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Hakan F. Oztop · Yasin Varol · Ioan Pop

Effects of wall conduction on natural convection in a porous triangular enclosure

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Abstract Conjugate natural convection in a two-dimensional triangular enclosure filled with a porous medium is examined in this article. It is assumed that the solid vertical wall is of finite conductivity and that the temperature of the inclined wall is lower than that of the vertical wall, while the horizontal wall is adiabatic. A finite difference method is used to solve the governing equations of convection and conduction for different parameters as Rayleigh number, width of the vertical solid wall, aspect ratio of the enclosure and thermal conductivity ratio between solid and porous media. It is found that heat transfer increases with increasing Rayleigh number and aspect ratio of the triangle, decreasing wall thickness and with the increase of the wall conductivity.

List of symbols

A	Aspect ratio, H/L
g	Gravitational acceleration
H	Height of triangle or cavity
hy'	Minimum height of partition
hy	Dimensionless minimum height of the partition, hy'/H
K	Permeability of the porous medium
k	Thermal conductivity ratio, k_s/k_f
L	Length of triangle or cavity
Nu	Mean Nusselt number
Nu_y	Local Nusselt number
Ra	Rayleigh number
T	Temperature
u, v	Velocity components in x, y directions
x, y	Cartesian coordinates
X, Y	Non-dimensional coordinates
w	Dimensionless weight of the solid wall, w'/L
w'	Weight of solid wall

H. F. Oztop
Department of Mechanical Engineering, Firat University, 23119 Elazig, Turkey

Y. Varol
Department of Mechanical Education, Firat University, 23119 Elazig, Turkey

I. Pop (✉)
Faculty of Mathematics, University of Cluj, CP 253, 3400 Cluj, Romania
E-mail: pop.ioan@yahoo.co.uk
Tel.: +40-264-594315
Fax: +40-264-591906

Greek letters

α_m	Thermal diffusivity of the porous medium
β	Thermal expansion coefficient
θ	Non-dimensional temperature
υ	Kinematic viscosity
ψ	Stream function
Ψ	Non-dimensional stream function

Subscript

c	Cold
f	Fluid
h	Hot
s	Solid

1 Introduction

Analysis of natural convection in enclosures filled with fluid-saturated porous media has been performed extensively in the recent years using different geometries, equation models and numerical techniques. Valuable references can be found in the books by Nield and Bejan [1] and Ingham and Pop [2]. The coupled conduction and natural convection process in enclosures occurs in numerous practical situations, such as insulation industry, high performance insulation for buildings, solar collectors, cold storage installations, and so on. In spite of these wide applications, the studies on conduction–natural convection analyses in porous enclosures are limited to square or rectangular enclosures.

Baytas et al. [3] studied the steady conjugate natural convection in a square cavity filled with a fluid-saturated porous medium. The bottom and top walls of the enclosures were chosen as thick solid material with finite thermal conductivity ratio. Mbaye et al. [4] performed both analytical and numerical studies on natural convection in an inclined porous layer bounded by a wall of finite thickness and conductivity. Further, Saeid [5,6] studied the effects of a conductive wall on natural convection heat transfer and fluid flow in single-sided [5] and two-sided [6] porous square enclosures. He observed that both thickness and thermal conductivity of the wall are extremely important parameters for temperature and flow field. Du and Bilgen [7] studied the problem of steady natural convection in a modified Trombe wall solar collector using a porous medium as an absorber at different boundary conditions. These conditions are: two isothermal vertical walls are at different temperatures, two horizontal walls are adiabatic and either uniform or non-uniform heat generating porous layers with orifices. The overall results indicate that the Darcy number and the geometrical parameters are the most important parameters affecting the performance of the system. Chang and Lin [8] performed a study to examine the natural convection in porous rectangular enclosures filled with non-Darcy fluids and bounded by solid walls. They found that thermal conductivities of the solids are the most important parameters on the flow and temperature fields. A review study on conjugate natural convection from different bodies or configurations in fluid saturated porous media has been carried out by Kimura et al. [9]. Further, Vaszi et al. [10] solved the problem of conjugate-free convection in a porous medium from a vertical plate and a vertical cylindrical fin. Liu et al. [11] performed a study on conjugate mixed convection over a plate fin embedded in a porous medium at different Prandtl numbers. They observed that the total heat transfer rate decreases with increasing convection–conduction parameter.

Natural convection in triangular enclosures filled with fluid-saturated porous media under different boundary conditions has been studied in several articles [10–16]. However, the thickness of the boundaries (walls) was neglected in these studies. Thus, only the convection regime was investigated. Therefore, the main aim of this article is to study numerically the problem of conduction–natural convection in a two-dimensional thick walled triangular enclosure filled with a porous medium. The previous work clearly indicates that there is as yet not any published study on conduction–natural convection in a triangular enclosure filled with a porous medium. Due to the lack of such results, the present study will be useful for thermal designers.

2 Physical description of system and mathematical model

Figure 1a shows the physical model, boundary conditions and coordinates for the considered geometry. It is a triangular enclosure with a thick vertical wall of finite thickness and thermal conductivity. The length of the bottom

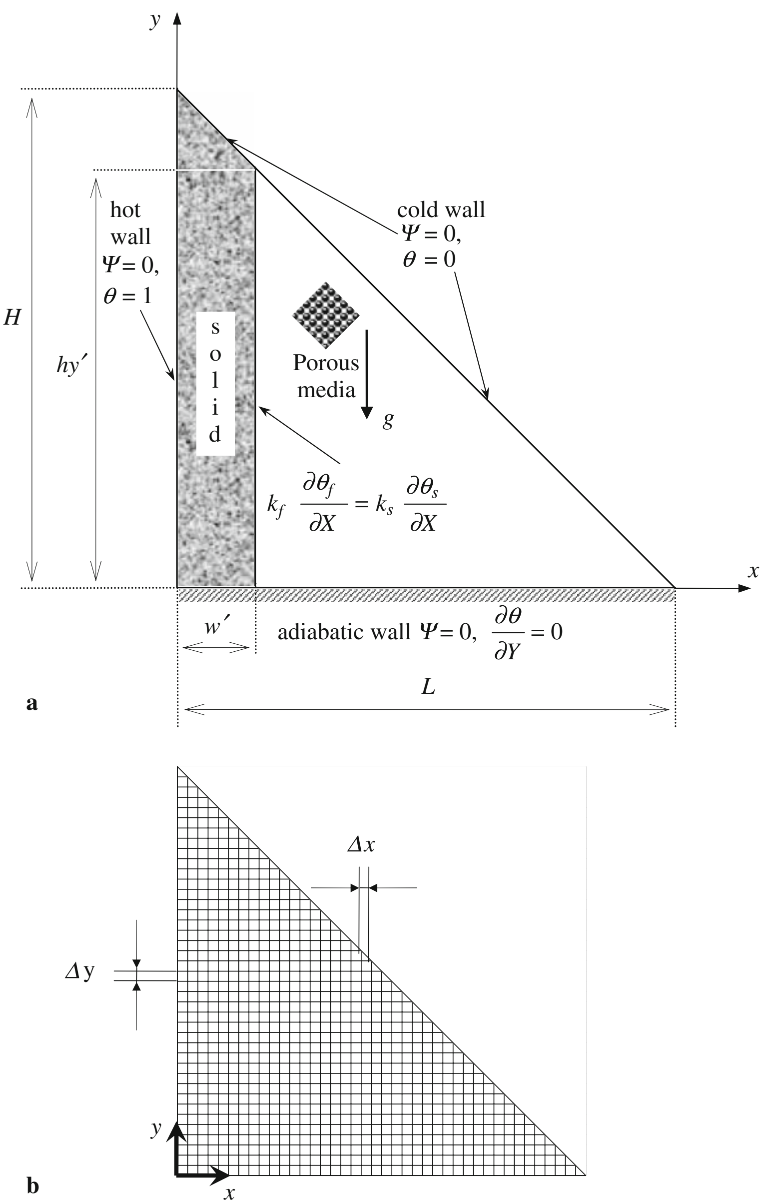

Fig. 1 **a** Schematical configuration and boundary conditions of the problem. **b** Computational domain and grid distribution

wall is L and the height of the vertical wall is H. It is assumed that the outside vertical wall temperature is higher than that of the inclined wall, while the bottom wall is insulated. Grid distribution is also shown in Fig. 1b. A regular grid was used in the system. The following assumptions are made: the properties of the fluid and the porous medium are constant; the cavity walls are impermeable; the Boussinesq approximation and the Darcy law model are valid; and the viscous drag and inertia terms of the momentum equations are negligible. With these assumptions, the dimensional governing equations as continuity, momentum, and energy can be written as follows:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

$$\frac{\partial u}{\partial y} - \frac{\partial v}{\partial x} = -\frac{g\beta K}{\upsilon}\frac{\partial T_f}{\partial x} \tag{2}$$

$$u\frac{\partial T_f}{\partial x} + v\frac{\partial T_f}{\partial y} = \alpha_m\left(\frac{\partial^2 T_f}{\partial x^2} + \frac{\partial^2 T_f}{\partial y^2}\right) \tag{3}$$

and the energy equation for the vertical solid wall is:

$$\frac{\partial^2 T_s}{\partial x^2} + \frac{\partial^2 T_s}{\partial y^2} = 0 \tag{4}$$

where u and υ are the velocity components along x and y axes, respectively, T_f is the fluid temperature, T_s is the temperature of the solid vertical wall, g is the acceleration due to gravity, K is the permeability of the porous medium, α_m is the effective thermal diffusivity of the porous medium, β is the thermal expansion coefficient and υ is the kinematic viscosity. Introducing the stream function ψ defined as

$$u = \frac{\partial \psi}{\partial y}, \quad \upsilon = -\frac{\partial \psi}{\partial x}, \tag{5}$$

Eqs. (1)–(4) can be written in non-dimensional form as

$$A^2 \frac{\partial^2 \Psi}{\partial X^2} + \frac{\partial^2 \Psi}{\partial Y^2} = -\ A\ Ra \frac{\partial \theta_f}{\partial X} \tag{6}$$

$$A \left(\frac{\partial \Psi}{\partial Y} \frac{\partial \theta_f}{\partial X} - \frac{\partial \Psi}{\partial X} \frac{\partial \theta_f}{\partial Y} \right) = A^2 \frac{\partial^2 \theta_f}{\partial X^2} + \frac{\partial^2 \theta_f}{\partial Y^2} \tag{7}$$

for the fluid-saturated porous medium and

$$A^2 \frac{\partial^2 \theta_s}{\partial X^2} + \frac{\partial^2 \theta_s}{\partial Y^2} = 0 \tag{8}$$

for the solid wall, respectively. Here, $A = H/L$ is the cavity ratio, $Ra = g\beta K\ (T_h - T_c)\ H/\alpha_m\ \upsilon$ is the Rayleigh number for the porous medium, and the non-dimensional quantities are defined as

$$X = \frac{x}{L}, \quad Y = \frac{y}{H}, \quad \Psi = \frac{\psi}{\alpha_m}, \quad \theta_f = \frac{T_f - T_c}{T_h - T_c}, \quad \theta_s = \frac{T_s - T_c}{T_h - T_c}. \tag{9}$$

The boundary conditions of Eqs. (6)–(8) are:
for all solid boundaries

$$\Psi = 0, \tag{10.1}$$

on the vertical wall (hot), $0 \leq Y \leq 1$

$$\theta_f = 1, \tag{10.2}$$

on the bottom wall (adiabatic), $0 \leq X \leq 1$

$$\frac{\partial \theta_f}{\partial Y} = 0, \tag{10.3}$$

on the inclined wall (cold)

$$\theta_f = 0, \tag{10.4}$$

and for the interface between solid and porous media

$$k_f \frac{\partial \theta_f}{\partial X} = k_s \frac{\partial \theta_s}{\partial X}. \tag{10.5}$$

The physical quantities of interest in this problem are the local Nusselt number Nu_y and the mean Nu Nusselt number, which are given by

$$Nu_y = \left(-\frac{\partial \theta_f}{\partial X} \right)_{X=w}, \quad Nu = \frac{1}{hy} \int_0^{hy} Nu_y dY \tag{11.1,2}$$

for the interface between the solid vertical wall and the porous medium.

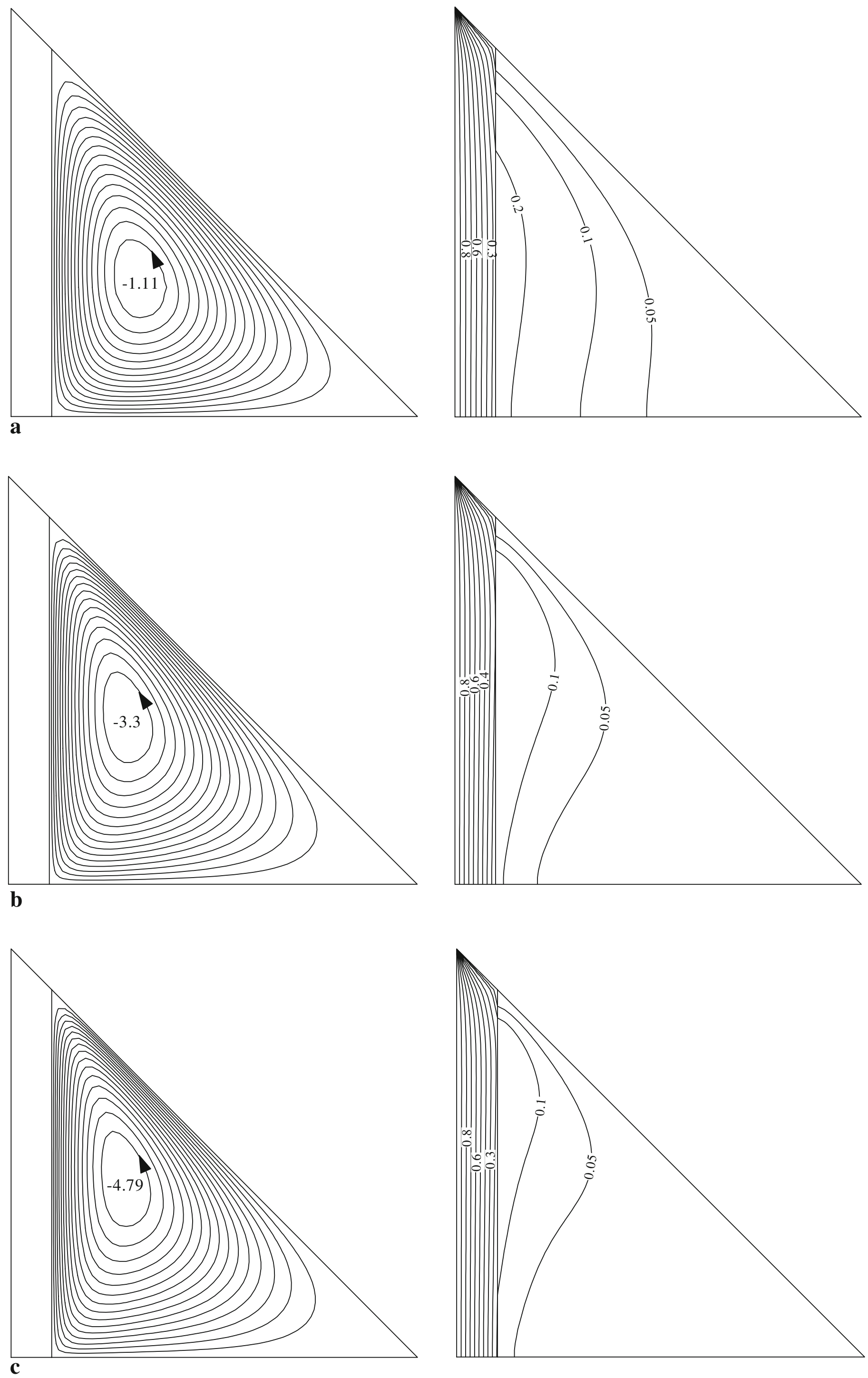

Fig. 2 Streamlines (*left*) and isotherms (*right*) for $A = 1$, $w = 0.1$ and $k = 0.1$: **a** $Ra = 100$; **b** $Ra = 500$; **c** $Ra = 1{,}000$

3 Numerical technique

Equations (6)–(8) subject to the boundary conditions (10) are integrated using the finite-difference method. Numerical simulations were carried out systematically to determine the effect of four main parameters of the problem, namely Rayleigh number Ra, thermal conductivity ratio k, thickness of the solid vertical wall w $(= w'/L)$ and aspect ratio A on the flow and heat transfer characteristics. The solution domain, therefore,

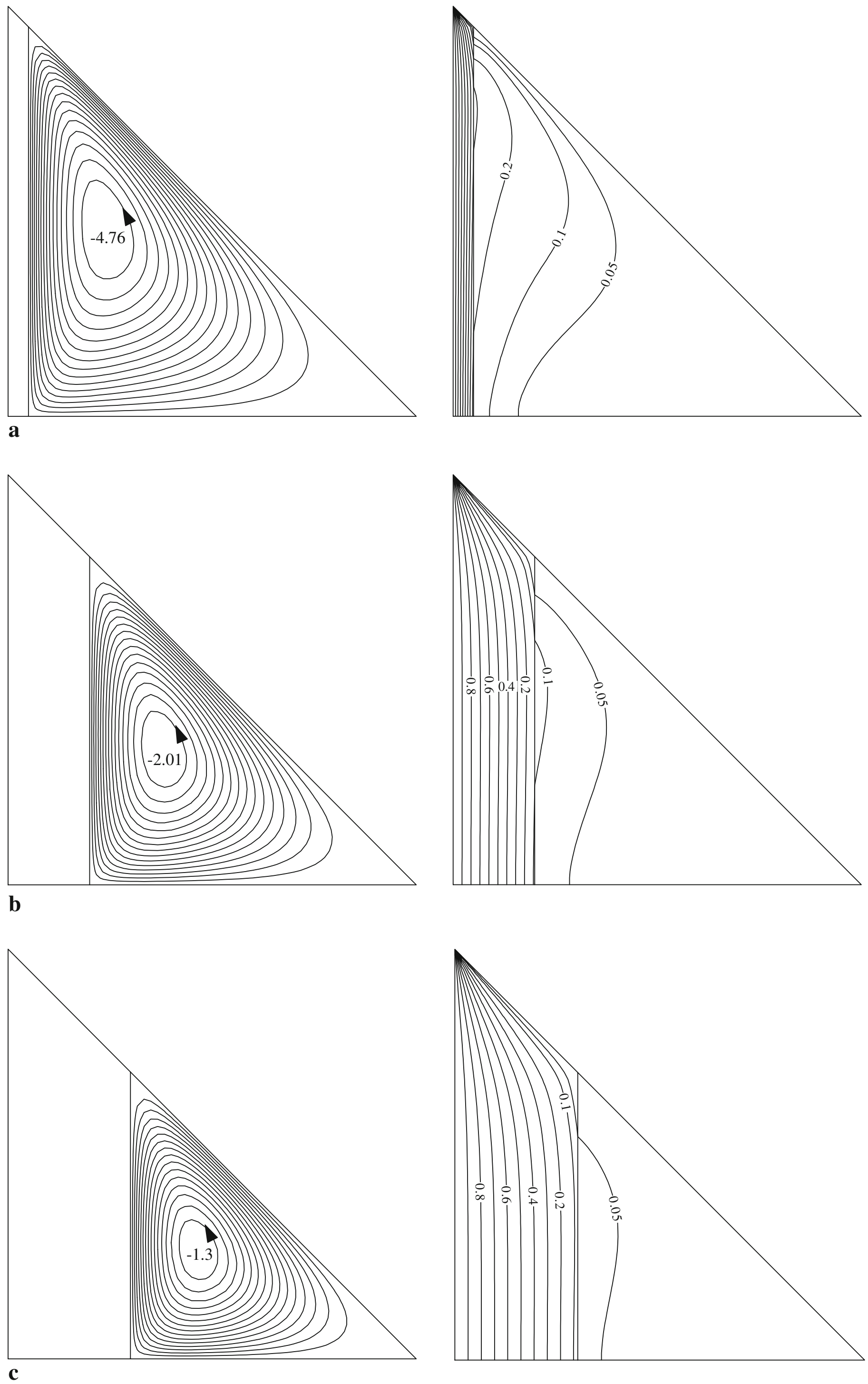

Fig. 3 Streamlines (*left*) and isotherms (*right*) for $A = 1$, *Ra* = 500 and $k = 0.1$: **a** $w = 0.05$; **b** $w = 0.2$; **c** $w = 0.3$

consists of grid points at which the equations are applied. The grid size was selected to be similar to that used by 61×61 for the square cavity with uniform grid spacing. The resulting algebraic equations were solved by Successive Under Relaxation (SUR) method. The iteration process is terminated under the following condition:

$$\sum_{i,j} \left|\phi_{i,j}^{m} - \phi_{i,j}^{m-1}\right| \Big/ \sum_{i,j} \left|\phi_{i,j}^{m}\right| \leq 10^{-5} \qquad (12)$$

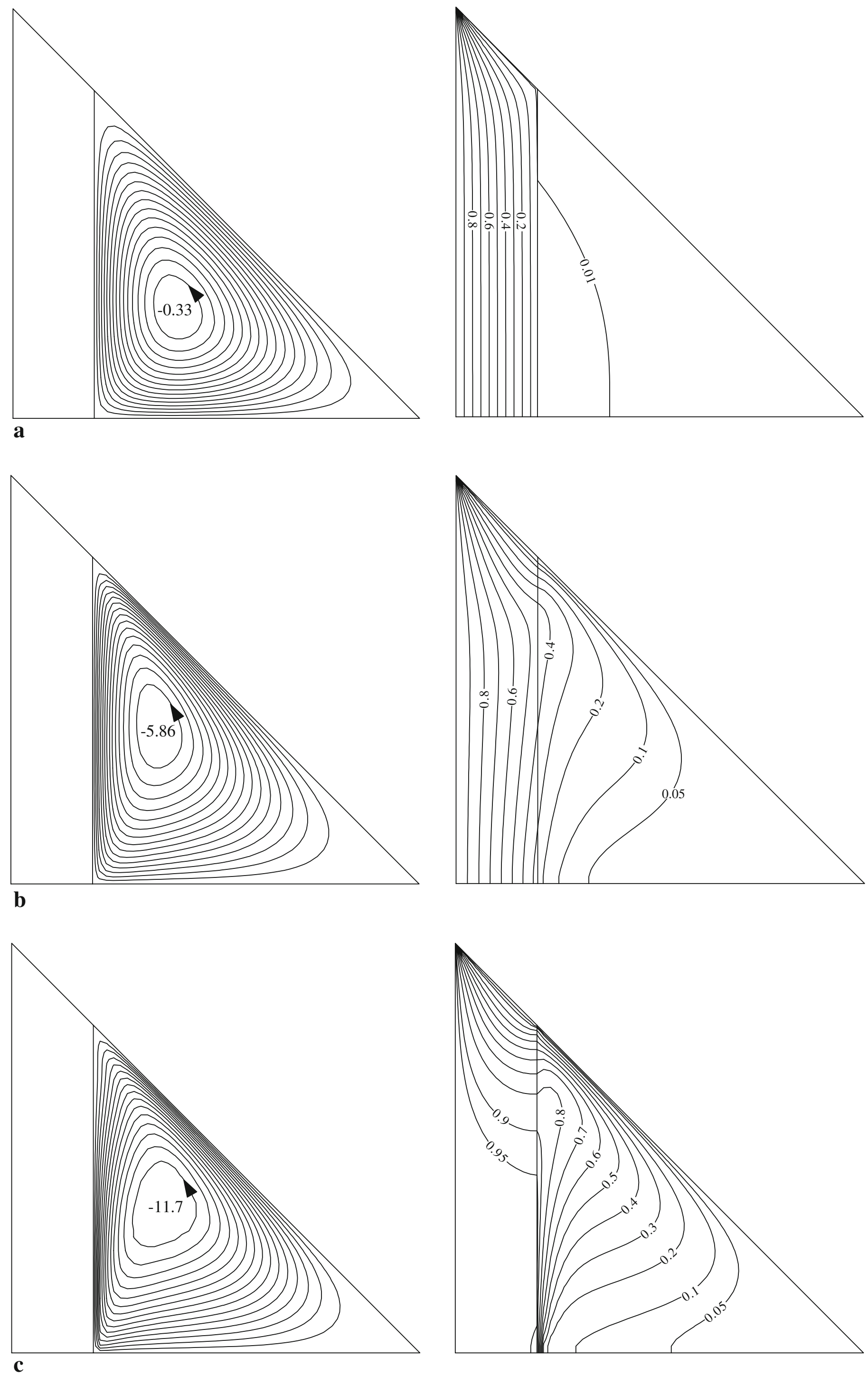

Fig. 4 Streamlines (*left*) and isotherms (*right*) for $A = 1$, $w = 0.2$ and *Ra* = 500: **a** $k = 0.01$; **b** $k = 1$; **c** $k = 100$

where m denotes the iteration step and ϕ stands for either θ_f, θ_s or Ψ. Due to lack of suitable results in the literature pertaining to the present configuration, the obtained results have been validated against the existing results for a square cavity filled with a porous medium. Thus, the comparison of the present results for the mean Nusselt number Nu, as defined by Eq. (12), with those from the open literature has been made for a value of $Ra = 1{,}000$. Comparison results can be found in our earlier publications as Varol et al. [12–15].

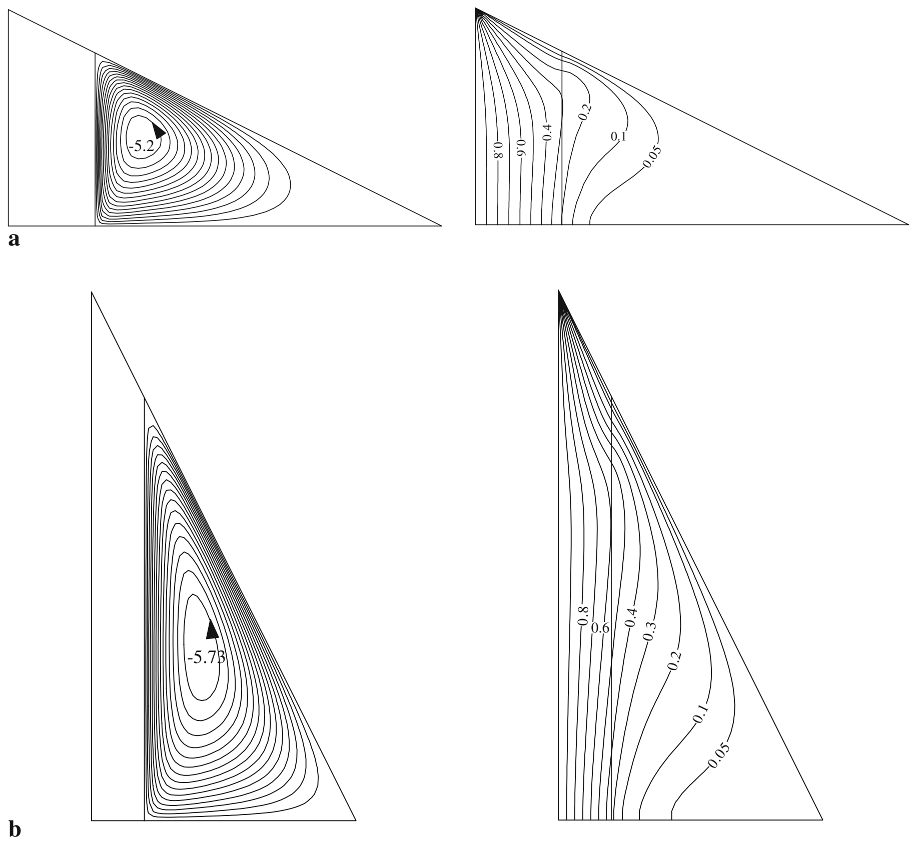

Fig. 5 Streamlines (*left*) and isotherms (*right*) for $k = 1$, $w = 0.2$ and $Ra = 500$: **a** $A = 0.5$; **b** $A = 2$

4 Results and discussion

A numerical study has been carried out to obtain the effects of a thick vertical wall with finite conductivity on natural convection in a porous triangular enclosure at different values of the parameters: aspect ratio, Rayleigh number, thickness of the wall and conductivity ratio.

Figure 2 is plotted to see the effects of the Rayleigh number on temperature and flow field with streamline (on the left) and isotherm (on the right), respectively. The governing parameters are taken as $A = 1$, $w = 0.1$ and $k = 0.1$. The figure shows that the thick wall behaves as a curtain for heat transfer. Thus, it prevents the heat transfer from hot wall to cold wall. The temperature gradient inside the solid wall is extremely high due to low thermal conductivity. A single circulation cell is formed in counterclockwise rotating direction. Convection becomes stronger with increasing Rayleigh number. Figure 3 shows the effect of the wall thickness parameter w on the temperature and flow fields. It is seen that w directly affects the volume of the triangular enclosure. Thus, the flow strength decreases with increasing w due to decreasing convection. In other words, more fluid is heated when a thin vertical wall is used. Effects of wall thermal conductivity ratio parameter k on temperature and flow field are presented in Fig. 4 for $A = 1$, $w = 0.2$ and $Ra = 500$. As can be seen from the figure, the thermal conductivity ratio is an extremely important parameter for the distribution of temperature and flow field inside the cavity. Convection heat transfer inside the triangular enclosure is enhanced with increasing thermal conductivity ratio due to the decrease of the temperature of the inner surface. For $k = 100$ the isotherms show a stratified flow within the enclosure with steep gradients near the vertical wall as can be seen from Fig. 4c. The aspect ratio of the enclosure is also an important parameter as can be concluded from the earlier studies [12–15]. To show the effects of this parameter, Fig. 5 is plotted for $k = 1$, $w = 0.2$ and $Ra = 500$.

Figure 6a illustrates the variation of the mean Nusselt number Nu with the Rayleigh number Ra for different values of the parameter w when $k = 0.1$ and $A = 1$. Higher heat transfer is obtained for thinner solid wall. The heat transfer regime is quasi-conductive for a thick wall. Thus, heat transfer becomes

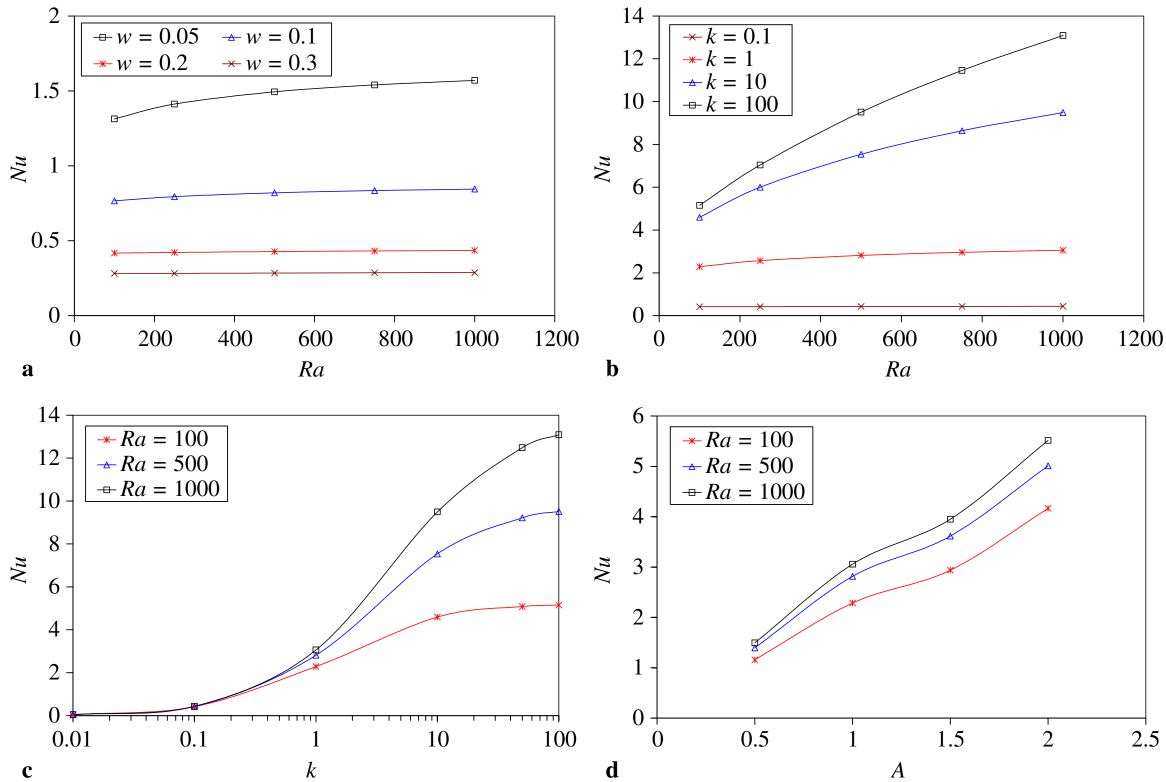

Fig. 6 Mean Nusselt number: **a** different thickness ratio of the vertical wall at $k = 0.1$, $A = 1$; **b** different conductivity ratios at $w = 0.2$, $A = 1$; **c** different Rayleigh numbers at $w = 0.2$, $A = 1$; **d** different aspect ratio at $w = 0.2$, $k = 1$

constant with increasing Rayleigh number. On the contrary, heat transfer increases with increasing Rayleigh number for $w = 0.05$ (thin wall). As indicated in Fig. 4 the value of thermal conductivity ratio affects the domination of the convection regime inside the enclosure as can be seen in Fig. 6b. Heat transfer increases with Rayleigh number for lower values of the thermal conductivity ratio. The conductive regime is dominant at $k = 0.1$ even in the triangular enclosure. Further, the effect of wall conductivity on the heat transfer is shown in Fig. 6c. It can be seen that heat transfer strongly depends on the thermal conductivity ratio and it increases with increasing thermal conductivity ratio. Heat transfer also increases with increasing aspect ratio parameter of the thick walled enclosure due to changing volume of the triangle. The aspect ratio becomes less effective especially at higher Rayleigh numbers as shown in Fig. 6d. Figure 7a is presented to see the effect of the Rayleigh number on the local Nusselt number along the porous side of the vertical wall for $k = 0.1$, $w = 0.05$ and $A = 1$. It shows that heat transfer decreases near the intersection point of the top of the solid wall and inclined wall of the enclosure. In that part, conductive heat transfer is dominant over the convection due to motionless fluid. The effect of the parameter w on the local Nusselt number is presented in Fig. 7b. As we mentioned above the volume of the triangle is directly related to the thickness of the solid wall. Thus, the distance of the porous side of the solid wall is decreased with the increase of the wall thickness. Higher values of Nu are obtained for the thinner wall ($w = 0.05$). At this value of w, heat transfer decreases from bottom to top of the solid wall. On the contrary, it becomes constant for higher value of w due to decreasing convection regime. Further, the effect of thermal conductivity k on the local Nusselt number is given in Fig. 7c for $Ra = 1{,}000$, $w = 0.1$ and $A = 1$. The figure shows that values of the local Nusselt number become constant at $k = 0.1$ due to the domination of conduction heat transfer regime. On the other hand, higher values of the local Nusselt are obtained for higher values of thermal conductivity. Its value becomes smaller at the intersection point of the solid and inclined wall due to motionless fluid. Finally, effects of the aspect ratio parameter A on the local Nusselt number are plotted in Fig. 7d. The figure shows that higher values of the local Nusselt number are formed for higher values of aspect ratio due to low affection of solid wall. The values of the local Nusselt number decrease along the wall and increase near the top wall.

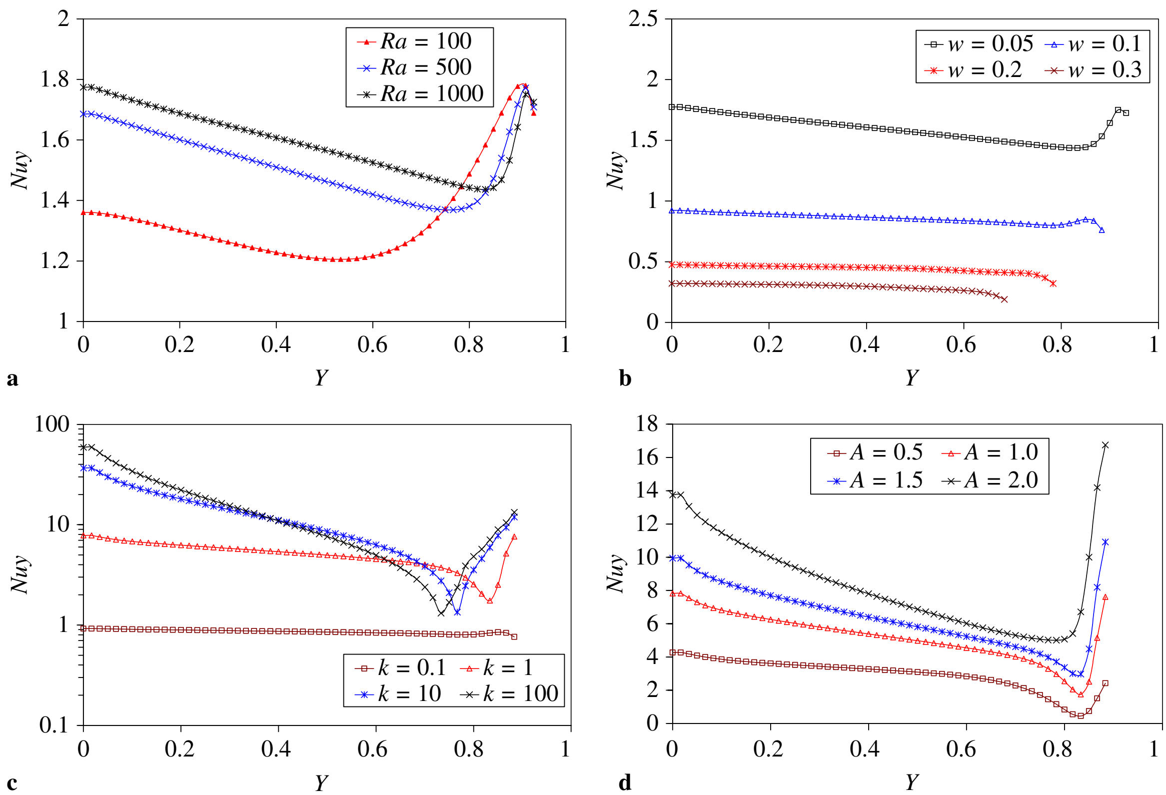

Fig. 7 Local Nusselt number variation: **a** different Rayleigh number at $k = 0.1$, $w = 0.05$, $A = 1$; **b** different thickness ratios of the vertical wall at $k = 0.1$, $Ra = 1{,}000$, $A = 1$; **c** different conductivity ratios at $Ra = 1{,}000$, $w = 0.1$, $A = 1$; **d** for different aspect ratios at $Ra = 1{,}000$, $w = 0.1$, $k = 1$

5 Conclusion

A numerical study of conduction–natural convection heat transfer for a thick walled triangular enclosure filled with a porous medium has been carried out. It is found that heat transfer increases with increasing aspect ratio A of the enclosure and that heat transfer is an increasing function of the Rayleigh number. For small values of the Rayleigh number the heat transfer regime is mainly conduction. Heat transfer is a decreasing function of thermal conductivity ratio. The conduction regime is dominant over the convection regime for higher dimensionless thickness of the solid wall. The strength of the circulation of the fluid-saturated porous medium is much higher for thin vertical walls of the enclosure. Finally, a solid vertical wall can be used as a control parameter for both heat transfer and flow fields.

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