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Moving antiplane shear crack in transversely isotropic magnetoelectroelastic media

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Abstract An antiplane shear strip crack moving uniformly in transversely isotopic magnetoelectroelastic media when subjected to representative non-constant crack-face loading conditions is studied. Readily calculable explicit closed-form representations are determined and discussed for the components of the stress, electric and magnetic fields created throughout the material. Representative numerical data are presented. Alternative boundary conditions for which corresponding analyses can be derived analogously are listed.

1 Introduction

There has been an increasing interest during the last few years in predicting both experimentally and analytically the coupled effects of magnetoelectroelastic composite materials. These have exciting possibilities of exploitation in a wide range of developments in high technological intelligent devices. A thorough understanding of the behavior of cracks and flaws within these media is thus clearly desirable.

Various studies of such stationary cracks subjected to uniform loads have been undertaken. For example, Gao et al. [1,2] and Gao et al. [3] used an extended Stroh-type formalism with complex variable potential functions to examine a single crack, collinear cracks and interfacial cracks, respectively, while in two very closely related papers Wang and Mai [4,5] develop energy functions, path-independent conservative integrals and complex variable solutions for an antiplane crack problem. Recently, Zhao et al. [6], and see also the useful references therein, considered a magnetoelectroelastic medium with an elliptical cavity again using complex eigenvector–eigenvalue techniques.

An antiplane crack moving constantly under permeable crack-face conditions within a magnetoelectroelastic material with uniform boundary conditions at infinity has been discussed using integral transform methods by Hu and Li [7]. On the other hand, Li [8] was concerned with the transient response of such a crack under sudden specially uniform impact loading, by reducing the problem to a Fredholm integral equation of the second kind using Fourier and Laplace transforms. It was necessary to solve the resulting equation numerically and to perform a numerical inversion of the Laplace transform to obtain dynamic field intensity factors in the time domain.

However, it is well-established that continuous distributions of dislocations can be used to model slit-like cracks in isotropic elastic media. In using a six-dimensional eigenvalue extended Stroh formalism for thereby studying cracks in general anisotropic elastic media, Barnett and Asaro [9] suggest that the technique of dislocation layers appears “to be more straightforward and to facilitate computational convenience”.

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The purpose of the present paper is to demonstrate that the so-called dislocation layer technique is indeed especially suited for extension in a particularly convenient and quite straightforward manner to derive analytically explicit versatile expressions for the components of the fields created by a mode III crack moving through a transversely isotropic magnetoelectroelastic medium under non-uniform crack-face loading conditions. The foundations of this current analysis are provided by the recent results of Liu et al. [10] for a dislocation moving uniformly through such materials, which are presented as an illustrative example of their more generalized plane problem.

In Sect. 2, the basic formulation of the situation under consideration is presented, together with the required components of the elastic, electric and magnetic fields around a moving magnetoelectroelastic screw dislocation. An analysis and derivation of the resulting fields created around a moving antiplane shear crack subject to specific illustrative boundary conditions are then given in Sect. 3. Some readily calculated representative numerical results for an electrically and magnetically impermeable crack are presented graphically.

Analogous analyses can readily be derived as desired for a range of alternative boundary conditions.

2 Physical and theoretical formulation

A homogeneous magnetoelectroelastic medium which is transversely isotropic in its response is supposed to be everywhere at rest and stress-free in a natural reference state initially with a uniform density ρ.

A loaded plane Griffith-type strip crack of width $2c$ moves through the material parallel to its axis in its own plane with a uniform velocity. At a time t, it is assumed that the crack occupies the region $y = 0$, $vt - c < x < vt + c$, $-\infty < z < \infty$ of the x–z plane, relative to a fixed system of rectangular Cartesian coordinates (x, y, z) with the z-axis chosen as a symmetry axis and v its speed of propagation. It is convenient to define a moving coordinate ξ by

$$\xi = x - vt. \tag{1}$$

The components σ_{ij}, ε_{ij}, E_i, H_i, D_i and B_i, where i, $j = x$, y or z, of the stress tensor σ, strain tensor ε, electric field vector E, magnetic field vector H, electric displacement vector D and magnetic induction vector B, respectively, within the medium are related by the constitutive equations, which in matrix notation take the forms

$$\begin{bmatrix} \sigma_{xx} \\ \sigma_{yy} \\ \sigma_{zz} \\ \sigma_{yz} \\ \sigma_{xz} \\ \sigma_{xy} \end{bmatrix} = \begin{bmatrix} c_{11} & c_{12} & c_{13} & 0 & 0 & 0 \\ c_{12} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & 2c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 2c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{11} - c_{12} \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{bmatrix} - \begin{bmatrix} 0 & 0 & e_{31} \\ 0 & 0 & e_{31} \\ 0 & 0 & e_{33} \\ 0 & e_{15} & 0 \\ e_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix}$$
$$- \begin{bmatrix} 0 & 0 & h_{31} \\ 0 & 0 & h_{31} \\ 0 & 0 & h_{33} \\ 0 & h_{15} & 0 \\ h_{15} & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}, \tag{2}$$

$$\begin{bmatrix} D_x \\ D_y \\ D_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 2e_{15} & 0 \\ 0 & 0 & 0 & 2e_{15} & 0 & 0 \\ e_{31} & e_{31} & e_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{bmatrix} + \begin{bmatrix} \varepsilon_{11} & 0 & 0 \\ 0 & \varepsilon_{11} & 0 \\ 0 & 0 & \varepsilon_{33} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} + \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}, \tag{3}$$

and

$$\begin{bmatrix} B_x \\ B_y \\ B_z \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 0 & 2h_{15} & 0 \\ 0 & 0 & 0 & 2h_{15} & 0 & 0 \\ h_{31} & h_{31} & h_{33} & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \varepsilon_{xx} \\ \varepsilon_{yy} \\ \varepsilon_{zz} \\ \varepsilon_{yz} \\ \varepsilon_{xz} \\ \varepsilon_{xy} \end{bmatrix} + \begin{bmatrix} \alpha_{11} & 0 & 0 \\ 0 & \alpha_{11} & 0 \\ 0 & 0 & \alpha_{33} \end{bmatrix} \begin{bmatrix} E_x \\ E_y \\ E_z \end{bmatrix} + \begin{bmatrix} \mu_{11} & 0 & 0 \\ 0 & \mu_{11} & 0 \\ 0 & 0 & \mu_{33} \end{bmatrix} \begin{bmatrix} H_x \\ H_y \\ H_z \end{bmatrix}. \tag{4}$$

Here, in the usual contracted Voigt's notation with i and j taking integer values, c_{ij}, e_{ij}, h_{ij}, ε_{ij}, α_{ij} and μ_{ij} are the elastic, piezoelectric, piezomagnetic, dielectric, magnetoelectric and magnetic permeability constants, respectively.

Two potentials, the electric potential, ϕ_e, and the magnetic potential, ϕ_m, can be defined which enable E and H to be expressed as

$$\mathbf{E} = -\nabla\phi_e, \quad \mathbf{H} = -\nabla\phi_m. \tag{5}$$

Non-constant mechanical, electric and magnetic loads, which yield a mode III antiplane deformation, are applied symmetrically to the two faces of the moving crack, with the medium remaining undisturbed at infinity. Recently, Wang and Mai [11] have discussed the applicability of specific conditions of electrically and/or magnetically permeable and/or impermeable crack faces. Eight such combinations of relevant boundary conditions, corresponding to those given by Hu and Li [12] for a magnetoelectroelastic strip, are for $|\xi| < c$:

$$\text{Case 1:}\ \sigma_{yz}(\xi,0) = \mathcal{T}(\xi), \quad D_y(\xi,0) = \mathcal{D}(\xi), \quad B_y(\xi,0) = \mathcal{B}(\xi), \tag{6}$$
$$\text{Case 2:}\ \varepsilon_{yz}(\xi,0) = \mathcal{S}(\xi), \quad D_y(\xi,0) = \mathcal{E}(\xi), \quad B_y(\xi,0) = \mathcal{B}(\xi), \tag{7}$$
$$\text{Case 3:}\ \sigma_{yz}(\xi,0) = \mathcal{T}(\xi), \quad E_y(\xi,0) = \mathcal{E}(\xi), \quad B_y(\xi,0) = \mathcal{B}(\xi), \tag{8}$$
$$\text{Case 4:}\ \varepsilon_{yz}(\xi,0) = \mathcal{S}(\xi), \quad D_y(\xi,0) = \mathcal{D}(\xi), \quad B_y(\xi,0) = \mathcal{B}(\xi), \tag{9}$$
$$\text{Case 5:}\ \sigma_{yz}(\xi,0) = \mathcal{T}(\xi), \quad D_y(\xi,0) = \mathcal{D}(\xi), \quad H_y(\xi,0) = \mathcal{H}(\xi), \tag{10}$$
$$\text{Case 6:}\ \varepsilon_{yz}(\xi,0) = \mathcal{S}(\xi), \quad E_y(\xi,0) = \mathcal{E}(\xi), \quad H_y(\xi,0) = \mathcal{H}(\xi), \tag{11}$$
$$\text{Case 7:}\ \sigma_{yz}(\xi,0) = \mathcal{T}(\xi), \quad E_y(\xi,0) = \mathcal{E}(\xi), \quad H_y(\xi,0) = \mathcal{H}(\xi), \tag{12}$$
$$\text{Case 8:}\ \varepsilon_{yz}(\xi,0) = \mathcal{S}(\xi), \quad D_y(\xi,0) = \mathcal{D}(\xi), \quad H_y(\xi,0) = \mathcal{H}(\xi), \tag{13}$$

with the non-uniform functions $\mathcal{T}$, $\mathcal{D}$, $\mathcal{B}$, $\mathcal{S}$, $\mathcal{E}$ and $\mathcal{H}$ specified.

As a particular illustration, the subsequent analysis in this paper is focused upon Case 1 which is prescribed by the conditions (6). The corresponding detailed manipulations which are necessary for studying the Cases 2–8 can then be developed analogously by an interested reader.

It is relevant to consider as an antecedent a "magnetoelectroelastic screw dislocation" moving in the material, which has not only the traditional discontinuity (the Burgers vector $\boldsymbol{b}$ of the elastic dislocation) in the elastic displacement $\boldsymbol{u}$ but also a discontinuity (the strength b_4 of the charge dipole line) in the electric potential ϕ_e and a discontinuity (the strength b_5 of the magnetic dipole line) in the magnetic potential ϕ_m. These jumps are given, respectively, for $\xi > 0$, by

$$\begin{aligned} &\mathbf{u}^{\mathrm{III}}(\xi,0+) - \mathbf{u}^{\mathrm{III}}(\xi,0-) = (0,0,-b), \\ &\phi_e^{\mathrm{III}}(\xi,0+) - \phi_e^{\mathrm{III}}(\xi,0-) = -b_4, \\ &\phi_m^{\mathrm{III}}(\xi,0+) - \phi_m^{\mathrm{III}}(\xi,0-) = -b_5. \end{aligned} \tag{14}$$

Throughout, the superscript III refers to quantities associated with a mode III deformation.

The generalized plane motion of a dislocation in an anisotropic elastic medium with piezoelectric, piezomagnetic and magnetoelectric effects has been considered by Liu et al. [10], who as an example present more explicit results for the transversely isotropic medium studied here. It is these which stimulated the current analysis.

With several material constants and parameters renamed and regrouped to highlight the important roles of, for example, κ_{em}, δ_{em}, μ_{em} and v_{em}, it can be deduced from the results of Liu et al. [10] that for a moving magnetoelectroelastic screw dislocation situated at the origin in a transversely isotropic medium

$$u_z^{\mathrm{III}}(\xi, y) = \frac{b}{2\pi}\tan^{-1}\left(\frac{\kappa_{em} y}{\xi}\right), \tag{15}$$

$$\phi_e^{\mathrm{III}}(\xi, y) = \frac{1}{2\pi}\left[b\frac{\mu_{11}e_{15} - \alpha_{11}h_{15}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}\tan^{-1}\left\{\frac{(\kappa_{em}-1)\xi y}{\xi^2 + \kappa_{em} y^2}\right\} + b_4\tan^{-1}\left(\frac{y}{\xi}\right)\right], \tag{16}$$

$$\phi_m^{\mathrm{III}}(\xi, y) = \frac{1}{2\pi}\left[b\frac{\varepsilon_{11}h_{15} - \alpha_{11}e_{15}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}\tan^{-1}\left\{\frac{(\kappa_{em}-1)\xi y}{\xi^2 + \kappa_{em} y^2}\right\} + b_5\tan^{-1}\left(\frac{y}{\xi}\right)\right], \tag{17}$$

where

$$\kappa_{em} = \sqrt{1 - \delta_{em}^2}, \quad \delta_{em}^2 = \frac{\rho v^2}{\mu_{em}} \tag{18}$$

with the magnetoelectroelastically stiffened elastic constant μ_{em} given by

$$\mu_{em} = c_{44} + \frac{e_{15}^2\mu_{11} + h_{15}^2\varepsilon_{11} - 2e_{15}h_{15}\alpha_{11}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}. \tag{19}$$

The corresponding non-zero components of the elastic, electric and magnetic fields then follow from the constitutive equations (2) to (4) in the form

$$\sigma_{xz}^{\mathrm{III}}(\xi, y) = -\frac{by}{2\pi}\left(\frac{\kappa_{em}\mu_{em}}{\xi^2 + \kappa_{em}^2 y^2} - \frac{\mu_{em} - c_{44}}{\xi^2 + y^2}\right) - \frac{b_4 y}{2\pi}\frac{e_{15}}{\xi^2 + y^2} - \frac{b_5 y}{2\pi}\frac{h_{15}}{\xi^2 + y^2}, \tag{20}$$

$$\sigma_{yz}^{\mathrm{III}}(\xi, y) = \frac{b\xi}{2\pi}\left(\frac{\kappa_{em}\mu_{em}}{\xi^2 + \kappa_{em}^2 y^2} - \frac{\mu_{em} - c_{44}}{\xi^2 + y^2}\right) + \frac{b_4\xi}{2\pi}\frac{e_{15}}{\xi^2 + y^2} + \frac{b_5\xi}{2\pi}\frac{h_{15}}{\xi^2 + y^2}, \tag{21}$$

$$\varepsilon_{xz}^{\mathrm{III}}(\xi, y) = -\frac{by}{4\pi}\frac{\kappa_{em}}{\xi^2 + \kappa_{em}^2 y^2}, \tag{22}$$

$$\varepsilon_{yz}^{\mathrm{III}}(\xi, y) = \frac{b\xi}{4\pi}\frac{\kappa_{em}}{\xi^2 + \kappa_{em}^2 y^2}, \tag{23}$$

$$E_x^{\mathrm{III}}(\xi, y) = \frac{by}{2\pi}\frac{\mu_{11}e_{15} - \alpha_{11}h_{15}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}\left(\frac{\kappa_{em}}{\xi^2 + \kappa_{em}^2 y^2} - \frac{1}{\xi^2 + y^2}\right) + \frac{b_4}{2\pi}\frac{y}{\xi^2 + y^2}, \tag{24}$$

$$E_y^{\mathrm{III}}(\xi, y) = -\frac{b\xi}{2\pi}\frac{\mu_{11}e_{15} - \alpha_{11}h_{15}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}\left(\frac{\kappa_{em}}{\xi^2 + \kappa_{em}^2 y^2} - \frac{1}{\xi^2 + y^2}\right) - \frac{b_4}{2\pi}\frac{\xi}{\xi^2 + y^2}, \tag{25}$$

$$H_x^{\mathrm{III}}(\xi, y) = \frac{by}{2\pi}\frac{\varepsilon_{11}h_{15} - \alpha_{11}e_{15}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}\left(\frac{\kappa_{em}}{\xi^2 + \kappa_{em}^2 y^2} - \frac{1}{\xi^2 + y^2}\right) + \frac{b_5}{2\pi}\frac{y}{\xi^2 + y^2}, \tag{26}$$

$$H_y^{\mathrm{III}}(\xi, y) = -\frac{b\xi}{2\pi}\frac{\varepsilon_{11}h_{15} - \alpha_{11}e_{15}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}\left(\frac{\kappa_{em}}{\xi^2 + \kappa_{em}^2 y^2} - \frac{1}{\xi^2 + y^2}\right) - \frac{b_5}{2\pi}\frac{\xi}{\xi^2 + y^2}, \tag{27}$$

$$D_x^{\mathrm{III}}(\xi, y) = -\frac{(be_{15} - b_4\varepsilon_{11} - b_5\alpha_{11})}{2\pi}\frac{y}{\xi^2 + y^2}, \tag{28}$$

$$D_y^{\mathrm{III}}(\xi, y) = \frac{(be_{15} - b_4\varepsilon_{11} - b_5\alpha_{11})}{2\pi}\frac{\xi}{\xi^2 + y^2}, \tag{29}$$

$$B_x^{\mathrm{III}}(\xi, y) = -\frac{(bh_{15} - b_4\alpha_{11} - b_5\mu_{11})}{2\pi}\frac{y}{\xi^2 + y^2}, \tag{30}$$

$$B_y^{\mathrm{III}}(\xi, y) = \frac{(bh_{15} - b_4\alpha_{11} - b_5\mu_{11})}{2\pi}\frac{\xi}{\xi^2 + y^2}. \tag{31}$$

On the boundary $y = 0$, where the boundary conditions (6) are to be imposed, it is crucial to observe that

$$\sigma_{yz}^{\mathrm{III}}(\xi, 0) = \frac{b\{c_{44} - (1 - \kappa_{em})\mu_{em}\}}{2\pi\xi} + \frac{b_4 e_{15}}{2\pi\xi} + \frac{b_5 h_{15}}{2\pi\xi}, \tag{32}$$

$$D_y^{\mathrm{III}}(\xi, 0) = \frac{b e_{15}}{2\pi\xi} - \frac{b_4 \varepsilon_{11}}{2\pi\xi} - \frac{b_5 \alpha_{11}}{2\pi\xi}, \tag{33}$$

$$B_y^{\mathrm{III}}(\xi, 0) = \frac{b h_{15}}{2\pi\xi} - \frac{b_4 \alpha_{11}}{2\pi\xi} - \frac{b_5 \mu_{11}}{2\pi\xi}. \tag{34}$$

The analogous expressions for $\varepsilon_{yz}^{\mathrm{III}}(\xi, 0)$, $E_y^{\mathrm{III}}(\xi, 0)$ and $H_y^{\mathrm{III}}(\xi, 0)$ which would be needed to consider correspondingly the Cases 2–8 in Eqs. (7)–(13) are similarly dependent upon $1/\xi$.

3 Antiplane shear crack

The well-established fundamental dislocation layer method, which was developed for a purely elastic material, whereby a loaded crack is analyzed by replacing it by an equivalent continuous planar distribution of dislocations is now developed correspondingly for a crack within a magnetoelectroelastic medium. The basic technique is adapted by spreading an array of magnetoelectroelastic screw dislocations of the type prescribed in Sect. 2 over the region $|\xi| < c$, $y = 0$, $-\infty < z < \infty$.

In order to model the mode III crack subject to conditions (6), the densities of the proposed distributions of elastic dislocations, charge dipole lines and magnetic dipole lines are taken to be $f(\xi)$, $f_4(\xi)$ and $f_5(\xi)$, respectively. Then, from Eqs. (32)–(34), it follows that the corresponding stress, electric displacement and magnetic induction components at a point on the ξ-axis are given by

$$\sigma_{yz}(\xi, 0) = \frac{b\{c_{44} - (1 - \kappa_{em})\mu_{em}\}}{2\pi} \int_{-c}^{c} \frac{f(\xi')}{\xi - \xi'} d\xi' + \frac{b_4 e_{15}}{2\pi} \int_{-c}^{c} \frac{f_4(\xi')}{\xi - \xi'} d\xi' + \frac{b_5 h_{15}}{2\pi} \int_{-c}^{c} \frac{f_5(\xi')}{\xi - \xi'} d\xi', \tag{35}$$

$$D_y(\xi, 0) = \frac{b e_{15}}{2\pi} \int_{-c}^{c} \frac{f(\xi')}{\xi - \xi'} d\xi' - \frac{b_4 \varepsilon_{11}}{2\pi} \int_{-c}^{c} \frac{f_4(\xi')}{\xi - \xi'} d\xi' - \frac{b_5 \alpha_{11}}{2\pi} \int_{-c}^{c} \frac{f_5(\xi')}{\xi - \xi'} d\xi', \tag{36}$$

$$B_y(\xi, 0) = \frac{b h_{15}}{2\pi} \int_{-c}^{c} \frac{f(\xi')}{\xi - \xi'} d\xi' - \frac{b_4 \alpha_{11}}{2\pi} \int_{-c}^{c} \frac{f_4(\xi')}{\xi - \xi'} d\xi' - \frac{b_5 \mu_{11}}{2\pi} \int_{-c}^{c} \frac{f_5(\xi')}{\xi - \xi'} d\xi'. \tag{37}$$

It is apparent from evaluating $\sigma_{yz}(\xi, 0)$, $D_y(\xi, 0)$ and $B_y(\xi, 0)$ from the Plemelj formulae that the integrals throughout Eqs. (35)–(37) should be interpreted as Cauchy principal value integrals. The system of three simultaneous equations which arises by equating the expressions (35)–(37) to the imposed boundary conditions (6) can be solved to yield

$$\int_{-c}^{c} \frac{f(\xi')}{\xi - \xi'} d\xi' = \frac{2\pi}{b(\varepsilon_{11}\mu_{11} - \alpha_{11}^2)\kappa_{em}\mu_{em}} \left\{(\varepsilon_{11}\mu_{11} - \alpha_{11}^2)\mathcal{T}(\xi) + (e_{15}\mu_{11} - h_{15}\alpha_{11})\mathcal{D}(\xi) - (e_{15}\alpha_{11} - h_{15}\varepsilon_{11})\mathcal{B}(\xi)\right\}, \tag{38}$$

$$\int_{-c}^{c} \frac{f_4(\xi')}{\xi - \xi'} d\xi' = \frac{2\pi}{b_4(\varepsilon_{11}\mu_{11} - \alpha_{11}^2)\kappa_{em}\mu_{em}} \{(e_{15}\mu_{11} - h_{15}\alpha_{11})\mathcal{T}(\xi) - \left[\mu_{11}\{c_{44} - (1 - \kappa_{em})\mu_{em}\} + h_{15}^2\right]\mathcal{D}(\xi) + [\alpha_{11}\{c_{44} - (1 - \kappa_{em})\mu_{em}\} + e_{15}h_{15}]\ \mathcal{B}(\xi)\}, \tag{39}$$

$$\int_{-c}^{c} \frac{f_5(\xi')}{\xi - \xi'} d\xi' = \frac{2\pi}{b_5(\varepsilon_{11}\mu_{11} - \alpha_{11}^2)\kappa_{em}\mu_{em}} \{-(e_{11}\alpha_{11} - h_{15}\varepsilon_{11})\mathcal{T}(\xi) + [\alpha_{11}\{c_{44} - (1 - \kappa_{em})\mu_{em}\} + e_{15}h_{15}]\ \mathcal{D}(\xi) - \left[\varepsilon_{11}\{c_{44} - (1 - \kappa_{em})\mu_{em}\} + e_{15}^2\right]\mathcal{B}(\xi)\right\}. \tag{40}$$

If the additional restrictions are imposed that the relative displacement, relative electric potential and relative magnetic potential all vanish at $\xi = \pm c$, the appropriate solutions for $f(\xi)$, $f_4(\xi)$ and $f_5(\xi)$ of these integral equations are deducible from the results of Muskhelishvili [13] and Gakhov [14] to be

$$f(\xi) = \frac{2}{\pi b(\varepsilon_{11}\mu_{11} - \alpha_{11}^2)\kappa_{em}\mu_{em}} \frac{1}{(c^2-\xi^2)^{\frac{1}{2}}} \int_{-c}^{c} \frac{(c^2-\xi'^2)^{\frac{1}{2}}}{\xi'-\xi} \{(\varepsilon_{11}\mu_{11}-\alpha_{11}^2)\mathcal{T}(\xi') + (e_{15}\mu_{11} - h_{15}\alpha_{11})\mathcal{D}(\xi') - (e_{15}\alpha_{11} - h_{15}\varepsilon_{11})\mathcal{B}(\xi')\} d\xi', \tag{41}$$

$$f_4(\xi) = \frac{2}{\pi b_4(\varepsilon_{11}\mu_{11} - \alpha_{11}^2)\kappa_{em}\mu_{em}} \frac{1}{(c^2-\xi^2)^{\frac{1}{2}}} \int_{-c}^{c} \frac{(c^2-\xi'^2)^{\frac{1}{2}}}{\xi'-\xi} \{(e_{15}\mu_{11} - h_{15}\alpha_{11})\mathcal{T}(\xi') - [\mu_{11}\{c_{44} - (1-\kappa_{em})\mu_{em}\} + h_{15}^2]\mathcal{D}(\xi') + [\alpha_{11}\{c_{44} - (1-\kappa_{em})\mu_{em}\} + e_{15}h_{15}]\mathcal{B}(\xi')\} d\xi', \tag{42}$$

$$f_5(\xi) = \frac{2}{\pi b_5(\varepsilon_{11}\mu_{11} - \alpha_{11}^2)\kappa_{em}\mu_{em}} \frac{1}{(c^2-\xi^2)^{\frac{1}{2}}} \int_{-c}^{c} \frac{(c^2-\xi'^2)^{\frac{1}{2}}}{\xi'-\xi} \{-(e_{15}\alpha_{11} - h_{15}\varepsilon_{11})\mathcal{T}(\xi') + [\alpha_{11}\{c_{44} - (1-\kappa_{em})\mu_{em}\} + e_{15}h_{15}]\mathcal{D}(\xi') - [\varepsilon_{11}\{c_{44} - (1-\kappa_{em})\mu_{em}\} + e_{15}^2]\mathcal{B}(\xi')\} d\xi'. \tag{43}$$

Having derived the above representations for the necessary densities $f(\xi)$, $f_4(\xi)$ and $f_5(\xi)$, all the components of the elastic, electric and magnetic fields can be directly calculated explicitly as desired from Eqs. (20)–(31) and (41)–(43).

To illustrate the type of detailed analyses which are necessary the expression for $\sigma_{yz}(\xi, y)$ is derived as a typical example. It is seen from Eq. (21) that

$$\sigma_{yz}(\xi,y) = \frac{b}{2\pi}\int_{-c}^{c}(\xi-\xi'')\left\{\frac{\kappa_{em}\mu_{em}}{(\xi-\xi'')^2+\kappa_{em}^2y^2} - \frac{\mu_{em}-c_{44}}{(\xi-\xi'')^2+y^2}\right\} f(\xi'')d\xi'' + \frac{b_4}{2\pi}\int_{-c}^{c}(\xi-\xi'')\frac{e_{15}}{(\xi-\xi'')^2+y^2} f_4(\xi'')d\xi'' + \frac{b_5}{2\pi}\int_{-c}^{c}(\xi-\xi'')\frac{h_{15}}{(\xi-\xi'')^2+y^2} f_5(\xi'')d\xi''. \tag{44}$$

Upon substitution of the expressions (41)–(43) this can be shown, after considerable algebraic manipulations, to take the form

$$\sigma_{yz}(\xi,y) = \frac{1}{\pi^2}\int_{-c}^{c}(c^2-\xi'^2)^{\frac{1}{2}}\mathcal{T}(\xi')\left[\int_{-c}^{c}\frac{(\xi-\xi'')d\xi''}{(c^2-\xi''^2)^{\frac{1}{2}}(\xi'-\xi'')\{(\xi-\xi'')^2+\kappa_{em}^2y^2\}}\right]d\xi' + \frac{1}{\pi^2}\frac{e_{15}\mu_{11}-h_{15}\alpha_{11}}{\varepsilon_{11}\mu_{11}-\alpha_{11}^2}\int_{-c}^{c}(c^2-\xi'^2)^{\frac{1}{2}}\mathcal{D}(\xi')\left[\int_{-c}^{c}\frac{(\xi-\xi'')}{(c^2-\xi''^2)^{\frac{1}{2}}(\xi'-\xi'')} \times\left\{\frac{1}{(\xi-\xi'')^2+\kappa_{em}^2y^2} - \frac{1}{(\xi-\xi'')^2+y^2}\right\}d\xi''\right]d\xi' - \frac{1}{\pi^2}\frac{e_{15}\alpha_{11}-h_{15}\varepsilon_{11}}{\varepsilon_{11}\mu_{11}-\alpha_{11}^2}\int_{-c}^{c}(c^2-\xi'^2)^{\frac{1}{2}}\mathcal{B}(\xi')\left[\int_{-c}^{c}\frac{(\xi-\xi'')}{(c^2-\xi''^2)^{\frac{1}{2}}(\xi'-\xi'')} \times\left\{\frac{1}{(\xi-\xi'')^2+\kappa_{em}^2y^2} - \frac{1}{(\xi-\xi'')^2+y^2}\right\}d\xi''\right]d\xi'. \tag{45}$$

For the sake of notational simplification, the functions $\mathcal{F}_k^F(\theta_k)$, $\mathcal{R}_k(\xi, y)$ and $\theta_k(\xi, y)$, for $k = \kappa_{em}$ and 1, and $F = \mathcal{T}$, $\mathcal{D}$ and $\mathcal{B}$, are now defined by

$$\mathcal{F}_k^F(\theta_k) = \frac{1}{\pi}\int_{-c}^{c} \frac{ky\cos\theta_k + (\xi - \xi')\sin\theta_k}{\mathcal{R}_k\left\{(\xi - \xi')^2 + k^2y^2\right\}}\left(c^2 - \xi'^2\right)^{\frac{1}{2}} F(\xi')d\xi', \tag{46}$$

$$\mathcal{R}_k e^{i\theta_k} = \left\{c^2 - (\xi + iky)^2\right\}^{\frac{1}{2}}, \tag{47}$$

with the branches of the square root function determined by choosing θ_k to be zero for $|\xi| < c$, $y = 0+$ and defined by analytic continuation elsewhere. Then it follows, by evaluating the expression (45) using the result (A.2) in the Appendix, that

$$\sigma_{yz}(\xi, y) = \mathcal{F}_{\kappa_{em}}^{\mathcal{T}}\left(\theta_{\kappa_{em}}\right) + \frac{e_{15}\mu_{11} - h_{15}\alpha_{11}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}\left\{\mathcal{F}_{\kappa_{em}}^{\mathcal{D}}\left(\theta_{\kappa_{em}}\right) - \mathcal{F}_1^{\mathcal{D}}(\theta_1)\right\}$$
$$-\frac{e_{15}\alpha_{11} - h_{15}\varepsilon_{11}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}\left\{\mathcal{F}_{\kappa_{em}}^{\mathcal{B}}\left(\theta_{\kappa_{em}}\right) - \mathcal{F}_1^{\mathcal{B}}(\theta_1)\right\}. \tag{48}$$

Analogously, Eqs. (20), (28)–(31), (A.1) and (A.2) lead to the representations

$$\sigma_{xz}(\xi, y) = -\frac{1}{\kappa_{em}}\mathcal{F}_{\kappa_{em}}^{\mathcal{T}}\left(\theta_{\kappa_{em}} - \frac{\pi}{2}\right) - \frac{e_{15}\mu_{11} - h_{15}\alpha_{11}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}\left\{\frac{1}{\kappa_{em}}\mathcal{F}_{\kappa_{em}}^{\mathcal{D}}\left(\theta_{\kappa_{em}} - \frac{\pi}{2}\right) - \mathcal{F}_1^{\mathcal{D}}\left(\theta_1 - \frac{\pi}{2}\right)\right\}$$
$$+\frac{e_{15}\alpha_{11} - h_{15}\varepsilon_{11}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}\left\{\frac{1}{\kappa_{em}}\mathcal{F}_{\kappa_{em}}^{\mathcal{B}}\left(\theta_{\kappa_{em}} - \frac{\pi}{2}\right) - \mathcal{F}_1^{\mathcal{B}}\left(\theta_1 - \frac{\pi}{2}\right)\right\}, \tag{49}$$

$$D_y(\xi, y) = \mathcal{F}_1^{\mathcal{D}}(\theta_1), \tag{50}$$

$$D_x(\xi, y) = -\mathcal{F}_1^{\mathcal{D}}\left(\theta_1 - \frac{\pi}{2}\right), \tag{51}$$

$$B_y(\xi, y) = \mathcal{F}_1^{\mathcal{B}}(\theta_1), \tag{52}$$

$$B_x(\xi, y) = -\mathcal{F}_1^{\mathcal{B}}\left(\theta_1 - \frac{\pi}{2}\right). \tag{53}$$

It is worthy of note at this stage that, with the general boundary conditions (6) imposed, Eqs. (48) and (49) exhibit explicitly that the stress components depend upon all of $\mathcal{T}$, $\mathcal{D}$ and $\mathcal{B}$ together with the magnetoelectroelastic material constants and the speed of the crack. On the other hand, however, Eqs. (50)–(53) show that the electric displacement components depend upon $\mathcal{D}$ alone, while the magnetic induction components are only dependent upon $\mathcal{B}$.

The important features of the distributions of these components near a crack tip become apparent by setting

$$\xi = c + r\cos\alpha, \quad y = r\sin\alpha, \tag{54}$$

and considering situations where $r \ll c$, into Eqs. (48)–(53). Then, from Eq. (47), the corresponding approximations to $\mathcal{R}_k$ and θ_k as $r \to 0$ can be shown to be

$$\mathcal{R}_k \sim \left\{2cr\left(\cos^2\alpha + k^2\sin^2\alpha\right)^{\frac{1}{2}}\right\}^{\frac{1}{2}}, \tag{55}$$

$$\theta_k \sim -(\pi - \Phi_k)/2, \tag{56}$$

with

$$\Phi_k = \tan^{-1}(k\tan\alpha), \tag{57}$$

where $\tan^{-1}(\ldots)$ is understood to indicate the principal value of the inverse tangent for $0 \le \alpha \le \pi/2$ and π plus the principal value for $\pi/2 \le \alpha \le \pi$. Substituting these into Eqs. (48)–(53) and (46), and putting

$$\Delta_{\kappa_{em}} = \left(\cos^2\alpha + \kappa_{em}^2\sin^2\alpha\right)^{\frac{1}{4}}, \tag{58}$$

yields

$$\sigma_{xz}(r,\alpha) \sim -\frac{1}{\kappa_{em}\Delta_{\kappa_{em}}}\sin\left(\frac{\Phi_{\kappa_{em}}}{2}\right)\frac{K_{\mathcal{T}}}{\sqrt{r}} - \frac{e_{15}\mu_{11} - h_{15}\alpha_{11}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}\left\{\frac{1}{\kappa_{em}\Delta_{\kappa_{em}}}\sin\left(\frac{\Phi_{\kappa_{em}}}{2}\right) - \sin\left(\frac{\alpha}{2}\right)\right\}\left(\frac{K_{\mathcal{D}}}{\sqrt{r}} - \frac{K_{\mathcal{B}}}{\sqrt{r}}\right), \tag{59}$$

$$\sigma_{yz}(r,\alpha) \sim \frac{1}{\Delta_{\kappa_{em}}}\cos\left(\frac{\Phi_{\kappa_{em}}}{2}\right)\frac{K_{\mathcal{T}}}{\sqrt{r}} + \frac{e_{15}\mu_{11} - h_{15}\alpha_{11}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}\left\{\frac{1}{\Delta_{\kappa_{em}}}\cos\left(\frac{\Phi_{\kappa_{em}}}{2}\right) - \cos\left(\frac{\alpha}{2}\right)\right\}\left(\frac{K_{\mathcal{D}}}{\sqrt{r}} - \frac{K_{\mathcal{B}}}{\sqrt{r}}\right), \tag{60}$$

$$D_x(r,\alpha) \sim -\sin\left(\frac{\alpha}{2}\right)\frac{K_{\mathcal{D}}}{\sqrt{r}}, \tag{61}$$

$$D_y(r,\alpha) \sim \cos\left(\frac{\alpha}{2}\right)\frac{K_{\mathcal{D}}}{\sqrt{r}}, \tag{62}$$

$$B_x(r,\alpha) \sim -\sin\left(\frac{\alpha}{2}\right)\frac{K_{\mathcal{B}}}{\sqrt{r}}, \tag{63}$$

$$B_y(r,\alpha) \sim \cos\left(\frac{\alpha}{2}\right)\frac{K_{\mathcal{B}}}{\sqrt{r}}, \tag{64}$$

as $r \to 0$. Here the field intensity factors, $K_{\mathcal{T}}$, $K_{\mathcal{D}}$ and $K_{\mathcal{B}}$, are defined for $F = \mathcal{T}, \mathcal{D}$ and $\mathcal{B}$ by

$$K_F = -\frac{1}{\pi\sqrt{2c}}\int_{-c}^{c}\left(\frac{c+\xi'}{c-\xi'}\right)^{\frac{1}{2}} F(\xi')d\xi'. \tag{65}$$

Near a crack tip, it then finally follows from Eqs. (59) and (60), (61) and (62), and (63) and (64) that the components $\sigma_{\alpha z}$, D_α and B_α of stress, electric displacement and magnetic induction are given by

$$\sigma_{\alpha z}(r,\alpha) \sim \frac{K_{\mathcal{T}}}{\sqrt{r}\Delta_{\kappa_{em}}}\left\{\frac{1}{\kappa_{em}}\sin\left(\frac{\Phi_{\kappa_{em}}}{2}\right)\sin\alpha + \cos\left(\frac{\Phi_{\kappa_{em}}}{2}\right)\cos\alpha\right\} + \frac{e_{15}\mu_{11} - h_{15}\alpha_{11}}{\varepsilon_{11}\mu_{11} - \alpha_{11}^2}\frac{K_{\mathcal{D}} - K_{\mathcal{B}}}{\sqrt{r}}\left[\frac{1}{\Delta_{\kappa_{em}}}\left\{\frac{1}{\kappa_{em}}\sin\left(\frac{\Phi_{\kappa_{em}}}{2}\right)\sin\alpha + \cos\left(\frac{\Phi_{\kappa_{em}}}{2}\right)\cos\alpha\right\} - \cos\left(\frac{\alpha}{2}\right)\right], \tag{66}$$

$$D_\alpha(r,\alpha) \sim \frac{K_{\mathcal{D}}}{\sqrt{r}}\cos\left(\frac{\alpha}{2}\right), \tag{67}$$

$$B_\alpha(r,\alpha) \sim \frac{K_{\mathcal{B}}}{\sqrt{r}}\cos\left(\frac{\alpha}{2}\right). \tag{68}$$

As in the analogous classical isotropic elastic situation, all the field components have been shown to exhibit a $1/\sqrt{r}$ crack-tip behavior and depend upon the imposed crack-face excitations only through the intensity factors defined by Eq. (65).

All the above results, as would be expected, are indeed in agreement when the magnetoelectric effects are removed with those deducible (by putting $c_{55} = c_{44}$) from the corresponding expressions derived by Tupholme [15] for cracks moving in purely elastic orthotropic crystals.

It is instructive to observe that in practice therefore the interesting magnitude of the stress concentration around the crack tip can be increased or decreased as desired by varying the applied mechanical, electric and magnetic loads depending upon the magnetoelectroelastic constants of the material.

Further, it should be noted from Eqs. (41)–(43) that the present analysis is not applicable when $\kappa_{em} = 0$. From the definitions (18), this value is attained when $\delta_{em} = 0$ which implies that the speed of the crack, v, equals that of the magnetoelectrically stiffened bulk shear wave speed, v_{em}, given by

$$v_{em} = \sqrt{\mu_{em}/\rho}. \tag{69}$$

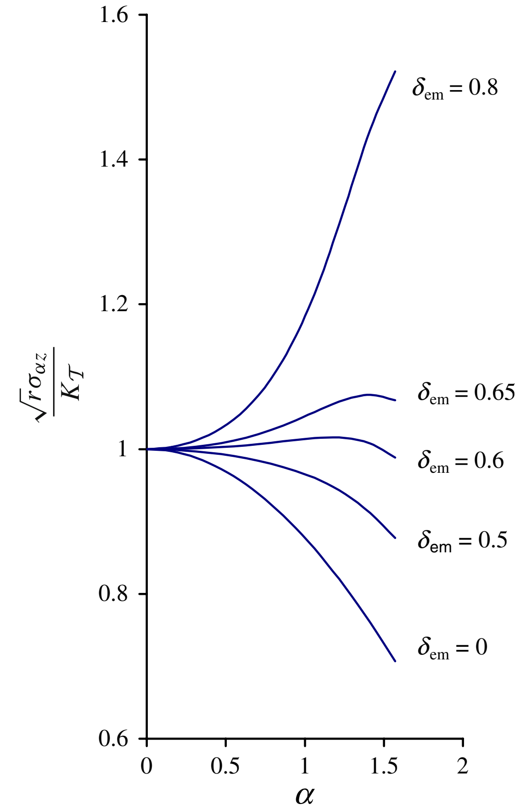

Fig. 1 Distribution of the scaled stress component $\sqrt{r}\sigma_{\alpha z}/K_{\mathcal{T}}$ around the crack tip for a range of valued of δ_{em}

The values of the relevant material moduli vary with the manufacturing process and have been discussed and presented by various authors, including for example Sue et al. [16], Hu and Li [7], Huang et al. [17] and Li [18]. Typical representative data are taken here to be $c_{44} = 4.53 \times 10^{10}\,\mathrm{Nm}^{-2}$, $e_{15} = 11.6\ \mathrm{Cm}^{-2}$, $h_{15} = 550\ \mathrm{NA}^{-1}\mathrm{m}^{-1}$, $\varepsilon_{11} = 0.8 \times 10^{-10}\ \mathrm{C}^2\mathrm{N}^{-1}\mathrm{m}^{-2}$, $\alpha_{11} = 0.5 \times 10^{-11}\ \mathrm{NsV}^{-1}\mathrm{C}^{-1}$, $\mu_{11} = -5.9 \times 10^{-4}\ \mathrm{Ns}^2\mathrm{C}^{-2}$, $\rho = 5.3 \times 10^3\ \mathrm{kg\ m}^{-3}$, for which, from Eqs. (19) and (69), the corresponding value of v_{em} is therefore about $18050\,\mathrm{ms}^{-1}$.

Numerical results illustrating the distribution of the scaled stress component $\sqrt{r}\sigma_{\alpha z}\big/K_{\mathcal{T}}$ around the crack tip, $\xi = c$, for a range of values of the scaled speed $v\big/v_{em}(= \delta_{em})$ in the commonly adopted illustrative case of an electrically and magnetically impermeable crack for which $\mathcal{D}(\xi) = \mathcal{B}(\xi) = 0$ can be deduced from the representation (66) and are presented graphically in Fig. 1. It can be seen that the stress component $\sigma_{\alpha z}$ develops a non-forward maximum, as the speed of the crack becomes sufficiently high. The critical speed at which this first occurs, for the above data, is given by $\delta_{em} \approx 0.58$ with the corresponding value of about $v = 10469\,\mathrm{ms}^{-1}$.

Appendix

Contour integrations can be used to verify that

$$\int_{-c}^{c} \frac{d\xi''}{\left(c^2 - \xi''^2\right)^{\frac{1}{2}} (\xi' - \xi'') \left\{(\xi - \xi'')^2 + k^2 y^2\right\}} = \frac{\pi \left\{ky \sin\Theta - (\xi - \xi') \cos\Theta\right\}}{yk\mathcal{R}\left\{(\xi - \xi')^2 + k^2 y^2\right\}}, \tag{A.1}$$

$$\int_{-c}^{c} \frac{(\xi - \xi'')d\xi''}{\left(c^2 - \xi''^2\right) (\xi' - \xi'') \left\{(\xi - \xi'')^2 + k^2 y^2\right\}} = \frac{\pi \left\{ky \cos\Theta + (\xi - \xi') \sin\Theta\right\}}{\mathcal{R}\left\{(\xi - \xi')^2 + k^2 y^2\right\}} \tag{A.2}$$

for constant k, where the branches of

$$\mathcal{R}e^{i\Theta} = \left\{c^2 - (\xi + iky)^2\right\}^{\frac{1}{2}}$$

are chosen similarly to those in Eq. (47).

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