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Generalized thermoelastic waves in spherical curved plates without energy dissipation

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Abstract In this article, the propagation of thermoelastic waves in orthotropic spherical curved plates subjected to stress-free, isothermal boundary conditions is investigated in the context of the Green–Naghdi (GN) generalized thermoelastic theory (without energy dissipation). The theoretical formulation is based on the linear GN thermoelastic theory. The coupled wave equation and heat conduction equation expressed by the displacement and temperature are obtained. By the Legendre orthogonal polynomial series expansion approach, the coupled controlling equations are solved. The convergence of the method is demonstrated through a numerical example. The dispersion curves of thermal modes and elastic modes are illustrated simultaneously. Dispersion curves of the corresponding purely elastic spherical plate are also shown to analyze the influence of thermoelasticity on elastic modes. The displacement, temperature and stress distributions of both elastic modes and thermal modes are calculated to show their differences. A thermoelastic spherical plate with a different ratio of radius to thickness is considered to show the influence of the ratio on the characteristics of thermoelastic waves.

1 Introduction

Because of its reliability and efficiency, ultrasonic nondestructive testing technique has been widely used for evaluating the quality and lifetime prediction of structures. Among the many ways of generating and detecting ultrasonic waves, laser ultrasonic technique has attracted great attention owing to its special noncontact evaluation of materials. When investigating the wave propagation generated by a laser, we have to consider the thermoelasticity of materials. Furthermore, in aerial and nuclear fields, the thermal effect on wave propagation cannot be ignored because of the extremely high velocity (giving rise to aerodynamic heating) and high temperature.

In recent decades, various generalized theories of thermoelasticity were developed to replace the classical theory of heat conduction in solids because the classical theory has an assumption of infinite speed, which is contrary to physical observation. Among these generalized thermoelastic theories, the LS (Lord–Shulman), GL (Green–Lindsay) and GN (Green–Naghdi) theories are the most important ones [1]. Based on the three generalized thermoelastic theories, guided thermoelastic waves were investigated by some researchers. Nayfeh and Nemat-Nasser [2], Sinha and Sinha [3], Sherief and Helmy [4] and Abd-alla and Al-dawy [5] studied isotropic thermoelastic Rayleigh waves. Massalas [6], Daimaruya and Naitoh [7], Massalas and Kalpakidis [8], Sharma et al. [9] and Verma and Hasebe [10] investigated the propagation of guided thermoelastic waves in isotropic plates. Verma and Hasebe [11,12], Al-Qahtani and Datta [1], Kumar and Kansal [13,14] and Al-Qahtani [15] considered the guided thermoelastic waves in anisotropic plates. Hawwa and Nayfeh [16,17]

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and Verma [18] investigated the propagation of thermoelastic waves in layered plates. Ponnusamy [19] studied the thermoelastic waves in solid cylinders. Sharma and Pathania [20] investigated the wave propagation in circumferential direction of transversely isotropic cylindrical curved plates.

The structures of the above research are half space, plane plate and hollow cylinder. As a common structure, the propagation of thermoelastic waves in hollow spheres has not been reported, but the elastic waves received much attention. Brekhovskikh [21] studied the surface wave propagation in solids with curved boundary, in which cylindrical and spherical boundaries were considered as special cases. Using shell-theory, Shah et al. [22] analyzed three-dimensional hollow spheres. Gaunaurd and Werby [23,24] derived dispersion curves for fluid loaded spherical shells. Kargl and Marston [25] also worked on the Lamb-like wave in isotropic spherical shells. Wang et al. [26] studied stress wave propagation in orthotropic laminated spherical shells subjected to arbitrary radial dynamic load by means of finite Hankel transforms and Laplace transforms. Towfighi and Kundu [27] and Yu et al. [28] studied wave propagation in anisotropic spherical curved plates. Yu et al. [29] also studied coupled electro-elastic spherical curved plates.

In this paper, the propagation of thermoelastic waves in spherical curved plates is studied on the basis of the GN generalized thermoelastic theory (without energy dissipation). The coupled controlling equations are solved by using the Legendre polynomial approach, which was developed by Lefebvre et al. [30] to solve the free-ultrasonic waves in multilayered plates and then in different waveguides [31–35]. The dispersion curves, displacement distributions, temperature distributions and stress distributions are illustrated. The influences of the ratio of radius to thickness on the wave characteristics are discussed. In this paper, stress-free, isothermal boundary conditions are assumed.

2 Theoretical formulation

Consider an orthotropic, thermoelastic hollow sphere with a thickness h. In the spherical coordinate system (θ, ϕ, r), a, b are the inner and outer radii, respectively.

The orthotropic thermoelastic constitutive equation in the spherical coordinate system can be written in the following form:

$$\begin{Bmatrix} T_{\theta\theta} \\ T_{\phi\phi} \\ T_{rr} \\ T_{r\phi} \\ T_{r\theta} \\ T_{\theta\phi} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & C_{13} & 0 & 0 & 0 \\ & C_{22} & C_{23} & 0 & 0 & 0 \\ & & C_{33} & 0 & 0 & 0 \\ & & & C_{44} & 0 & 0 \\ & \text{symmetry} & & & C_{55} & 0 \\ & & & & & C_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_{\theta\theta} \\ \varepsilon_{\phi\phi} \\ \varepsilon_{rr} \\ 2\varepsilon_{r\phi} \\ 2\varepsilon_{r\theta} \\ 2\varepsilon_{\theta\phi} \end{Bmatrix} - \begin{bmatrix} \beta_1 \\ \beta_2 \\ \beta_3 \\ 0 \\ 0 \\ 0 \end{bmatrix} \{T\}, \tag{1}$$

where T_{ij}, ε_{ij} are the stress and strain; C_{ij}, β_i are the elastic and volume expanding coefficients; T is the temperature change above the uniform reference temperature "T_0".

The relationship between the strain and the displacement can be expressed as

$$\varepsilon_{\theta\theta} = \frac{1}{r}\frac{\partial u_\theta}{\partial \theta} + \frac{u_r}{r}, \quad \varepsilon_{\phi\phi} = \frac{1}{r\sin\theta}\frac{\partial u_\phi}{\partial \phi} + \frac{u_r}{r} + \frac{\cot\theta}{r}u_\theta, \quad \varepsilon_{rr} = \frac{\partial u_r}{\partial r}, \quad \varepsilon_{r\theta} = \frac{1}{2}\left(\frac{1}{r}\frac{\partial u_r}{\partial \theta} + \frac{\partial u_\theta}{\partial r} - \frac{u_\theta}{r}\right),$$
$$\varepsilon_{r\phi} = \frac{1}{2r}\left(\frac{1}{\sin\theta}\frac{\partial u_r}{\partial \phi} - u_\phi\right) + \frac{1}{2}\frac{\partial u_\phi}{\partial r}, \quad \varepsilon_{\theta\phi} = \frac{1}{2r}\left(\frac{1}{\sin\theta}\frac{\partial u_\theta}{\partial \phi} + \frac{\partial u_\phi}{\partial \theta} - u_\phi \cot\theta\right), \tag{2}$$

where u_i are the elastic displacements.

For the wave propagation considered in this paper, the body forces and heat sources are assumed to be zero. Thus, the dynamic equation for the thermoelastic plate based on the GN generalized thermoelastic theory is governed by

$$\begin{aligned} &\frac{\partial T_{rr}}{\partial r} + \frac{1}{r}\frac{\partial T_{r\theta}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial T_{r\phi}}{\partial \phi} + \frac{2T_{rr} + T_{r\theta}\cot\theta - T_{\theta\theta} - T_{\phi\phi}}{r} = \rho\frac{\partial^2 u_r}{\partial t^2}, \\ &\frac{\partial T_{r\theta}}{\partial r} + \frac{1}{r}\frac{\partial T_{\theta\theta}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial T_{\theta\phi}}{\partial \phi} + \frac{3T_{r\theta} + \cot\theta\left(T_{\theta\theta} - T_{\phi\phi}\right)}{r} = \rho\frac{\partial^2 u_\theta}{\partial t^2}, \\ &\frac{\partial T_{r\phi}}{\partial r} + \frac{1}{r}\frac{\partial T_{\theta\phi}}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial T_{\phi\phi}}{\partial \phi} + \frac{3T_{r\phi} + 2T_{\theta\phi}\cot\theta}{r} = \rho\frac{\partial^2 u_\phi}{\partial t^2}, \end{aligned} \tag{3a}$$

$$K_3\frac{\partial^2 T}{\partial r^2}+K_1\frac{1}{r}\left(\frac{\partial T}{\partial r}+\frac{1}{r}\frac{\partial^2 T}{\partial\theta^2}\right)+K_2\frac{1}{r}\left(\frac{\partial T}{\partial r}+\frac{1}{r\sin^2\theta}\frac{\partial^2 T}{\partial\phi^2}+\frac{\cot\theta}{r}\frac{\partial^2 T}{\partial\theta^2}\right)$$
$$-T_0\frac{\partial^2}{\partial t^2}\left(\beta_3\varepsilon_{rr}+\beta_1\varepsilon_{\theta\theta}+\beta_2\varepsilon_{\phi\phi}\right)=\rho C_e\frac{\partial^2 T}{\partial t^2}. \tag{3b}$$

Here, Eq. (3a) is the elastic wave equation and Eq. (3b) is the heat conduction equation; K_i is the material constant characteristic of the theory (not the coefficients of thermal conductivity) and C_e is the specific heat at constant strain.

By introducing the rectangular window function $\pi(r)$

$$\pi(r)=\begin{cases}1, & a\le r\le b,\\ 0, & \text{elsewhere},\end{cases}$$

the stress-free boundary conditions ($T_{rr}=T_{r\phi}=T_{r\theta}=0$ at $r=a, r=b$) are automatically incorporated in the constitutive relations:

$$\begin{Bmatrix}T_{\theta\theta}\\T_{\phi\phi}\\T_{rr}\\T_{r\phi}\\T_{r\theta}\\T_{\theta\phi}\end{Bmatrix}=\begin{bmatrix}C_{11}&C_{12}&C_{13}&0&0&0\\ &C_{22}&C_{23}&0&0&0\\ & &C_{33}&0&0&0\\ & & &C_{44}&0&0\\ &\text{symmetry}& & &C_{55}&0\\ & & & & &C_{66}\end{bmatrix}\begin{Bmatrix}\varepsilon_{\theta\theta}\\\varepsilon_{\phi\phi}\\\varepsilon_{rr}\\2\varepsilon_{r\phi}\\2\varepsilon_{r\theta}\\2\varepsilon_{\theta\phi}\end{Bmatrix}\pi(r)-\begin{bmatrix}\beta_1\\\beta_2\\\beta_3\\0\\0\\0\end{bmatrix}\{T\}\pi(r). \tag{4}$$

For a free harmonic wave propagating in a spherical curved plate, the wave front on the surface of a spherical shell is assumed to be toroidal [25,27]. In addition, to study wave propagation from point A to B in a spherical plate segment, the two points A and B can always be aligned along the equator of a sphere by adjusting the positions of the north and south poles. Therefore, to study the wave propagation between two points in a spherical plate segment, it is sufficient to solve the governing equations for $\theta=\pi/2$ only. Thus the propagating wave is independent of θ. Then the temperature change and the displacement components of this toroidal wave can be written as follows:

$$u_r(r,\theta,\phi,t)=\exp(ikb\phi-i\omega t)U(r), \tag{5a}$$
$$u_\theta(r,\theta,\phi,t)=\exp(ikb\phi-i\omega t)V(r), \tag{5b}$$
$$u_\phi(r,\theta,\phi,t)=\exp(ikb\phi-i\omega t)W(r), \tag{5c}$$
$$T(r,\theta,\phi,t)=\exp(ikb\phi-i\omega t)X(r). \tag{5d}$$

$U(r)$, $V(r)$, $W(r)$ represent the amplitude of vibration in the r,θ,ϕ directions, respectively, and $X(r)$ represents the amplitudes of temperature change. k is the magnitude of the wave vector in the propagation direction, and ω is the angular frequency.

Substituting Eqs. (2), (4), (5) into Eq. (3), the governing differential equations in terms of displacement components and temperature change can be obtained:

$$\left[r^2C_{33}U''+2rC_{33}U'+rikb\,(C_{23}+C_{44})W'+\left(C_{13}+C_{23}-C_{11}-C_{22}-2C_{12}-k^2b^2C_{44}\right)U\right.$$
$$\left.+\,ikb\,(C_{23}-C_{12}-C_{22}-C_{44})W-r^2\beta_3X'-r\,(2\beta_3-\beta_2-\beta_1)\,X\right]\pi(r)$$
$$+(\delta(r-a)-\delta(r-b))\left[r^2C_{33}U'+r\,(C_{13}+C_{23})\,U+rikbC_{23}W-r^2\beta_3X\right]=-\rho r^2\omega^2U, \tag{6a}$$
$$\left[r^2C_{55}V''+2rC_{55}V'-\left(2C_{55}+C_{12}+k^2b^2C_{66}\right)V\right]\pi(r)+(\delta(r-a)-\delta(r-b))\left(r^2C_{55}V'+r-C_{55}V\right)$$
$$=-\rho r^2\omega^2V, \tag{6b}$$
$$\left[r^2C_{44}W''+rikb\,(C_{23}+C_{44})\,U'+2r\,(C_{44}+ikbC_{24})\,W'+ikb\,(2C_{44}+C_{12}+C_{22})\,U\right.$$
$$\left.+\,(C_{66}-2C_{44}-k^2b^2\,C_{22})\,W-\beta_2rikbX\right]\pi(r) \tag{6c}$$
$$+(\delta(r-a)-\delta(r-b))\left(r^2C_{44}W'+rikbC_{44}U-rC_{44}W\right)=-\rho r^2\omega^2W,$$
$$r^2K_3X''+K_1rX'+K_2\left(rX'-k^2b^2X\right)+T_0\omega^2\left(r^2\beta_3U'+r\beta_1U+\beta_2rikbW+\beta_2rU\right)$$
$$=-\rho r^2C_e\omega^2X. \tag{6d}$$

Here, Eq. (6b) is independent of the other three equations. In fact, Eq. (6b) represents the propagating SH waves. Equations (6a) and (6c) control the propagating Lamb-like waves and are coupled with the heat conduction equation. In this paper, only the coupled thermoelastic Lamb-like waves are considered.

Multiplying by ρr, Eq. (6d) can be written as follows:

$$\begin{aligned} &T_0\left[r\beta_3\cdot\rho r^2\omega^2U'+(\beta_1+\beta)_2\cdot\rho r^2\omega^2U+\beta_2ikb\cdot\rho r^2\omega^2W\right]\\ &+\rho r\left[r^2K_3X''+K_1rX'+K_2\left(rX'-k^2b^2X\right)\right]=-\rho r\cdot\rho r^2C_e\omega^2X. \end{aligned} \tag{7}$$

Here

$$\rho r^2U'=\left(\rho r^2U\right)'-2\rho rU. \tag{8}$$

So, we can eliminate the terms $\rho r^2\omega^2U'$, $\rho r^2\omega^2U$ and $\rho r^2\omega^2W$ in Eq. (7) through Eqs. (6a) and (6b). Then Eq. (6d) becomes

$$\begin{aligned} &\rho r\left(r^2K_3X''+rK_1X'-k^2b^2K_1X\right)-T_0r\beta_3\left\{\left[r^2C_{33}U''+2rC_{33}U'+rikb\left(C_{23}+C_{44}\right)W'\right.\right.-r^2\beta_3X'\\ &+\left(C_{13}+C_{23}-C_{11}-C_{22}-2C_{12}-k^2b^2C_{44}\right)U+ikb\left(C_{23}-C_{12}-C_{22}-C_{44}\right)W-2r\beta_3X\\ &-r\left(\beta_2+\beta_1\right)X]\pi(r)+\left(\delta(r-a)-\delta(r-b)\right)\left[r^2C_{33}U'+r\left(C_{13}+C_{23}\right)U+rikbC_{23}W-r^2\beta_3X\right]\}'\\ &-T_0\left(\beta_1+\beta_2-2\beta_3\right)\left\{\left[r^2C_{33}U''+2rC_{33}U'+rikb\left(C_{23}+C_{44}\right)W'-r^2\beta_3X'\right.\right.-r\left(\beta_2+\beta_1\right)X\\ &+\left(C_{13}+C_{23}-C_{11}-C_{22}-2C_{12}-k^2b^2C_{44}\right)U+ikb\left(C_{23}-C_{12}-C_{22}-C_{44}\right)W-2r\beta_3X]\pi(r)\\ &+\left(\delta(r-a)-\delta(r-b)\right)\left[r^2C_{33}U'+r\left(C_{13}+C_{23}\right)U+rikbC_{23}W-r^2\beta_3X\right]\}\\ &-T_0ikb\beta_1\left\{\left[r^2C_{44}W''+rikb\left(C_{23}+C_{44}\right)U'+2r\left(C_{44}+ikbC_{24}\right)W'+ikb\left(2C_{44}+C_{12}+C_{22}\right)U\right.\right.\\ &+\left.\left(C_{66}-2C_{44}-k^2b^2C_{22}\right)W-\beta_2rikbX\right]\pi(r)\\ &+\left(\delta(r-a)-\delta(r-b)\right)\left(r^2C_{44}W'+rikbC_{44}U-rC_{44}W\right)=-\rho r\cdot\rho r^2C_e\omega^2X. \end{aligned} \tag{9}$$

To obtain the solution of the thermoelastic waves controlled by the coupled equations (6a), (6c) and (9), we expand $U(r)$ and $W(r)$ to Legendre orthogonal polynomial series as

$$U(r)=\sum_{m=0}^{\infty}p_m^1Q_m(r),\quad W(r)=\sum_{m=0}^{\infty}p_m^2Q_m(r), \tag{10}$$

and expand $X(r)$ to

$$X(r)=(r-a)(r-b)\sum_{m=0}^{\infty}p_m^3Q_m(r). \tag{11}$$

Obviously, Eq. (11) automatically satisfies the isothermal boundary conditions ($T=0$ at $r=a, r=b$). Here $p_m^i (i=1,2,3)$ are the expansion coefficients and

$$Q_m(r)=\sqrt{\frac{2m+1}{(b-a)}}P_m\left(\frac{2r-(b+a)}{(b-a)}\right)$$

with P_m being the mth Legendre polynomial. Theoretically, m runs from 0 to ∞. In practice, the summation over the polynomials in Eqs. (10) and (11) can be halted at some finite value $m=M$, when higher order terms become essentially negligible.

Equations (6a), (6c) and (9) are multiplied by $Q_j(r)$ with j running from 0 to M. Then integrating over r from a to b gives the following $3(M+1)$ equations:

$$A_{11}^{j,m}p_m^1+A_{12}^{j,m}p_m^2+A_{13}^{j,m}p_m^3=-\omega^2\cdot M_m^jp_m^1, \tag{12a}$$
$$A_{21}^{j,m}p_m^1+A_{22}^{j,m}p_m^2+A_{23}^{j,m}p_m^3=-\omega^2\cdot M_m^jp_m^2, \tag{12b}$$
$$A_{31}^{j,m}p_m^1+A_{32}^{j,m}p_m^2+A_{33}^{j,m}p_m^3=-\omega^2\cdot\rho rC_e\cdot MT_m^jp_m^3, \tag{12c}$$

Table 1 The material properties of Si_3N_4

Property	C_{11}	C_{13}	C_{33}	C_{55}	β_1	β_3	C_e	ρ
Si_3N_4	574	127	433	108	3.22	2.71	670	3.2

Units: C_{ij} ($10^9\,\mathrm{N\,m^{-2}}$), β_i ($10^6\,\mathrm{N\,deg^{-1}\,m^{-2}}$), C_e (J Kg deg $\mathrm{m^{-1}}$), ρ ($10^3\,\mathrm{kg\,m^{-3}}$)

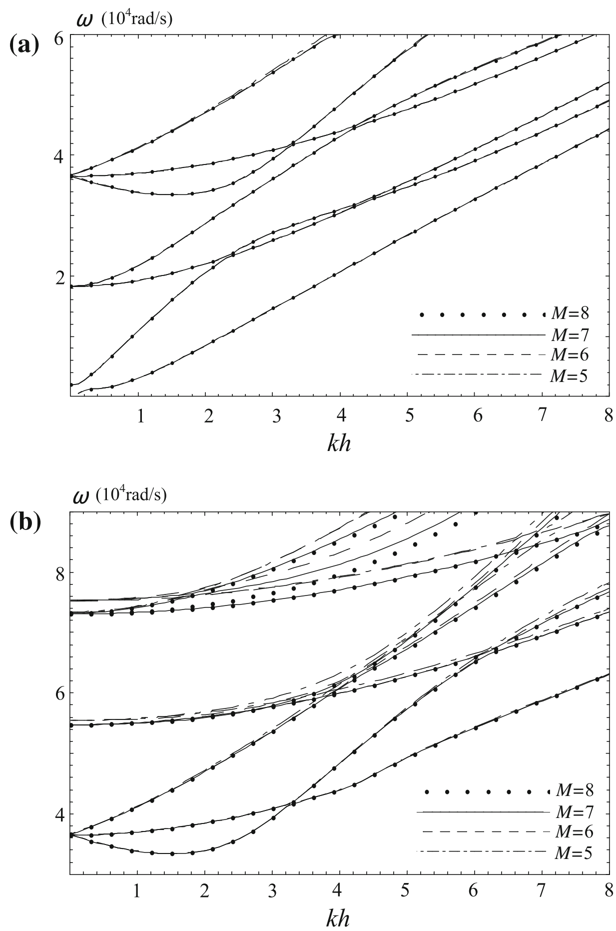

Fig. 1 Dispersion curves for the thermoelastic spherical plate for various "M": **a** mode 1–7; **b** mode 5–12

where summation over repeated index m is implied with m ranging from 0 to M. $A^{j,m}_{\alpha\beta}$ ($\alpha, \beta = 1, 2, 3$) and M^j_m are the elements of a non-symmetric matrix. They can be obtained according to Eqs. (6) and (9).

Equations (12) can be written as

$$\begin{bmatrix} A^{j,m}_{11} & A^{j,m}_{12} & A^{j,m}_{13} \\ A^{j,m}_{21} & A^{j,m}_{22} & A^{j,m}_{22} \\ A^{j,m}_{31} & A^{j,m}_{32} & A^{j,m}_{33} \end{bmatrix} \begin{Bmatrix} p^1_m \\ p^2_m \\ p^3_m \end{Bmatrix} = -\omega^2 \begin{bmatrix} M^j_m & 0 & 0 \\ 0 & M^j_m & 0 \\ 0 & 0 & \rho r C_e \cdot MT^j_m \end{bmatrix} \begin{Bmatrix} p^1_m \\ p^2_m \\ p^3_m \end{Bmatrix}. \tag{13}$$

So, Eq. (13) yields a form of the eigenvalue problem. The eigenvalue ω^2 gives the angular frequency of the thermoelastic wave and eigenvectors $p^i_m (i = 1, 2, 3)$ show the displacement and temperature distributions. The phase velocity can be obtained according to $Vph = \omega/k$. The complex matrix Eq. (13) can be solved numerically making use of standard computer programs for the diagonalization of non-symmetric square matrices. The matrix dimensionality in both members of Eq. (13) is $(3M + 3) \times (3M + 3)$. So $3(M + 1)$ eigenmodes are generated from the order M of the expansion. The solutions to be accepted are those eigenmodes for which convergence is obtained as M is increased. We determine that the eigenvalues obtained are

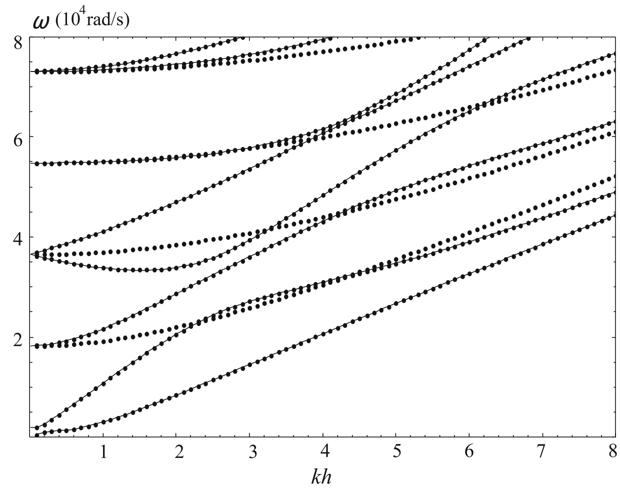

Fig. 2 Frequency spectra for the spherical plate: *solid line* purely elastic plate, *dotted line* thermoelastic plate

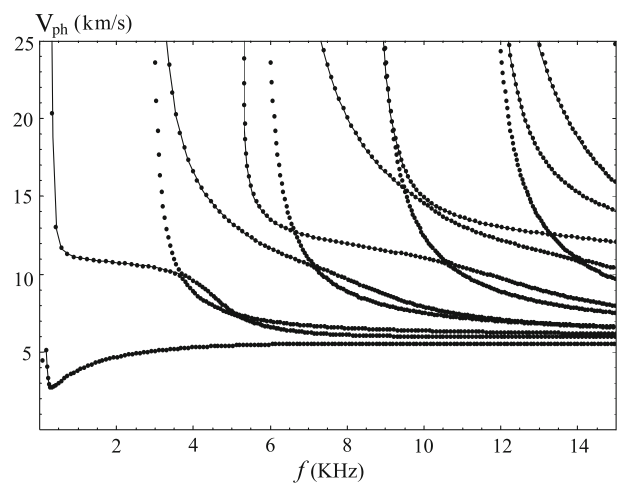

Fig. 3 Phase velocity spectra for the spherical plate: *solid line* purely elastic plate, *dotted line* thermoelastic plate

converged solutions when a further increase in the matrix dimension does not result in a significant change in the eigenvalue. Calculation of stresses is a straightforward subsequent step: after evaluating the displacement and temperature, the stress distributions can be immediately calculated from Eqs. (1) and (2).

3 Numerical results

Based on the foregoing formulations, a computer program has been written using Mathematica to calculate the dispersion curves and displacement, temperature, stress distributions for the thermoelastic spherical plate composed of Si_3N_4 with inner radius $a = 9$ m and thickness $h = 1$ m. Their material constants are listed in Table 1 except for K_i, the material constant characteristics of the theory. We take their values as $K_1 = C_e C_{11}/4$ and $K_3 = C_e C_{33}/4$ [36].

3.1 Convergence of the problem

In order to observe the convergence of the problem, the thermoelastic wave dispersion curves are calculated when $M = 5, 6, 7$ and 8, respectively, as shown in Fig. 1. Figure 1a shows the dispersion curves of the first

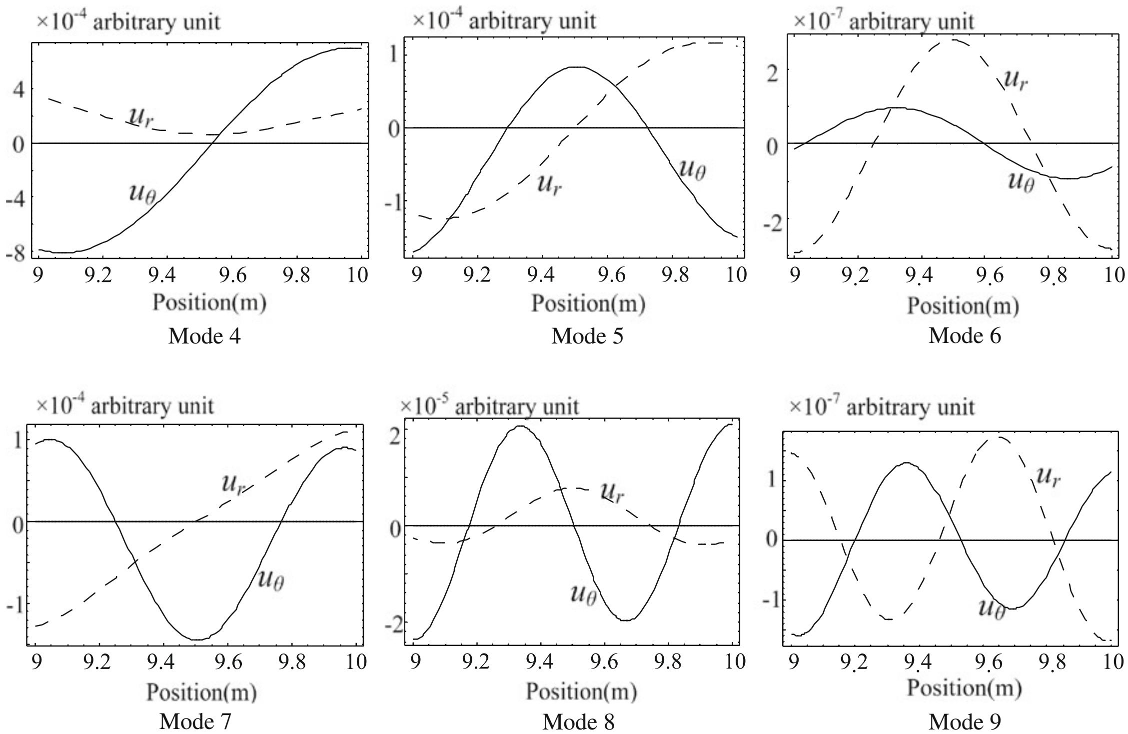

Fig. 4 Displacement distributions of the thermoelastic spherical plate at $kh = 1.2$

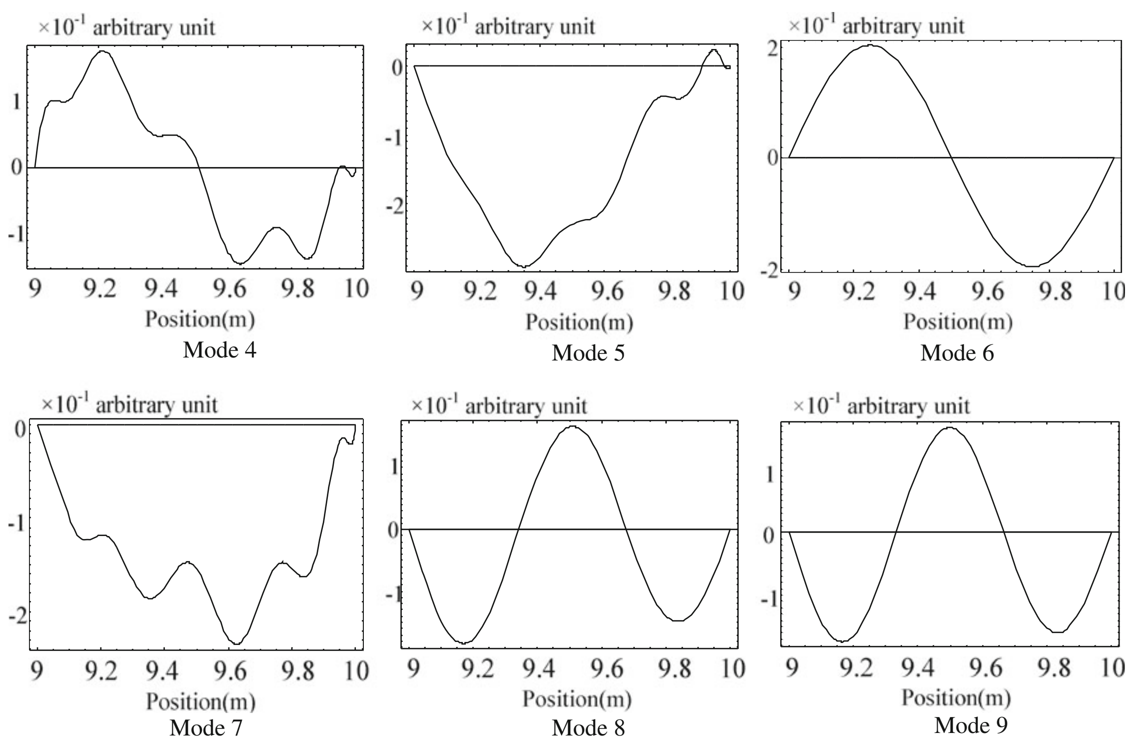

Fig. 5 Temperature change distributions of the thermoelastic spherical plate at $kh = 1.2$

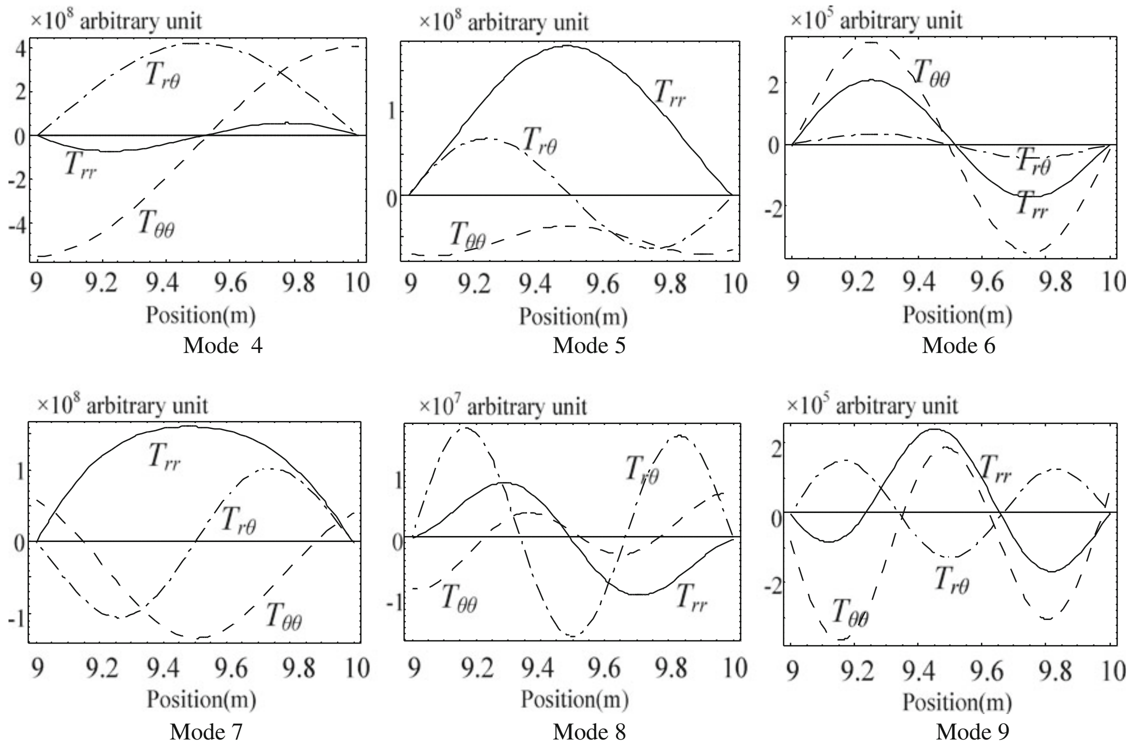

Fig. 6 Stress distributions of the thermoelastic spherical plate at $kh = 1.2$

seven modes and Fig. 1b shows the dispersion curves of mode 5 to mode 12. It can be seen that for the first six modes, the dispersion curves are coincident when $M = 5$, 6, 7 and 8, which indicates that the first six modes are convergent when $M = 5$. Similarly, according to Fig. 1, it can be concluded that the first eight modes are convergent when $M = 6$ and that the first ten modes are convergent when $M = 7$. So, we assume that at least the first $M + 1$ modes are convergent.

For all the unmentioned calculations, the series expansions Eqs. (10) and (11) are truncated at $M = 10$.

3.2 Wave characteristics in the thermoelastic spherical plate

The frequency spectra and phase velocity spectra for the thermoelastic spherical plate are shown in Figs. 2 and 3, in which the solid lines are the dispersion curves of the purely elastic plate and the dotted lines are the dispersion curves of the thermoelastic plate. It can be seen that the dispersion curves of some modes in the thermoelastic plate are almost coincident with those in the purely elastic plate. Obviously, these modes are elastic modes and they are little influenced by the thermoelasticity. The other modes in the thermoelastic plate are thermal modes. They do not exist in the pure elastic plate. According to Eqs. (6a) and (6c), the value of elastic modes is determined by the elastic constants C_{ij} and volume expanding coefficients β_i. The influence of thermoelasticity on elastic modes comes from the volume expanding coefficients β_i. From table 1, the elastic constants are about ten thousand times of the volume expanding coefficients, which leads to the influence of thermoelasticity being little. Furthermore, the thermal modes have similar dispersive behavior as the elastic modes. Their phase velocities decrease with the increase of frequency, and eventually congregate together with the elastic modes.

Figures 4, 5 and 6 show the displacement distributions, temperature change distributions and stress distributions of the mode 4–9 at $kh = 1.2$, of which the mode 6 and 9 are thermal modes and the other four modes are elastic modes. It can be checked on these figures that, the stress T_{rr}, $T_{r\phi}$ and the temperature change T are zero on the inner and outer surfaces of the curved plate. More attention should be paid to the fact that the displacement and stress amplitudes of thermal modes are far less than those of elastic modes, but the temperature change amplitudes of thermal modes and elastic modes are similar.

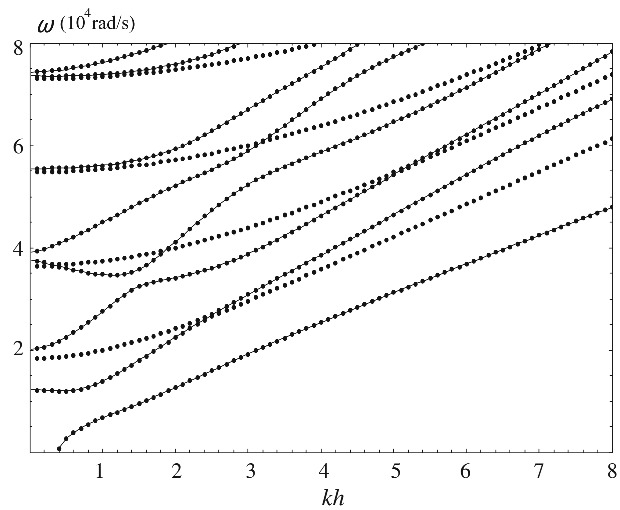

Fig. 7 Frequency spectra for the spherical plate with small ratio of radius to thickness: *solid line* purely elastic plate, *dotted line* thermoelastic plate

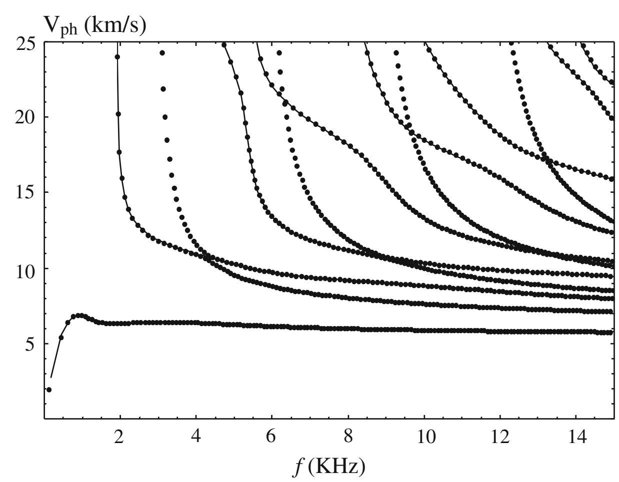

Fig. 8 Phase velocity spectra for the spherical plate with small ratio of radius to thickness: *solid line* purely elastic plate, *dotted line* thermoelastic plate

3.3 The influence of the ratio of radius to thickness

In this section, another thermoelastic spherical curved plate with small ratio of radius to thickness is considered. Its material and thickness of the curved plate is the same to the first example, but its outer radius is changed to 2 m. The ratio of the outer radius to thickness is defined as η.

The calculated dispersion curves are shown in Figs. 7 and 8. Compared with Figs. 2 and 3, it can be seen that the ratio has considerable influence both on elastic modes and on thermal modes. From the dispersion curves, the influence on elastic modes and on thermal modes is similar. The influence of the ratio on the elastic wave dispersion curves is well known. Here, our interest lies in how the influence on the elastic modes is different from that on the thermal. Figures 9, 10 and 11 show the displacement distributions, temperature change distributions and stress distributions of the spherical curved plate with small η, of which the mode 6 and 9 are thermal modes and the other four modes are elastic modes. The displacement and stress amplitudes of thermal modes are much smaller than those of elastic modes, which is similar to the large ratio.

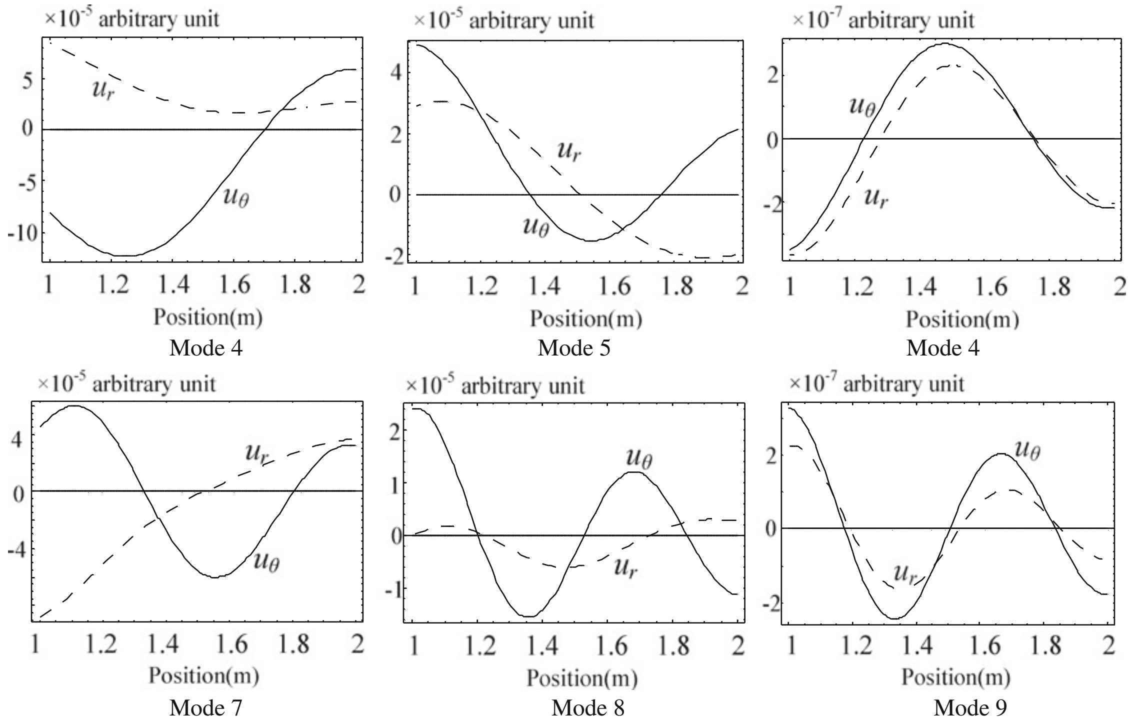

Fig. 9 Displacement distributions of the thermoelastic spherical plate with small ratio of radius to thickness at $kh = 1.2$

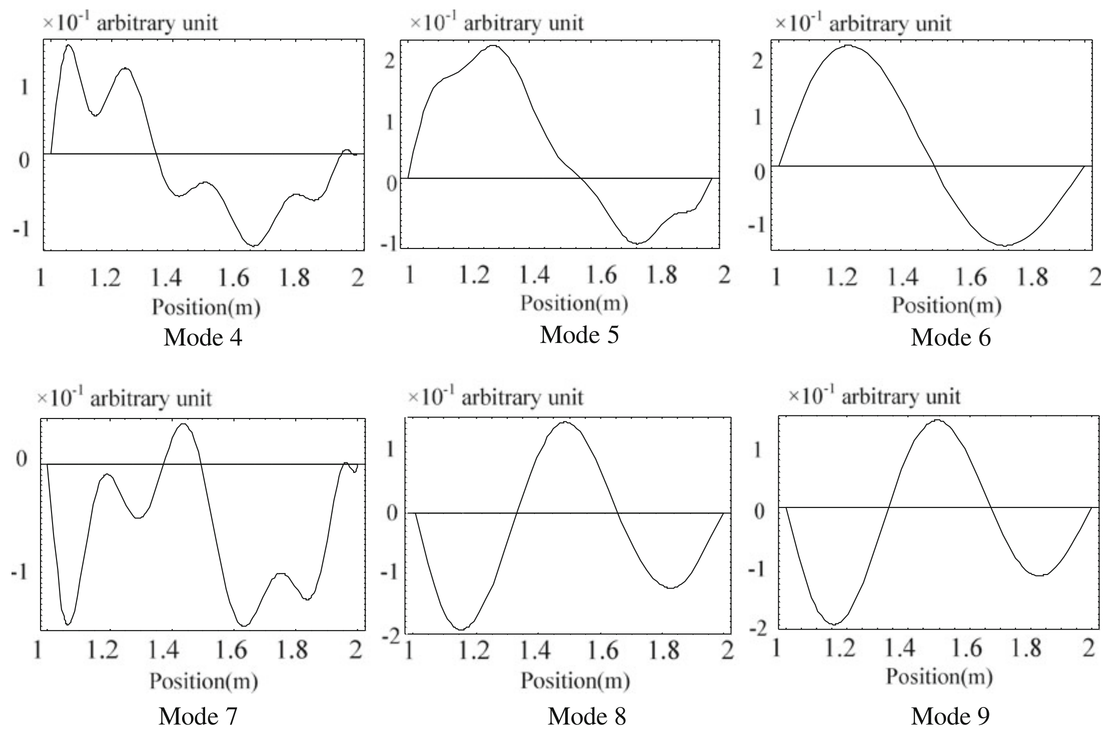

Fig. 10 Temperature change distributions of the thermoelastic spherical plate with small ratio of radius to thickness at $kh = 1.2$

In contrast to the large ratio, the displacement and stress amplitudes of the elastic modes decrease while those of the thermal modes increase. The temperature amplitude does not vary considerably with the decrease of the ratio.

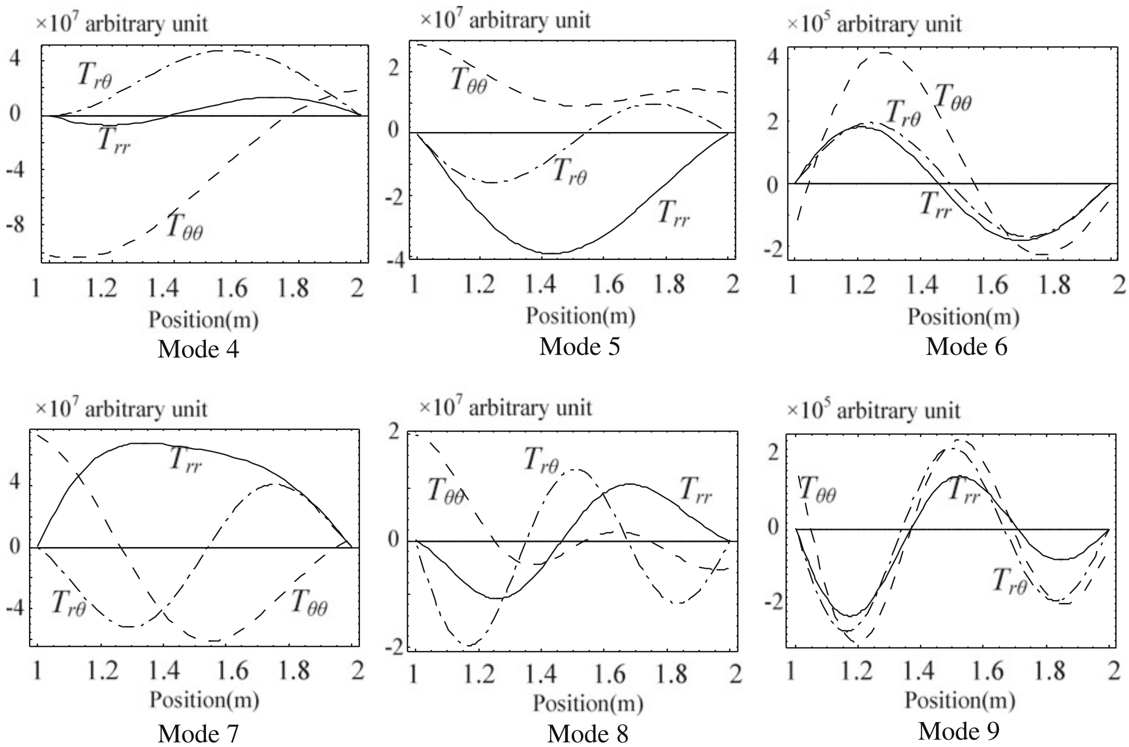

Fig. 11 Stress distributions of the thermoelastic spherical plate with small ratio of radius to thickness at $kh = 1.2$

4 Conclusions

In the context of the generalized GN thermoelastic theory, the wave characteristics in the spherical curved plate are discussed. Based on the calculated results, the following conclusions can be drawn:

(a) For the wave propagation in the orthotropic thermoelastic spherical curved plate, the independent SH wave is not influenced by the thermoelasticity.
(b) For the propagating thermoelastic waves in the spherical curved plate, the thermoelasticity has little influence on the elastic modes, while the displacements and stresses generated by thermal modes are far less than those by elastic modes.
(c) The ratio of radius to thickness has obvious influence both on elastic modes and on thermal modes. As the ratio decreases, the displacement and stress of the elastic modes decrease, but those of thermal modes increase.

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References

1. Al-Qahtani, H., Datta, S.: Thermoelastic waves in an anisotropic infinite plate. J. Appl. Phys. **96**, 3645–3658 (2004)
2. Nayfeh, A.H., Nemat-Nasser, S.: Thermoelastic waves in solids with thermal relaxations. Acta Mech. **12**, 53–69 (1971)
3. Sinha, H., Sinha, S.B.: Velocity of Rayleigh waves with thermal relaxation in time. Acta Mech. **23**, 159–166 (1975)
4. Sherief, H.H., Helmy, K.A.: A two-dimensional generalized thermoelasticity problem for a half space. J. Therm. Stress. **22**, 897–910 (1999)
5. Abd-alla, A.E.N., Al-dawy, A.A.S.: Thermal relaxation times effect on Rayleigh waves in generalized thermoelastic media. J. Therm. Stress. **24**, 367–381 (2001)
6. Massalas, C.V.: Thermoelastic waves in a thin plate. Acta Mech. **65**, 51–61 (1986)
7. Daimaruya, M., Naitoh, M.: Dispersion and energy dissipation of thermoelastic waves in a plate. J. Sound Vib. **117**, 511–518 (1987)
8. Massalas, C.V., Kalpakidis, V.K.: Thermoelastic waves in a waveguide. Int. J. Eng. Sci. **25**, 1207–1218 (1987)

9. Sharma, J.N., Singh, D., Kumar, R.: Generalized thermoelastic waves in homogeneous isotropic plates. J. Acoust. Soc. Am. **108**, 848–851 (2000)
10. Verma, K.L., Hasebe, N.: Dispersion of thermoelastic waves in a plate with and without energy dissipation. Int. J. Thermophys. **22**(3), 957–978 (2001)
11. Verma, K.L., Hasebe, N.: Wave propagation in plates of general anisotropic media in generalized thermoelasticity. Int. J. Eng. Sci. **39**, 1739–1763 (2001)
12. Verma, K.L., Hasebe, N.: Wave propagation in transversely isotropic plates in generalized thermoelasticity. Arch. Appl. Mech. **72**, 470–482 (2002)
13. Rajneesh, K., Tarun, K.: Propagation of Lamb waves in transversely isotropic thermoelastic diffusive plate. Int. J. Solids Struct. **45**, 5890–5913 (2008)
14. Rajneesh, K., Tarun, K.: Rayleigh-LambWaves in transversely isotropic thermoelastic diffusive layer. Int. J. Thermophys. doi:10.1007/s10765-008-0522-x
15. Al-Qahtani, H.: Extensional thermoelastic waves in transversely isotropic plate according to a higher order theory. Acta Mech. doi:10.1007/s00707-008-0130-7
16. Hawwa, M.A., Nayfeh, A.H.: The general problem of thermoelastic waves in anisotropic periodically laminated composites. Compos. Eng. **5**(12), 1499–1517 (1995)
17. Hawwa, M.A., Nayfeh, A.H.: Thermoelastic waves in a laminated composite with a second sound effect. J. Appl. Phys. **80**(5), 2733–2738 (1996)
18. Verma, K.L.: On the propagation of waves in layered anisotropic media in generalized thermoelasticity. Int. J. Eng. Sci. **40**, 2077–2096 (2002)
19. Ponnusamy, P.: Wave propagation in a generalized thermoelastic solid cylinder of arbitrary cross-section. Int. J. Solids Struct. **44**, 5336–5348 (2007)
20. Sharma, J.N., Pathania, V.: Generalized thermoelastic wave propagation in circumferential direction of transversely isotropic cylindrical curved plates. J. Sound Vib. **281**, 1117–1131 (2005)
21. Brekhovskikh, L.M.: Surface waves confined to the curvature of the boundary in solid. Sov. Phys. Acoust. **13**, 462–472 (1968)
22. Shah, A.H., Ramakrishnan, C.V., Datta, S.K.: Three-dimensional and shell-theory analysis of elastic waves in a hollow sphere. ASME J. Appl. Mech. **36**, 431–439 (1969)
23. Gaunaurd, G.C., Werby, M.F.: Similarities between various Lamb waves in submerged spherical shells, and Rayleigh waves in elastic spheres and flat half-spaces. J. Acoust. Soc. Am. **89**, 2731–2739 (1991)
24. Gaunaurd, G.C., Werby, M.F.: Sound scattering by resonantly excited, fluid loaded, elastic spherical shells. J. Acoust. Soc. Am. **90**, 2536–2550 (1991)
25. Kargl, S.G., Marston, P.L.: Ray synthesis of lamb wave contributions to the total scattering cross section for an elastic spherical shell. J. Acoust. Soc. Am. **88**, 1103–1113 (1990)
26. Wang, X., Lu, G., Guillow, S.R.: Stress wave propagation in orthotropic laminated thick-walled spherical shells. Int. J. Solids Struct. **39**, 4027–4037 (2002)
27. Towfighi, S., Kundu, T.: Elastic wave propagation in anisotropic spherical curved plates. Int. J. Solids Struct. **40**, 5495–5510 (2003)
28. Yu, J.G., Wu, B., Huo, H.L., He, C.F.: Characteristics of guided waves in anisotropic spherical curved plates. Wave Motion **44**(4), 271–281 (2007)
29. Yu, J.G., Wu, B., Huo, H.L., He, C.F.: Wave propagation in functionally graded piezoelectric spherical curved plates. Phys. Status Solidi (B) **244**(9), 3377–3389 (2007)
30. Lefebvre, J.E., Zhang, V., Gazalet, J., Gryba, T.: Legendre polynomial approach for modeling free-ultrasonic waves in multilayered plates. J. Appl. Phys. **85**, 3419–3427 (1999)
31. Jiangong, Y., Qiujuan, M.: Circumferential wave in functionally graded piezoelectric cylindrical curved plates. Acta Mech. **198**(3–4), 171–190 (2008)
32. Yu, J.G., Wu, B., He, C.F.: Characteristics of guided waves in graded spherical curved plates. Int. J. Solids Struct. **44** (11–12), 3627–3637 (2007)
33. Wu, B., Yu, J.G., He, C.F.: Wave propagation in non-homogeneous magneto-electro-elasitc plates. J. Sound Vib. **317** (1–2), 250–264 (2008)
34. Yu, J.G., Wu, B.: Circumferential wave in magneto-electro-elastic functionally graded cylindrical curved plates. Eur. J. Mech. A/Solids **28**(3), 560–568 (2009)
35. Jiangong, Y., Bin, W.: Chen Guoqiang. Wave characteristics in functionally graded piezoelectric hollow cylinders. Arch. Appl. Mech. **79**(9), 807–824 (2009)
36. Sharma, J.N., Pathania, V.: Generalized thermoelastic waves in anisotropic plates sandwiched between liquid layers. J. Sound Vib. **278**(1–2), 383–411 (2004)