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MHD boundary-layer flow of a micropolar fluid past a wedge with variable wall temperature

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Summary. The steady laminar MHD boundary-layer flow past a wedge immersed in an incompressible micropolar fluid in the presence of a variable magnetic field is investigated. The governing partial differential equations are transformed to the ordinary differential equations using similarity variables, and then solved numerically using a finite-difference scheme known as the Keller-box method. Numerical results show that the micropolar fluids display drag reduction and consequently reduce the heat transfer rate at the surface, compared to the Newtonian fluids. The opposite trends are observed for the effects of the magnetic field on the fluid flow and heat transfer characteristics.

Nomenclature

a, b	positive constants
A	non-dimensional constant of integration
$B(x)$	magnetic field
B_0	uniform magnetic field
C_f	skin friction coefficient
f	dimensionless stream function
Gr_x	local Grashof number
h	dimensionless microrotation or angular velocity
i	dimensionless microinertia
j	microinertia
k	thermal conductivity
K	material parameter
m	Falkner-Skan power-law parameter
M	magnetic parameter
n	temperature exponent parameter
N	component of the microrotation vector normal to the xy-plane
Nu_x	local Nusselt number
Pr	Prandtl number
q_w	heat transfer from the surface of the wedge
Re_x	local Reynolds number
T	fluid temperature
$T_w(x)$	wedge temperature
T_∞	ambient temperature
u, v	velocity components along the x- and y-directions, respectively

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$U(x)$	free stream velocity
x, y	Cartesian coordinates along the surface and normal to it, respectively

Greek symbols

α	thermal diffusivity
β	Hartree pressure gradient parameter
γ	spin-gradient viscosity
η	similarity variable
θ	dimensionless temperature
κ	vortex viscosity
μ	dynamic viscosity
ν	kinematic viscosity
ρ	fluid density
σ	electrical conductivity
τ_w	skin friction from the surface of the wedge
ψ	stream function

Subscripts

w	condition at the surface of the wedge
∞	condition at infinity

Superscript

$'$	differentiation with respect to η or $\hat{\eta}$

1 Introduction

Research interest in the flows of micropolar fluids has increased substantially over the past decades due to the occurrence of these fluids in industrial processes. In the history of fluid mechanics, Eringen [1], [2] is a pioneering researcher who has formulated the theory of micropolar fluids. This theory takes into account the effect arising from the local structure and micro-motions of the fluid elements, and is able to describe the behavior of the polymeric additives, animal blood, lubricants, liquid crystals, dirty oils, solutions of colloidal suspensions, etc. The theory requires that one must solve an additional transport equation representing the principle of conservation of local angular momentum, as well as the usual transport equations for the conservation of mass and momentum.

A thorough review of micropolar fluid mechanics was given by Ariman et al. [3], [4]. Recently, Kim [5] and Kim and Kim [6] have considered the steady boundary-layer flow of a micropolar fluid past a fixed wedge with constant surface temperature and constant surface heat flux, respectively. The similarity variables found by Falkner and Skan [7] were employed to reduce the governing partial differential equations to ordinary differential equations. Unfortunately, the angular momentum equation was not correctly derived so that the results of these papers [5], [6] are inaccurate. Therefore, the objective of this paper is to improve and extend the work of Kim [5] by considering the effect of variable magnetic field on the fluid flow and heat transfer characteristics for a fixed wedge with variable surface temperature. The effect of a transverse magnetic field on a porous wedge placed symmetrically with respect to the flow direction in a non-Newtonian fluid has been considered by Hady and Hassanien [8]. Watanabe and Pop [9] presented the numerical results of MHD free convection flow over a wedge in the presence of a magnetic field, while Kafoussias and Nanousis [10] investigated the MHD laminar boundary-layer flow over a permeable wedge. Both of these papers considered a wedge immersed in a Newtonian fluid. Later, Yih [11] extended the work of Watanabe and Pop [12], by considering the MHD forced convection flow adjacent to a non-isothermal wedge. The former considered MHD boundary-layer flow over a flat plate in the presence of a transverse magnetic field. The effects of a constant magnetic field on the fluid flow and heat transfer characteristics were also considered in [13]–[16], while those of variable magnetic field were studied in [17]–[25]. Lykoudis [17] studied the natural convection adjacent to a vertical hot plate surrounded by an electrically conducting fluid in the presence of a magnetic field acting in the direction perpendicular to the induced movement caused by the buoyant forces, and found that similarity solutions exist when the intensity of the magnetic field changes with $x^{-1/4}$, where x is the coordinate measured in the direction of the flow. The existence of similarity solutions was then established by the experiment, as reported in [18].

2 Formulation of the problem

Consider the steady laminar boundary-layer flow past a wedge in an electrically conducting micropolar fluid in the presence of a magnetic field $B(x)$ applied in the normal direction to the walls of the wedge, as shown in Fig. 1. The induced magnetic field is assumed to be small. This implies a small magnetic Reynolds number $Re_m = \mu_0\sigma\, U(x) \ll 1$, where μ_0 is the magnetic permeability, σ is the electrical conductivity and $U(x)$ is the velocity outside the boundary-layer (inviscid flow). Under these assumptions, the boundary-layer equations are

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0, \tag{1}$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{dU}{dx} + \left(\frac{\mu+\kappa}{\rho}\right)\frac{\partial^2 u}{\partial y^2} + \frac{\kappa}{\rho}\frac{\partial N}{\partial y} + \frac{\sigma B^2(x)}{\rho}(U-u), \tag{2}$$

$$\rho j\left(u\frac{\partial N}{\partial x} + v\frac{\partial N}{\partial y}\right) = \frac{\partial}{\partial y}\left(\gamma\frac{\partial N}{\partial y}\right) - \kappa\left(2N + \frac{\partial u}{\partial y}\right), \tag{3}$$

$$u\frac{\partial j}{\partial x} + v\frac{\partial j}{\partial y} = 0, \tag{4}$$

$$u\frac{\partial T}{\partial x} + v\frac{\partial T}{\partial y} = \alpha\frac{\partial^2 T}{\partial y^2}, \tag{5}$$

where the x- and y-axes are measured along the surface of the wedge and normal to it, respectively, u and v are respectively the velocity components along the x- and y-axes, and the other quantities are defined in the Nomenclature. However, it should be mentioned that μ, κ, ρ and α are the fixed parameters, while j, N and γ are the quantities depending on (x, y). We shall assume that the boundary conditions of these equations are of the following form:

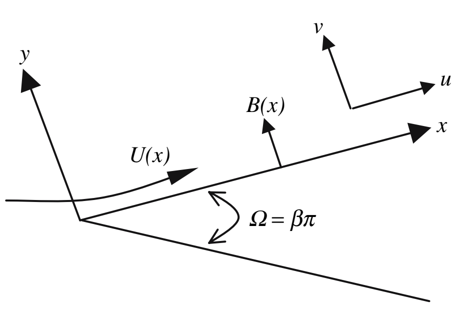

Fig. 1. Physical model and coordinate system

$$u=0,\quad v=0,\quad j=0,\quad N=-\frac{1}{2}\frac{\partial u}{\partial y},\quad T=T_w(x)\quad \text{at}\quad y=0,$$
$$u\to U(x),\quad N\to 0,\quad T\to T_\infty\quad \text{as}\quad y\to\infty. \tag{6}$$

We also assume that the external velocity of the fluid and the temperature of the wedge are $U(x) = ax^m$ and $T_w(x) = T_\infty + bx^n$, respectively. Here, $m = \beta/(2 - \beta)$, and β is the Hartree pressure gradient parameter which corresponds to $\beta = \Omega/\pi$ for a total angle Ω of the wedge, n is the temperature exponent parameter, and a and b are positive constants. The induced magnetic field and the Hall effects are neglected. We notice that $0 \le m \le 1$ with $m = 0$ for the boundary-layer flow over a stationary flat plate (Blasius problem) and $m = 1$ for the flow near the stagnation point on an infinite wall, while $n = 0$ corresponds to a constant wedge temperature. In order to obtain similarity solutions of the problem described by Eqs. (1)–(6), we assume that the variable magnetic field $B(x)$ is of the form $B(x) = B_0x^{(m-1)/2}$. This form of $B(x)$ has also been used by Cobble [19], [20], Helmy [21], Anjali Devi and Thiyagarajan [23], Chiam [24], and very recently by Hoernel [25] in their MHD flow problems past moving or fixed flat plates. Following Ahmadi [26] and Kline [27], we assume that the spin-gradient viscosity γ can be defined as

$$\gamma(x,y)=(\mu+\kappa/2)j(x,y)=\mu(1+K/2)j(x,y), \tag{7}$$

where $K = \kappa/\mu$ denotes the dimensionless viscosity ratio and is called the material parameter. This assumption is invoked to allow the field of equations predicts the correct behavior in the limiting case when the microstructure effects become negligible and the total spin N reduces to the angular velocity (see [26] or [28]). Equation (7) has also been used by many researchers, such as, for example, Gorla [29] and Ishak et al. [30] to study different problems of convective flow of micropolar fluids. It is stated by Ahmadi [26] that for a non-constant microinertia, it is possible using Eq. (7) to find similar and self-similar solutions for a large number of problems of micropolar fluids. It is also worth mentioning that the case $K = 0$ describes the classical Navier-Stokes equations for a viscous and incompressible fluid.

Following Kim [5] and Falkner and Skan [7], we introduce now the following similarity variables:

$$\psi(x,y)=\left[\frac{2vxU(x)}{m+1}\right]^{1/2}f(\eta),\quad N(x,y)=U(x)\left[\frac{(m+1)U(x)}{2vx}\right]^{1/2}h(\eta),$$
$$j(x,y)=\frac{2vx}{(m+1)U(x)}i(\eta),\quad \eta=\left[\frac{(m+1)U(x)}{2vx}\right]^{1/2}y,\quad \theta(\eta)=\frac{T-T_\infty}{T_w-T_\infty}, \tag{8}$$

where v is the kinematic viscosity and ψ is the stream function defined in the usual way as $u = \partial\psi/\partial y$ and $v = -\partial\psi/\partial x$, so as to identically satisfy Eq. (1). Substituting (8) into Eqs. (2)–(5), we get the following ordinary differential equations:

$$(1+K)f'''+ff''+\frac{2m}{m+1}\left(1-f'^2\right)+Kh'+M(1-f')=0, \tag{9}$$

$$(1+K/2)(ih')'+i\left(fh'-\frac{3m-1}{m+1}f'h\right)-K(2h+f'')=0, \tag{10}$$

$$(1-m)f'i-\frac{m+1}{2}fi'=0, \tag{11}$$

$$\frac{1}{Pr}\theta''+f\theta'-\frac{2n}{m+1}f'\theta=0, \tag{12}$$

subject to the boundary conditions

$$f(0) = 0, \quad f'(0) = 0, \quad i(0) = 0, \quad h(0) = -\frac{1}{2}f''(0), \quad \theta(0) = 1,$$
$$f'(\infty) \to 1, \quad h(\infty) \to 0, \quad \theta(\infty) \to 0, \tag{13}$$

where primes denote differentiation with respect to η, $M = 2\sigma\, B_0^2/[a\rho(m+1)]$ is the magnetic parameter and $Pr = \nu/\alpha$ is the Prandtl number. If we integrate Eq. (11) subjected to (13), we get

$$i = Af^{2(1-m)/(1+m)}, \tag{14}$$

where A is a non-dimensional constant of integration. We notice that Eqs. (9)–(11) were also derived by Kim [5]. However, Eq. (10) was wrongly derived in Kim [5] because Eq. (14) in his paper contained the extra term $[m/(m+1)]\eta f'h'$.

The physical quantities of interest are the skin friction coefficient and the local Nusselt number, which are, respectively, defined as

$$C_f = \frac{\tau_w}{\rho U^2(x)/2}, \quad Nu_x = \frac{xq_w}{k(T_w - T_\infty)}, \tag{15}$$

where the skin friction τ_w and the heat transfer from the plate q_w are defined as

$$\tau_w = \left[(\mu + \kappa)\frac{\partial u}{\partial y} + \kappa N\right]_{y=0}, \quad q_w = -k\left(\frac{\partial T}{\partial y}\right)_{y=0}. \tag{16}$$

Using the variables (8), we get

$$\frac{1}{2}C_f Re_x^{1/2} = \sqrt{\frac{m+1}{2}}\left(1 + \frac{K}{2}\right)f''(0), \quad Nu_x/Re_x^{1/2} = -\sqrt{\frac{m+1}{2}}\theta'(0), \tag{17}$$

where $Re_x = U(x)x/\nu$ is the local Reynolds number.

2.1 Flat plate problem

In this case, $m = 0$ and thus, Eqs. (9) and (10) reduce to

$$(1 + K)f''' + ff'' + Kh' + M(1 - f') = 0, \tag{18}$$

$$(1 + K/2)(ih')' + i(fh' + f'h) - K(2h + f'') = 0, \tag{19}$$

subject to the boundary conditions (13). Also, Eq. (14) becomes

$$i = Af^2. \tag{20}$$

The solution of Eqs. (18)–(20) subject to the boundary conditions (13) in the absence of the transverse magnetic field ($M = 0$) can be found in [26]. If $K \neq 0$, but $A = 0$, i.e., $i = 0$, from Eq. (19), we find

$$h = -\frac{1}{2}f'', \tag{21}$$

that is, gyration is identical to the angular velocity. Then Eq. (18) becomes

$$(1 + K/2)f''' + ff'' + M(1 - f') = 0. \tag{22}$$

Following Rees and Bassom [31], we take

$$\hat{f}(\hat{\eta}) = (1 + K/2)^{-1/2}f(\eta), \quad \hat{\eta} = (1 + K/2)^{-1/2}\eta, \tag{23}$$

and Eq. (22) reduces to

$$\hat{f}''' + \hat{f}\hat{f}'' + M(1 - \hat{f}') = 0, \tag{24}$$

with the boundary conditions

$$\hat{f}(0) = 0, \quad \hat{f}'(0) = 0, \quad \hat{f}'(\infty) \to 1, \tag{25}$$

where now primes denote differentiation with respect to $\hat{\eta}$. The problem (24) and (25) describes the MHD boundary-layer flow of a Newtonian fluid over a flat plate in the presence of an applied magnetic field, which was first studied by Rossow [32]. The skin friction coefficient given by Eq. (17) now becomes

$$\frac{1}{2} C_f Re_x^{1/2} = \sqrt{\frac{m+1}{2}} \left(1 + \frac{K}{2}\right)^{-1/2} \hat{f}''(0). \tag{26}$$

2.2 Wedge problem

The problem when $M = 0$ (absent of magnetic field) and $n = 0$ (isothermal wall) has been considered by Kim [5] but his equation was not adequately derived. Thus, we cannot compare our results with the results reported by Kim [5]. Furthermore, if $A = 0$, i.e., $i = 0$, it can be easily shown that on using (23), Eqs. (9) and (10) reduce to

$$\hat{f}''' + \hat{f}\hat{f}'' + \beta(1 - \hat{f}'^2) + M(1 - \hat{f}') = 0, \tag{27}$$

where $\beta = 2m/(m + 1)$, subject to the boundary conditions (25). This equation has also been derived by Soundalgekar et al. [33].

3 Results and discussion

Equations (9), (10), (12) and (14) subject to the boundary conditions (13) are solved numerically using a finite-difference scheme known as the Keller-box method as described in the book by Cebeci and Bradshaw [34], for several values of the parameters K, M, m and n. As in [5], we consider only the values of A and Pr unity, except for comparison with previously reported cases. This value of non-dimensional constant of integration A was also used by Ahmadi [26]. In order to verify the accuracy of the present method used on the simulation model, the results are compared with those cases reported by Yih [11], Cebeci and Bradshaw [34], Chamkha et al. [35] and Lin and Lin [36], as shown in Tables 1 and 2, and the comparisons are found to be in a very good agreement. Therefore, the developed code can be used with great confident to study the problem considered in this paper.

Figures 2–4 display the dimensionless velocity profiles $f'(\eta)$ for various values of m, M and K, respectively, while the other parameters are fixed. It is observed that the velocity $f'(\eta)$ increases with m and M but decreases with K. These results show that increasing the wedge angle as well as the magnetic field is to accelerate the velocity, while increasing the material parameter K is to decelerate it. Further, the boundary-layer thickness decreases with an increase in m or M which in turn increases the velocity gradient at the surface ($\eta = 0$), and hence produces an increase in the skin friction coefficient. This observation is consistent with the values of $f''(0)$ shown in Tables 1 and 2. The opposite trend is observed for the effect of K. These figures also show that the boundary conditions (13) are satisfied, which support the validity of the numerical results obtained.

Variation of the non-dimensional temperature $\theta(\eta)$ with η for different values of m, M and K are displayed in Figs. 5 and 6, respectively. The temperature gradient at the surface, which represents

Table 1. Values of $C_f Re_x^{1/2}$ for various values of K, M and m

K	M	m	Yih [11]	Cebeci and Bradshaw [34]	Chamkha et al. [35]	Present results
0	0	0	0.332057	0.33206	0.332206	0.3321
		1/3	0.757448	0.75745	0.757586	0.7575
		1	1.232588	1.232710	1.232710	1.2326
1	1	0				0.9599
		1/3				1.3983
		1				0.8670

Table 2. Values of $Nu_x/Re_x^{1/2}$ for various values of K, M and Pr when m = 0 and n = 0

K	M	Pr	Yih [11]	Lin and Lin [36]	Present results
0	0	0.0001	0.005590	0.005588	0.0056
		0.001	0.017316	0.017316	0.0173
		0.01	0.051589	0.051590	0.0516
		0.1	0.140034	0.140032	0.1400
		1	0.332057	0.332057	0.3321
		10	0.728141	0.728148	0.7281
		100	1.571831	1.57186	1.5719
		1000	3.387083	3.38710	3.3871
		10000	7.297402	7.29742	7.2974
1	1	1			0.3719

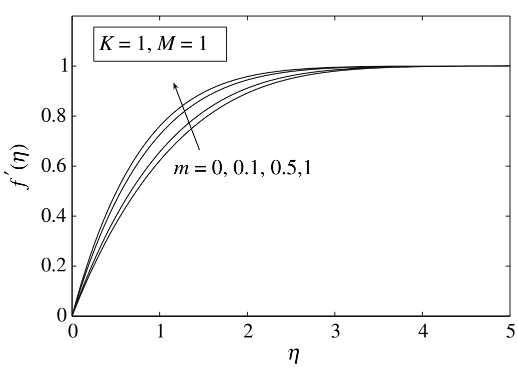

Fig. 2. Velocity profiles $f'(\eta)$ for various values of M with K = 1 and M = 1

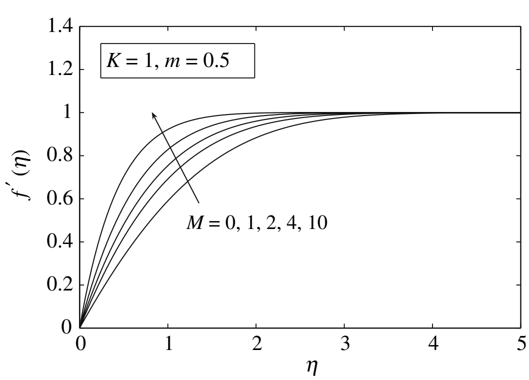

Fig. 3. Velocity profiles $f'(\eta)$ for various values of M with K = 1 and m = 0.5 (Ω = 120°)

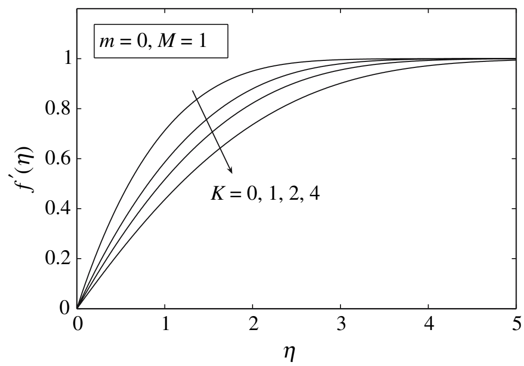

Fig. 4. Velocity profiles $f'(\eta)$ for various values of K with $M = 1$ and $m = 0$ (flat plate)

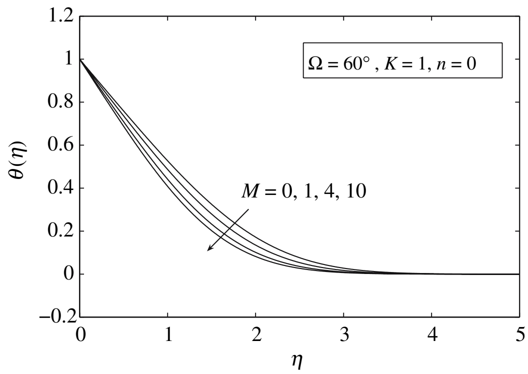

Fig. 5. Temperature profiles $\theta(\eta)$ for various values of M when $m = 0.2$ ($\Omega = 60°$), $K = 1$ and $n = 0$ (isothermal wall)

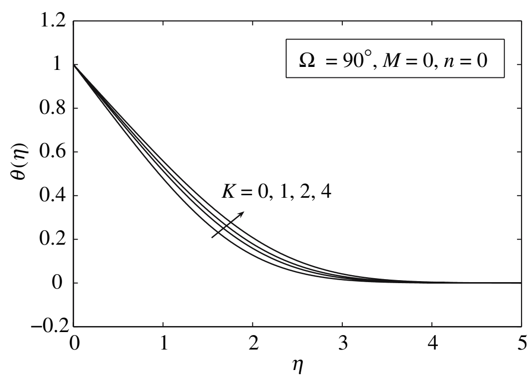

Fig. 6. Temperature profiles $\theta(\eta)$ for various values of K when $m = 1/3$ ($\Omega = 90°$), $M = 0$ and $n = 0$

the heat transfer rate increases with m, and M, but it decreases with K. The same trend is observed as the variation of the velocity profiles. This result indicates that increasing the skin friction coefficient is to increase the heat transfer rate at the surface. Influence of n over the dimensionless temperature $\theta(\eta)$ is shown graphically in Fig. 7. It is evident from this figure that the effect of n is to reduce the temperature, but increases the thermal boundary-layer thickness, which in turn gives rise to the temperature gradient at the surface. Thus, the heat transfer rate at the surface increases with n.

The sample of microrotation profiles for various values of M when $m = 0.2$ ($\Omega = 60°$) and $K = 1$ is displayed in Fig. 8. It is observed that the absolute value of the dimensionless microrotation or angular velocity $|h(\eta)|$ continuously decreases with η and becomes zero far away from the surface, which satisfies the boundary conditions (13). As expected, the microrotation effects are more

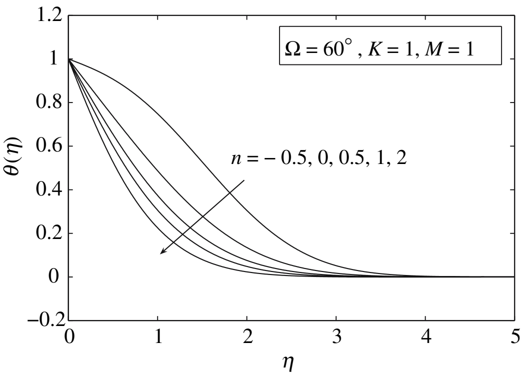

Fig. 7. Temperature profiles $\theta(\eta)$ for various values of n when $m = 0.2$ ($\Omega = 60°$), $K = 1$ and $M = 1$

dominant near the wall. Also, $|h(\eta)|$ increases as M increases in the vicinity of the wedge but the reverse happens as one moves away from it.

Figure 9 displays the variation of the skin friction coefficient in terms of $f''(0)$ against the wedge angle Ω for different values of M and K. It is observed that $f''(0)$ increases with an increase in M, but it decreases as K increases. These results are consistent with the velocity gradient shown in Figs. 3 and 4. The same result was reported by Yih [11] for the Newtonian fluid, concerning the effect of the magnetic parameter on the skin friction coefficient.

The variation of the local Nusselt number, which represents the heat transfer rate at the surface, in terms of $-\theta'(0)$ are presented in Figs. 10 and 11 for $n = 0$ and $n = 1$, respectively. For a fixed wedge angle, $-\theta'(0)$ decreases as the material parameter K increases. This result is due to the fact that the micromotion in micropolar fluid reduces the drag reduction, and consequently decreases the heat transfer rate at the surface. For $n = 0$ (isothermal wedge), the value of $-\theta'(0)$ increases monotonically with an increase of the wedge angle, for both $M = 0$ and $M = 1$, but different features are observed for $n = 1$. For $M = 1$, $-\theta'(0)$ decreases monotonically with Ω, whereas it increases to a certain value and then decreases monotonically for $M = 0$. Thus, the magnetic field gives more influence to the heat transfer rate of a variable temperature wedge compared to an isothermal wedge. Finally, as reported by Yih [11], the magnetic parameter M increases the local Nusselt number.

4 Conclusions

We have theoretically studied the problem of steady two-dimensional laminar fluid flow past a fixed wedge immersed in an electrically conducting micropolar fluid. The governing partial differential

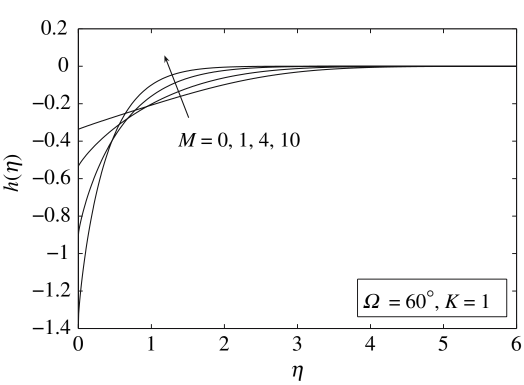

Fig. 8. Microrotation profiles $h(\eta)$ for various values of M when $m = 0.2$ ($\Omega = 60°$) and $K = 1$

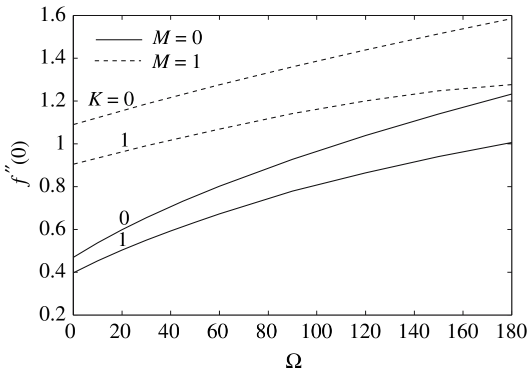

Fig. 9. Skin friction coefficient $f''(0)$ as a function of Ω for $M = 0, 1$ and $K = 0, 1$

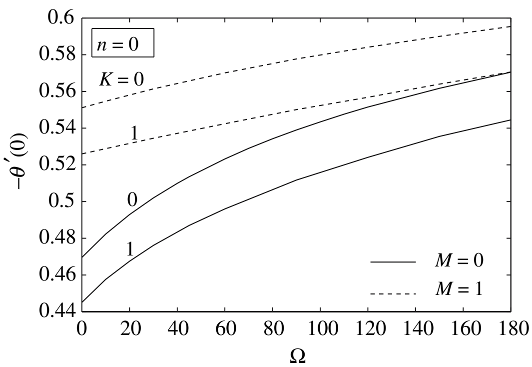

Fig. 10. Local Nusselt number $-\theta'(0)$ as a function of Ω for $M = 0, 1$ and $K = 0, 1$ when $n = 0$

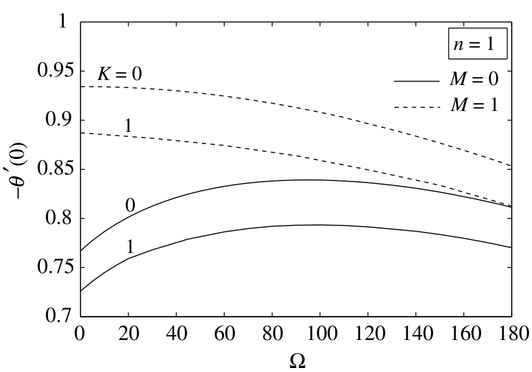

Fig. 11. Local Nusselt number $-\theta'(0)$ as a function of Ω for $M = 0, 1$ and $K = 0, 1$ when $n = 1$

equations were transformed using suitable variables to get the ordinary differential equations, and then solved numerically using an implicit finite-difference scheme known as the Keller-box method. Numerical results for the velocity, temperature and microrotation profiles as well as the skin friction coefficient and the local Nusselt number for various values of the material parameter K, Falkner-Skan power-law parameter m, magnetic parameter M and temperature exponent parameter n, while the dimensionless constant A and the Prandtl number Pr are fixed to be unity (the same as in [5]), were obtained and have been illustrated in graphical forms. The numerical values of the skin friction coefficient and the local Nusselt number for some values of the parameters were also obtained and favorable comparisons with previously published cases of the problem were performed. The numerical results showed that micropolar fluids display drag reduction compared to the classical

Newtonian fluid ($K = 0$), and consequently reduce the heat transfer rate at the surface. The opposite trends were observed for the effects of the transverse magnetic field on the fluid flow and heat transfer characteristics. The skin friction coefficient increases as the wedge angle (i.e. m) increases. Increasing n is to increase the heat transfer rate at the surface.

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