-II and GN-III admit propagation of thermal signals of finite speeds [17]. However, GN-II does not sustain propagation of magneto-thermo-elastic waves which undergo attenuation and dispersion [18]. An important feature of GN-III theory is that it accommodates dissipation of thermal energy due to the presence of a thermal damping term. In the context of a linearized version of this theory (Green and Naghdi [16,17]), a theorem on the uniqueness of solutions has been established by Chandrasekhariah [19,20]. Chandrashekhariah [21] have studied one-dimensional thermal wave propagation in a half-space based on the GN model due to the sudden exposure of the temperature to the boundary using the Laplace transform method. Chandrasekhariah and Srinath [22] have studied thermo-elastic interactions caused by a continuous heat source in a homogeneous isotropic unbounded thermo-elastic body by employing the linear theory of thermo-elasticity without energy dissipation (TEWOED).

Thermo-elastic interactions with energy dissipation in an infinite solid with distributed periodically varying heat sources have been studied by Banik et al. [23] and for functionally graded material without energy dissipation have been studied by Mallik and Kanoria [24]. Das and Kanoria [25] have studied magneto-thermo-elastic interaction in a functionally graded isotropic unbounded medium due to the presence of periodically varying heat sources. Kar and Kanoria [26,27] have analyzed thermo-elastic interactions with energy dissipation in a transversely isotropic thin circular disk and in an unbounded body with a spherical hole. Mallik and Kanoria [28] have solved a two-dimensional problem for a transversely isotropic generalized thick plate with spatially varying heat source. Das and Kanoria [29] described the magneto-thermo-elastic wave propagation in an unbounded perfectly conducting elastic solid with energy dissipation. Islam and Kanoria [30] have studied the dynamical response in a two-dimensional transversely isotropic thick plate due to a heat source. Islam et al. [31] have discussed the study of dynamical response in a two-dimensional transversely isotropic thick plate with spatially varying heat sources and body forces. The generalized thermo-elastic problem of a spherical shell under a thermal shock has been solved by Kar and Kanoria [32].

The generalized thermo-elasticity theory with dual-phase-lag effect has been developed by Tzou [33] and Chandrasekharaiah [34]. Tzou [33] introduced two-phase lags to both the heat flux vector and temperature gradient. According to this model, the classical Fourier's law $\vec{q} = -K\vec{\nabla}T$ has been replaced by $\vec{q}(P, t+\tau_q) = -K\vec{\nabla}T(P, t+\tau_T)$, where the temperature gradient $\vec{\nabla}T$ at a point P of the material at time $t+\tau_T$ corresponds to the heat flux vector $\vec{q}$ at the same point at time $t+\tau_q$. The delay time τ_T is interpreted as that caused by the microstructural interactions and is called the phase lag of the temperature gradient. The other delay time τ_q is interpreted as the relaxation time due to the fast transient effects of the thermal inertia and is called the phase lag of the heat flux. For $\tau_q = \tau_T \neq 0$, this is identical with classical Fourier's law. If $\tau_q = \tau$ and $\tau_T = 0$, Tzou [33] refers to the model as single-phase-lag model. Roychoudhuri [35] studied one-dimensional thermo-elastic wave propagation in an elastic half-space in the context of a dual-phase-lag model. The effect of three-phase lags on generalized thermo-elasticity for an infinite medium with cylindrical cavity has been studied by Kumar and Mukhopadhyay [36].

The next generalization is known as three-phase-lag thermo-elasticity which is due to Roychoudhuri [37]. According to this model, $\vec{q}(P, t+\tau_q) = -[K\vec{\nabla}T(P, t+\tau_T) + K^*\vec{\nabla}\nu(P, t+\tau_\nu)$, where $\vec{\nabla}\nu(\dot{\nu} = T)$ is the thermal displacement gradient and τ_ν is the phase lag for the thermal displacement gradient. Quintanilla and Racke [38] have discussed the theory of heat conduction models with three-phase lags. Kar and Kanoria [39] have discussed thermo-elastic interaction due to a step input of temperature on the boundaries of a functionally graded orthotropic hollow sphere in the context of linear theories of generalized thermo-elasticity.

The purpose of the present work is to study magneto-thermo-elastic interaction due to the presence of periodically varying heat sources in a perfectly conducting medium in the context of the linear theory of generalized thermo-elasticity (GN-II, GN-III and 3P lag models). The governing equations are expressed in Laplace-Fourier transform domain. The solution for displacement, temperature, stress and strain in the Laplace transform domain is obtained by taking the Fourier inversion, which is carried out by using residual calculus, where the poles of the integrand are obtained numerically in the complex domain by using Laguerre's method. The inversion of the Laplace transform is computed numerically by using a method based on the Fourier series expansion technique [40]. The results obtained theoretically have been computed numerically and are presented graphically to show the comparison of results of the above theories and also the effect of the magnetic field and damping coefficient on the physical quantities.

2 Basic equations

For a perfectly conducting medium, the constitutive equations are

$$\sigma_{ij} = 2\mu e_{ij} + [\lambda\Delta - \gamma(T - T_0)]\delta_{ij} \tag{1}$$

where

$$e_{ij} = \frac{1}{2}(u_{i,j} + u_{j,i}), \quad \Delta = e_{ii}. \tag{2}$$

The stress equations of motion in the presence of body forces F_i are

$$\sigma_{ij,j} + F_i = \rho \ddot{u}_i. \tag{3}$$

The heat equation corresponding to generalized thermo-elasticity for the three-phase-lag model with energy dissipation in the presence of a heat source is

$$\begin{aligned}&\rho c_v \left(\ddot{T} + \tau_q \dddot{T} + \frac{1}{2}\tau_q{}^2 \ddddot{T} \right) + \gamma T_0 \left(\ddot{\Delta} + \tau_q \dddot{\Delta} + \frac{1}{2}\tau_q{}^2 \ddddot{\Delta} \right) \\ &= K(\nabla^2 \dot{T} + \tau_T \nabla^2 \ddot{T}) + K^*(\nabla^2 T + \tau_v \nabla^2 \dot{T}) + \rho \dot{Q}\end{aligned} \tag{4}$$

where $\gamma = (3\lambda + 2\mu)\alpha_t$, K is the thermal conductivity, K^* the material constant, τ_T the delay time caused by the microstructural interactions and is called the phase lag of the temperature gradient, τ_q the delay time due to the fast transient effects of thermal inertia and is called the phase lag of the heat flux and τ_v the phase lag for the thermal displacement gradient.

3 Formulation of the problem

We now consider an unbounded, perfectly conducting thermo-elastic medium at a uniform reference temperature T_0 in the presence of periodically varying heat sources distributed over a plane area. We shall consider a one-dimensional disturbance of the medium, so that the displacement vector $\mathbf{u}$ and temperature field T can be expressed in the following form:

$$\mathbf{u} = (u(x,t), 0, 0), \tag{5}$$

$$T = T(x,t). \tag{6}$$

The electromagnetic field is governed by Maxwell's equations (in the absence of the displacement current and charge density) as

$$\text{curl } \mathbf{H} = \mathbf{J}, \quad \text{curl } \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}, \quad \text{div } \mathbf{B} = 0, \quad \mathbf{B} = \mu_e \mathbf{H}. \tag{7.1–4}$$

The generalized Ohm's law in the deformable continuum is

$$\mathbf{J} = \sigma(\mathbf{E} + \dot{\mathbf{u}} \times \mathbf{B}), \tag{8}$$

where the small effect of a temperature gradient on the conduction current $\mathbf{J}$ is neglected.

In the context of the linear theory of generalized thermo-elasticity based on the three-phase-lag model, the equation of motion, heat equation and constitutive equation can be written as

$$(\lambda + 2\mu)\frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} + F_x = \rho \frac{\partial^2 u}{\partial t^2} \tag{9}$$

where

$$\mathbf{F} = (\mathbf{J} \times \mathbf{B}), \quad \mathbf{F} = (F_x, F_y, F_z),$$

$$\begin{aligned}&K\left(\frac{\partial^3 T}{\partial x^2 \partial t} + \tau_T \frac{\partial^4 T}{\partial x^2 \partial t^2}\right) + K^*\left(\frac{\partial^2 T}{\partial x^2} + \tau_\gamma \frac{\partial^3 T}{\partial x^2 \partial t}\right) + \rho \dot{Q} \\ &= \rho c_v \left(\frac{\partial^2 T}{\partial t^2} + \tau_q \frac{\partial^3 T}{\partial t^3} + \frac{1}{2}\tau_q{}^2 \frac{\partial^4 T}{\partial t^4}\right) + \gamma T_0 \left(\frac{\partial^3 T}{\partial t^2 \partial x} + \tau_q \frac{\partial^4 u}{\partial t^3 \partial x} + \frac{1}{2}\tau_q{}^2 \frac{\partial^5 u}{\partial t^4 \partial x}\right),\end{aligned} \tag{10}$$

$$\tau_{xx} = (\lambda + 2\mu)e_{xx} - \gamma(T - T_0) \tag{11}$$

where

$$e_{xx} = \frac{\partial u}{\partial x}. \tag{12}$$

We set $\mathbf{H} = \mathbf{H_0} + \mathbf{h}$, where $\mathbf{H_0} = (0, 0, H_0)$. The perturbed magnetic field $\mathbf{h}$ is so small that the product of $\mathbf{h}$ and $\mathbf{u}$ and their derivatives can be neglected for linearization of the field equations.

We assume that all the vector and scalar functions depend only on the spatial coordinate x and time t and are independent of the y and z coordinates.

Equation (7.1) gives

$$J_x = 0, \quad J_y = -\frac{\partial H_z}{\partial x}, \quad J_z = \frac{\partial H_y}{\partial x}, \tag{13}$$

where $\mathbf{J} = (J_x, J_y, J_z)$, $\mathbf{H} = (H_x, H_y, H_z)$.

Equation (7.2) yields

$$\frac{\partial H_x}{\partial t} = 0, \quad \frac{\partial E_z}{\partial x} = \mu_e \frac{\partial H_y}{\partial t}, \quad \frac{\partial E_y}{\partial x} = -\mu_e \frac{\partial H_z}{\partial t}, \quad \mathbf{E} = (E_x, E_y, E_z). \tag{14}$$

Equation (7.3) gives $\dfrac{\partial h_x}{\partial x} = 0$, which implies that $h_x = 0$, since initially no perturbed field is applied along the x-axis.

The modified Ohm's law gives

$$J_x = \sigma E_x, \quad J_y = \sigma \left[E_y - \mu_e H_z \frac{\partial u}{\partial t} \right], \quad J_z = \sigma \left[E_z + \mu_e H_y \frac{\partial u}{\partial t} \right]. \tag{15}$$

Now $J_x = 0$ implies $E_x = 0$.

By eliminating J_x, J_y, J_z and using Eqs. (7), (8), and (15), we get

$$\frac{\partial H_z}{\partial t} = \nu_H \frac{\partial^2 H_z}{\partial x^2} - \frac{\partial}{\partial x}\left(H_z \frac{\partial u}{\partial t} \right), \tag{16}$$

$$\frac{\partial H_y}{\partial t} = \nu_H \frac{\partial^2 H_y}{\partial x^2} - \frac{\partial}{\partial x}\left(H_y \frac{\partial u}{\partial t} \right), \tag{17}$$

where $\nu_H = (\sigma \mu_e)^{-1}$ is called the magnetic viscosity.

Equation (9) reduces to

$$(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} - \frac{\partial}{\partial x} \left[\frac{1}{2} \mu_e ({H_y}^2 + {H_z}^2) \right] = \rho \frac{\partial^2 u}{\partial t^2}. \tag{18}$$

We set $H_z = H_0 + h_z$ where the perturbed magnetic field h_z is small compared to the strong initial magnetic field H_0. Then, from Eqs. (16), (17) and (18) after linearization, we get

$$\frac{\partial h_z}{\partial t} = \nu_H \frac{\partial^2 h_z}{\partial x^2} - H_0 \frac{\partial^2 u}{\partial x \partial t}, \quad \frac{\partial h_y}{\partial t} = \nu_H \frac{\partial^2 h_y}{\partial x^2} \tag{19.1, 2}$$

and

$$(\lambda + 2\mu) \frac{\partial^2 u}{\partial x^2} - \gamma \frac{\partial T}{\partial x} - \mu_e H_0 \frac{\partial h_z}{\partial x} = \rho \frac{\partial^2 u}{\partial t^2}. \tag{20}$$

Now for a perfect electrical conductor, $\nu_H \longrightarrow 0$ as $\sigma \longrightarrow \infty$. Equation (19.1) leads to $h_z = -H_0 \dfrac{\partial u}{\partial x}$, since there is no perturbation at ∞. Then, Eq. (20) reduces to

$$c_1{}^2 (1 + R_H) - \frac{\gamma}{\rho} \frac{\partial T}{\partial x} = \frac{\partial^2 u}{\partial t^2} \tag{21}$$

where $R_H = \frac{\mu_e H_0^2}{\rho c_1^2} = \frac{v_A^2}{c_1^2}$, $c_1 = \sqrt{\frac{\lambda + 2\mu}{\rho}}$, and $v_A = \sqrt{\frac{\mu_e}{\rho}} H_0$ is the Alf'ven wave velocity of the medium. The coefficient R_H represents the effect of an external magnetic field in the thermo-elastic processes proceeding in the body.

We introduce the following dimensionless quantities:

$$x' = \frac{x}{l}, \quad u' = \frac{\lambda + 2\mu}{\gamma T_0 l} u, \quad t' = \frac{c_1 t}{l}, \quad \theta = \frac{T - T_0}{T_0}, \quad \tau'_{x'x'} = \frac{\tau_{xx}}{\gamma T_0}, \quad e'_{x'x'} = e_{xx},$$
$$1 + R_H = R_{\mathrm{M}}^2, \quad \tau_T{}' = \frac{c_1 \tau_T}{l}, \quad \tau_v{}' = \frac{c_1 \tau_v}{l}, \quad \tau_q{}' = \frac{c_1 \tau_q}{l},$$

where $l =$ some standard length and $c_1 = \sqrt{\frac{\lambda+2\mu}{\rho}}$ is the standard speed, and omitting primes, Eqs. (10), (11), (12) and (21) can be re-written in dimensionless form as

$$R_{\mathrm{M}}^2 \frac{\partial^2 u}{\partial x^2} - \frac{\partial \theta}{\partial x} = \frac{\partial^2 u}{\partial t^2}, \tag{22}$$

$$\left(1 + \tau_v \frac{\partial}{\partial t}\right)\left[c_{\mathrm{T}}^2 \frac{\partial^2 \theta}{\partial x^2}\right] + \left(1 + \tau_T \frac{\partial}{\partial t}\right)\left[\kappa_0 \frac{\partial^2}{\partial x^2}\left(\frac{\partial \theta}{\partial t}\right)\right] + Q_0 = \left(1 + \tau_q \frac{\partial}{\partial t} + \frac{1}{2}\tau_q^2 \frac{\partial^2}{\partial t^2}\right)$$
$$\times \left[\frac{\partial^2 \theta}{\partial t^2} + \epsilon_{\mathrm{T}} \frac{\partial^2}{\partial t^2}\left(\frac{\partial u}{\partial x}\right)\right], \tag{23}$$

$$\tau_{xx} = \frac{\partial u}{\partial x} - \theta, \tag{24}$$

$$e_{xx} = \frac{\gamma T_0}{\lambda + 2\mu} \frac{\partial u}{\partial x} \tag{25}$$

where

$$c_{\mathrm{T}}^2 = \frac{K^*}{\rho c_v c_1^2}, \quad \epsilon_{\mathrm{T}} = \frac{\gamma^2 T_0}{(\lambda + 2\mu)\rho c_v}, \quad \kappa_0 = \frac{K}{\rho c_v c_1 l}, \quad Q_0 = \frac{l}{T_0 c_v c_1} \frac{\partial Q}{\partial t}.$$

We assume that the medium is initially at rest. The undisturbed state is maintained at a reference temperature. Then, we have

$$u(x, 0) = \dot{u}(x, 0) = \theta(x, 0) = \dot{\theta}(x, 0) = 0. \tag{26}$$

4 Method of solution

Let us define the Laplace-Fourier double transform of the function $g(x, t)$ by

$$\bar{g}(x, p) = \int_0^\infty g(x, t) \mathrm{e}^{-pt} \mathrm{d}t, \quad Re(p) > 0$$

$$\bar{g}(\alpha, p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} \bar{g}(x, p) \mathrm{e}^{\mathrm{i}\alpha x} \mathrm{d}x.$$

Applying the Laplace-Fourier double integral transform to Eqs. (22)–(25), we obtain:

$$(R_M^2 \alpha^2 + p^2)\hat{\bar{u}}(\alpha, p) = \mathrm{i}\alpha \hat{\bar{\theta}}(\alpha, p)\ , \tag{27}$$

$$\left[\{(1 + \tau_v p)c_{\mathrm{T}}^2 + (1 + \tau_T p)p\kappa_0\}\alpha^2 + \left(1 + p\tau_q + \frac{1}{2}\tau_q^2 p^2\right) p^2\right] \hat{\bar{\theta}}(\alpha, p)$$
$$= \mathrm{i}\epsilon_{\mathrm{T}} \alpha p^2 (1 + p\tau_q + \frac{1}{2}\tau_q^2 p^2)\hat{\bar{u}}(\alpha, p) + \hat{\bar{Q}}_0, \tag{28}$$

$$\hat{\bar{\tau}}_{xx}(\alpha, p) = -\mathrm{i}\alpha \hat{\bar{u}}(\alpha, p) - \hat{\bar{\theta}}(\alpha, p), \tag{29}$$

$$\hat{\bar{e}}_{xx}(\alpha, p) = -\mathrm{i}\alpha \beta_1 \hat{\bar{u}}(\alpha, p) \tag{30}$$

where $\beta_1 = \frac{\gamma T_0}{\lambda+2\mu}$.

Solving Eqs. (27) and (28) for $\hat{\bar{u}}(\alpha, p)$ and $\hat{\bar{\theta}}(\alpha, p)$, we get

$$\hat{\bar{u}}(\alpha, p) = \frac{\mathrm{i}\alpha \hat{\bar{Q}}_0}{M(\alpha)}, \tag{31}$$

$$\hat{\bar{\theta}}(\alpha, p) = \frac{(R_\mathrm{M}^2\alpha^2 + p^2)\hat{\bar{Q}}_0}{M(\alpha)}, \tag{32}$$

where

$$\begin{aligned} M(\alpha) &= \{(1+\tau_\nu p)c_\mathrm{T}^2 + (1+\tau_T p)p\kappa_0\}R_\mathrm{M}^2\alpha^4 + \alpha^2\left[p^2\left\{p(1+\tau_T p)\kappa_0 + (1+\tau_\nu p)c_\mathrm{T}^2\right.\right. \\ &\quad \left.\left. + R_\mathrm{M}^2\left(1 + p\tau_q + \frac{1}{2}\tau_q^2 p^2\right) + \epsilon_\mathrm{T}\left(1 + p\tau_q + \frac{1}{2}\tau_q^2 p^2\right)\right\}\right] + p^4\left(1 + p\tau_q + \frac{1}{2}\tau_q^2 p^2\right) \\ &= R_\mathrm{M}^2\{c_\mathrm{T}^2(1+\tau_\nu p) + p(1+\tau_T p)\kappa_0\}(\alpha-\alpha_1)(\alpha-\alpha_2)(\alpha-\alpha_3)(\alpha-\alpha_4). \end{aligned} \tag{33}$$

Now the expressions for the stress and strain in the Laplace-Fourier transform domain can be obtained from Eqs. (29) and (30), using Eqs. (31) and (32),

$$\begin{aligned} \hat{\bar{\tau}}_{xx}(\alpha, p) &= \frac{\alpha^2\hat{\bar{Q}}_0}{M(\alpha)} - \frac{(R_M^2\alpha^2 + p^2)\hat{\bar{Q}}_0}{M(\alpha)} \\ &= \frac{[\alpha^2(1-R_\mathrm{M}^2) - p^2]\hat{\bar{Q}}_0}{M(\alpha)}, \end{aligned} \tag{34}$$

$$\hat{\bar{e}}_{xx}(\alpha, p) = \frac{\alpha^2\beta_1\hat{\bar{Q}}_0}{M(\alpha)} . \tag{35}$$

Thus, the solution for the displacement, temperature, stress and strain in the Laplace transform domain can be obtained in terms of the following four integrals:

$$\bar{u}(x, p) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{\mathrm{i}\alpha\hat{\bar{Q}}_0}{M(\alpha)}\mathrm{e}^{-\mathrm{i}\alpha x}\,\mathrm{d}\alpha, \tag{36}$$

$$\bar{\theta}(x, p) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{(R_\mathrm{M}^2\alpha^2 + p^2)\hat{\bar{Q}}_0}{M(\alpha)}\mathrm{e}^{-\mathrm{i}\alpha x}\,\mathrm{d}\alpha, \tag{37}$$

$$\bar{\tau}_{xx}(x, p) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{[\alpha^2(1-R_\mathrm{M}^2) - p^2]\hat{\bar{Q}}_0}{M(\alpha)}\mathrm{e}^{-\mathrm{i}\alpha x}\,\mathrm{d}\alpha, \tag{38}$$

$$\bar{e}_{xx}(x, p) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty}\frac{\alpha^2\beta_1\hat{\bar{Q}}_0}{M(\alpha)}\mathrm{e}^{-\mathrm{i}\alpha x}\,\mathrm{d}\alpha. \tag{39}$$

4.1 Periodically varying heat source

Now let us assume that the heat source is distributed over the plane $x = 0$ in the following form:

$$\begin{aligned} Q_0 &= Q_0^*\delta(x)\sin\frac{\pi t}{\tau}, \qquad 0 \le t \le \tau, \\ &= 0, \qquad t > \tau. \end{aligned} \tag{40}$$

Then,

$$\hat{\bar{Q}}_0 = \frac{Q_0^*\pi\tau(1+\mathrm{e}^{-p\tau})}{\sqrt{2\pi}(\pi^2 + p^2\tau^2)}. \tag{41}$$

Thus, the expressions for the displacement, temperature, stress and strain in the Laplace transform domain take the following form:

$$\bar{u}(x,p)=\int_{-\infty}^{\infty}\frac{\mathrm{i}\alpha Q_0^*\tau(1+\mathrm{e}^{-p\tau})}{2(\pi^2+p^2\tau^2)M(\alpha)}\mathrm{e}^{-\mathrm{i}\alpha x}\,\mathrm{d}\alpha, \tag{42}$$

$$\bar{\theta}(x,p)=\int_{-\infty}^{\infty}\frac{(R_\mathrm{M}^2\alpha^2+p^2)Q_0^*\tau(1+\mathrm{e}^{-p\tau})}{2(\pi^2+p^2\tau^2)M(\alpha)}\mathrm{e}^{-\mathrm{i}\alpha x}\,\mathrm{d}\alpha, \tag{43}$$

$$\bar{\tau}_{xx}(x,p)=\int_{-\infty}^{\infty}\frac{[\alpha^2(1-R_\mathrm{M}^2)-p^2]Q_0^*\tau(1+\mathrm{e}^{-p\tau})}{2(\pi^2+p^2\tau^2)M(\alpha)}\mathrm{e}^{-\mathrm{i}\alpha x}\,\mathrm{d}\alpha, \tag{44}$$

$$\bar{e}_{xx}(x,p)=\int_{-\infty}^{\infty}\frac{\alpha^2\beta_1 Q_0^*\tau(1+\mathrm{e}^{-p\tau})}{2(\pi^2+p^2\tau^2)M(\alpha)}\mathrm{e}^{-\mathrm{i}\alpha x}\,\mathrm{d}\alpha. \tag{45}$$

Applying contour integration to Eqs. (42)–(45), we obtain

$$\begin{aligned}\bar{u}(x,p)&=-\frac{\mathrm{i}Q_0^*\pi\tau(1+\mathrm{e}^{-p\tau})}{R_\mathrm{M}^2(\pi^2+p^2\tau^2)N(p)}\sum_{\substack{j=1\\ \mathrm{Im}(\alpha_j)<0}}^{4}\mathrm{i}\alpha_j A_j\mathrm{e}^{-\mathrm{i}\alpha_j x}\quad\text{for }\ x>0\\ &=\frac{\mathrm{i}Q_0^*\pi\tau(1+\mathrm{e}^{-p\tau})}{R_\mathrm{M}^2(\pi^2+p^2\tau^2)N(p)}\sum_{\substack{j=1\\ \mathrm{Im}(\alpha_j)>0}}^{4}\mathrm{i}\alpha_j A_j\mathrm{e}^{-\mathrm{i}\alpha_j x}\quad\text{for }\ x<0,\end{aligned} \tag{46}$$

$$\begin{aligned}\bar{\theta}(x,p)&=-\frac{\mathrm{i}Q_0^*\pi\tau(1+\mathrm{e}^{-p\tau})}{R_\mathrm{M}^2(\pi^2+p^2\tau^2)N(p)}\sum_{\substack{j=1\\ \mathrm{Im}(\alpha_j)<0}}^{4}(R_\mathrm{M}^2\alpha_j^2+p^2)A_j\mathrm{e}^{-\mathrm{i}\alpha_j x}\quad\text{for }\ x>0\\ &=\frac{\mathrm{i}Q_0^*\pi\tau(1+\mathrm{e}^{-p\tau})}{R_\mathrm{M}^2(\pi^2+p^2\tau^2)N(p)}\sum_{\substack{j=1\\ \mathrm{Im}(\alpha_j)>0}}^{4}(R_\mathrm{M}^2\alpha_j^2+p^2)A_j\mathrm{e}^{-\mathrm{i}\alpha_j x}\quad\text{for }\ x<0,\end{aligned} \tag{47}$$

$$\begin{aligned}\bar{\tau}_{xx}(x,p)&=-\frac{\mathrm{i}Q_0^*\pi\tau(1+\mathrm{e}^{-p\tau})}{R_\mathrm{M}^2(\pi^2+p^2\tau^2)N(p)}\sum_{\substack{j=1\\ \mathrm{Im}(\alpha_j)<0}}^{4}[\alpha^2(1-R_\mathrm{M}^2)-p^2]A_j\mathrm{e}^{-\mathrm{i}\alpha_j x}\quad\text{for }\ x>0\\ &=\frac{\mathrm{i}Q_0^*\pi\tau(1+\mathrm{e}^{-p\tau})}{R_\mathrm{M}^2(\pi^2+p^2\tau^2)N(p)}\sum_{\substack{j=1\\ \mathrm{Im}(\alpha_j)>0}}^{4}[\alpha^2(1-R_\mathrm{M}^2)-p^2]A_j\mathrm{e}^{-\mathrm{i}\alpha_j x}\quad\text{for }\ x<0,\end{aligned} \tag{48}$$

$$\begin{aligned}\bar{e}_{xx}(x,p)&=-\frac{\mathrm{i}\beta_1 Q_0^*\pi\tau(1+\mathrm{e}^{-p\tau})}{R_\mathrm{M}^2(\pi^2+p^2\tau^2)N(p)}\sum_{\substack{j=1\\ \mathrm{Im}(\alpha_j)<0}}^{4}\alpha_j^2A_j\mathrm{e}^{-\mathrm{i}\alpha_j x}\quad\text{for }\ x>0\\ &=\frac{\mathrm{i}\beta_1 Q_0^*\pi\tau(1+\mathrm{e}^{-p\tau})}{R_\mathrm{M}^2(\pi^2+p^2\tau^2)N(p)}\sum_{\substack{j=1\\ \mathrm{Im}(\alpha_j)>0}}^{4}\alpha_j^2A_j\mathrm{e}^{-\mathrm{i}\alpha_j x}\quad\text{for }\ x<0\end{aligned} \tag{49}$$

where A_j's are given by

$$A_j = \prod_{\substack{n=1 \\ n \neq j}}^{4} \frac{1}{(\alpha_j - \alpha_n)} \tag{50}$$

and

$$N(p) = \{c_{\mathrm{T}}^2(1 + \tau_\nu p) + p(1 + \tau_T p)\kappa_0\}.$$

5 Inversion of Laplace transform

It is difficult to find the inverse Laplace transform of the complicated solution for the displacement, temperature, stress and strain in the Laplace transform domain. So we have to resort to numerical computation. We now outline the numerical procedure to solve the problem. Let $\bar{f}(x, p)$ be the Laplace transform of a function $f(x, t)$.

Then, the inversion formula for the Laplace transform can be written as

$$f(x, t) = \frac{1}{2\pi \mathrm{i}} \int_{d-\mathrm{i}\infty}^{d+\mathrm{i}\infty} \mathrm{e}^{pt} \bar{f}(x, p)\, \mathrm{d}p \tag{51}$$

where d is an arbitrary real number larger than the real parts of all singularities of $\bar{f}(x, p)$.

Taking $p = d + \mathrm{i}w$, the preceding integral takes the form

$$f(x, t) = \frac{\mathrm{e}^{dt}}{2\pi} \int_{-\infty}^{\infty} f(x, d + \mathrm{i}w)\, \mathrm{d}w\ . \tag{52}$$

Expanding the function $h(x, t) = \mathrm{e}^{-dt} f(x, t)$ in a Fourier series in the interval $[0, 2T]$, we obtain the approximate formula [40]

$$f(x, t) = f_\infty(x, t) + E_{\mathrm{D}} \tag{53}$$

where

$$f_\infty(x, t) = \frac{1}{2}c_0 + \sum_{k=1}^{\infty} c_K \quad 0 \leq t \leq 2T, \tag{54}$$

$$c_k = \frac{\mathrm{e}^{dt}}{T}\left[\mathrm{e}^{\frac{\mathrm{i}k\pi t}{T}} \bar{f}\left(x, d + \frac{\mathrm{i}k\pi t}{T}\right)\right]. \tag{55}$$

The discretization error E_{D} can be made arbitrarily small by choosing d large enough [40]. Since the infinite series in Eq. (54) can be summed up to a finite number N of terms, the approximate value $f(x, t)$ becomes

$$f_N(x, t) = \frac{1}{2}c_0 + \sum_{k=1}^{N} c_k, \quad 0 \leq t \leq 2T. \tag{56}$$

Using the preceding formula to evaluate $f(x, t)$, we introduce a truncation error E_{T} that must be added to the discretization error to produce the total approximation error.

Two methods are used to reduce the total error. First, the 'Korrektur' method is applied to reduce the discretization error. Next, the ϵ-algorithm is used to accelerate convergence [40].

The Korrektur method uses the following formula to evaluate the function $f(x, t)$:

$$f(x, t) = f_\infty(x, t) - \mathrm{e}^{-2dT} f_\infty(x, 2T + t) + E'_{\mathrm{D}}, \tag{57}$$

where the discretization error $|E'_{\mathrm{D}}| \leq |E_{\mathrm{D}}|$. Thus, the approximate value of $f(x,t)$ becomes

$$f_{NK}(x,t) = f_N(x,t) - \mathrm{e}^{-2dT} f_{N'}(x, 2T+t), \tag{58}$$

where N' is an integer such that $N' < N$.

We shall now describe the ϵ-algorithm that is used to accelerate the convergence of the series in Eq. (56). Let $N = 2q+1$, where q is a natural number and $s_m = \sum_{k=1}^{m} c_k$ is the sequence of the partial sum of the series in Eq. (56).

We define ϵ-sequence by

$$\epsilon_{0,m} = 0, \quad \epsilon_{1,m} = s_m$$

and

$$\epsilon_{p+1,m} = \epsilon_{p-1,m} + \frac{1}{\epsilon_{p,m+1} - \epsilon_{p,m}}, \quad p = 1, 2, 3, \ldots.$$

It can be shown that [40] the sequence $\epsilon_{1,1}, \epsilon_{3,1}, \epsilon_{5,1}, \ldots, \epsilon_{N,1}$ converges to $f(x,t) + E_{\mathrm{D}} - \dfrac{c_0}{2}$ faster than the sequence of partial sums s_m, $m = 1, 2, 3, \ldots$

The actual procedure used to invert the Laplace transform consists of using Eq. (58) together with the ϵ-algorithm. The values of d and T are chosen according to the criteria outlined by Honig and Hirdes [40].

6 Numerical results and discussion

To get the solution for the thermal displacement, temperature, stress and strain in the space-time domain, we have to apply the Laplace inversion formula to Eqs. (46)–(49), respectively. This has been done numerically using a method based on the Fourier series expansion technique mentioned above. To get the roots of the polynomial $M(\alpha)$ in the complex domain, we have used Laguerre's method. The numerical code has been prepared using Fortran 77 programming language. For computational purpose, a copper-like material with a material constant [41] has been taken into consideration,

$$\begin{aligned} &\epsilon_{\mathrm{T}} = 0.0168, \quad \lambda = 1.387 \times 10^{11} \quad \mathrm{Nm}^{-2}, \quad \mu = 0.448 \times 10^{11} \quad \mathrm{Nm}^{-2}, \\ &\alpha_{\mathrm{t}} = 16.7 \times 10^{-6}\ \mathrm{K}^{-1}, \quad c_1 = 1\ \mathrm{m\,s}^{-1}. \end{aligned}$$

Also, we have taken $Q_0^* = 1$, $\tau = 1$ and $c_{\mathrm{T}} = 2$ so the faster wave is the thermal wave.

The relaxation time parameters are taken as $\tau_q = 0.001$, $\tau_T = 0.05$, $\tau_\nu = 0.05$, which agree with the stability condition in [38].

We now present our results to compare the thermal displacement, temperature, stress and strain in the case of the TEWOED (GN-II model), TEWED (GN-III model) and three-phase-lag model (3P model) for an unbounded elastic medium in the form of the graphs (Figs. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16). Now the Figures (Figs. 1, 2, 3, Figs. 5, 6, 7, Figs. 9, 10, 11 and Figs. 13, 14, 15) are plotted against distance x for $t = 0.4$, and the Figures (Figs. 4, 8, 12 and 16) are plotted against time t for $x = 0.3$.

Figure 1 depicts the variation of the thermal displacements (u) versus distance (x). It is observed that the displacement increases for $0.0 \leq x \leq 0.3$ and then decreases and ultimately tends to zero for $x \geq 0.9$ in the presence of a magnetic field (WMF) by taking $R_{\mathrm{M}} = 2.0$, and it is also observed that displacement increases for $0.0 \leq x \leq 0.2$ and then decreases and ultimately goes to zero in the absence of a magnetic field (WOMF) by taking $R_{\mathrm{M}} = 1.0$ for GN-II model, i.e., $\kappa_0 = 0.0$. For the case of WOMF, the result complies with that of Roychoudhuri and Dutta [41] where they have used the analytical method. Also in the case when $\kappa_0 = 1.2$, both for GN-III model and 3P model, u increases first, then decreases and ultimately approaches to zero as before with the increase of x for WMF ($R_{\mathrm{M}} = 2.0$) as well as WOMF ($R_{\mathrm{M}} = 1.0$). As may be seen from the figure the rate of decay is slower in the case of 3P model than that of GN-III model and that is again slower than that of GN-II model both for WMF and WOMF, this implies that $\kappa_0 = 1.2$ corresponds to a slower rate of decay than the case when $\kappa_0 = 0.0$ in the absence of a magnetic field ($R_{\mathrm{M}} = 1.0$). This result agrees with that of Banik et al. [23]. The magnitude of the displacement u is large in the GN-II model in comparison to GN-III model and 3P model, but the qualitative behavior is almost the same for all the three models.

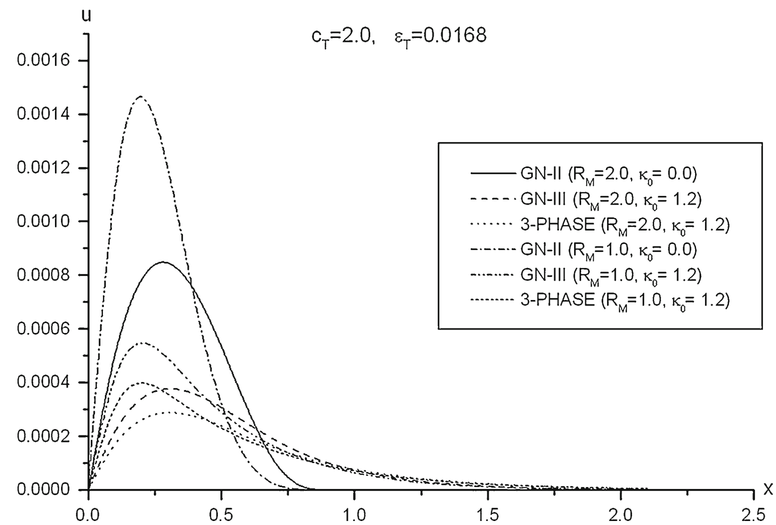

Fig. 1 Variation of displacement u with distance x

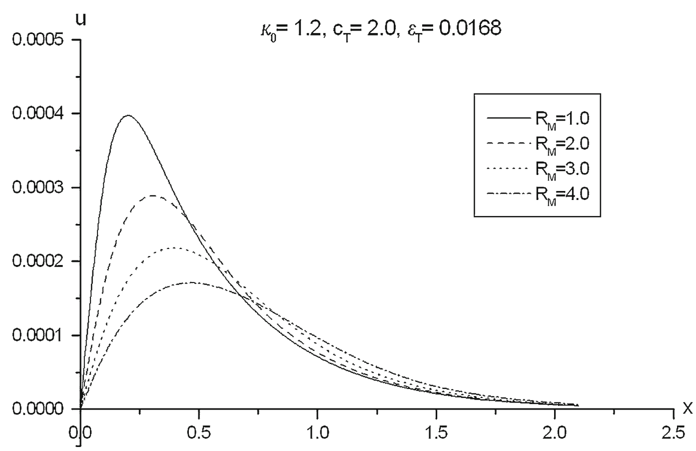

Fig. 2 Variation of displacement u with distance x in the three-phase-lag model

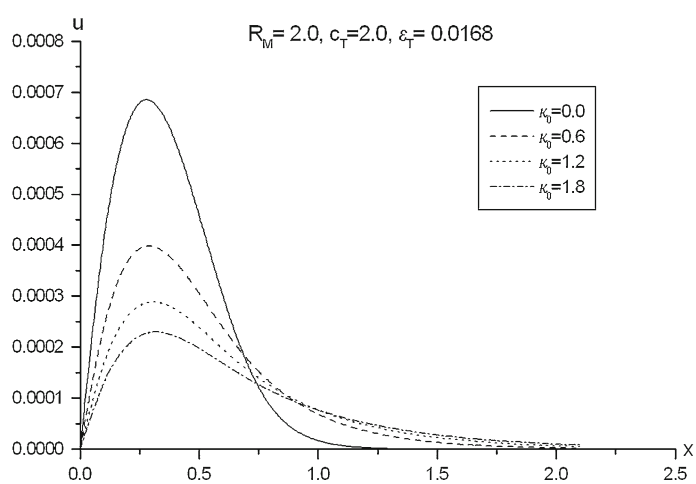

Fig. 3 Variation of displacement u with distance x for the three-phase-lag model

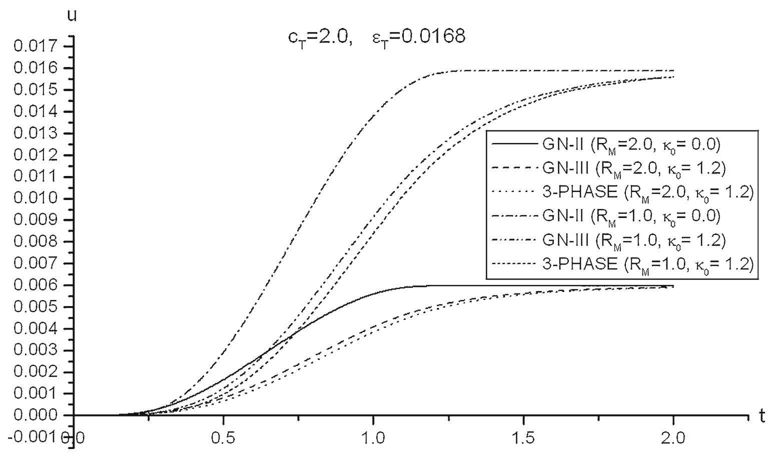

Fig. 4 Variation of displacement u with time t

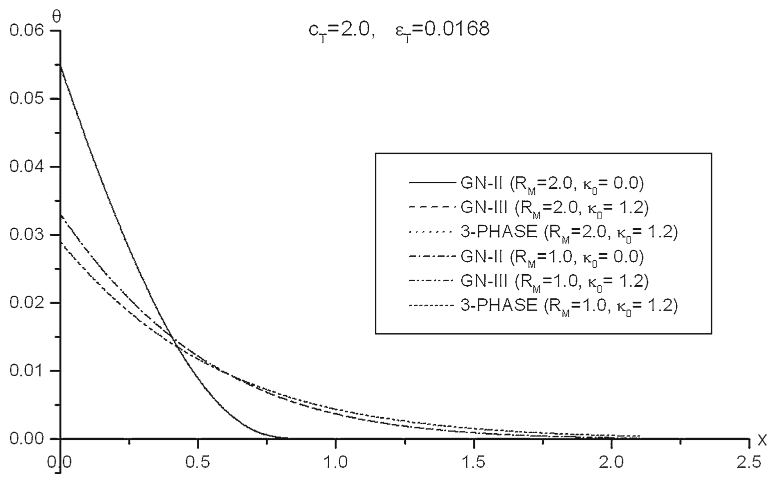

Fig. 5 Variation of temperature θ with distance x

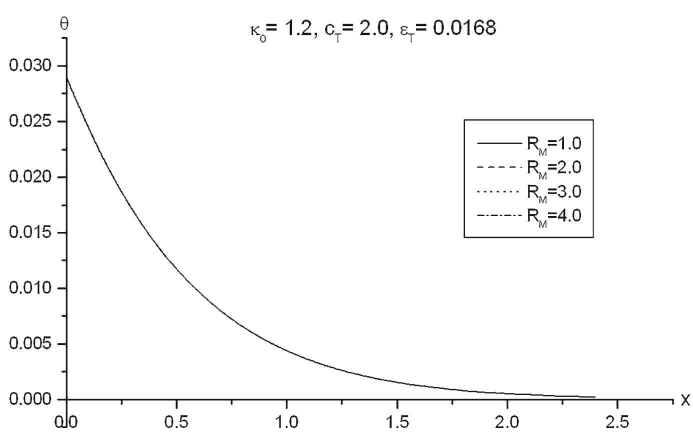

Fig. 6 Variation of temperature θ with distance x in the three-phase-lag model

Figure 2 depicts the variation of the displacement (u) with distance (x) taking $R_{\mathrm{M}} = 1, 2, 3, 4$ and keeping the damping coefficient $\kappa_0 = 1.2$ where we have considered the 3P model. Here also a similar qualitative behavior observed as in the case of Fig. 1. But one important thing observed here is that with the increase in the magnetic field the magnitude of displacement decreases, which is quite plausible.

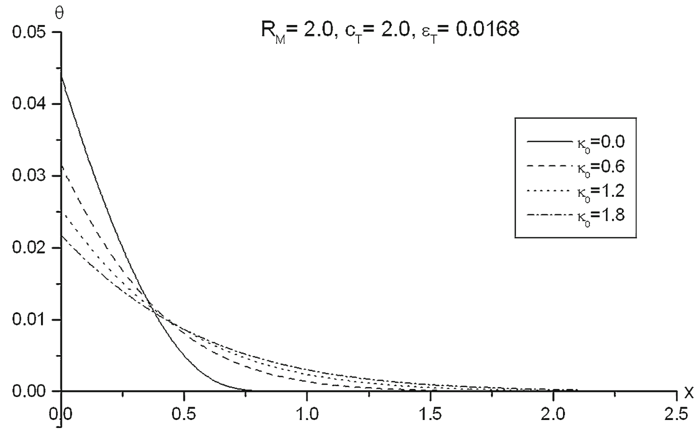

Fig. 7 Variation of temperature θ with distance x in the three-phase-lag model

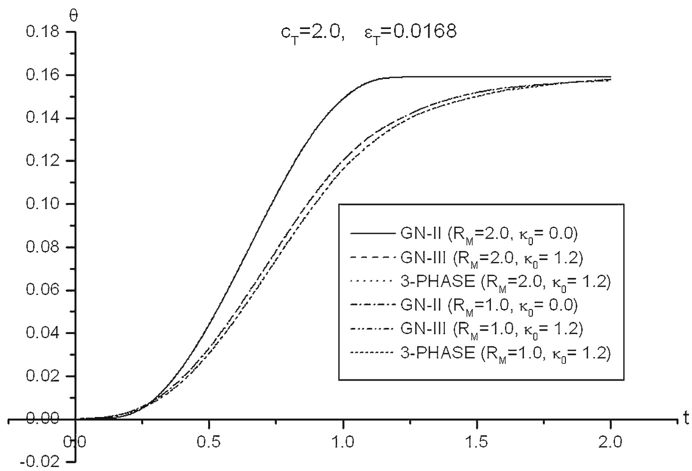

Fig. 8 Variation of temperature θ with time t

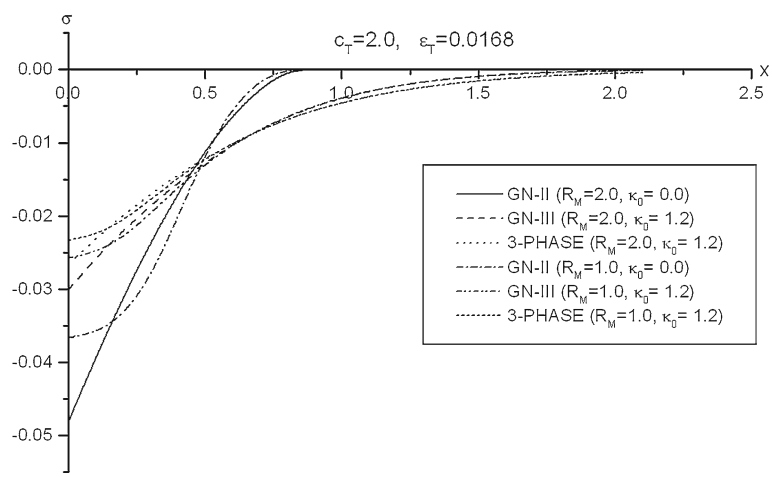

Fig. 9 Variation of stress σ with distance x

Figure 3 is plotted to show the variation of displacement (u) with distance (x) for $R_M = 2.0$, where we have again considered the 3P model. Figure 3 depicts the effect of the damping coefficient on the displacement. Now it is observed that as the damping coefficient increases the rate of decay of the displacement becomes slow.

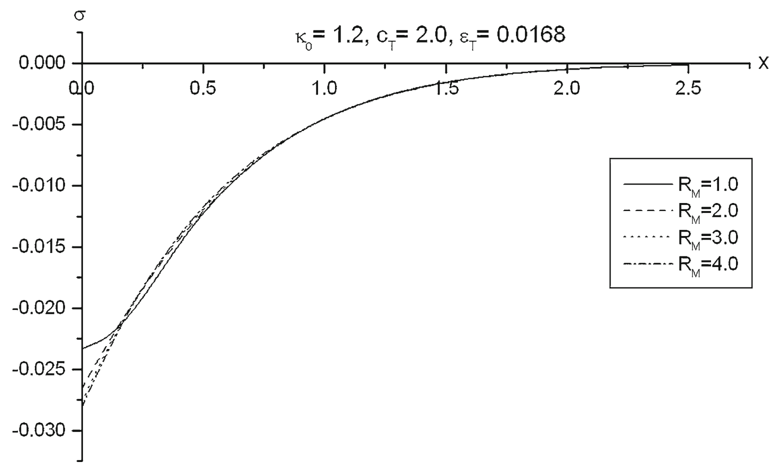

Fig. 10 Variation of stress σ with distance x in the three-phase-lag model

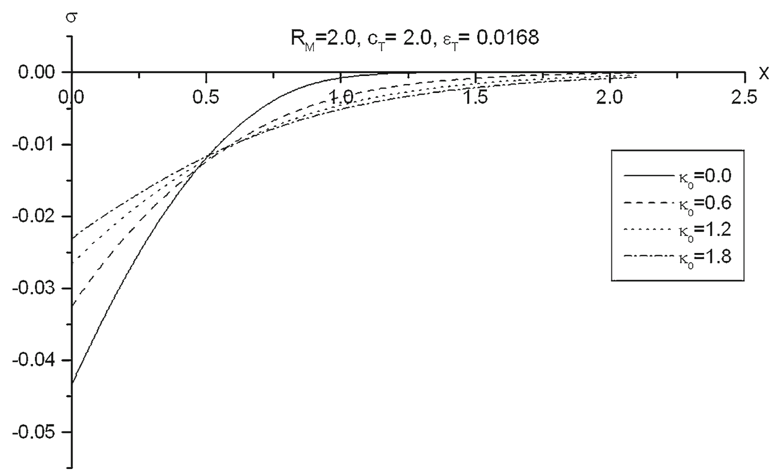

Fig. 11 Variation of stress σ with distance x in the three-phase-lag model

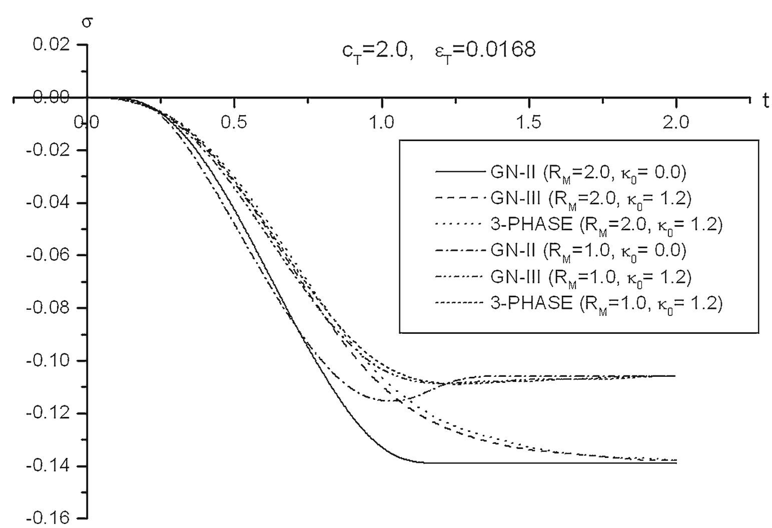

Fig. 12 Variation of stress σ with time t

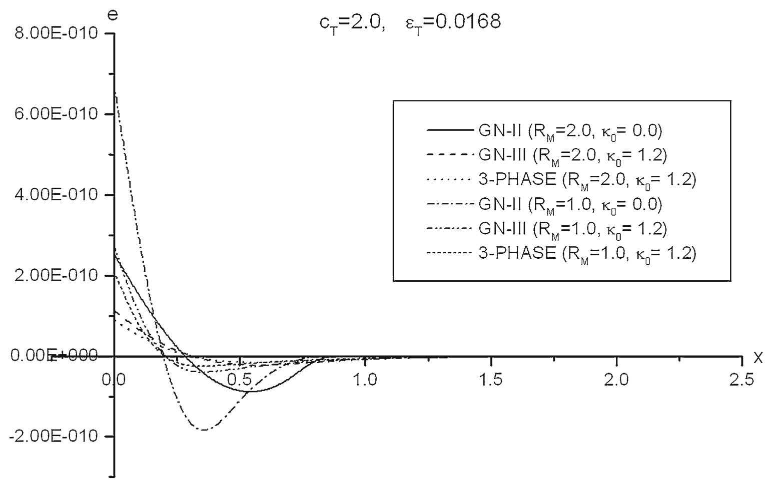

Fig. 13 Variation of strain e with distance x

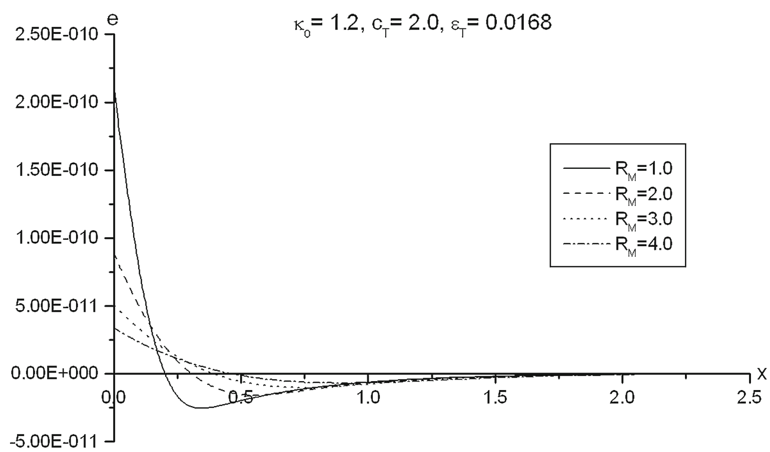

Fig. 14 Variation of strain e with distance x in the three-phase-lag model

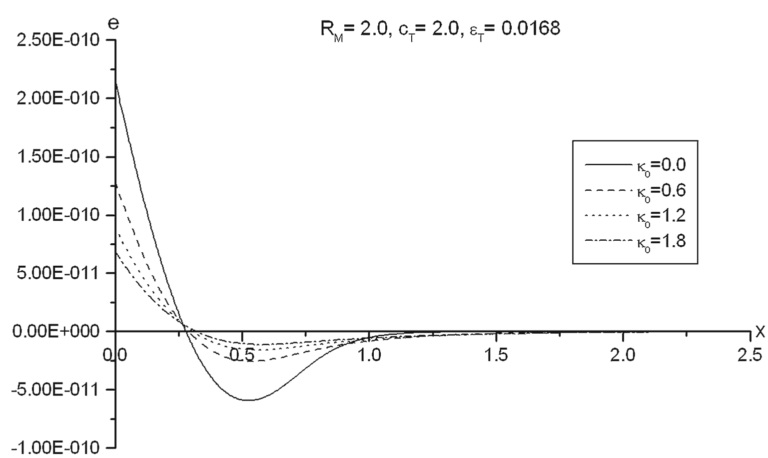

Fig. 15 Variation of strain e with distance x in the three-phase-lag model

Figure 4 represents the variation of displacement (u) against time (t) for $x = 0.3$. It is observed that the displacement increases first and then reaches a constant value with the increase in time t in the presence of a magnetic field (WMF) by taking $R_M = 2.0$, and it is also observed that in the absence of a magnetic field (WOMF), i.e., for $R_M = 1.0$, the displacement shows the same qualitative behavior for GN-II model

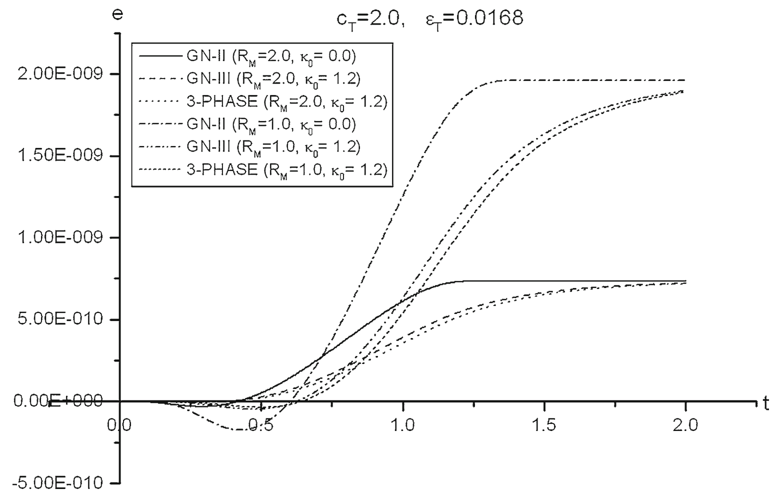

Fig. 16 Variation of strain e with distance x

($\kappa_0 = 0.0$). Also, in the case when $\kappa_0 = 1.2$, for both GN-III model and 3P model, u increases first and then ultimately approaches to a constant value as before with the increase of t for WMF ($R_{\mathrm{M}} = 2.0$) and WOMF ($R_{\mathrm{M}} = 1.0$). But one important thing is observed here that in the presence of a magnetic field the displacement is smaller in the case of 3P model than that of GN-III model which is again smaller than that of GN-II model. It is also seen from this Figure that with the increase in magnetic field the magnitude of displacement decreases for all the three models. It is observed that the time to reach the steady state for GN-II model for WMF and WOMF is faster than for the other two models, which is quite plausible since for GN-II model there is no such dissipation of energy.

Figure 5 depicts the variation of temperature (θ) with distance (x) for WMF ($R_{\mathrm{M}} = 2.0$) and WOMF ($R_{\mathrm{M}} = 1.0$). Here, it can be observed that temperature decreases with the increase in distance and finally goes to zero for GN-II ($\kappa_0 = 0.0$), GN-III ($\kappa_0 = 1.2$) and 3P model ($\kappa_0 = 1.2$). In the case of 3P model, the rate of decay is slower than that of GN-III model and that is again slower than that of GN-II model both for WMF and WOMF. From this figure, it can also be observed that there is no such effect of the magnetic field on temperature.

Figure 6 shows the variation of temperature (θ) with distance (x) for the various values of the magnetic field keeping $\kappa_0 = 1.2$. Here, we have considered the 3P model. This Figure depicts that there is no such effect of the magnetic field on temperature for this model.

Figure 7 is plotted to show the variation of temperature (θ) versus distance (x) for 3P model in the presence of a magnetic field ($R_{\mathrm{M}} = 2.0$). It is observed from the Figure that temperature decreases with the increase in distance and finally goes to zero for all values of the damping coefficient but as the damping coefficient increases, the rate of decay decreases.

Figure 8 depicts the variation of temperature (θ) with time (t) for WMF ($R_{\mathrm{M}} = 2.0$) and WOMF ($R_{\mathrm{M}} = 1.0$). Here, it is observed that temperature increases for $0.0 \leq x \leq 1.1$ and then approaches a steady state for GN-II ($\kappa_0 = 0.0$) model by taking $R_{\mathrm{M}} = 2.0$ and $R_{\mathrm{M}} = 1.0$. For GN-III model and 3P model, the displacement shows the same nature for WMF and WOMF. From this Figure, it can also be observed that there is no such effect of the magnetic field on temperature. Here also the time to reach the steady state for GN-II model for WMF and WOMF is faster than for the other two models, which is quite plausible since for GN-II model there is no such dissipation of energy as it is in Fig. 4.

Figure 9 exhibits the space variation (x) of stress (σ) in the presence of a magnetic field ($R_{\mathrm{M}} = 2.0$) and also in the absence of a magnetic field ($R_{\mathrm{M}} = 1.0$). It is observed that stress is compressive in nature, and the magnitude is maximum near the boundary. Here, the rate of decay is faster in the case of GN-II model than in case of GN-III model, which is again faster than 3P model both for $R_{\mathrm{M}} = 2.0$ and $R_{\mathrm{M}} = 1.0$.

Figures 10 and 11 are plotted to show the variation of stress (σ) against distance (x) for 3P model taking various values of the magnetic field and the damping coefficient, respectively. From Fig. 10, it is observed that with the increase in the magnetic field the magnitude of stress increases near the boundary, and from Fig. 11 it is observed that by increasing the value of the damping coefficient the magnitude of stress decreases near the boundary, but the rate of decay is reversed in nature.

Figure 12 depicts the variation of stress (σ) with time (t) for GN-II, GN-III and 3P model in the presence of a magnetic field ($R_{\mathrm{M}} = 2.0$) and in the absence of a magnetic field ($R_{\mathrm{M}} = 1.0$). It is observed that the magnitudes of stress are large in the case of TEWED (GN-II) theory in comparison with the rest of the theories. This Figure also shows that by increasing magnetic field the damping of stress is also increasing until σ reaches a constant value.

Figure 13 gives the variation of strain (e) against distance (x) in the presence of a magnetic field ($R_{\mathrm{M}} = 2.0$) and in the absence of a magnetic field ($R_{\mathrm{M}} = 1.0$). From this figure, we can show for GN-II model ($\kappa_0 = 0.0$) that strain is positive up to a distance $x = 0.2$, and for GN-III model, ($\kappa_0 = 1.2$) and 3P model ($\kappa_0 = 1.2$) strain is positive up to a distance $x = 0.3$, and then, it is negative and finally diminishes to zero for all the three models in the case of WMF ($R_{\mathrm{M}} = 2.0$). Now in the absence of a magnetic field (WOMF, i.e., $R_{\mathrm{M}} = 1.0$), the magnitude of strain is larger near the boundary than that of WMF for all the three models. The qualitative behavior is nearly the same.

Figure 14 depicts the variation of strain (e) with distance (x) for 3P model taking $\kappa_0 = 1.2$ and $R_{\mathrm{M}} = 1, 2, 3$ and 4. In this Figure, it can be observed that with the increase in the magnetic field the magnitude of strain decreases near the boundary and ultimately approaches to zero as distance increases, which is quite plausible since the periodic disturbance is given on the boundary.

Figure 15 is plotted to show the variation of the strain (e) versus distance (x) in the presence of a magnetic field ($R_{\mathrm{M}} = 2.0$) where we have considered the three-phase-lag model. Here, the effect of the damping coefficient on strain is such that for all values of κ_0 strain is positive first, then remains negative and finally goes to zero.

Figure 16 depicts the variation of strain (e) with time (t). This figure shows that for GN-II, GN-III and 3P model strain is negative first, then increases and ultimately reaches a steady state both for WMF ($R_{\mathrm{M}} = 2.0$) and WOMF ($R_{\mathrm{M}} = 1.0$). In Figs. 1, 5, 9, 13 when there is no such magnetic field ($R_{\mathrm{M}} = 1.0$) but there is a dissipation of energy ($\kappa_0 = 1.2$), the result agrees with that of Banik et al. [23] and when $R_{\mathrm{M}} = 1.0$ and $\kappa_0 = 0.0$, the result is confirmed by that of Roychoudhuri and Dutta [41] in which the closed-form solution of the problem has been derived.

Acknowledgments We are grateful to Professor S. C. Bose of the Department of Applied Mathematics, University of Calcutta, for his kind help and guidance in the preparation of the paper. We also express our sincere thanks to the reviewers for their valuable suggestions for the improvement of the paper.

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