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Blasius and Sakiadis problems in nanofluids

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Abstract The classical problems of forced convection boundary layer flow and heat transfer past a semi-infinite static flat plate (Blasius problem) and past a moving semi-infinite flat plate (Sakiadis problem) using nanofluids are theoretically studied. The similarity equations are solved numerically for three types of metallic or nonmetallic nanoparticles such as copper (Cu), alumina (Al_2O_3), and titania (TiO_2) in the base fluid of water with the Prandtl number $\mathrm{Pr} = 6.2$ to investigate the effect of the solid volume fraction parameter φ of the nanofluids. Also, the case of conventional or regular fluid ($\varphi = 0$) with $\mathrm{Pr} = 0.7$ is considered for comparison with known results from the open literature. The comparison shows excellent agreement. The skin friction coefficient, Nusselt number, and the velocity and temperature profiles are presented and discussed in detail. It is found that the solid volume fraction affects the fluid flow and heat transfer characteristics.

1 Introduction

Nanofluids are a new kind of heat transfer fluids containing a small quantity of nanosized particles (usually less than 100 nm) that are uniformly and stably suspended in a liquid. These nanofluids appear to have a very high thermal conductivity and may be able to meet the rising demand as an efficient heat transfer agent [1]. This characteristic feature of high thermal conductivity has been observed by Masuda et al. [2]. Nanotechnology has been widely used in the industry since materials with sizes of nanometers possess unique physical and chemical properties. Nanoparticles used in nanofluids have been made out of many materials by physical and chemical synthesis processes. Typical physical methods include the mechanical grinding method and the inert-gas-condensation technique [3]. Choi [4] is the first who used the term nanofluids to refer to a fluid with suspended nanoparticles. Choi et al. [5] showed that the addition of small amount (less than 1% by volume) of nanoparticles to conventional heat transfer liquids increased the thermal conductivity of the fluid up to approximately two times. On the other side, Buongiorno [6] noted that the nanoparticle absolute velocity can be viewed as the sum of the base fluid velocity and a relative velocity (that he calls the slip velocity). He has shown that in the absence of turbulent effects, it is the Brownian diffusion and the thermophoresis that

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are important and he has written down conservation equations based on these two effects. However, several other works (Keblinski et al. [7,8] and Prasher et al. [9]) proposed alternative mechanisms for the "abnormal increase" in the thermal conductivity and viscosity of the nanofluids than those proposed by Buongiorno [6]. For instance, by comparing the Brinkman model and experimental data that are supplied by Maiga et al. [10] and Polidori et al. [11], one may find an augmentation of the dynamic viscosity of ∼30% at a 4% particle loading. More recently, Popa et al. [12] were making a comparison between the Maxwell model and the experimental data provided by Mintsa et al. [13] for thermal conductivity of nanofluids. Popa et al. [12] find that Maxwell's model strongly overestimates the thermal conductivity of the nanofluid. There are also several numerical and experimental studies on the forced and natural convection using nanofluids related with differentially heated enclosures, and we mention here those by Khanafer et al. [14], Maiga et al. [10,15], Abu-Nada [16], Hwang et al. [17], Jou and Tzeng [18], Tiwari and Das [19], Oztop and Abu-Nada [20], and Muthtamilselvan et al. [1]. The book by Das et al. [21] and the recent review paper by Kakaç and Pramunjaroenkij [22] present excellent collection of up to now published papers on nanofluids.

However, studies on boundary layer flows using nanofluids are very limited. To the best of our knowledge, there are only the papers by Nield and Kuznetsov [23], which are dealing with the classical problems of natural convective boundary layer flow in a porous medium saturated by a nanofluid, known as Cheng-Minkowycz's problem [24], and Kuznetsov and Nield [25], for the problem of natural convective boundary layer flow of a nanofluid past a vertical semi-infinite flat plate, known as Pohlhausen-Kuiken-Bejan's problem after Kuznetsov and Nield [25]. The model used by these authors for the nanofluid incorporates the effects of Brownian motion and thermophoresis as proposed by Buongiorno [6]. The similarity solutions obtained for both problems depend on a Lewis number, a buoyancy-ratio number, a Brownian motion number, and a thermophoresis number. The aim of the present paper is, therefore, to extend the classical forced convection boundary layer flow past a static semi-infinite flat plate (Blasius problem [26]) and, respectively, the forced convection boundary layer flow past a moving flat plate (Sakiadis problem [27]) to the case of a nanofluid using the model of Tiwari and Das [19]. Three different nanofluids made of Cu, Al_2O_3, and TiO_2 are tested to investigate the effect of the solid volume fraction parameter φ of the nanofluid on the flow and heat transfer characteristics. The case of conventional or regular fluid ($\varphi = 0$) with $\mathrm{Pr} = 0.7$ is also considered for comparison with known results from the available literature.

2 Basic equations

We consider the steady two-dimensional boundary layer flow past a fixed (Blasius [26]) or past a moving (Sakiadis [27]) semi-infinite flat plate in a water-based nanofluid containing different types of nanoparticles: Cu, Al_2O_3, and TiO_2. The nanofluid is assumed incompressible, the flow is assumed to be laminar, and the viscous dissipation and radiation effects are neglected. It is also assumed that the base fluid (i.e. water) and the nanoparticles are in thermal equilibrium and no slip occurs between them. The thermophysical properties of the nanofluids are given in Table 1 (taken from Oztop and Abu-Nada [20]). Under these assumptions and following the model equations of a nanofluid proposed by Tiwari and Das [19], the basic continuity, momentum, and energy equations in the vectorial form for the steady-state flow are

$$\nabla \cdot \mathbf{V} = 0, \tag{1}$$

$$(\mathbf{V} \cdot \nabla)\mathbf{V} = -\frac{1}{\rho_{\mathrm{nf}}}\nabla p + \frac{\mu_{\mathrm{nf}}}{\rho_{\mathrm{nf}}}\nabla^2 \mathbf{V}, \tag{2}$$

$$(\mathbf{V} \cdot \nabla)T = \alpha_{\mathrm{nf}}\nabla^2 T, \tag{3}$$

Table 1 Thermophysical properties of the fluid phase (water) and nanoparticles [20]

Physical properties	Fluid phase (water)	Cu	Al_2O_3	TiO_2
C_p(J/kgK)	4179	385	765	686.2
ρ(kg/m^3)	997.1	8933	3970	4250
k(W/mK)	0.613	400	40	8.9538

where $\mathbf{V}$ is the velocity vector, T is the temperature of the nanofluid, p is the pressure of the nanofluid, $\mu_{\rm nf}$ is the dynamic viscosity of the nanofluid, $\rho_{\rm nf}$ is the density of the nanofluid, and $\alpha_{\rm nf}$ is the thermal diffusivity of the nanofluid, which are given by

$$\mu_{\rm nf} = \frac{\mu_{\rm f}}{(1-\varphi)^{2.5}}, \quad \rho_{\rm nf} = (1-\varphi)\rho_{\rm f} + \varphi\rho_{\rm s}, \quad \alpha_{\rm nf} = \frac{k_{\rm nf}}{(\rho C_p)_{\rm nf}},$$
$$\frac{k_{\rm nf}}{k_{\rm f}} = \frac{(k_{\rm s}+2k_{\rm f}) - 2\varphi(k_{\rm f}-k_{\rm s})}{(k_{\rm s}+2k_{\rm f}) + \varphi(k_{\rm f}-k_{\rm s})}, \quad (\rho C_p)_{\rm nf} = (1-\varphi)(\rho C_p)_{\rm f} + \varphi(\rho C_p)_{\rm s}. \tag{4}$$

Here, $k_{\rm nf}$ is the thermal conductivity of the nanofluid, φ is the solid volume fraction parameter of the nanofluid, $\rho_{\rm f}$ is the reference density of the fluid fraction, $\rho_{\rm s}$ is the reference density of the solid fraction, $\mu_{\rm f}$ is the viscosity of the fluid fraction, $k_{\rm f}$ is the thermal conductivity of the fluid fraction, and $k_{\rm s}$ is the thermal conductivity of the solid volume fraction. Further, $(\rho C_p)_{\rm nf}$ is the heat capacitance of the nanofluid as expressed by Xuan and Li [28], Khanafer et al. [14], and Abu-Nada [16], where C_p is the specific heat at constant pressure. It is worth mentioning that the viscosity $\mu_{\rm nf}$ of the nanofluid can be approximated as viscosity of the base fluid $\mu_{\rm f}$ containing dilute suspension of fine spherical particles and is given by Brinkman [29]. The effective thermal conductivity of the nanofluid $k_{\rm nf}$ is approximated by the Maxwell-Garnetts model, which is found to be appropriate for studying heat transfer enhancement using nanofluids (Maiga et al. [15] and Abu-Nada [16]). Other several models for $k_{\rm nf}$ can be found in Kakaç and Pramunjaroenkij [22] and Abu-Nada [30].

We consider a Cartesian coordinate system (x, y), where x and y are the coordinates measured along the plate and normal to it, respectively, and assume that the flow takes place at $y \geq 0$. It is also assumed that the constant temperature of the flat plate is T_w and that of the ambient nanofluid is T_∞. Under the boundary layer approximations and the fact that this flow is one of zero pressure gradient, the basic equations (1–3) can be written in the Cartesian coordinates x and y as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{5}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\mu_{\rm nf}}{\rho_{\rm nf}}\frac{\partial^2 u}{\partial y^2}, \tag{6}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha_{\rm nf}\frac{\partial^2 T}{\partial y^2}. \tag{7}$$

We assume that these equations are subject to the boundary conditions

i) Blasius problem

$$v = 0, \; u = 0 \quad \text{at } y = 0, \qquad u = U \quad \text{as;} \quad y \to \infty, \tag{8a}$$

ii) Sakiadis problem

$$v = 0, \; u = U \quad \text{at } y = 0, \qquad u = 0 \quad \text{as } y \to \infty, \tag{8b}$$

where u and v are the velocity components along the axes x and y, and U is the constant velocity of the free stream (inviscid flow) or that of a moving flat plate. The boundary conditions for the energy equation (7) are

$$T = T_w \quad \text{at } y = 0, \qquad T = T_\infty \quad \text{as } y \to \infty. \tag{9}$$

We look for a similarity solution of Eqs. (5–7) with the boundary conditions (8) and (9) of the following form:

$$\psi = (U\nu_{\rm f}x)^{1/2} f(\eta), \quad \theta(\eta) = (T-T_\infty)/(T_w-T_\infty), \quad \eta = (U/\nu_{\rm f}x)^{1/2}\, y, \tag{10}$$

where $\nu_{\rm f}$ is the kinematic viscosity of the fluid fraction and ψ is the stream function that is defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$. On substituting Eqs. (10) into (6) and (7), we obtain the following uncoupled ordinary differential equations:

$$\frac{1}{(1-\varphi)^{2.5}\,(1-\varphi + \varphi\rho_{\rm s}/\rho_{\rm f})} f''' + \frac{1}{2} f f'' = 0, \tag{11}$$

$$\frac{1}{\Pr}\frac{k_{\rm nf}/k_{\rm f}}{[1-\varphi + \varphi(\rho C_p)_{\rm s}/(\rho C_p)_{\rm f}]}\theta'' + \frac{1}{2} f\theta' = 0 \tag{12}$$

subject to the boundary conditions

i) Blasius problem

$$f(0) = 0, \quad f'(0) = 0, \quad f'(\infty) = 1, \tag{13a}$$

ii) Sakiadis problem

$$f(0) = 0, \quad f'(0) = 1, \quad f'(\infty) = 0, \tag{13b}$$

and for the energy Eq. (12), we have

$$\theta(0) = 1, \quad \theta(\infty) = 0, \tag{14}$$

where primes denote differentiation with respect to the similarity variable η and $\Pr = \nu_f/\alpha_f$ is the Prandtl number. It is worth mentioning that Eq. (11) reduces to that derived by Blasius [26] or Sakiadis [27] when $\varphi = 0$ (regular fluid).

Quantities of practical interest in this study are the skin friction coefficient C_f and the local Nusselt number Nu, which are defined as

$$C_f = \frac{\tau_w}{\rho_f U^2}, \quad Nu = \frac{x q_w}{k_f(T_w - T_\infty)}, \tag{15}$$

where τ_w is the skin friction or the shear stress and q_w is the heat flux from the plate which are given by

$$\tau_w = \mu_{nf}\left(\frac{\partial u}{\partial y}\right)_{y=0}, \quad q_w = -k_{nf}\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{16}$$

Substituting Eqs. (10) into (15) and (16), we obtain

$$Re_x^{1/2} C_f = \frac{1}{(1-\varphi)^{2.5}} f''(0), \quad Re_x^{-1/2} Nu = -\frac{k_{nf}}{k_f}\theta'(0), \tag{17}$$

where $Re_x = Ux/\nu_f$ is the local Reynolds number.

3 Results and discussion

Numerical solutions to the governing ordinary differential equations (11) and (12) with the boundary conditions (13) and (14) are obtained using the shooting method. The effect of the solid volume fraction of nanofluid φ and the Prandtl number Pr are analyzed for three different nanofluids Cu-water, Al_2O_3-water, and TiO_2-water as working fluids with $\Pr = 6.2$ [20]. The effect of solid volume fraction φ is investigated, as in [20], in the range of $0 \le \varphi \le 0.2$. However, Muthtamilselvan et al. [1] considered that φ varies in the range of $0\% \le \varphi \le 8\%$, Abu-Nada [30] in the range of $0\% \le \varphi \le 9\%$, and Ghasemi and Aminossadati

Table 2 Values of $Re_x^{1/2} C_f$ for the Blasius problem

φ	Present			Blasius [26]
	Cu-water	Al_2O_3-water	TiO_2-water	
0	0.3321	0.3321	0.3321	0.3321
0.002	0.3355	0.3339	0.3340	
0.004	0.3390	0.3357	0.3359	
0.008	0.3459	0.3394	0.3398	
0.01	0.3494	0.3412	0.3417	
0.012	0.3528	0.3431	0.3436	
0.014	0.3563	0.3449	0.3456	
0.016	0.3597	0.3468	0.3476	
0.018	0.3632	0.3487	0.3495	
0.02	0.3667	0.3506	0.3515	
0.1	0.5076	0.4316	0.4362	
0.2	0.7066	0.5545	0.5642	

Table 3 Values of $-Re_x^{1/2}C_f$ for the Sakiadis problem

φ	Present			Sakiadis [27]
	Cu-water	Al_2O_3-water	TiO_2-water	
0	0.4446	0.4446	0.4446	0.44375
0.002	0.4492	0.4470	0.4471	
0.004	0.4538	0.4494	0.4497	
0.008	0.4630	0.4544	0.4548	
0.01	0.4676	0.4568	0.4574	
0.012	0.4722	0.4593	0.4600	
0.014	0.4768	0.4618	0.4626	
0.016	0.4814	0.4643	0.4653	
0.018	0.4860	0.4668	0.4679	
0.02	0.4906	0.4693	0.4705	
0.1	0.6788	0.5778	0.5840	
0.2	0.9446	0.7428	0.7556	

Table 4 Values of $f''(0)$ and $\theta'(0)$ for the Sakiadis problem with $\varphi = 0$ (regular fluid) and $\Pr = 0.7$ (air)

Author(s)	$-f''(0)$	$-\theta'(0)$
Sakiadis [27]	0.44375	
Tsou et al. [32]	0.444	0.3492
Takhar et al. [33]	0.4439	0.3508
Pop et al. [34]	0.4445517	0.3507366
Pantokratoras [35]	0.4438	0.3500
Andersson and Aarseth [36]	0.4445516	0.3541247
Present	0.4445516	0.3541247

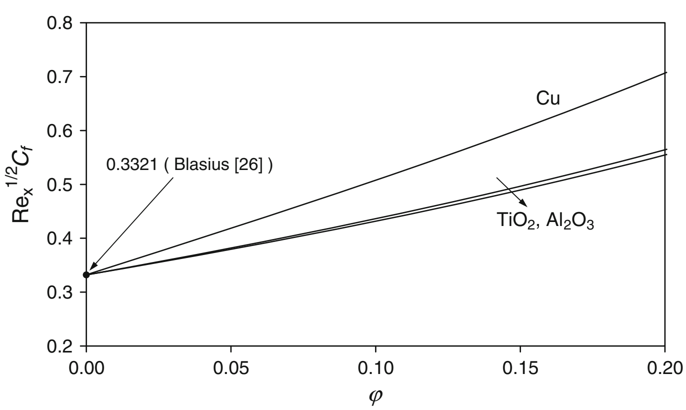

Fig. 1 Variation of the skin friction coefficient with φ for the Blasius problem

[31] in the range of $0\% \leq \varphi \leq 5\%$. In order to assess the accuracy of the present results, comparison with the existing solution in literature for the special case of a Newtonian or regular fluid ($\varphi = 0$) with those of Blasius [26] and Sakiadis [27] when $\Pr = 6.2$ (water) is shown in Tables 2 and 3. Further, Table 4 compares the present results for $\Pr = 0.7$ (air) with those of Sakiadis [27], Tsou et al. [32], Takhar et al. [33], Pop et al. [34], Pantokratoras [35], and Andersson and Aarseth [36]. It is seen that the present results are in excellent agreement with those reported by the above-mentioned authors. Therefore, we are confident that our results are accurate for the conductivity and viscosity models considered in the modeling of the present problem and this is a reinforcement to study this problem further. On the other hand, this provides credibilty to the accuracies of the present results for the conductivity and viscosity models considered in the modeling of the present problem and they can therefore be used with some confidence by other researchers. However, this does not confirm that the conductivity and viscosity models used here are the most appropriate. Other such models can be found in Kakaç and Pramunjaroenkij [22], Abu-Nada [30], and Ghasemi and Aminossadati [31].

Figures 1–4 present the variation of the skin friction coefficient $Re_x^{1/2}C_f$ and the Nusselt number $Re_x^{-1/2}Nu$ with the nanofluid volume fraction φ of the nanoparticles for Cu-water, Al_2O_3-water, and TiO_2-water as

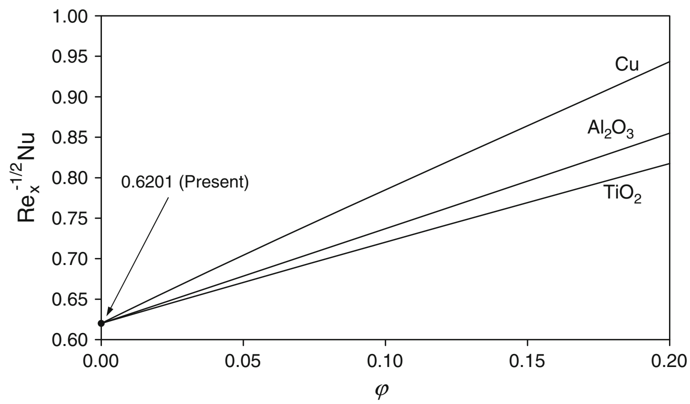

Fig. 2 Variation of the Nusselt number with φ for the Blasius problem

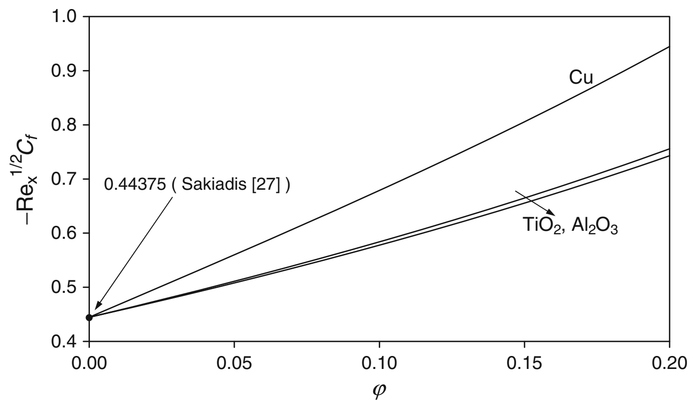

Fig. 3 Variation of the skin friction coefficient with φ for the Sakiadis problem

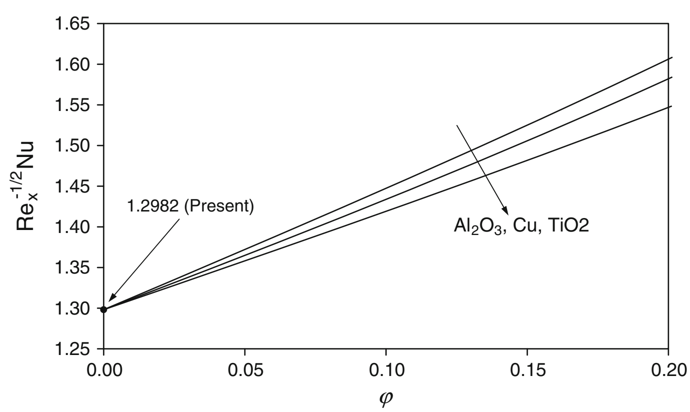

Fig. 4 Variation of the Nusselt number with φ for the Sakiadis problem

working fluids for both Blasius and Sakiadis problems. It is seen that both the skin friction coefficient and the Nusselt number increase when increasing the parameter φ. Thus, more particles are suspended and thermal conductivity of nanoparticles increases. Further, both the Tables 2 and 3 and Figs. 1 and 3 show that the lowest skin friction coefficient is obtained for Al_2O_3, while the lowest value of the Nusselt number is obtained for TiO_2, as can be seen from Figs. 2 and 4, because TiO_2 has the lowest thermal conductivity compared with Cu and Al_2O_3. However, the difference between the values of the skin friction coefficient for TiO_2 and Al_2O_3 is very small, as can be seen from Figs. 1 and 3. The thermal conductivity of Al_2O_3 is approximately one-tenth of Cu, as given in Table 1. Figures 2 and 4 show that the highest values of the Nusselt number are obtained for

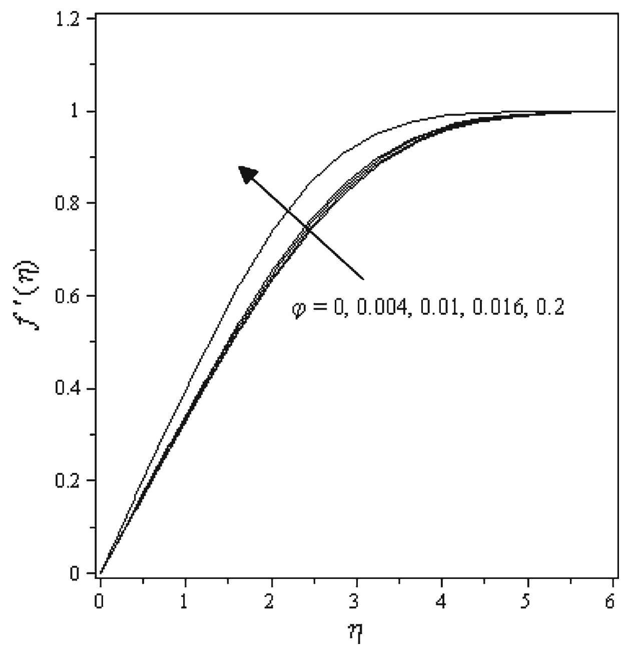

Fig. 5 Velocity profiles for various φ for the Blasius problem with Cu-water as working fluid

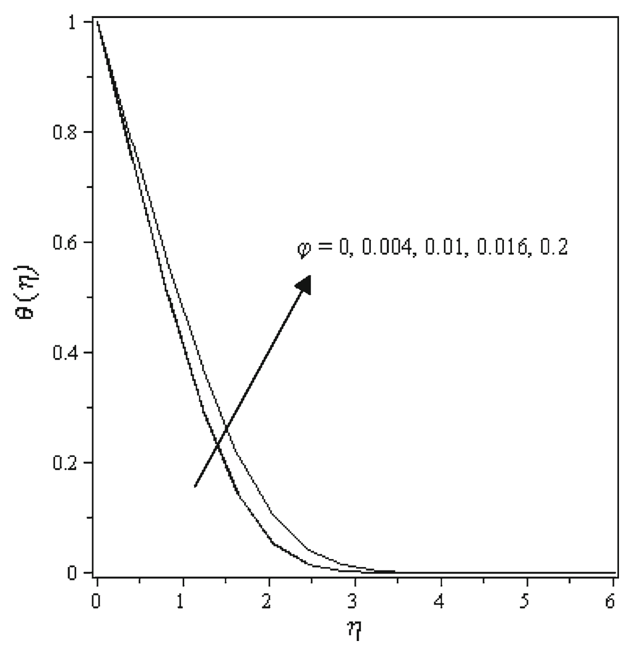

Fig. 6 Temperature profiles for various φ for the Blasius problem with Cu-water as working fluid

Cu and the lowest one for TiO_2. On the other hand, Figs. 2 and 4 indicate that more fluid is heated for higher values of volume fraction φ of nanoparticles.

Further, Figs. 5–8 show the velocity and temperature profiles for both the Blasius [26] and Sakiadis [27] problems for some values of the parameter φ ($0 \le \varphi \le 0.2$) when $\Pr = 6.2$ (water). In order to save space, we consider here results only for the case Cu-water as working fluid. As shown from these figures, both the velocity and temperature profiles are not significantly sensitive to the volume fraction parameter φ. This is explained by looking at Eq. (4), where the Brinkman formula shows that the viscosity of the nanofluid is only sensitive to the parameter φ and not influenced by the type of nanoparticles. On the other hand, Figs. 5 and 7 show that the momentum boundary layer increases with φ for the Blasius problem, while it decreases with φ

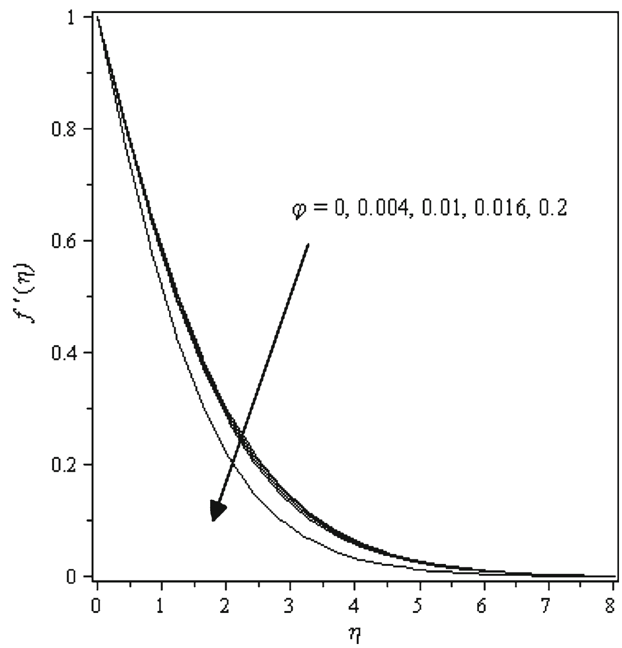

Fig. 7 Velocity profiles for various φ for the Sakiadis problem with Cu-water as working fluid

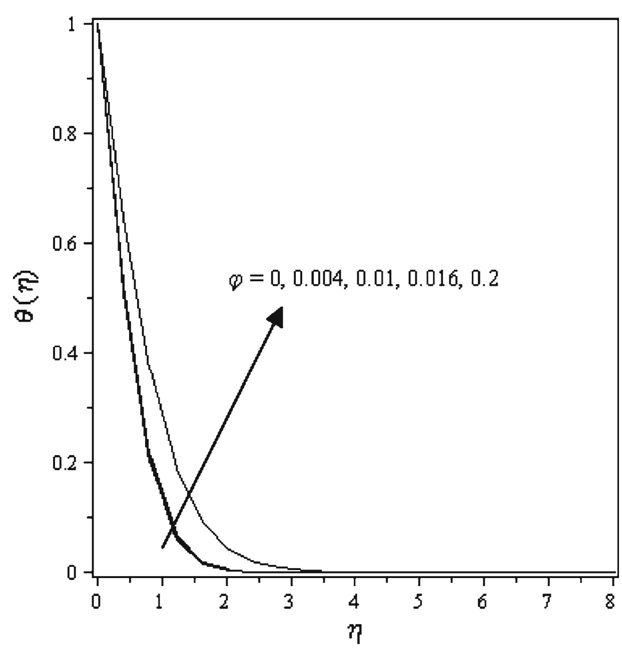

Fig. 8 Temperature profiles for various φ for the Sakiadis problem with Cu-water as working fluid

for the Sakiadis problem. However, Figs. 6 and 8 show that the thermal boundary layer thickness increases with an increase in the parameter φ because of the increase in the local Nusselt number, as can be seen from Figs. 2 and 4. This result is also supported by Oztop and Abu-Nada [20]. Therefore, nanofluids are capable to substantially change the flow and heat transfer characteristics in these two problems considered.

4 Conclusions

The problems of forced convection boundary layer flow and heat transfer past a static semi-infinite flat plate (Blasius problem [26]) and past a moving semi-infinite flat plate (Sakiadis problem [27]) in the presence of nanofluids are considered using the model proposed by Tiwari and Das [19]. Results for various parametric conditions are presented and discussed. Some important points can be found in the obtained results, such as:

- the inclusion of nanoparticles into the base water fluid has produced an increase in the skin friction and heat transfer (Nusselt number) coefficients, which increases appreciably with an increase in nanoparticle volume fraction. The addition of nanoparticles showed an improvement in the heat transfer rate;
- nanofluids are capable to change the velocity and temperature profiles in the boundary layer;
- the type of nanofluid is a key factor for heat transfer enhancement;
- the highest values for both skin friction coefficient and Nusselt number are obtained when using Cu nanoparticles in the base fluid of water with the Prandtl number $Pr = 6.2$, as can be seen from Figs. 2 to 4.

As has been mentioned by Muthtamilselvan et al. [1], the study of the topic of nanofluids is still at an early stage so that complementary works are needed to understand the flow and heat transfer characteristics of nanofluids and identify new and unique applications for these fluids.

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