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Online detection of the breathing crack using an adaptive tracking technique

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Summary. Early detection of structural damage is an important goal of any structural health monitoring system. Among numerous data analysis techniques, those which are used for online damage detection have received considerable attention recently, although the problem of online detection in continuous structures, for example beams, is quite challenging. In this paper, it is shown how the type, the size and the location of breathing cracks are identified online with the use of the records which are gathered from a continuous beam. For determining the existence of a breathing crack in a beam, its vibrating behavior is simulated. The algorithm of the least square estimation with the use of adaptive tracking is employed for identification purposes. This algorithm is capable of detecting the abrupt changes in problem parameters and traces its variations. With the use of reducing domain algorithm, this identification method shows better results and can detect the breathing crack in beams more efficiently. Finally, it is shown that with the use of sufficient mode shapes the method is capable of identifying the breathing crack in beams and frames. The efficiency of the proposed algorithm is shown through some case studies.

1 Introduction

Fatigue cracks often exist in structural members due to repeated loading and can lead to structural failure. For this reason, methods allowing early detection and localization of cracks have been the subject of intensive investigation in the last two decades. As a result, a variety of analytical, numerical and experimental methods are developed. Among the techniques employed for crack detection are vibration-based methods, which offer effective and fast means for detecting the fatigue cracks in structures. An important issue in breathing crack detection is that a breathing crack shows nonlinear dynamic behavior. Consequently, it seems appropriate to study this type of behavior of breathing cracks for developing more accurate vibration-based detection techniques. In most cases, it is needed to identify the breathing crack when it occurs. Structural system identification can be divided into two categories; namely, online and offline identifications. For the online identification of the changes of structural parameters, time domain analyses have been used with some success, including the methods of least-square estimation [1]–[3] and the filter approaches, including the extended Kalman filter [4]–[6], $H\infty$ filter [7] and Monte Carlo filter [8]. To date, the online detection of the changes of structural parameters due to nonlinear damages such as a breathing crack in continuous systems is still a challenging problem.

Several researchers have addressed the problem of a beam with a breathing crack. Ostachowicz and Krawczuck [9] studied the forced vibrations of a cantilever beam with a breathing crack. They used the harmonic balance technique to solve the equation of motion. The periodically varying stiffness was simulated with a square wave function with a fundamental frequency equal to the forcing frequency. Shen and Chu [10] simulated the dynamic response of simply supported beams with a closing crack using a bilinear equation of motion for each mode, to determine the variation in the response spectrum due to the presence of the crack. The same authors obtained a closed form solution for the response of a bilinear oscillator [11]. Using this approach, they investigated the behavior of a cracked beam under low frequency excitation. Abraham and Brandon [12] also developed a model to predict the vibration of a beam with a breathing crack. They related the two segments of a beam, separated by the crack, using time varying connection matrices. The connection matrices were expanded in Fourier series. Despite the progress made towards the modeling of a breathing crack, work remains to be done to detect an opening and closing crack and consequently track its variation in time. The aim of the present paper is to identify a continuous structure with a breathing crack online by employing the concept of new nonlinear least square estimation, namely adaptive tracking in the time domain.

2 Modeling a continuous beam with breathing crack

2.1 Modeling a breathing crack

The accuracy of an analytical determination of vibration characteristics of a beam with a breathing crack depends mainly on the crack model. A wide spectrum of such models can be found in the literature: a crack was modeled by a spring [13], elastic hinge [14], cut-out [15], a pair of concentrated couples [16], a zone with reduced Young's modulus [17], or its effect was taken into account by semi-empirical functions describing stress and strain distribution by the volume of the cracked beam [18]. Chati et al. [19] and Bovsunovsky and Matveev [20] modeled the process of crack opening and closing by means of a piecewise-linear system. The approach to crack modeling presented here is based on the assumption that the beam is of constant cross-section along its length, and the crack is modeled by a zone with reduced moment of inertia which changes with time to show the breathing crack behavior. The dimensions of this zone are determined from energy criteria [14]. It is presumed that such a crack model is physically justified in as much as the decrease in moment of inertia and the shift of the neutral axis in the cross-section with a crack in fact take place.

The reduction of the cross-sectional moment of inertia of the cracked beam causes the change in strain energy. The change in the strain energy of the equivalent beam due to reduction of the cross-sectional moment of inertia in a small region 2 $\bar{x}$ with its midpoint at x_D, (the place of crack), is equal to

$$\Delta U_1 = U_{\text{inact}} - U_{\text{damage}} = \int_{x_D-\bar{x}}^{x_D+\bar{x}} \frac{M_{x_D}^2}{2E}\left(\frac{1}{I_0} - \frac{1}{I_D}\right) dx = \frac{M_{x_D}^2}{E}\bar{x}\left(\frac{1}{I_0} - \frac{1}{I_D}\right) \tag{1}$$

if the change in bending moment $M(x)$ along this equivalent segment is neglected. I_D is the reduced cross-sectional moment of inertia. In the linearly elastic body, the change of strain energy due to the presence of crack of the mode I deformation, for the plane stress problems will be as follows [21]:

$$\Delta U_2 = \frac{b}{E}\int_0^a K_I^2 da. \tag{2}$$

Here, the expression for the stress intensity factor obtained by Tada for the case of pure bending of a cracked strip is used:

$$K_I = \frac{4.4M}{bh^{3/2}E}\left[\left(1-\frac{a}{h}\right)^{-3} - \left(1-\frac{a}{h}\right)^3\right]^{1/2}, \tag{3}$$

where a, h and b are the crack length, the beam height and width, respectively.

In the frequency range, the strain wavelength is greater by several orders of magnitude than the crack size, the elastic field in its neighborhood can be considered as quasi-static [20]. This makes it possible to neglect the influence of the dynamic effect on the stress intensity factor. Substitution of Eq. (3) into Eq. (2) results in

$$\Delta U_2 = \frac{4.41M^2}{bh^2E}\frac{\left(1-\frac{a}{h}\right)^6 - 3\left(1-\frac{a}{h}\right)^2 + 2}{\left(1-\frac{a}{h}\right)^2}. \tag{4}$$

The energy criterion for equivalence of the cracked beam and its model has the form

$$\Delta U_1 = \Delta U_2. \tag{5}$$

From Eq. (5) and Eqs. (4) and (1), the parameter $d(x)$ with respect to crack length a is derived as

$$\alpha = \frac{4.41}{12}\frac{h}{\bar{x}}\left(\frac{a}{h}\right)^2\frac{12 - 20\frac{a}{h} + 15\left(\frac{a}{h}\right)^2 - 6\left(\frac{a}{h}\right)^3 + \left(\frac{a}{h}\right)^4}{\left(1-\frac{a}{h}\right)^2}, \tag{6}$$

$$d(x) = \frac{\alpha}{1+\alpha}. \tag{7}$$

The parameter γ is used for modeling the opening and closing behavior of the crack in the beam as follows:

$$\gamma = \begin{cases} 1 & \text{when the crack is open,} \\ 0 & \text{when the crack is closed.} \end{cases} \tag{8}$$

If the curvature has a tendency to close the crack, the breathing crack is in its closed phase, and if it tends to open the crack, the crack is in open phase. Therefore, with the use of the parameter γ, the crack behavior can be described in its two vibration phases. The changes of the reduced equivalent moment of inertia of the beam can be expressed as

$$I_d = I(1-\gamma d(x)). \tag{9}$$

As much as the bending stiffness of the beam is concerned, Eq. (9) can be written as follows in terms of Young's modulus:

$$E_d = E(1-\gamma d(x)). \tag{10}$$

2.2 Dynamic response of a damaged beam

Assume that the damages in a beam are uniform through the thickness of the beam (i.e., thickness-through damages), the intact Young's modulus E can be replaced by the effective

Young's modulus E_d to derive the dynamic equation of motion for the beams in the damaged state as follows:

$$\frac{\partial^2}{\partial x^2}\left(EI\frac{\partial^2 w}{\partial x^2}\right) - \frac{\partial^2}{\partial x^2}\left(EI_D\frac{\partial^2 w}{\partial x^2}\right) + \rho A\ddot{w} = f(x,t), \tag{11}$$

where EI_D is the effective reduction of bending stiffness due to the presence of the damage, and is as follows:

$$EI_D = \int_A Ed(x)y^2 dA. \tag{12}$$

Using the normal modes of the intact beam, forced vibration response of the crack beam can be obtained by superposing M normal modes as

$$w(x,t) = \sum_{m=1}^{M} W_m(x)\overline{q}_m(t). \tag{13}$$

Substituting Eq. (13) into Eq. (11) and applying the orthogonality property of mode shapes as

$$\int_0^L \rho A W_m W_n dx = \delta_{mn}, \tag{14}$$

$$\int_0^L EIW''_m W''_n dx = \Omega_m^2 \delta_{mn} \tag{15}$$

yields the modal equations for the damaged beam as follows:

$$\ddot{\bar{q}}_m + \Omega_m^2 \bar{q}_m - \sum_{n=1}^{M} \lambda_{mn}\bar{q}_n = f_m(t) \quad (m = 1, 2, \ldots, M). \tag{16}$$

The matrix λ_{mn} reflects the influence of the damage and is defined as

$$\lambda_{mn} = \gamma EI \int_0^L d(x) W''_m W''_n dx. \tag{17}$$

Consider a harmonic point forces applied at $x = x_{Fk}$ $(k = 1 \ldots K)$ as

$$f(x,t) = \sum_{k=1}^{K} F_{0k}\delta(x - x_{Fk})e^{i\omega_k t}, \tag{18}$$

where K is the number of applied forces, F_{0k} is the amplitude of the k-th harmonic point force, and ω_k is the k-th excitation (circular) frequency. Then $f_m(t)$ can be written as

$$f_m(t) = \sum_{k=1}^{K} W_m(x_{Fk})F_{0k}e^{i\omega_k t}. \tag{19}$$

If the equivalent damage is considered to be thickness-through damage of magnitude D between 0 and 1, uniformly distributed over the small span $2\,\bar{x}$ and its midpoint at x_D, then $d(x)$ is a *piecewise uniform* thickness-through damage and can be represented by

$$d(x) = D\{H[x - (x_D - \bar{x})] - H[x - (x_D + \bar{x})]\}, \tag{20}$$

where $H(x)$ is Heaviside's unit function. Substituting Eq. (20) into Eq. (17) yields

$$\lambda_{mn} = \left(EI \int_{x_D - \bar{x}}^{x_D + \bar{x}} W''_m W''_n dx\right)\gamma D \equiv k_{mn} D'. \tag{21}$$

If there are N equivalent crack zones in the beam, Eq. (21) can be represented as:

$$K_{mn}^{j} = EI \int_{x_{D_j}-\bar{x}_j}^{x_{D_j}+\bar{x}_j} W''_m W''_n dx, \tag{22}$$

$$\lambda_{mn} = \sum_{j=1}^{N} K_{mn}^{j} D'_j, \tag{23}$$

$$D'_j = \gamma_j D_j, \tag{24}$$

where N is the number of damage detection zones, and $2\bar{x}$, x_{D_j} and D_j represent the magnitude, location and size of the piecewise uniform damage over the j-th damage detection zone, respectively.

2.3 Least-square estimation with adaptive tracking

Considering M modes, the equation of motion of the cracked beam in modal coordinate can be expressed as Eq. (16). With some modification, this equation can be rewritten as

$$\Delta\ddot{q}_m - \boldsymbol{\Omega}_m^2 q_m = \sum_{n=1}^{M} (\lambda_{mn} - \delta_{mn}\boldsymbol{\Omega}_n^2)\bar{q}_n. \tag{25}$$

If the right part of the above equation is considered as f_m,

$$f_m = \Delta\ddot{q}_m + \boldsymbol{\Omega}_m^2 \Delta q_m. \tag{26}$$

The observation equation associated with the equation of motion, (Eq. 25), can be written as

$$\mathbf{T}\left[\bar{\mathbf{Q}}; t\right]\mathbf{D} + \boldsymbol{\varepsilon}(t) = \mathbf{G}(t), \tag{27}$$

in which $\mathbf{D} = [D_1 D_2 \cdots D_n]^T$ is an unknown parameter vector at time t; $\bar{\mathbf{Q}}$ is defined as $\bar{\mathbf{Q}} = \left[\bar{\mathbf{q}}^{\mathrm{T}} \quad \dot{\bar{\mathbf{q}}}^{\mathrm{T}}\right]^{\mathrm{T}}$, where $\ddot{\bar{\mathbf{q}}}$ is the measured modal acceleration response vector; $\bar{\mathbf{q}}$ is the modal measured displacement vector; $\mathbf{G}(t)$ is an m-vector defined as Eq. (26) and obtained from excitation and displacement vector, $\boldsymbol{\varepsilon}(t)$ is m-proposed model noise vector contributed by the measurement noise and possible model error, and $\mathbf{T}$ is an $m \times n$ data matrix.

At the time instant $t = t_k = k\Delta t$ with Δt being the sampling interval, Eq. (27) can be written as

$$\mathbf{T_k}\mathbf{D}_k + \boldsymbol{\varepsilon}_k = \mathbf{G}_k, \tag{28}$$

in which $\mathbf{T}_k = \mathbf{T}\left(\bar{\mathbf{Q}}_k, t_k\right)$ is the $m \times n$ observation matrix at t_k, $\mathbf{G}_k = \mathbf{G}(t_k)$, $\boldsymbol{\varepsilon}_k = \boldsymbol{\varepsilon}(t_k)$ and $\mathbf{D}_k = [D_1(t_k)\, D_2(t_k) \ldots D_n(t_k)\,]^T$ is the unknown parametric whose entries are equivalent damage magnitudes of the considered zones in the beam. Combining all the equations of (28) for k time instants (from 1 to k), and assuming that $\mathbf{D}_k$ is a constant vector, i.e. $\mathbf{D}_k$ does not vary with respect to time, one obtains

$$\bar{\mathbf{T}}_k\mathbf{D}_k + \mathbf{E}_k = \bar{\mathbf{G}}_k, \tag{29}$$

where $\bar{\mathbf{T}}_k = \left[\mathbf{T}_1^T, \mathbf{T}_2^T, \ldots, \mathbf{T}_k^T\right]$, $\mathbf{E}_k = \left[\boldsymbol{\varepsilon}_1^T, \boldsymbol{\varepsilon}_2^T, \ldots, \boldsymbol{\varepsilon}_k^T\right]$ and $\bar{\mathbf{G}}_k = \left[\mathbf{G}_1^T, \mathbf{G}_1^T, \ldots, \mathbf{G}_k^T\right]$. Let $\hat{\mathbf{D}}_k$ be the estimate of $\mathbf{D}_k$ at $t_k = (k)\Delta t$ and consider an objective function as

$$J\left[\hat{\mathbf{D}}_k\mathbf{W}\right] = \left[\bar{\mathbf{G}}_k - \bar{\mathbf{T}}_k\hat{\mathbf{D}}_k\right]^T\mathbf{W}\left[\bar{\mathbf{G}}_k - \bar{\mathbf{T}}_k\hat{\mathbf{D}}_k\right], \tag{30}$$

where W is a $km \times km$ weighting matrix. Minimization of $J\left[\hat{\mathbf{D}}_k \mathbf{W}\right]$ yields

$$\hat{\mathbf{D}}_k = \left[\bar{\mathbf{T}}_k^T \mathbf{W} \mathbf{T}_k\right]^{-1} \bar{\mathbf{T}}_k^T \mathbf{W} \mathbf{G}_k. \tag{31}$$

With $\mathbf{W} = \mathbf{I}$, the recursive solution for $\hat{\mathbf{D}}_k$ is obtained from Eq. (31) as [22]

$$\hat{\mathbf{D}}_k = \hat{\mathbf{D}}_{k-1} + \mathbf{K}_k(\bar{\mathbf{Q}}_k)\left[\mathbf{F}_k - \mathbf{T}_k(\bar{\mathbf{Q}}_k)\hat{\mathbf{D}}_{k-1}\right], \tag{32}$$

$$\mathbf{K}_k(\bar{\mathbf{Q}}_k) = \mathbf{P}_{k-1}\mathbf{T}_k^T(\bar{\mathbf{Q}}_k)\left[\mathbf{I} + \mathbf{T}_k(\bar{\mathbf{Q}}_k) \times \mathbf{P}_{k-1}\mathbf{T}_k^T(\bar{\mathbf{Q}}_k)\right]^{-1}, \tag{33}$$

$$\mathbf{P}_{k-1} = \mathbf{P}_{k-2} - \mathbf{K}_{k-1}(\bar{\mathbf{Q}}_{k-1})\mathbf{T}_{k-1}(\bar{\mathbf{Q}}_{k-1})\mathbf{P}_{k-2}, \tag{34}$$

where $\mathbf{K}_k =$ the least square estimation gain matrix, and $\mathbf{P}_{k-1}^T = \mathbf{P}_{k-1} > 0$ is the adaptation gain matrix. In Eq. (34), k starts from 2 and $\mathbf{P}_0$ is estimated.

The recursive solution $\hat{\mathbf{D}}_k$ in Eqs. (32)–(34) is derived from the constant parametric vector $\mathbf{D}_k$. For identifying the breathing cracks which have time-varying parameters, it is necessary to use a method that is capable of tracking the time changes of the parameters. Here, the method of adaptive tracking technique is employed for the identification of time-varying cracks. In order to track the variation of each parameter, say the j-th equivalent piecewise uniform damage, the estimation error $\left[\hat{D}_j(k) - D_j(k)\right]$ is expressed by $\lambda_j(k)\left[\hat{D}_j(k) - D_j(k)\right]$, where $\lambda_j(k)$ is determined from the measured data at t_{k-1}. Thus only the noise causes the residual error, and the contribution due to the parametric variation is eliminated. Hence, the above modification for the estimation error is reflected in the P_{k-1} matrix of Eq. (33), changing it into $\mathbf{\Lambda}_k \mathbf{P}_{k-1} \mathbf{\Lambda}_k^T$, where $\mathbf{\Lambda}_k$ is a diagonal matrix formed by $\lambda_j(k)$ s. The recursive solution for the time-varying parameters, $\hat{\mathbf{D}}_k$, is obtained from Eqs. (32)–(34) as [3]

$$\hat{\mathbf{D}}_k = \hat{\mathbf{D}}_{k-1} + \mathbf{K}_k(\bar{\mathbf{Q}}_k)\left[\mathbf{F}_k - \mathbf{T}_k(\bar{\mathbf{Q}}_k)\hat{\mathbf{D}}_{k-1}\right], \tag{35}$$

$$\mathbf{K}_k(\bar{\mathbf{Q}}_k) = \left(\mathbf{\Lambda}_k\mathbf{P}_{k-1}\mathbf{\Lambda}_k^T\right)\mathbf{T}_k^T(\bar{\mathbf{Q}}_k)\left[\mathbf{I} + \mathbf{T}_k(\bar{\mathbf{Q}}_k) \times \left(\mathbf{\Lambda}_k\mathbf{P}_{k-1}\mathbf{\Lambda}_k^T\right)\mathbf{T}_k^T(\bar{\mathbf{Q}}_k)\right]^{-1}, \tag{36}$$

$$\mathbf{P}_{k-1} = \left(\mathbf{\Lambda}_{k-1}\mathbf{P}_{k-2}\mathbf{\Lambda}_{k-1}^T\right) - \mathbf{K}_{k-1}(\bar{\mathbf{Q}}_{k-1})\mathbf{T}_{k-1}(\bar{\mathbf{Q}}_{k-1})\left(\mathbf{\Lambda}_{k-1}\mathbf{P}_{k-2}\mathbf{\Lambda}_{k-1}^T\right). \tag{37}$$

In Eqs. (36) and (37), $\mathbf{\Lambda}_k$ is a diagonal matrix known as the adaptive factor matrix, i.e., $\mathbf{\Lambda}_k = \mathrm{diag}[\lambda_1(k), \lambda_2(k), \ldots, \lambda_n(k)]$, where $\lambda_j(k)$ is referred to as the adaptive factor for the estimated parameter $D_j(t_k)$.

The adaptive factor matrix $\mathbf{\Lambda}_k$ is proposed to be determined by adapting the previous data at $\mathbf{t}_k$. Thus,

$$\bar{\boldsymbol{\mu}}_k = \mathbf{F}_k - \mathbf{T}_k\hat{\mathbf{D}}_k, \tag{38}$$

$$\boldsymbol{\mu}_k = \mathbf{F}_k - \mathbf{T}_k\hat{\mathbf{D}}_{k-1}, \tag{39}$$

in which $\bar{\boldsymbol{\mu}}_k$ is the m-residual error vector and $\boldsymbol{\mu}_k$ is the m-predicted output error vector from previous data. The substitution of Eq. (35) in Eq. (39) and the formation of the covariance matrix results in

$$E[\bar{\boldsymbol{\mu}}_k\bar{\boldsymbol{\mu}}_k^T] = (\mathbf{I} - \mathbf{T}_k\mathbf{K}_k)E[\boldsymbol{\mu}_k\boldsymbol{\mu}_k^T](\mathbf{I} - \mathbf{T}_k\mathbf{K}_k)^T. \tag{40}$$

If the covariance matrix of the predicted output, $E[\boldsymbol{\mu}_k\boldsymbol{\mu}_k^T]$, is estimated and given by V_k and as the $\hat{\mathbf{D}}_k$ approaches $\mathbf{D}_k$, Eq. (40) can be written as

$$\mathbf{V}_k - \left[\mathbf{I} + \mathbf{T}_k\left(\mathbf{\Lambda}_k\mathbf{P}_{k-1}\mathbf{\Lambda}_k^T\right)\mathbf{T}_k^T\right]\boldsymbol{\sigma}_k^2\left[\mathbf{I} + \mathbf{T}_k\left(\mathbf{\Lambda}_k\mathbf{P}_{k-1}\mathbf{\Lambda}_k^T\right)\mathbf{T}_k^T\right]^T = 0, \tag{41}$$

in which $\boldsymbol{\sigma}_k^2$ is an $m \times m$ diagonal matrix with diagonal elements as σ^2 which is the variance of the model noises as

$$E[\bar{\boldsymbol{\mu}}_k\bar{\boldsymbol{\mu}}_k^T] = E[\boldsymbol{\varepsilon}_k\boldsymbol{\varepsilon}_k^T] = \boldsymbol{\sigma}_k^2. \tag{42}$$

Since only one sample of measurements has been gathered from the beam, the ensemble average of quantities is approximated by recent data temporal average, and in each time more data is employed for calculating the average. Therefore one can define

$$\boldsymbol{\sigma}_k^2 = \frac{1}{s-1}\sum_{i=k-s}^{k-1}\bar{\boldsymbol{\mu}}_i\bar{\boldsymbol{\mu}}_i^T \quad s \le k-1. \tag{43}$$

As any parameter varies, $\mathbf{V}_k$ increases and with the use of a fading factor the importance of current data can be reflected. Therefore, according to Xia et al. [23], $\mathbf{V}_k$ is

$$\mathbf{V}_k = \frac{\left[\boldsymbol{\mu}_k\boldsymbol{\mu}_k^T + \sum_{i=1}^{k-1}\left(v^{k-i}\prod_{j=i}^{k-1}\bar{\lambda}_j^{-1}\right)\boldsymbol{\mu}_i\boldsymbol{\mu}_i^T\right]}{\left[1 + \sum_{i=1}^{k-1}v^{k-i}\prod_{j=i}^{k-1}\bar{\lambda}_j^{-1}\right]}, \tag{44}$$

in which $\bar{\lambda}_j^{-1}$ is the adaptive factor which is higher than 1, and v is the factor between 0 and 1, which is assumed as 0.9 in this study.

The solution of Eq. (41) can be obtained by minimizing the following objective function:

$$J\left[\hat{\mathbf{D}}_k(\mathbf{\Lambda}_k)\right] = \sum_{j=1}^{N}\left|\frac{\hat{D}_j(k) - \hat{D}_j(k-1)}{\hat{D}_j(k-1)}\right|, \tag{45}$$

in which $\hat{D}_j(k-1)$ is the estimated j-th parameter at t_{k-1}, and $\hat{D}_j(k)$ is the estimated j-th parameter at t_k. This form of objective function is considered according to the idea that the optimal solution should follow the most direct path from $\hat{D}_j(k-1)$ to $\hat{D}_j(k)$ in order to reduce the unnecessary oscillations of parametric estimations. In order to deal with the constraint obtained in Eq. (41), an optimal solution of Eq. (43) subjected to the following constraint is considered:

$$\left\|\mathbf{V}_k - \left[\mathbf{I} + \mathbf{T}_k\left(\mathbf{\Lambda}_k\mathbf{P}_{k-1}\mathbf{\Lambda}_k^T\right)\bar{\mathbf{T}}_k^T\right]\boldsymbol{\sigma}_k^2\left[\mathbf{I} + \mathbf{T}_k\left(\mathbf{\Lambda}_k\mathbf{P}_{k-1}\mathbf{\Lambda}_k^T\right)\bar{\mathbf{T}}_k^T\right]^T\right\| \le \delta, \tag{46}$$

in which $\|\|$ is the Euclidean norm of the matrix, and δ is a small constant.

2.4 Reduced domain algorithm

For better parameter identification, a reduced domain search algorithm is used [25]. In each time the number of unknown quantities is equal to that of DDZs, and the matrix k_{mn}^j requires definite integrals only over the damaged zones. Thus, instead of examining the entire domain of the problem to search out damages (i.e., full-domain method), one can reduce the domain of the problem in advance by removing damage-free zones to examine only the reduced domain of the problem (i.e., reduced-domain method). The reduced-domain method will not degrade the accuracy of damage identification results at all. In order to realize the reduced-domain method, however, one should know the locations and the size of the damage-free zones in advance. Unfortunately, this is impracticable for most cases. Thus, one needs a method to search for damage-free zones in the process of damage identification analysis. In this paper, the following three-step method is introduced:

Step 1: Divide the length of the beam into N DDZs and use the adaptive tracking technique to predict N unknown damages $\mathbf{D}_j$ for N DDZs. The first prediction results are represented by D_j (first step) $(j = 1, 2, \ldots k, \ldots N)$.
Step 2: Divide each DDZ of the first step into M sub-DDZs to have a total $(M \times N)$ sub-DDZs and use the adaptive tracking technique to re-predict $(M \times N)$ unknown damages for $(M \times N)$ sub-DDZs. The second prediction results are represented by Dij (second step) $(i = 1, 2, \ldots k, \ldots M$ and $j = 1, 2, \ldots k, \ldots N)$.
Step 3: If Dij (second step) < Dij (first step), conclude that the i-th sub-DDZ within the j-th DDZ is damage-free. Otherwise, the sub-DDZ is suspected of damage.

Once crack-free zones of a structure are searched out and removed from the domain of the problem by using the present method, it is possible to put $D = 0$ for all the removed damage-free zones and to conduct damage identification only for the remaining domain, which is the reduced-domain method of damage identification. By iteratively using the reduced-domain method, all damage-free zones can be removed from the original domain of the problem to obtain the damaged zones.

3 Numerical results and discussions

In order to illustrate the proposed method, a special crack is placed in the structure and its response is gathered. Then using an adaptive tracking algorithm, the crack is identified. From Eq. (25) it is obvious that for identification the mode shapes and natural frequencies of intact and damage structure and the damage structure responses are needed. For intact structure the natural frequencies and mode shapes can be computed numerically or analytically from Eq. (11) in which the material and structural properties are for the refined intact structure model. The term *refined* refers to a good agreement between the measured and analytical modal parameters. When the responses of the structure are measured experimentally, it is liable to be contaminated by various measurement noises. Thus, using the method of [25], an e% random noise is added to the response of the structure, which is computed analytically or numerically, to represent the measurement noise in measured responses. As an example:

$$\bar{\ddot{y}}(\mathbf{X_p}) = \ddot{y}(\mathbf{X_p})\left(1 + \frac{\mathbf{e}}{100} \times \mathbf{f}\right), \tag{47}$$

where $\bar{\ddot{y}}(\mathbf{X_p})$ is the acceleration response simulated to include the measurement noises, and the random function which is uniformly distributed, with the mean equal to 0 and variance equal to 1.

In order to measure the accuracy of the method, a root mean squared of the differences between the real and predicted parameters is used and the *damage identification error* is defined as:

$$\begin{aligned} \mathrm{DIE}(t) &= \sqrt{\frac{1}{L}\int_0^L \left[d^{\mathrm{Pred.}}(x,t) - d^{\mathrm{True}}(x,t)\right]^2 \mathrm{d}x} \\ &= \sqrt{\frac{1}{L}\sum_j^N 2\bar{x}_j \left[D_j^{\mathrm{Pred.}}(t) - D_j^{\mathrm{True}}(t)\right]^2}, \end{aligned} \tag{48}$$

where L is the total length of the beam or the structure and the superscripts 'True' and 'Pred.' indicate the true and predicted damage states, respectively. As the value of $\mathrm{DIE}(t)$ becomes smaller, the predicted damage state gets closer to the true one. Here, for showing the identification process, the mean of DIE, namely $\mathrm{DIE}_{\mathrm{avg.}}$ in time is used which is the parameter employed for comparison of the accuracy of each step of reduced-domain algorithm.

As an illustrative example, a cantilever uniform beam is considered. The beam has a length $L = 1\,\mathrm{m}$, the intact bending stiffness $EI = 10\,\mathrm{Nm}^2$, and the mass density per length of the beam is taken as $\rho A = 0.35\,\mathrm{kg/m}$.

One of the most important factors for successful crack identification in beams is the selection of excitation and location of exerting forces. As shown in [25], the error of identification is highly dependent of the beam oscillation frequencies. It is also shown that the most reliable damage identification can be obtained when the excitation frequencies are chosen close to the natural frequencies of the beam. It should be mentioned that if the damage location coincides with a node of special mode shape, the excitation of the beam with that frequencies leads to incorrect assessment. Thus, in this paper the excitation frequencies are chosen not only near the natural frequencies, but are also selected from some mode shapes which do not have any node within the most candidate-damaged zones.

3.1 Breathing crack identification

For identifying the location and extent of breathing cracks, the previous example is considered and its accelerations and displacements are measured in the same number of locations as the number of mode shapes which is chosen in the modal equation, and the velocity is computed from numerical integration from acceleration. At first, it is assumed that the beam is intact and its damage parameters (equivalent damage of the considered spans of beam) have a small perturbation that can be assumed as zero. Therefore, for distinguishing the start of crack breathing, appropriate criteria should be chosen. In this paper it is assumed that until $D_{\mathrm{avg.\ recent}\ n\ \mathrm{sample}} > 0.005$ with n being chosen from the excitation frequencies, the beam is intact and all the damage parameters are assumed to be zero. First the equivalent damage magnitudes for N damage spans are predicted in time t and if all values are higher than the considered damage threshold, then this procedure is redone for $M \times N$ damage span. By comparing the two predicted damage magnitudes, if the damage of a special span is higher in the second step than in the first step, then it is kept despite it is eliminated from the search domain and its equivalent damage magnitude is assumed zero. This procedure is continued until the equivalent damage of crack in the proposed time is calculated with reasonable accuracy.

In each of the following cases, the assumed values and the special considerations are mentioned:

3.2 Identification of a beam with a single breathing crack

A single crack with the depth $a = 0.5$ h is considered in the middle of a beam with a length $L = 1$ m, where h is the beam depth. The bending stiffness is 10 Nm^2 and the mass per length is $\rho\grave{a} = 0.35\,\mathrm{kg/m}$. For exciting the beam, three harmonic forces with $\omega_1 = 20\,\mathrm{rad/s}$, $\omega_2 = 120\,\mathrm{rad/s}$, and $\omega_3 = 650\,\mathrm{rad/s}$ act at the end of beam and have no phase differences. Eight sensors are used to measure the displacements and the accelerations throughout the beam. For studying the crack (a breathing crack which starts breathing at $t = 2$ s) the identification process has two steps: first, the beam is divided to 5 probable damage segments, and then the beam is divided to 10 damage segments. 2% white noise contamination is added to measure the records for modeling the unpredictable errors. The following parameters are used as a feed to the adaptive tracking algorithm:

$$\mathbf{P}_{\mathbf{0}_5} = 10^6 \mathbf{I}_5, \quad \mathbf{P}_{\mathbf{0}_{10}} = 10^6 \mathbf{I}_{10} \quad \text{and} \quad \kappa = 5.0.$$

Before the time $t = 0.2$ s, the algorithm showed random parameter variation which was then canceled because of non-adequate measurement from the beam. The procedure of detecting the crack in its opening phase and the mean damage magnitude are shown in Fig. 1.

3.3 Identification of a beam with two breathing cracks

Two cracks with the depths of $a_1 = 0.3$ h and $a_2 = 0.4$ h are considered to be placed in the beam at $0.35L$ and $0.55L$, respectively, where h is the beam depth. The bending stiffness is taken as 10 Nm^2 and the mass per length is $\rho A = 0.35$ kg/m. For exciting the beam, three

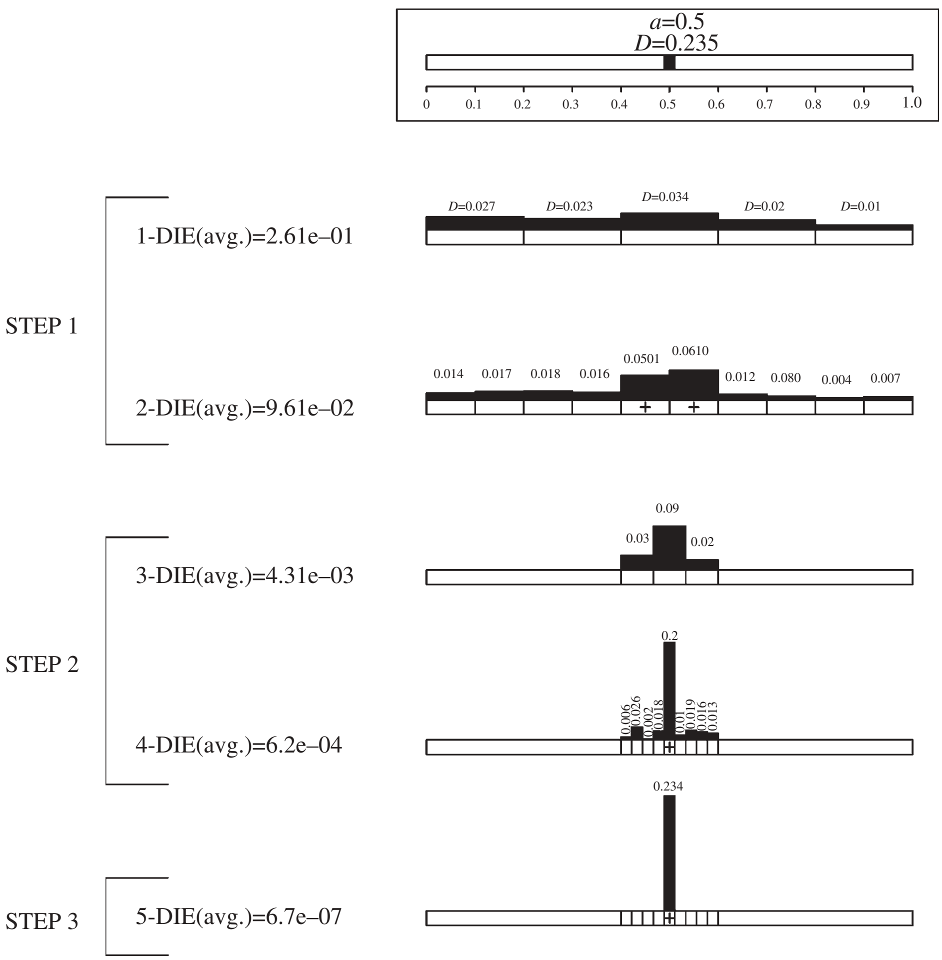

Fig. 1. The results of the identification for a single crack at the middle of a beam and different steps of the process

harmonic forces with $\omega_1 = 20$ rad/s, $\omega_2 = 120$ rad/s and $\omega_3 = 650$ rad/s acting at the end of the beam are considered having no phase differences. Eight sensors are used to measure the displacements and accelerations throughout the beam. For studying the crack, a crack which is placed at 0.55 L is breathing after the vibration of the beam, and another crack which is placed at 0.35 L starts breathing at $t = 2$s. The identification process has two steps: first, the beam is divided into 5 probable damage segments and then the beam is divided to 10 damage segments. 2% white noise contamination is added to the measured records to model the unpredictable errors. The following parameters are used as a feed to the adaptive tracking algorithm:

$$\mathbf{P}_{\mathbf{0}_5} = 10^6\mathbf{I}_5, \quad \mathbf{P}_{\mathbf{0}_{10}} = 10^6\mathbf{I}_{10} \quad \text{and} \quad \kappa = 7.0.$$

As an example, the procedure of detecting the crack in its opening phase and the mean damage magnitude are shown in Fig. 2, and the variation of the identified equivalent damage

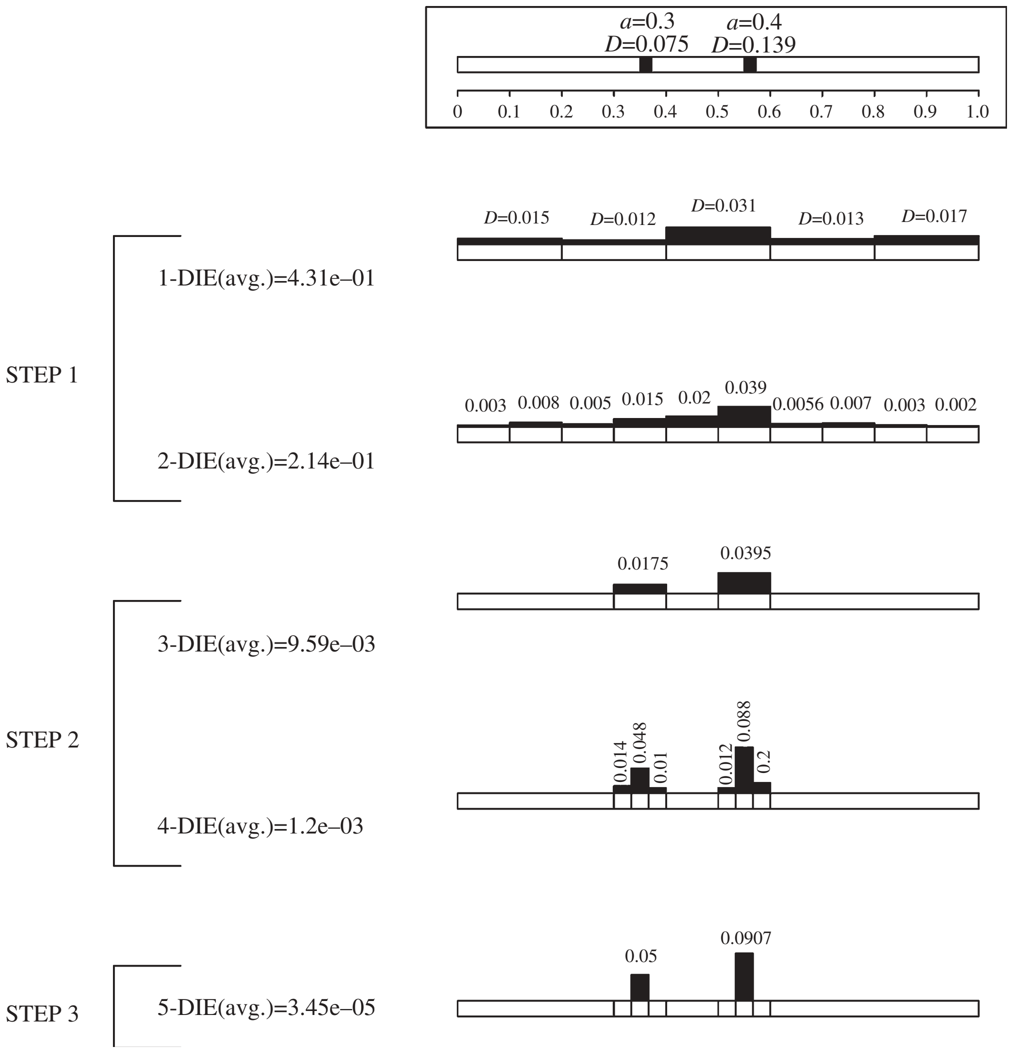

Fig. 2. The result of identification for a beam with two breathing cracks

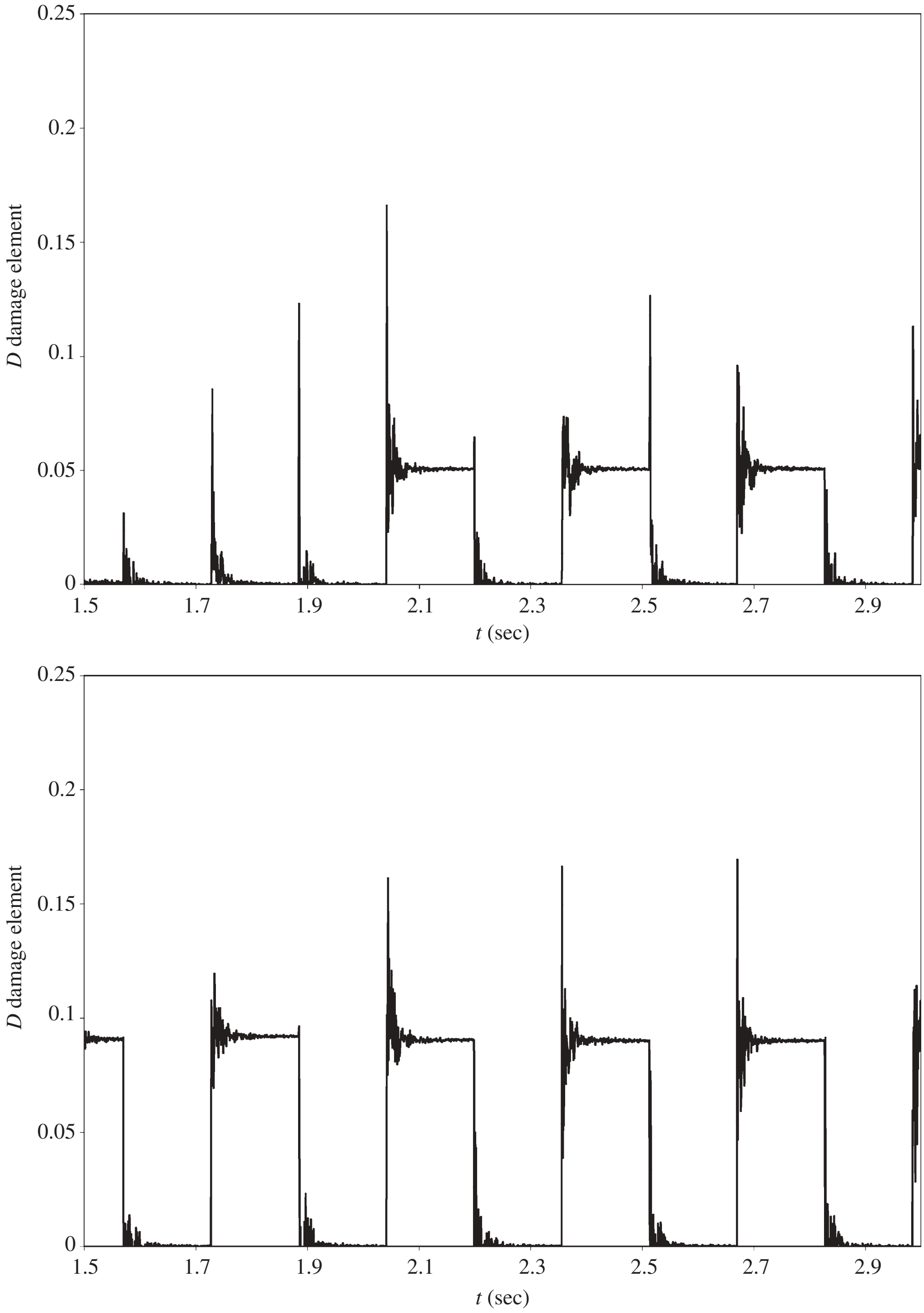

Fig. 3. The variation of damage magnitude with time, in elements containing cracks (the last step of the identification process)

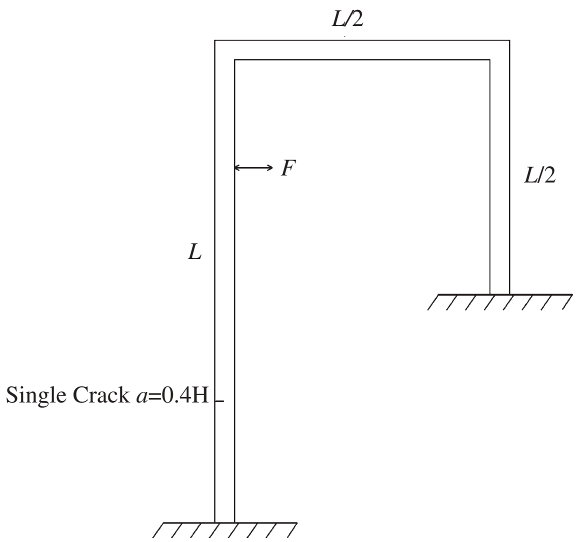

Fig. 4. Scheme of the frame and its crack position

magnitude with time, for the two segments with the corresponding cracks, are presented in Fig. 3.

3.4 Identification of a U-shaped frame with a breathing crack

In this example, a U-shaped frame is considered and a single crack is placed in 0.25 L from the fix support in the right leg which is start breathing at $t = 2.18$ s with the depth of 0.4 H (Fig. 4). The frame properties are:

L = 1.7 m, B = 0.025 m, H = 0.019 m, and $\rho = 7800$ kg/m^3.

By placing 12 sensors in the frame and using 12 modes of the frame, in two processes, the crack is identified, and with the use of the reduce-domain algorithm the required precision is achieved. In order to simulate the noise in a real measurement, the computed displacements and accelerations are contaminated by 2% white noise according to Eq. (47). Also for the excitation of the frame, four harmonic loads with no phase differences are excreted to frame at point O (Fig. 5) with the frequencies $\omega = 50$, 100, 150, 200 Hz. The following parameters are used in the adaptive tracking algorithm:

$$\kappa = 7.0 \quad \text{and} \quad \mathbf{P}_0 = 10^8\mathbf{I}.$$

The result of identification and its error are shown in Fig. 5. In this example using the reduced-domain algorithm, it becomes possible to identify the crack more precisely, and by tracking changes of equivalent damage of cracked segment, the behavior of the crack can be monitored as shown in Fig. 6.

4 Concluding remarks

An adaptive tracking technique, based on the least square estimation, has been proposed to identify the time-varying crack in continuous structures such as an Euler-Bernoulli beam and frames. With a similar formulation, it is possible to use this technique in other continuous structures such as the Timoshenko beam and plate. An adaptive tracking condition to identify

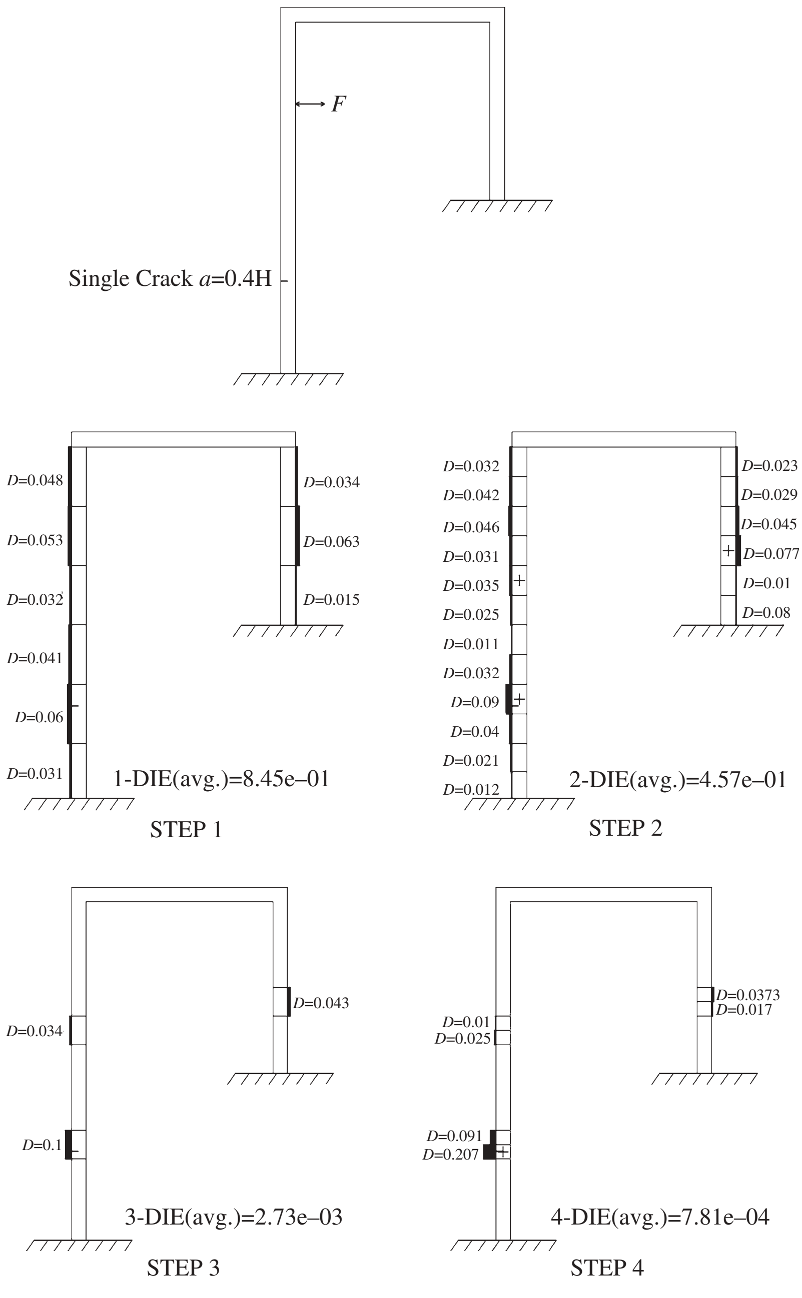

Fig. 5. The result of the U-shaped frame identification

the changes of a system parameter has been derived for the least square estimation approach, and a solution is presented using the minimum variation principle leading to the constrained optimization algorithm. A special case of time-varying crack is a breathing crack that can be detected by the use of the proposed identification procedure, and its nonlinear behavior can be detected. With the use of the proposed dynamic equation of motion of cracked beams, the estimation of modal data of the cracked structure is eliminated, and the domain or size of the

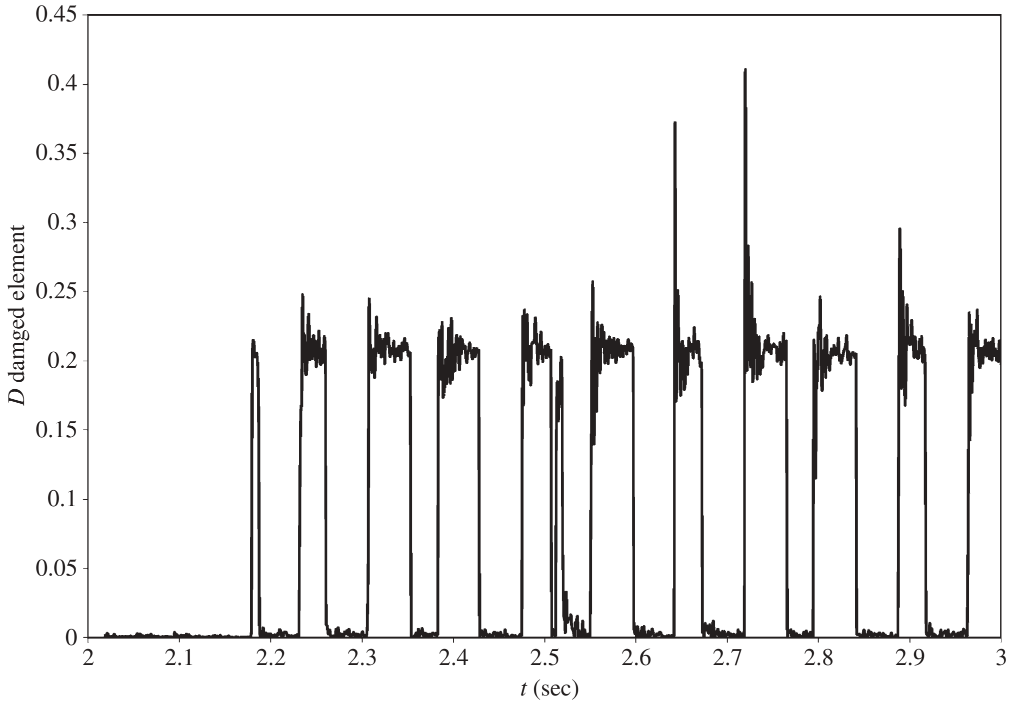

Fig. 6. The variation of equivalent damage with time for an element of a U-shaped frame containing a crack (the last step of the identification process)

problem is drastically reduced by iteratively using the reduced domain method, introduced in this paper.

Using a proper model of breathing crack, the feasibility of the present structural damage identification method is verified through some numerically simulated damage identification tests. In general, the reduced-domain method is found to provide the most reliable damage identification. Further study in the modeling of the breathing crack and identifying its parameters is required for a higher noise level of the sensors.

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