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Efficient modeling of smart piezoelectric composite laminates: a review

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Abstract Current research issues in the development of efficient analysis models and their efficient numerical implementation for smart piezoelectric laminated structures are discussed in this paper. The improved zigzag theories with a layerwise quadratic variation of electric potential have emerged as the best compromise between accuracy and cost for hybrid composite, sandwich and FGM beams and plates. The concept of associating surface potentials to electric nodes and internal potentials to physical nodes is very effective in modeling the equipotential electroded surfaces. Unified formulations for shear and extension mode actuation, and modeling of piezoelectric composite actuators and sensors are discussed. Future challenge lies in developing efficient theories capable of predicting the interlaminar transverse shear stresses in hybrid laminates directly from the constitutive equations.

1 Introduction

The design of structures involved in the advanced technology including the next generation of aerospace vehicles, automobiles, robotics, medical instruments and sports products is undergoing a sea change in order to make them ultra-reliable with a near-zero incidence of failure and to fulfill stringent performance requirements under a challenging environment. This has led to the concept of multifunctional structures wherein multiple properties of materials are exploited in such a way that besides its major designated functionality, the same structural component can accomplish at least one more task, such as the active vibration control, health monitoring etc. of the host structure. Composite and sandwich laminates integrated with embedded or surface-mounted piezoelectric sensors and actuators (hereafter called hybrid laminates) to introduce self-sensing and actuation capabilities, form a very important part of this new generation of smart structures.

The piezoelectricity, which is an electromechanical phenomenon that couples mechanical and electric fields, was first discovered in quartz by Pierre and Jacques Curie in 1880 [18]. The research on the use of piezoelectric materials as distributed sensors and actuators for smart structural system was initiated [34] about a hundred years later with the advent of polymeric piezoelectric materials [4]. Beginning with the pioneering book of Tiersten [149] on piezoelectric plate vibration, several books [43,109,115,127,155,156] have appeared on the mechanics of piezoelectric and smart plates and shells. For large scale structural control applications such as in aerospace structures, monolithic piezoelectric actuators and sensors suffer from certain shortcomings with regard to tailorable anisotropic actuation, i.e. directional actuation, robustness against damage during use and handling, ability to cover the entire structure for distributed actuation and sensing, and conformability to curved structural members such as shells and tubes. Recent developments of the piezoelectric fiber reinforced

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composite (PFRC) with high strength (maximum strain of 2,300 microstrain), toughness, operating range (−1,500 to 2,800 V), life (200 million cycles), conformability to curved shell surfaces and broadband have addressed these concerns and widened the scope of application [159] of piezoelectric laminates in modern structures and systems.

Due to the presence of high layerwise inhomogeneity in mechanical and electric properties in the hybrid laminates, modeling of structures made of such laminates requires special attention. Finite element (FE) method provides a very powerful tool for the analysis of such smart structures of general geometries. The most accurate global laminate level as well as local layerwise response of hybrid laminated structures can be obtained by the exact analytical solution based on the three-dimensional (3D) piezoelasticity. In these solutions, no adhoc assumptions are made on the variations of the field variables (displacements u_i, electric potential ϕ, stress components σ_{ij}, electric displacements D_i) across the thickness. The equations of motion, the charge balance equation, the constitutive equations of 3D piezoelasticity, and the strain-displacement and electric field-electric potential relations are solved analytically satisfying the boundary conditions and the continuity conditions at layer interfaces exactly at all points. However, such 3D analytical solutions can be obtained only for few specific geometries and boundary conditions. 3D finite elements of piezoelectric solids that are now available in commercial FE packages can provide a correct distribution of the field variables. But use of the solid elements not only results in a very large problem size which becomes intractable for dynamics and control applications, but also encounters challenges for thin piezoelectric layers. Also, the requirement of re-meshing to modify the laminate configuration (lay-up) significantly restricts its use for optimization design problems. Therefore, efficient and accurate 2D laminate theories, and their generalized finite elements able to easily manipulate any modification of laminate lay-up are essential for dynamics, control and optimization of smart structures. Predictions of the response of hybrid laminated structures integrated with PFRC sensors and actuators would require accurate determination of effective properties of PFRC, where the poling and the electric field directions are perpendicular to the fibers oriented in the plane of the structures. An appropriate micromechanical model is needed for this purpose.

Delamination starting from a stress-free edge are known to be the main mode of failure in laminated composite structures [91]. In presence of material and geometric discontinuities at the free edges, a truly three-dimensional (3D) stress field arises at the layer interfaces in the vicinity of the free edges of laminated plates/shells under mechanical and/or hygrothermal loading. In view of weak transverse normal and shear strengths in composites, this stress concentration may eventually lead to premature interlaminar failure (delamination). Due to the coupling effects of electric and pyroelectric fields in hybrid piezoelectric laminates, the free-edge effect, also known as the boundary layer effect, becomes a more complex phenomenon. Accurate estimation of the free-edge stress field of both elastic and hybrid piezoelectric composite and sandwich plates is thus an essential component in composite and smart structure engineering, for achieving reliable design.

This paper presents the state-of-the-art in the development of efficient analysis models and their efficient numerical implementation for hybrid piezoelectric laminated plate-type structures with special regard to dynamics and control, free-edge stresses and micomechanical behavior. The future challenges in these areas are also identified.

2 Three-dimensional piezoelasticity solutions

These are solutions where no simplified hypothesis is made on the variations of the field variables, namely, the displacements, stresses, electric potential and electric displacements, along the thickness coordinate. The equations of motion, constitutive equations of linear 3D electrothermoelasticity, the charge balance equation, strain-displacement relation and the electric field-potential relations at all field points are solved exactly satisfying the equilibrium and continuity/jump conditions at the layer interfaces and the boundary conditions. A review of 3D analytical approaches of multilayered and functionally graded piezoelectric plates and shells has been presented recently by Wu et al. [173].

Beginning with the pioneering work of Pagano [108] providing an exact 3D elasticity solution of simply-supported cross-ply composite plates, such solutions for hybrid plates and shells have attracted researcher's attention for a long time. It is mainly because they serve as useful benchmarks for assessing various 2D theories of piezoelectric laminated plates and shells. These 3D solutions also provide insight into the complex coupled electromechanical behavior of hybrid laminated plates and shells and serve as a reference for making approximate assumptions for the distributions of response entities across the thickness, leading to the development of advanced plate and shell theories. For simply supported rectangular plates, a double Fourier series expansion of the solutions in the in-plane coordinates transform the governing partial differential equations into a system of ordinary differential equations (ODEs) in the thickness coordinate z with constant coefficients (independent of z). Its general solution is constructed using the classical Pagano type approach or using the state space approach. 3D exact piezoelasticity solutions for simply supported rectangular hybrid piezoelectric plates have been presented for static electro-mechanical loading by Ray et al. [122], Heyliger [48,49], Bisegna and Maceri [16], Lee and Jiang [90], Cheng et al. [27] and Vel and Batra [164,165], for thermoelectromechanical loading by Xu et al. [175] and Kapuria et al. [74], for free vibration response by Heyliger and Saravanos [50], and for harmonic steady state response by Kapuria and Achary [63] using Pagano type direct solution and state space approaches.

Exact 3D solutions for the generalized plane strain problems of piezoelectric and magnetoelastic angle-ply flat panels in cylindrical bending have been presented for static electrothermomechanical response [35] and steady state forced harmonic response [88]. For simply supported finite-length cross-ply cylindrical shells and angle-ply infinite cylindrical panels, the governing equations of 3D piezoelasticity reduce to first order ODEs of variable coefficients, which prevent a straight forward close-form solution unlike in case of constant coefficients. An approximate 3D elasticity solution for free vibration response of simply supported doubly curved shallow orthotropic laminated shells was presented by Bhimaraddi [15] by employing a successive approximation or layerwise method. In this method, each layer of the laminate is divided into a number of sublayers and variable coefficients of the governing differential equations are approximated to be constants in each sublayer with their values corresponding to the middle surface of the sublayer. Soldatos and his coworkers [138,181] used a similar approach to obtain free vibration response of simply-supported isotropic and cross-ply composite circular cylindrical shells of revolution and panels.

Exact 3D piezoelasticity solutions for finite simply supported cross-ply cylindrical shells and panels with radially polarized piezoelectric layers have been presented for static response by Xu and Noor [174], and Kapuria et al. [72,73], using the modified Frobenius method [17] for the solution. In this method, the solution is constructed as a product of an exponential function and a power series in the thickness coordinate, whereas in the conventional Frobenius method [82], the solution is assumed as a product of radial coordinate raised to a power (r^{λ}) and a power series in that coordinate. It can be readily seen that in the modified method, a one term solution in the power series ensures the exact solution for the case of constant coefficients, which is however, not the case for the conventional Frobenius or power series method. Consequently, this method yields much faster convergence compared to the other two methods. Also, the modified method has not led to multiple roots for any of the numerical studies conducted earlier for shells under static loadings. Employing the same method, Dumir et al. [36] obtained static electrothermomechanical response of simply-supported angle-ply hybrid cylindrical panels in cylindrical bending. Very recently, Kapuria et al. [78] using the same method presented the free vibration and steady state response under harmonic electromechanical loading of simply-supported angle-ply hybrid cylindrical panels in cylindrical bending. This method involves no approximations of the kind used in [15,138,181].

3 Three-dimensional solutions for plates with arbitrary boundary conditions

As stated earlier, exact 3D elasticity and piezoelasticity solutions of elastic and hybrid composite plates and shells can be obtained for simply supported and electrically grounded boundary conditions. However, the simply supported edges do not exhibit the well-known singular effects observed near clamped and traction-free edges. 3D analytical solutions for laminates with these edge conditions and/or when the edges are electrically in contact with a low-permittivity medium like air wherein the normal component of electric displacement vanishes, are necessary for assessing the accuracy of 2D theories in predicting edge effects. But no analytical 3D solutions exist for elastic as well as hybrid laminated plates with these edge conditions, which satisfy all the boundary and interface continuity conditions exactly at all points, which is important specially for the accurate prediction of stresses and displacements near the edges.

Few approximate analytical solutions have been presented for such cases. Fan and Sheng [39] presented an analytical 3D solution for bending of laminated orthotropic plates with clamped edges by combining the 3D solutions of simply supported plates such that the in-plane displacements are suppressed at locations corresponding to the top and bottom surfaces of each sublayer along the clamped edges, thereby satisfying the clamped boundary conditions in approximate sense at discrete locations [180]. The same procedure was employed for the case of bending of laminated plates with free edges [38]. Ye [179] adopted a similar approach to obtain the 3D analytical solutions for free vibration of clamped cross-ply laminated rectangular plates with clamped boundary, wherein the condition of zero in-plane displacements at the clamped edges was satisfied at discrete points using the Lagrange multiplier method. Vel and Batra [163] used the Eshelby-Stroh formalism for generalized plane-strain case of anisotropic elasticity to obtain a series solution for the cylindrical bending of laminated elastic plates subjected to arbitrary boundary conditions. However, the interface continuity and the boundary conditions are satisfied not exactly but in the sense of Fourier series, resulting in an infinite system of equations in an infinite number of unknowns. They [161] also generalized the Eshelby-Stroh formalism to obtain a similar solution for the rectangular anisotropic laminated plates under Levy type boundary conditions. This method has been extended to multilayered piezoelectric plates under cylindrical bending [162] and general bending [164] with arbitrary edge conditions. Here also, the boundary and interface continuity conditions are satisfied in the approximate sense of Fourier series, yielding an infinite system of equations. In such an approach, the accuracy of the solution depends on the number of terms considered in the series and a higher number of terms leads to increased computational efforts. The last two remain the only 3D approximate analytical solutions for bending of a hybrid piezoelectric plate to date. No 3D analytical solution exists for the free vibration of a hybrid plate subjected to non-simply supported boundary conditions.

In recent years, semi-analytical methods [180] have been proposed to analyse laminated elastic composite structures having general geometry and boundary conditions. In these methods, the variations of the field variables in the in-plane coordinates are approximated using the traditional numerical methods such as the finite element method, finite difference method and differential quadrature method, while the variations in the thickness coordinate is obtained through the analytical solution of a system of ODEs and by employing the transfer matrix (recursive) approach. The ODEs with constant coefficients are usually solved by the state space approach. A 3D semi-analytical solution based on finite element discretisation in in-plane coordinates was presented by Guiping and Limin [45] for the thermal stress analysis of laminated composite plates, who reported very limited results only for validation. Similar semi-analytical finite elements (SSFEM) in conjunction with the state space method (SSM) for static analysis of elastic laminated plates have been presented also by Sheng and Ye [134,135] and Attallah et al. [3] using mixed variational principle and by Kant et al. [58] using the strong Bubnov-Galerkin weighted residual statements.

A 3D semi-analytical method (SSDQM) combining the state space method (SSM) in the thickness direction with the differential quadrature method (DQM) in in-plane direction was developed by Chen et al. [26] to study the free vibration response of cross-ply laminated beams. The SSDQM was subsequently used by the same group of researchers to determine free vibration characteristics of laminated cross-ply plates in cylindrical bending [24], rectangular cross-ply plates [25] under Levy-type boundary conditions and angle-ply plates in cylindrical bending [97]. The same method has been extended to laminated composites plates with general edge constraints [98]. A 3D semi-analytical solution for static and free vibration analysis of hybrid piezoelectric plates has been presented by Qing et al. [116] using SSFEM. In view of the approximations in-plane coordinates, the semi-analytical methods would encounter challenges in predicting accurately the stress and displacements near the non-simply supported edges, because of sharp gradients and possible presence of singularity which would also lead to high computational cost and possibly numerical instabilities [98].

4 Efficient 2D Modeling

A 2D laminate theory for hybrid plate/shell structures should ideally have the following attributes: (1) To be computationally efficient (number of primary variables being independent of the number of layers), (2) To yield accurate response for moderately thick to thin laminates, and be robust (valid for any laminate configurations and loading conditions), (3) To consider two-way electromechanical coupling, (4) To account for the layerwise variation of in-plane displacements, (5) To consider the transverse normal deformability in presence of potential and thermal fields, and (6) To satisfy the continuity of transverse stresses at layer interfaces. As a better alternative to the 3D piezoelasticity approach, coupled layerwise theories (LWTs) and their finite elements, considering layerwise linear or quadratic variation for electric potential, in-plane displacements and also transverse displacement in some cases, have been presented [112,146,130]. These theories yield accurate results, but suffer from an excessive number of displacement/stress variables in proportion to the number of layers.

On the other hand, the most computationally efficient 2D laminate theories are the equivalent single layer (ESL) theories where the displacements are assumed to follow a single global variation across the entire laminate thickness, irrespective of the material properties of the layers. The classical laminate theory (CLT), first order shear deformation theory (FSDT), the refined third order theory (TOT) of Reddy [123] and higher order theories (HOT) belong to this class, which differ from each other by the degree of polynomial functions adopted for approximating the displacements across the thickness coordinate. The earlier studies did not consider the stiffness of the piezoelectric layers and incorporated the piezoelectric effects in terms of effective forces and moments due to induced strain of piezoelectric actuators [22,33,168,177]. Uncoupled CLT considering the stiffness of the piezoelectric layers but neglecting the direct piezoelectric effect, i.e., without considering the electric potential variables as independent field variables, has been applied to hybrid plates by Tzou [154], Lee [89], Hwang and Park [52] and Sekouri et al. [132]. This theory neglects shear deformation. The uncoupled FSDT, which incorporates the shear deformation effect, has been applied without electromechanical coupling for active vibration control of hybrid plates by Chandrashekhara and Agarwal [21]. These uncoupled theories have been extended to the thermal loading by Tauchert [147], Rao and Sunar [120], Tzou and Howard [157], Tzou and Ye [158], and Sunar and Rao [142] for the CLT and by Jonnalagadda et al. [56] and Kapuria and Dumir [66] for the FSDT. A coupled CLT considering the electric potentials as independent state variables has been employed by Fernandes and Pouget [40] for static and free vibration response of a laminated structure containing piezoelectric actuators in which the variation of electric potential is taken quadratic across the piezoelectric layers. Coupled FSDT has been applied to hybrid plates by several researchers [140,66,113]. The coupled FSDT including the direct piezoelectric effect and pyroelectric effect for thermal loading has been applied to hybrid plates by Ishihara and Noda [54] and Krommer and Irschik [85]. In the coupled third order theory (TOT) for hybrid piezoelectric plates [23,42,44,102,133,148,188], transverse shear strain components are zero at the bottom and the top, but the shear traction–free conditions are not satisfied if the bottom or the top layer is a piezoelectric layer having non-zero in-plane electric field components E_x, E_y. Moreover, the TOTs presented in Refs. [44,133,188] consider an assumed linear or higher order [44] temperature profile across thickness and not the actual temperature profile based on heat conduction equation, which most often has a layerwise distribution. Kapuria and Achary [61] have recently developed a coupled consistent third order theory (CTOT) for hybrid piezoelectric plates under thermoelectromechanical load in which the shear traction–free conditions are exactly satisfied and the temperature field is approximated as sublayerwise piecewise linear to model the actual temperature profile as obtained from the solution of the heat conduction equations. The ESL theories wherein a global expansion of displacements across the thickness is used, can not account for the layerwise distortion of the lines normal to the mid-surface as observed from the 3D exact piezoelasticity solutions. These theories violate the slope discontinuity of in-plane displacements and continuity of transverse shear stresses at the layer interfaces, yielding inaccurate global (deflection, in-plane stresses, etc.) as well as local (transverse stresses) response of moderately thick and even thinner laminates.

As the best possible compromise between accuracy and computation efficiency, zigzag theories (ZIGT) have been proposed, wherein the assumptions of displacements are the same as in the LWT with additional quadratic and cubic or trigonometric global variation across thickness for the in-plane displacements. But the number of variables is reduced to that of the corresponding ESL theory by enforcing transverse shear continuity conditions at layer interfaces, and shear traction free conditions at the top and bottom surfaces. Kapuria [59,60] and Kapuria and Achary [62,64] extended the efficient zigzag theory of elastic laminated plates [32,136] to the fully coupled electromechanical response of hybrid piezoelectric beams and plates, wherein the in-plane displacements are approximated as layerwise linear like the LWT with additional third order global variation across thickness, and the transverse displacement is approximated to account for the transverse normal strain contributed by the electric and thermal fields. The number of displacement variables is reduced to only five, like FSDT and TOT, by enforcing transverse shear continuity conditions at layer interfaces, and shear traction–free conditions at top and bottom surfaces exactly. This theory has been found to yield very accurate results for global static and dynamic response of hybrid plates with highly inhomogeneous composite as well as sandwich substrates. Kim et al. [81] and Cho and Ho [31] have also presented coupled zigzag theories for hybrid plates under thermoelectric load. But, the continuity conditions of transverse shear stresses at the layer interfaces and the shear traction-free conditions at the top and bottom surfaces are not satisfied for non-zero in-plane electric field components, which may be induced due to piezoelectric coupling or may be applied through segmented piezoelectric actuator layers.

Approximation for the variation of the electric potential ϕ across the piezoelectric layer is an important issue. Exact 3D piezoelasticity solutions reveal [59] that the electric potential ϕ follows a nearly quadratic variation across piezoelectric layers and the assumption of a linear variation for ϕ can cause a significant error in the computed response [144,160]. While most of the theories consider a linear variation of ϕ across the layer, a sublayerwise linear variation is considered in [59,60,62,130], in which the piezoelectric layer can be divided into an arbitrary number of sublayers. Such a description can yield a fairly accurate prediction of ϕ [59], but it results in a large number of unknown potentials and may still give erroneous results for the

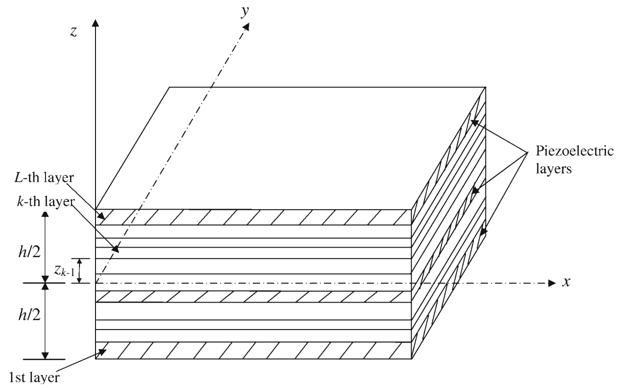

Fig. 1 Geometry of a hybrid plate segment

transverse electric displacement and in-plane normal stresses in the piezoelectric layers. Higher order approximations of the electric potential have been adopted by Rogacheva [127] for single-layer piezoelectrics, and by Krommer [84] and Vasques and Rodrigues [160] for layered smart beams, wherein approximations are made on the axial component of the electric field or the electric displacement. Polit and Bruant [113] considered a layerwise quadratic variation of ϕ in a coupled FSDT formulation for hybrid plates. Kapuria and Kulkarni [70] presented an improved zigzag theory (IZIGT) by including a quadratic variation for electric potential across the piezoelectric layers.

A brief description of displacement and electric potential field approximation in this theory is presented in the following paragraphs. Consider a hybrid angle-ply composite plate (Fig. 1) with some piezoelectric sensor/actuator layers of class mm2 symmetry type, with poling along the thickness direction z. The electric potential ϕ is assumed to follow a piecewise quadratic variation between n_ϕ points at $z = z_\phi^j$ across the thickness:

$$\phi(x, y, z, t) = \Psi_\phi^j(z)\phi^j(x, y, t) + \Psi_c^l(z)\phi_c^l(x, y, t), \tag{1}$$

where $j = 1, 2, \ldots, n_\phi$ and $l = 1, 2, \ldots, n_\phi - 1$. The summation convention is used for repeated indices j and l. $\phi^j(x, y, t)$ is the electric potential at $z = z_\phi^j$ and $\phi_c^l(x, y, t)$ denotes the quadratic component of the electric potential at $z = (z_\phi^l + z_\phi^{l+1})/2$. $\Psi_\phi^j(z)$ and $\Psi_c^l(z)$ are the piecewise linear and quadratic functions, respectively:

$$\Psi_\phi^j(z) = \left\{ \begin{array}{ll} 0 & \text{if } z \le z_\phi^{j-1} \quad \text{or} \quad \text{if } z \ge z_\phi^{j+1} \\ (z - z_\phi^{j-1})/(z_\phi^j - z_\phi^{j-1}) & \text{if } z_\phi^{j-1} < z < z_\phi^j \\ (z_\phi^{j+1} - z)/(z_\phi^{j+1} - z_\phi^j) & \text{if } z_\phi^j < z < z_\phi^{j+1} \end{array} \right\}, \tag{2}$$

$$\Psi_c^l(z) = \left\{ \begin{array}{ll} 4(z_\phi^{l+1} - z)(z - z_\phi^l)/(z_\phi^{l+1} - z_\phi^l)^2 & \text{if } z_\phi^l \le z \le z_\phi^{l+1} \\ 0 & \text{otherwise} \end{array} \right\}.$$

The temperature field $T(x, y, z, t)$ for the hybrid plate can be solved either analytically for some geometries or by the finite element method. For the 2D models, T is approximated as piecewise linear between n_T points at z_T^p, $p = 1, 2, \ldots, n_T$ across the thickness h with $z_T^1 = z_0$, $z_T^{n_T} = z_L$, in terms of its values at these points:

$$T(x, y, z, t) = \Psi_T^p(z)T^p(x, y, t), \tag{3}$$

where $T^p(x, y, t) = T(x, y, z_T^p, t)$. $\Psi_T^p(z)$ are linear interpolation functions for T, and the summation convention is used with index p taking values $1, 2, \ldots, n_T$. n_T can differ from L with $n_T \ge L$ and is determined by the accuracy required of T. The choice of the number of points z_T^p for each layer would depend on the variation of the temperature gradient across the thickness of the layer.

The transverse displacement w is approximated by accounting for the transverse normal strain due to electric field and thermoelectric field. Thus, integrating $\varepsilon_z = w_{,z} \simeq -\bar{d}_{33}\phi_{,z} + \bar{\alpha}_3 T$ yields

$$w(x,y,z,t) = w_0(x,y,t) - \{\bar{\Psi}^j_\phi(z)\phi^j(x,y,t) + \bar{\Psi}^l_c(z)\phi^l_c(x,y,t)\} + \bar{\Psi}^p_T(z)T^p(x,y,t), \quad (4)$$

where $\bar{\Psi}^j_\phi(z) = \int_0^z \bar{d}_{33}\Psi^j_{\phi,z}(z)\,dz$, $\bar{\Psi}^l_c = \int_0^z \bar{d}_{33}\Psi^l_{c,z}(z)\,dz$, $\bar{\Psi}^p_T = \int_0^z \bar{\alpha}_3\Psi^p_T(z)\,dz$.

The in-plane displacements u_x and u_y are assumed to follow a global third order variation combined with a layerwise linear variation, which are finally expressed in terms of five displacement variables by imposing the continuity of transverse shear stresses at layer interfaces and zero shear traction at top and bottom surfaces:

$$u(x,y,z,t) = u_0(x,y,t) - zw_{0_d}(x,y,t) + R^k(z)\psi_0(x,y,t) + \bar{R}^{kp}(z)T^p_d, \quad (5)$$

where

$$u = \begin{bmatrix} u_x \\ u_y \end{bmatrix}, \quad w_{0_d} = \begin{bmatrix} w_{0,x} \\ w_{0,y} \end{bmatrix}, \quad u_0 = \begin{bmatrix} u_{0x} \\ u_{0y} \end{bmatrix}, \quad \psi_0 = \begin{bmatrix} \psi_{0x} \\ \psi_{0y} \end{bmatrix}, \quad T^p_d = \begin{bmatrix} T^p_{,x} \\ T^p_{,y} \end{bmatrix}. \quad (6)$$

u_0 denotes the translation components at the reference surface ($z = 0$) and ψ_0 is related to its shear rotations. $R^k(z)$ is a 2×2 matrix of layerwise functions of z of the form

$$\begin{aligned} R^k(z) &= \hat{R}^k_1 + z\hat{R}^k_2 + z^2\hat{R}_3 + z^3\hat{R}_4, \\ \bar{R}^{kp}(z) &= \hat{R}^{kp}_1 + z\hat{R}^{kp}_2 + z^2\hat{R}^p_3 + z^3\hat{R}^p_4, \end{aligned} \quad (7)$$

where $\hat{R}^k_1$, $\hat{R}^k_2$, $\hat{R}_3$, $\hat{R}_4$, $\hat{R}^{kp}_1$, $\hat{R}^{kp}_2$, $\hat{R}^p_3$ and $\hat{R}^p_4$ are 2×2 coefficient matrices which depend on the material properties and the lay-ups. The IZIGT has been tested in comparison with exact 3D solutions for static electrothermomechanical [60,64], dynamic [62] and buckling response [65] of simply-supported hybrid plates and has been found to be very accurate for composite, sandwich and highly in-homogeneous test laminates. The results are far superior to those predicted by the ESL theories with the same number of primary variables.

The ZIGTs including the one in [64], however, are unable to accurately predict the transverse shear stresses directly from the constitutive equations. This can be seen from Fig. 2, which reveals that the prediction of τ_{zx} by the ZIGT from the constitutive equation is poor for pressure load and poorer for potential loading. These can be accurately obtained only by integrating the 3D equations of momentum. The integration requires computation of higher order derivatives of displacements, which poses difficulties in the finite element implementation and is still a great concern. This issue was apparently first addressed for elastic composite plates by Li and Liu [95], who proposed a global-local theory (GLT) wherein local layerwise terms up to third order are combined with the global third order variations of in-plane displacements using double-superposition hypothesis. The continuity of the local displacement terms at the layer interfaces is satisfied in two groups, and conditions on

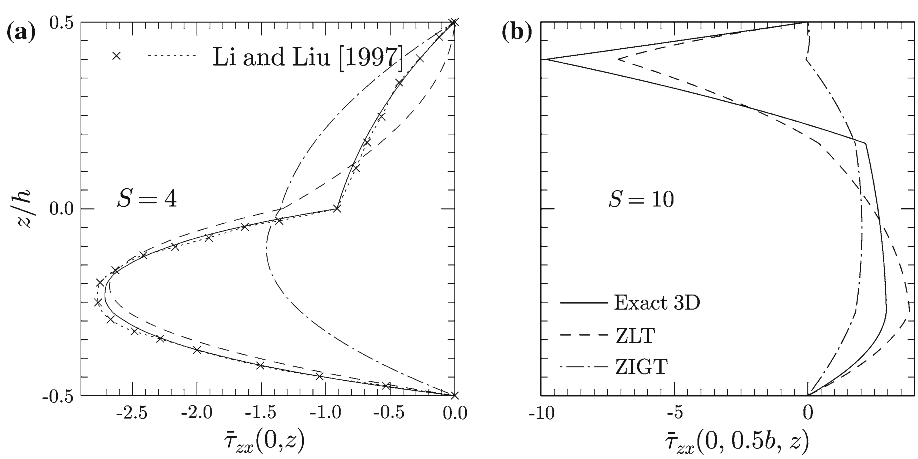

Fig. 2 Through-the-thickness variation of transverse stress $\bar{\tau}_{zx}$ for **a** elastic two-layer [0°/90°] composite panel under pressure load and **b** square four layer [0°/90°/90°/0°] plate under potential loading [71]

the transverse shear stresses are satisfied for all terms together to reduce the number of unknown displacement variables to eleven. The theory was shown to predict the transverse stresses accurately from constitutive equations without using any postprocessing method. The theory was later generalized for m-th order global variation by Zhen and Wanji [187]. Li and Liu [95] and Zhen and Wanji [187], however, have not presented a variational development of the theory that would result in governing differential equations of motion and the variationally consistent boundary conditions.

A static analysis of cross-ply laminated piezoelectric plates has also been performed by Zhen and Wanji [186] using a triangular finite element which uses a combination of the GLT of Li and Liu [95] for mechanical variables and a piecewise linear electric field. The formulation, however, violates the conditions on transverse shear stresses in presence of non-zero in-plane electric field, and does not consider layerwise variations of deflection due to electric transverse normal strain. No results have been presented in [186] for hybrid plates with finite thickness of piezoelectric layers under electric potential loading. Very recently, Kapuria and Nath [71] have presented a zigzag-local theory (ZLT) for hybrid plates by adding local second and third order terms to the ZIGT approximations [60,64] of in-plane displacements, with the aim of predicting transverse shear stresses directly from the constitutive equations. By satisfying the continuity of in-plane displacements at layer interfaces for each of the local terms separately and enforcing the conditions on transverse shear stresses, the number of primary displacement variables is reduced to nine. The inclusion of local higher-order terms in the expression of in-plane displacements enables better prediction of not only transverse stresses, but also in-plane displacements and other stresses. Of particular significance is the much improved prediction of transverse shear stresses at the interface between the host substrate and the actuated layer directly from constitutive equations, for which the existing ZIGT yields very poor results. The prediction of deflection for the electric potential load case by the present theory is also very accurate and superior to the ZIGT.

5 Efficient FE modeling of hybrid plates

Detailed reviews of FE modeling of piezoelectric hybrid laminates have been reported [7,99,169]. The third order zigzag theories, which satisfy the continuity conditions of transverse shear stresses at the layer interfaces and the shear traction-free conditions at the top and bottom surfaces, require C^1-continuity of the transverse displacement w along the inter-element boundary, for the finite element formulation. This condition can be conveniently implemented in rectangular elements, but it poses difficulties for general conforming quadrilateral elements. This causes serious hinderance in implementing these otherwise good theories in general purpose finite element programming environments. The problem of satisfying the C^1 continuity requirement for w was circumvented successfully for isotropic thin plates by Jeychandrabose et al. [55], who developed a four-node improved discrete Kirchhoff quadrilateral (IDKQ) element for static bending analysis of thin isotropic plates by imposing the Kirchhoff hypothesis of the classical plate theory at certain discrete points in the interior and on the boundary of the element. The IDKQ technique was successfully applied by Kapuria and Kulkarni [68,86] to TOT and ZIGT for elastic anisotropic laminated plates. Utilizing the same interpolation technique for w, a quadrilateral plate element based on the IZIGT was developed in [69,70] for accurate dynamic analysis of hybrid smart plates of arbitrary laminate and geometric configurations.

The piezoelectric actuators and sensors are always electroded with metallic coating, which renders their surfaces equipotential. The available finite elements consider electric potential degrees of freedom (DOF) as nodal or element variables. In such cases, to model the equipotential condition on the electroded surfaces, it would be necessary to impose the constraints of equality on the electric DOF of the nodes/elements on the same electroded surface or to average them as an approximation. Besides, in case of elemental electric DOF, the in-plane electric field, which is induced due to direct piezoelectric effect, can not be accounted for. Sze and Yao [143] presented a solid element wherein the electric potential is considered linear across the piezoelectric layer and electric DOF corresponding to a piezoelectric patch are attached to a separate electric node instead of the physical nodes. But this approach can not account for the in-plane electric fields, as all the electric DOF are associated with electric nodes.

A new hybrid approach is proposed in [69,70] wherein the potential DOF ϕ^j associated with the electroded surfaces of the piezoelectric patches in a plate section are attached to an electric node, which can be connected to several elements. The electric node does not have any physical coordinates unlike the physical nodes. The potential DOF ϕ^l corresponding to the quadratic component of the potential distribution are associated with the physical nodes to allow for the in-plane electric field. This concept not only eliminates the need of imposing equality constraints on the electric DOF for equipotential condition but also results in a significant reduction in the number of electric DOF, while allowing for the in-plane electric field. The new element has been shown to be superior to other available elements [107,126,151] for smart plates in terms of accuracy and computational efficiency for static, free vibration and transient response of smart composite and sandwich plates. The numerical study showed it is possible to apply a non-uniform potential distribution over a piezoelectric actuator surface by segmenting the surface into small electrodes with applied calculated at the center of the electrode. Such segmentation leads to deflection and stresses which are very close to the continuous distribution case. The number of segments in an electroded sensor surface can have an appreciable effect on the deflection and natural frequencies. The computation time for transient analysis using the present element is about $\frac{1}{290}$th of that required for the 3D FE analysis in ABAQUS. Due to its accuracy and efficiency, the element is suitable for use in dynamic and control applications involving smart plates.

6 Unified formulation for extension and shear mode actuation mechanisms

Most of the existing refined models incorporating advanced kinematics and two-way electromechanical coupling consider the extension actuation mechanism (EAM) of the piezoelectric elements through the d_{31}/d_{32} effect. The piezoelectric materials, when constrained and poled perpendicularly to the applied electric field, undergo transverse shear deformation through the d_{15}, d_{24} constants, which is known as the shear actuation mechanism (SAM). The use of shear mode actuators for adaptive structures was first investigated by Sun and Zhang [141] who showed that shear actuators are subjected to much lower stresses compared to the extension actuators for the same electric field. All subsequent publications reported on the analysis of SAM beams [8,9,117,118,152] use a three-layer sandwich beam model, which is rather restrictive for modeling general laminates with EAM and SAM actuators to be placed at any arbitrary location across the thickness. A unified version of the efficient zigzag theory capable of modeling actuators and sensors with arbitrary poling direction, placed at arbitrary location across the thickness, has been presented by Kapuria and Hagedorn [67] for smart hybrid SAM-EAM beams. The model can accurately predict both the quadratic and cubic variations of electric potential in the extension and shear modes, respectively, as observed from 2D piezoelasticity solutions. It was shown that for a cantilever, an extension actuator yields maximum tip deflection when placed at the clamped end, whereas a shear actuator should be placed at some distance away from the clamped end for maximum effect. This theory needs to be extended for SAM-EAM plates and shells.

7 Use of PFRC actuators/sensors

PFRCs are emerging as better substitution for monolithic piezoceramics as distributed actuators and sensors in smart structures because of their directional actuation, robustness against damage during use and handling, ability to cover the entire structure and conformability to curved structural members such as shells and tubes. To apply the 3D and 2D laminate theories described above for analysis of smart laminated structures integrated with PFRC actuators and sensors, the effective lamina level properties of the PFRC are needed. This calls for an appropriate micromechanics model which can predict the effective properties of a unidirectional PFRC material based on the properties of its constituents. A number of micromechanics models that have been proposed for the determination of effective properties of piezoelectric composites based on Voigt type iso-field [20,137], Mori-Tanaka [37], self-consistent [37,93] and asymptotic homogenization [94,128] methods, consider poling and electric field directions parallel to the fiber axis, causing a d_{33} effect. While such composites with the piezoelectric fibers oriented along the thickness direction (Fig. 3a) are applied in ultrasonic transducers, for structural applications, the stiff piezo-ceramic fibers must be oriented in the plane of the structures (Fig. 3b). Hence, the above models can not be employed for structural control applications.

Very few studies have been reported on the micromechanics model for PFRC with poling and electric field directions perpendicular to the fiber, the first one being that of Bent [10], who also proposed interdigited electrode piezoelectric fiber composites (IDEPFC) for improved actuation performance [11]. The iso-field method was employed to predict the effective electromechanical properties. In this work, even though a general methodology for calculation of effective material properties for the 3D stress field was briefly outlined, detailed closed form solutions and the results for effective material properties were presented by considering uniaxial stress fields [12]. In the iso-field approach, the effective properties for the 3D case are obtained for representative volume elements (RVE) of two possible connectivities for the piezoelectric fiber and matrix phases: models A and B with material connectivity on x_1–x_2 and x_1–x_3 planes respectively, as shown in

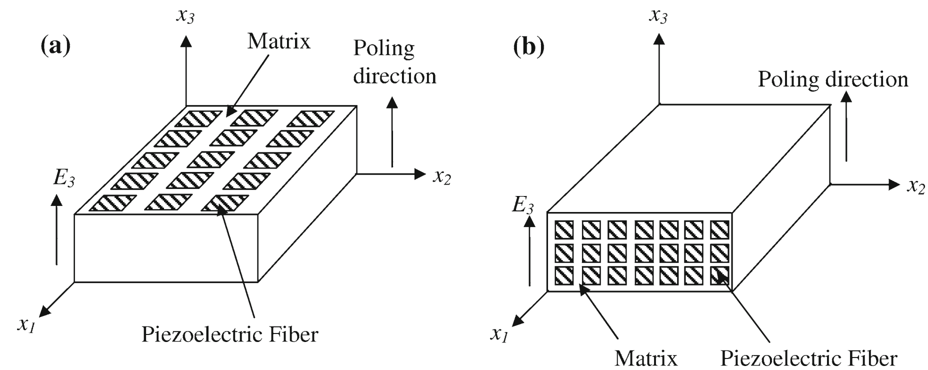

Fig. 3 Schematic representation of PFRC with poling and electric field directions **a** parallel and **b** normal to fiber direction

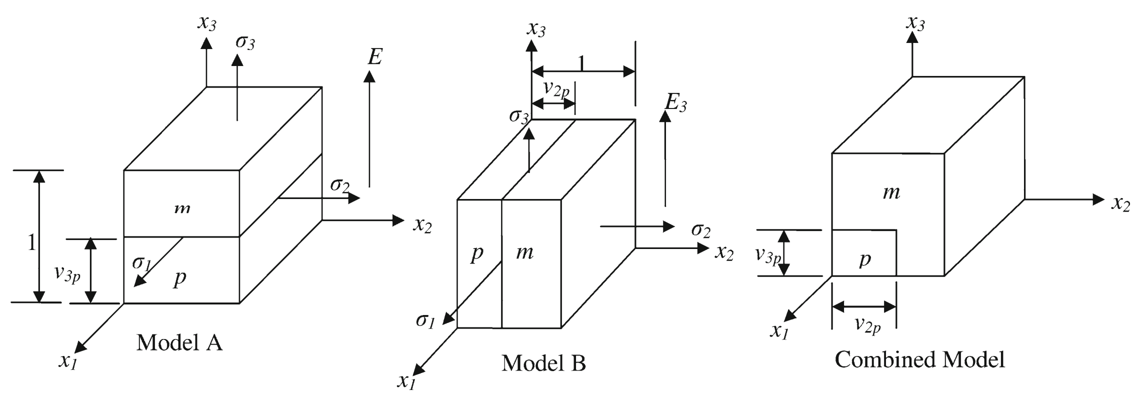

Fig. 4 Representative volume elements for iso-field model (p-piezo-ceramic, m-matrix)

Fig. 4. The strain and electric field components parallel to the connecting plane of two phases are assumed to be uniform and the same in both phases (iso-field condition) and iso-stress and iso-electric displacement conditions are assumed to exist along normal to the connecting plane. For the case of combined model (AB), the material properties of the piezoelectric fiber phase in model B are replaced with the effective properties from model A. Typical plots for variation of the effective piezoelectric strain constants d^e_{31} and d^e_{32} with the ratio of dielectric constants ϵ^p_{33} and ϵ^m_{33} for the piezoelectric fibers and matrix are shown in Fig. 5. It is revealed that d^e_{ij} is maximum for $\epsilon^p_{33}/\epsilon^m_{33} = 1$ and reduces drastically with the increase in this ratio, as happens for the currently available materials. Development of suitable matrix materials with ϵ^m_{33} of the order of ϵ^p_{33} would be extremely useful in enhancing the effectiveness of PFRCs. Mallik and Ray [100] presented a simpler model using the uniform fields concept, but in this formulation, the electric field is assumed to be the same in both piezoelectric fiber and elastic matrix, which is not achievable by applying a potential difference across the lamina due to the large difference in their dielectric constants. The uniform electric field assumption is valid only for $\epsilon^p_{33}/\epsilon^m_{33} = 1$, which is not possible for materials available so far. The formulation, thus, gives unrealistically high values of effective piezoelectric strain constants. The same formulation was extended to the electrothermomechanical case by Kumar and Chakraborty [87]. It appears that the iso-field 3D model has not been extended so far for computing the effective electrothermomechanical properties of PFRCs.

8 Hybrid functionally graded structures

Functionally graded materials (FGMs) with a gradient compositional variation of its constituents from one surface to the other provide an elegant solution to the problem of high transverse shear stresses that are induced when two dissimilar materials with a large difference in material properties are bonded. The gradation of the composition can be achieved as a continuous variation or a discrete step-wise variation. It is possible to combine an FGM substrate with piezoelectric/PFRC actuators and sensors to develop high-performance adaptive structures. Also functionally graded piezoelectric actuators and sensors can be used to reduce the interfacial

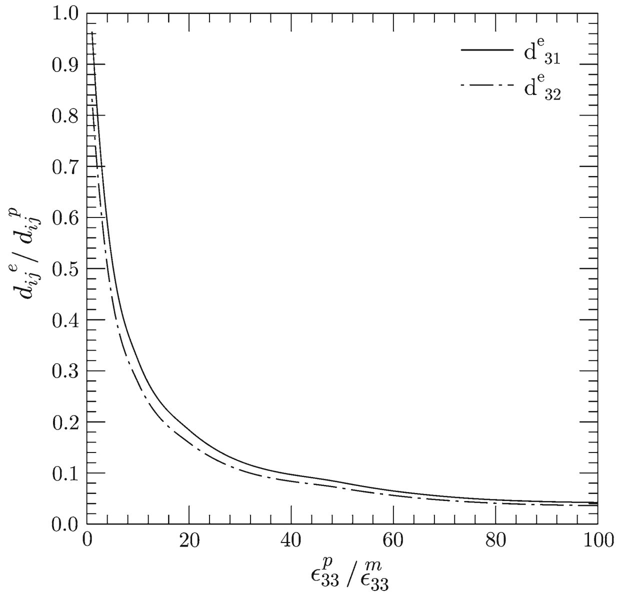

Fig. 5 Effect of dielectric ratio ($\epsilon_{33}^{p}/\epsilon_{33}^{m}$) on effective piezoelectric strain constants d_{31}^{e} and d_{32}^{e} for fiber volume fraction $v_f = 0.6$ (Fiber = PZT-5H; Matrix = Epoxy)

shear stresses. Various ESL theories such as the FSDT and TOT have been applied for dynamic analysis of hybrid FGM plates and shells. The classical plate theory (CPT) [172,178] and FSDT [114,124] have been employed for static bending, free vibration and transient response of FGM rectangular and circular plates using a power law variation for the volume fractions and employing a linear rule of mixtures (ROM) [80] for computing all effective material properties. Many researchers [19,125,176] have developed theories for FGM structures based on assumed variation (power law/exponential law) of material properties across the thickness. In actual practice, the variation of the material composition across the thickness is known and the effective material properties of the two-phase system need to be calculated based on an appropriate model. The accuracy of predicted response will depend on both kinematic modeling and the micromechanics model for estimating effective material properties. Usually, an FGM is regarded as a special kind of composite of two materials, with graded composition, and hence various models proposed for composites are applied to predict the variation of the effective material properties with varying composition. Among the various averaging approaches proposed, the approximate approaches, namely, the self-consistent model of Hill [51], the mean field micromechanics models of Mori and Tanaka [105] and Wakashima and Tsukamoto [166], the simple rule of mixtures [80], the modified rule of mixtures (MROM) [150] and the unit cell model [121] provide simple and convenient ways for predicting the overall response of two phase composite materials. The Kerner's model [80] and the Schapery model [131] are the two simplest models that have been proposed for the estimation of the coefficient of thermal expansion (CTE) of the metal-matrix composite. The Wakashima-Tsukamoto model (W-T-M) [166,171] and the Turner model [153] have been proposed for the CTE of macroscopically isotropic dual-phase composite and aligned fiber composite, respectively. Cho and Ha [28] examined the applicability of some of the above models (ROM, MROM and W-T-M) for estimating averaged elastic and thermal properties for an FGM by comparing the result with the finite element discretization approach. In a recent study, Bhattacharyya et al. [13] established through experimental study the validity of MROM for estimating the effective elastic modulus of Al/SiC and Ni/Al_2O_3 FGM systems. Kapuria et al. [75,76] developed a 1D theory for layered FGM beams using the zigzag theory in conjunction with MROM for estimating effective elastic modulus and W-T-M for the effective CTE. The accuracy of the predicted response was established by comparing with experimental results for thermomechanical static deflection and natural frequencies of Al/SiC and Ni/Al_2O_3 FGM systems, prepared through powder metallurgy and thermal spraying techniques, respectively. The characterization of the two FGM systems [14] revealed that the micro-hardness variation across the interfaces between two graded layers is consistent with the analytically obtained jump in the in-plane stresses at the interface. The theory was extended to piezoelectric FGM beams by the same authors [77].

9 3D Solutions for Free-Edge Stresses

It is now well established from experimental as well as theoretical investigations that the presence of material and geometric discontinuities at the free edges in composite laminates results in a truly 3D and, at least on a theoretical basis, similar stress fields at the layer interfaces in the vicinity of free edges. It is commonly referred to as the free-edge effect and also as boundary-layer effect, which decays rapidly away from the laminates edges. Due to piezoelectric and pyroelectric coupling effects in hybrid piezoelectric laminates, the free-edge effects become a more complex phenomena. In view of a weak transverse normal and shear interfacial strengths in composites, these localized interlaminar stresses may result in premature interlaminar failure, or delamination in the vicinity of free edges. The latter is known to be the main mode of failure of laminated composite structures [91]. Thus, an accurate estimation of interlaminar stresses and deformation at the free edges is of critical importance in assuming the design of elastic as well as hybrid composite structures. This explains the enormity of the scientific effort that has been invested on this topic over the last 40 years since the work of Hayashi [46] who reported the existence of maximum interlaminar transverse shear stress at the free edge of a three-ply cross ply laminate under tension. The interlaminar normal stress, however, is neglected in the formulation of Hayashi [46]. The first accurate description of the 3D elasticity problem of the free-edge effect was presented by Pipes and Pagano [111] for a symmetric laminate strip under uniaxial tension. No exact solution is known to exist for these governing equations of 3D elasticity satisfying all boundary and interface continuity conditions exactly at all points, hence, various approximate methods have been adopted. Detailed reviews of the adopted methodologies so far for the determination of the free-edge field can be found in [57,103,104]. The various methods reported can be broadly classified into (a) close-form solutions based on approximate 2D laminate theories and (b) numerical approaches based on 3D elasticity as well as approximate 2D theories. The approximate 2D theories employed for close-form solutions of the free-edge problem are

(1) displacement based equivalent single-layer (ESL) theories (e.g. by Krishnamurty and Kumar [83], Becker [5,6]),
(2) displacement based layerwise theories (e.g. by Zhu and Lam [189], Nosier and Bahrami [106], Tahani and Nosier [145]),
(3) stress based ESL theories (e.g. by Flanagan [41]), and
(4) stress based layerwise theories (e.g. by Kassapoglou [79], Yin [183], Cho and Kim [29]).

In the 2D theories, the thickness variation of the displacement/stress variables are assumed a priori and also, the boundary conditions at the free edges are generally satisfied only in an integral sense. Being a 3D stress concentration problem with possible singularity, any approximation and assumption on the variation of state variables will have significant bearing on the accuracy. In view of the difficulty in developing analytical solutions of 3D elasticity, numerical methods have been employed to determine the 3D stress field at the free edges of elastic composite under tension, bending and twisting mechanical loading and thermal loading. The various methods developed are (1) finite difference method (FDM), (2) displacement based general purpose finite element method (FEM) and (3) special purpose finite element methods. Pipes and Pagno [111] presented the first numerical solution of 3D elasticity equations for free-edge stress field using the FDM for a laminate in uniaxial tension, which was extended to bending by Salamon [129]. Altus et al. [1] presented a full 3D finite difference scheme for an angle-ply laminate under uniaxial tension, disregarding the assumption of constant stress field along axial coordinate. The FDM was soon followed by a general purpose displacement based finite element solution, utilized by e.g. Herakovich [47], Wang and Crossman [167], Raju and Crews [119], Lin [96], Lessard et al. [92], Yi and Hilton [182].

Special purpose finite elements have been developed to deal with the stress singularity by Wang and Yuan [170], Stolarski and Chiang [139], Icardi and Bertetto [53], and Phillips et al. [110]. Recently, an approximate close form solution of the 3D elasticity equations for free-edge and cracking effect in cross-ply and angle-ply composite laminates under extension and thermal loading has been presented by Zhang et al. [184,185]. In this formulation, a Fourier series approximation is used in the in-plane transverse direction, while an exact analytical solution is obtained for the variations in the thickness coordinate, using a state space approach. As a result, the free-edge conditions are satisfied only approximately in the sense of Fourier series at a discrete number of points along the edge thickness. This approximation along with the very nature of Fourier series approximation near the ends is likely to yield inaccurate prediction of structures near the free edge. An iterative approximate method for the 3D elasticity solution of free-edge stresses of composite laminates under extension, bending, twisting and thermal loadings has been presented by Cho and Kim [30] using Lekhnitskii stress functions, however, the free-edge conditions and the displacement continuity conditions at the interfaces satisfied in an integral sense, as stated earlier.

In spite of the continuous quest for the development improved solutions for free-edge stress field in laminates, no accurate analytical 3D elasticity solution satisfying all boundary and interface conditions exactly at all points exists so far, against which the accuracy of the other 3D or 2D approximate methods can be assessed. In the context of smart piezoelectric hybrid plates, the only study that has been reported so far on the free-edge stresses is the displacement based 3D finite element analysis of hybrid symmetric composite laminates under uniaxial tension as presented by Mannini and Gaudenzi [101] and Artel and Becker [2].

10 Future challenges

Based on the previous discussion, the following issues emerge as the future challenges before the solid mechanics community engaged in smart composite structure technology development:

1. Development of efficient 2D laminate theories for hybrid laminates which can accurately predict the interlaminar transverse shear and normal stresses is the need of the hour, since these stresses are the predominant causes of failure (delamination) in such laminates subjected to electro-thermo-mechanical loading. The theories presented in [71,186] are welcome developments, but a lot more need to be done.
2. Efficient and robust finite elements for hybrid plates and shells integrated with electroded and segmented piezoelectric monolithic and PFRC sensors and actuators are needed to be developed for dynamics and control applications, based on the new theories.
3. Benchmark 3D solutions for computation of free-edge stresses in hybrid laminated structures are also long overdue problems waiting to be addressed. Such solutions would lead to better understanding of the boundary effect phenomena in hybrid laminates, resulting in more reliable design and also will be useful in assessing the accuracy of approximate numerical solutions. Three-dimensional solutions for hybrid plates and shells with non-simply supported boundary conditions are also equally important for assessing the accuracy of the 2D laminate theories more robustly.
4. Development of matrix (polymer) materials with dielectric constants of the order of piezoelectric materials or other means of ensuring uniform electric field conditions in PFRC will go a long way in enhancing the applicability of PFRC sensors and actuators in control applications.
5. The other challenges include incorporating non-linear piezoelectric coupling, developing efficient models for hybrid SAM-EAM shells etc.

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