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Kawashima condition and acceleration waves for binary nonreacting mixtures

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Summary. The existence of global smooth solutions for a binary nonreacting mixture of Grad type is proved by using the Kawashima condition. The propagation of acceleration waves is also investigated. The characteristic speeds and the amplitude equation are derived.

1 Introduction

In the general theory of a hyperbolic system of balance laws, the requirement of a convex entropy density leads to the locally well-posedness of the initial value problem, that is the system has a unique local (in time) smooth solution for smooth initial data, provided that the fluxes and the productions are smooth enough. However, these smooth solutions may develop singularities, shocks or blowup in a finite time so that the existence of the global smooth solutions is not ensured.

On the other hand, the presence of production terms may lead to the existence of global smooth solutions owing to the competition between the dissipation and the hyperbolicity. More precisely, if the dissipation dominates the hyperbolicity, we expect that the smooth solutions exist for all time and converge to a constant state. This kind of system is called dissipative.

One way to identify whether a hyperbolic system with productions is dissipative is given by the so-called Kawashima condition, firstly introduced by Shizuta and Kawashima [1], [2], or genuine coupling [3]. Roughly speaking, if we consider a general one-dimensional hyperbolic system of balance laws

$$\partial_t \boldsymbol{F}^{(0)}(\boldsymbol{U}) + \partial_x \boldsymbol{F}(\boldsymbol{U}) = \boldsymbol{f}(\boldsymbol{U}), \tag{1}$$

the Kawashima condition asserts that in the equilibrium manifold any characteristic eigenvector $\boldsymbol{d}$ is not in the null space of $\nabla_{\boldsymbol{U}} \boldsymbol{f}$, that is

$$\nabla_{\boldsymbol{U}} \boldsymbol{f} \cdot \boldsymbol{d}|_{eq} \neq \boldsymbol{0}. \tag{2}$$

For a strictly dissipative system satisfying the Kawashima condition Hanouzet and Natalini [1] have proved the global existence of smooth solutions. This result has been generalized by Yong [4] in the case of n-dimensional systems. Recently, Ruggeri [5], [6] has analyzed the existence of global smooth solutions for a binary mixture of Euler fluids. He has proved that the

Kawashima condition is satisfied only in presence of chemical reactions, while in absence of them, the global existence remains an open problem.

In our opinion this is due to the fact that in mixtures of Euler fluids, viscosity and heat conduction are both neglected so that the dissipative effects are due only to the chemical reactions and to the diffusion. These two dissipative effects prevail over the hyperbolicity but, if chemical reactions are also neglected, the dissipation is not sufficiently strong to dominate the hyperbolicity, and consequently the Kawashima condition is violated.

Therefore, it seems reasonable to us to investigate the existence of global smooth solutions for binary mixtures of ideal gases taking into account the effects of viscosity and of heat conduction. In fact, by using the model proposed by Heckl and Müller [7] in the context of Grad's theory, we prove that the Kawashima condition holds also in absence of chemical reactions. On the other hand, Ruggeri [8] has proved that the Kawashima condition is satisfied for an ideal viscous heat conductive gas described by the Grad's theory.

Since the Kawashima condition has an interpretation in terms of the acceleration waves [9], [10], finally we analyze the propagation of these waves in a binary nonreacting mixture of ideal gases.

2 Field equations

Let us consider a binary nonreacting mixture of ideal monatomic gases, described by the model proposed in [7]. This model, based upon the Grad's theory, contains an extended set of variables. In fact, in addition to the classical ones, also the diffusion flux and the concentration of one constituent as well as the stress deviators and the heat fluxes of both constituents are assumed as field variables.

We restrict our analysis to the one-dimensional case so that the field equations describing a binary nonreacting mixture are given by

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0, \tag{3.1}$$

$$\frac{\partial(\rho c)}{\partial t} + \frac{\partial[\rho c(v+u)]}{\partial x} = 0, \tag{3.2}$$

$$\frac{\partial(\rho v)}{\partial t} + \frac{\partial}{\partial x}\left[\rho\left(\frac{k}{m_1}c + \frac{k}{m_2}(1-c)\right)T + \sigma^{(1)} + \sigma^{(2)} + \rho\left(v^2 + \frac{c}{1-c}u^2\right)\right] = 0, \tag{3.3}$$

$$\frac{\partial[\rho c(v+u)]}{\partial t} + \frac{\partial}{\partial x}\left[\rho c\frac{k}{m_1}T + \sigma^{(1)} + \rho c(v+u)^2\right] = -\rho^2 cZu, \tag{3.4}$$

$$\begin{aligned}\frac{\partial}{\partial t}&\left[3\rho\left(\frac{k}{m_1}c + \frac{k}{m_2}(1-c)\right)T + \rho\left(v^2 + \frac{c}{1-c}u^2\right)\right] + \frac{\partial}{\partial x}\left[2\left(q^{(1)} + q^{(2)}\right) + 5\rho c\left(\frac{k}{m_1} - \frac{k}{m_2}\right)Tu\right.\\ &+ 5\rho\left(\frac{k}{m_1}c + \frac{k}{m_2}(1-c)\right)Tv + 2\sigma^{(1)}(v+u) + 2\sigma^{(2)}\left(v - \frac{c}{1-c}u\right)\\ &\left. + \rho\frac{c}{1-c}u^2\left(2v + \frac{1-2c}{1-c}u\right) + \rho v\left(v^2 + \frac{c}{1-c}u^2\right)\right] = 0,\end{aligned} \tag{3.5}$$

$$\frac{\partial}{\partial t}\left[\sigma^{(1)}+\frac{2}{3}\rho c(v+u)^2\right]+\frac{\partial}{\partial x}\left\{\frac{8}{15}q^{(1)}+\frac{1}{3}\left[4\rho c\frac{k}{m_1}T+7\sigma^{(1)}\right](v+u)+\frac{2}{3}\rho c(v+u)^3\right\}$$

$$=\frac{\rho c}{m_1}\left[H^{(1)}\sigma^{(1)}+\frac{1-c}{c}H^{(2)}\sigma^{(1)}+H^{(3)}\sigma^{(2)}\right]-\frac{4}{3}\rho^2cZu(v+u)+\frac{2}{3}\frac{\rho^2}{m_1}\frac{c}{1-c}H^{(3)}u^2, \tag{3.6}$$

$$\frac{\partial}{\partial t}\left[\sigma^{(2)}+\frac{2}{3}\rho(1-c)\left(v-\frac{c}{1-c}u\right)^2\right]+\frac{\partial}{\partial x}\left\{\frac{8}{15}q^{(2)}+\frac{1}{3}\left[4\rho(1-c)\frac{k}{m_2}T+7\sigma^{(2)}\right]\left(v-\frac{c}{1-c}u\right)\right.$$

$$\left.+\frac{2}{3}\rho(1-c)\left(v-\frac{c}{1-c}u\right)^3\right\}=\frac{\rho(1-c)}{m_2}\left[H^{(3)}\sigma^{(1)}+H^{(4)}\sigma^{(2)}+\frac{c}{1-c}H^{(5)}\sigma^{(2)}\right]$$

$$+\frac{4}{3}\rho^2cZu\left(v-\frac{c}{1-c}u\right)+\frac{2}{3}\frac{\rho^2}{m_2}\frac{c}{1-c}H^{(3)}u^2, \tag{3.7}$$

$$\frac{\partial}{\partial t}\left\{q^{(1)}+\left[\frac{5}{2}\rho c\frac{k}{m_1}T+\sigma^{(1)}\right](v+u)+\frac{1}{2}\rho c(v+u)^3\right\}+\frac{\partial}{\partial x}\left\{\frac{1}{2}\frac{k}{m_1}T\left[5\rho c\frac{k}{m_1}T+7\sigma^{(1)}\right]\right.$$

$$\left.+\frac{16}{5}q^{(1)}(v+u).+\left[4\rho c\frac{k}{m_1}T+\frac{5}{2}\sigma^{(1)}\right](v+u)^2+\frac{1}{2}\rho c(v+u)^4\right\}$$

$$=\frac{\rho c}{m_1^2}\left[K^{(1)}q^{(1)}+\frac{1-c}{c}K^{(2)}q^{(1)}+K^{(3)}q^{(2)}\right]+\frac{\rho c}{m_1}\left[H^{(1)}\sigma^{(1)}+\frac{1-c}{c}H^{(2)}\sigma^{(1)}+H^{(3)}\sigma^{(2)}\right](v+u)$$

$$-\frac{3}{2}\rho^2cZu(v+u)^2-\frac{5}{2}\rho^2cZ\frac{k}{m_1}Tu+\frac{1}{3}\frac{\rho^2}{m_1}\frac{c}{1-c}\hat{Z}(v+u)u^2$$

$$-\frac{\rho}{m_1^2}\left[m_1^2Z\sigma^{(1)}+\frac{c}{1-c}K^{(3)}\sigma^{(2)}+2\frac{\rho c}{(1-c)^2}\frac{m_1^2m_2^2}{(m_1+m_2)^2}Zu^2\right]u, \tag{3.8}$$

$$\frac{\partial}{\partial t}\left\{q^{(2)}+\left[\frac{5}{2}\rho(1-c)\frac{k}{m_2}T+\sigma^{(2)}\right]\left(v-\frac{c}{1-c}u\right)+\frac{1}{2}\rho(1-c)\left(v-\frac{c}{1-c}u\right)^3\right\}$$

$$+\frac{\partial}{\partial x}\left\{\frac{1}{2}\frac{k}{m_2}T\left[5\rho(1-c)\frac{k}{m_2}T+7\sigma^{(2)}\right]+\frac{16}{5}q^{(2)}\left(v-\frac{c}{1-c}u\right)\right.$$

$$\left.+\left[4\rho(1-c)\frac{k}{m_2}T+\frac{5}{2}\sigma^{(2)}\right]\left(v-\frac{c}{1-c}u\right)^2+\frac{1}{2}\rho(1-c)\left(v-\frac{c}{1-c}u\right)^4\right\}=\frac{\rho(1-c)}{m_2^2}$$

$$\times\left[K^{(3)}q^{(1)}+K^{(4)}q^{(2)}+\frac{c}{1-c}K^{(5)}q^{(2)}\right]+\frac{\rho(1-c)}{m_2}\left[H^{(3)}\sigma^{(1)}+H^{(4)}\sigma^{(2)}+\frac{c}{1-c}H^{(5)}\sigma^{(2)}\right]$$

$$\times\left(v-\frac{c}{1-c}u\right)+\frac{3}{2}\rho^2cZu\left(v-\frac{c}{1-c}u\right)^2+\frac{5}{2}\rho^2cZ\frac{k}{m_2}Tu+\frac{1}{3}\frac{\rho^2}{m_2}\frac{c}{1-c}\hat{Z}\left(v-\frac{c}{1-c}u\right)u^2$$

$$+\frac{\rho}{m_2^2}\left[K^{(3)}\sigma^{(1)}+\frac{c}{1-c}m_2^2Z\sigma^{(2)}+2\frac{\rho c}{(1-c)^2}\frac{m_1^2m_2^2}{(m_1+m_2)^2}Zu^2\right]u, \tag{3.9}$$

where

$\rho = \rho_1 + \rho_2$ is the density of the mixture,
ρ_1 and ρ_2 are the densities of the two constituents,
$v = \frac{\rho_1 v^{(1)} + \rho_2 v^{(2)}}{\rho}$ is the barycentric velocity,
$v^{(1)}$ and $v^{(2)}$ are the velocities of the two constituents,
$c = \frac{\rho_1}{\rho}$ is the concentration of the first constituent,
$u = v^{(1)} - v$ is the diffusion velocity of the first constituent,
k is the Boltzmann constant,
m_1 and m_2 are the two molecular masses,
T is the temperature of the two constituents,
$\sigma^{(1)}$ and $\sigma^{(2)}$ are the components of the deviatoric part
of the stress tensors of the two constituents,
$q^{(1)}$ and $q^{(2)}$ are the components of the heat fluxes of the two constituents,

while the production terms read as follows:

$$\begin{aligned}
&Z = \frac{4\pi}{m_1+m_2} Y_1^{12}, \qquad \hat{Z} = 2H^{(3)} + 5\frac{m_1 m_2}{m_1+m_2} Z,\\
&H^{(1)} = 6\pi\left(Y_2^{11} - Y_1^{11}\right), \quad H^{(2)} = \frac{4\pi m_1}{(m_1+m_2)^2}\left(3m_2 Y_2^{12} - (2m_1 + 3m_2) Y_1^{12}\right),\\
&H^{(3)} = \frac{4\pi m_1 m_2}{(m_1+m_2)^2}\left(3Y_2^{12} - Y_1^{12}\right), \quad H^{(5)} = \frac{4\pi m_2}{(m_1+m_2)^2}\left(3m_1 Y_2^{12} - (3m_1 + 2m_2) Y_1^{12}\right),\\
&H^{(4)} = 6\pi\left(Y_2^{22} - Y_1^{22}\right), \quad K^{(2)} = \frac{4\pi m_1^2}{(m_1+m_2)^3}\left(4m_1 m_2 Y_2^{12} - \left(3m_1^2 + 4m_1 m_2 + m_2^2\right) Y_1^{12}\right),\\
&K^{(1)} = 4\pi m_1\left(Y_2^{11} - Y_1^{11}\right), \quad K^{(4)} = 4\pi m_2\left(Y_2^{22} - Y_1^{22}\right),\\
&K^{(3)} = \frac{16\pi m_1^2 m_2^2}{(m_1+m_2)^3} Y_2^{12}, \quad K^{(5)} = \frac{4\pi m_2^2}{(m_1+m_2)^3}\left(4m_1 m_2 Y_2^{12} - \left(m_1^2 + 4m_1 m_2 + 3m_2^2\right) Y_1^{12}\right).
\end{aligned} \tag{4}$$

The constants Y_i depend on the strength of the interaction between particles of the two constituents and they are defined as [7]

$$Y_1^{\alpha\beta} = \int_0^{\frac{\pi}{2}} f_{\alpha\beta} \sin\theta_{\beta\alpha} \cos^2\theta_{\beta\alpha} d\theta_{\beta\alpha},$$

$$Y_2^{\alpha\beta} = \int_0^{\frac{\pi}{2}} f_{\alpha\beta} \sin\theta_{\beta\alpha} \cos^4\theta_{\beta\alpha} d\theta_{\beta\alpha},$$

where $\theta_{\beta\alpha} = (\pi - \chi_{\beta\alpha})/2$, with $\chi_{\beta\alpha}$ the scattering angle, while $f_{\alpha\beta} = \sigma_{\alpha\beta} g^{\alpha\beta}$, with $\sigma_{\alpha\beta}$ the effective cross section for $\alpha\beta$ -scattering into the solid angle element $\sin\theta_{\beta\alpha} d\theta_{\beta\alpha} d\varepsilon$ and $g^{\alpha\beta} = c^{\alpha} - c^{\beta}$ the relative velocity of molecules α and β . In the case of Maxwellian molecules, considered therein, $f_{\alpha\beta}$ is a function of $\theta_{\beta\alpha}$ alone.

The system under consideration (3) is a set of nine equations in the nine field variables, that is

$$\boldsymbol{U} = \left(\rho, c, v, u, T, \sigma^{(1)}, \sigma^{(2)}, q^{(1)}, q^{(2)}\right)^T. \tag{5}$$

In particular, Eqs. (3.1, 3, 5) represent the conservation laws of mass, momentum and energy of the whole mixture, while (3.2, 4) are the balance laws of mass and momentum of the first constituent, and (3.6–9) are the balance laws of the stress deviators and the heat fluxes of the two constituents, respectively.

More precisely, system (3) belongs to the class of hyperbolic conservative systems (1) with dissipation due to the presence of production terms, which can be transformed into the quasi-linear form

$$A^{(0)}(\boldsymbol{U})\boldsymbol{U}_t + A(\boldsymbol{U})\boldsymbol{U}_x = \boldsymbol{f}(\boldsymbol{U}), \tag{6}$$

with $A^{(0)} = \frac{\partial \boldsymbol{F}^{(0)}}{\partial \boldsymbol{U}}$ and $A = \frac{\partial \boldsymbol{F}}{\partial \boldsymbol{U}}$.

On the other hand, since $A^{(0)}$ is non singular, system (6) may be recast into the normal form

$$\boldsymbol{U}_t + \hat{A}(\boldsymbol{U})\boldsymbol{U}_x = \hat{\boldsymbol{f}}(\boldsymbol{U}), \tag{7}$$

with $\hat{A} = \left(A^0\right)^{-1} A$ and $\hat{\boldsymbol{f}} = \left(A^0\right)^{-1} \boldsymbol{f}$.

3 Entropy law and structure of the system

System (3) is an hyperbolic system compatible with the entropy law

$$\frac{\partial(\rho\eta)}{\partial t} + \frac{\partial}{\partial x}(\rho\eta v + \phi) = \Sigma, \tag{8}$$

where

$\rho\eta = \rho_1\eta_1 + \rho_2\eta_2$ is the entropy density of the mixture, (9.1)

$$\eta_\alpha = \frac{3}{2}\frac{k}{m_\alpha}\ln T - \frac{k}{m_\alpha}\ln\rho_\alpha - \frac{1}{4}\frac{\sigma^{(\alpha)}\sigma^{(\alpha)}}{\frac{k}{m_\alpha}\rho_\alpha^2 T^2} - \frac{1}{5}\frac{q^{(\alpha)}q^{(\alpha)}}{\left(\frac{k}{m_\alpha}\right)^2\rho_\alpha^2 T^3}$$

is the entropy density of the constituent α, (9.2)

$\phi = \rho_1(\eta_1 - \eta_2)u + \phi^{(1)} + \phi^{(2)}$ is the entropy flux of the mixture, (9.3)

$\phi^{(\alpha)} = \dfrac{q^{(\alpha)}}{T} - \dfrac{2}{5}\dfrac{\sigma^{(\alpha)}q^{(\alpha)}}{\frac{k}{m_\alpha}\rho_\alpha T^2}$ is the entropy flux of the constituent α, (9.4)

$$\begin{aligned}\Sigma = {} & \frac{\rho^2}{T}\frac{c}{1-c}Zu^2 - \frac{1}{2kT^2}\left[\left(H^{(1)} + \frac{1-c}{c}H^{(2)}\right)\sigma^{(1)}\sigma^{(1)} + 2H^{(3)}\sigma^{(1)}\sigma^{(2)}\right. \\ & \left. + \left(H^{(4)} + \frac{c}{1-c}H^{(5)}\right)\sigma^{(2)}\sigma^{(2)}\right] - \frac{2}{5k^2T^3}\left[\left(K^{(1)} + \frac{1-c}{c}K^{(2)}\right)q^{(1)}q^{(1)}\right. \\ & \left. + 2K^{(3)}q^{(1)}q^{(2)} + \left(K^{(4)} + \frac{c}{1-c}K^{(5)}\right)q^{(2)}q^{(2)}\right] \end{aligned} \tag{9.5}$$

is the entropy production of the mixture.

Due to the complex structure of the system under consideration, we limit our analysis to mixtures in which the difference between molecular masses of the two constituents is negligible, namely $m_1 \approx m_2 = m$. This "equal masses assumption" is the same considered in [5] for mixtures of Euler fluids and, for this particular case, it was proved that the first and the second sound are uncoupled [11].

Let us show that $h = -\rho\eta$ is a convex function with respect to $\boldsymbol{F}^{(0)}$ at least in a neighborhood of the equilibrium state. In fact, by considering h as a function of the densities $\boldsymbol{F}^{(0)}$, the Hessian matrix $\frac{\partial^2 h}{\partial \boldsymbol{F}^{(0)} \partial \boldsymbol{F}^{(0)}}$ evaluated in equilibrium state

$$\boldsymbol{U}_0 = (\rho_0, c_0, 0, 0, T_0, 0, 0, 0, 0)^T \tag{10}$$

reads

$$\left.\frac{\partial^2 h}{\partial \boldsymbol{F}^{(0)} \partial \boldsymbol{F}^{(0)}}\right|_{\mathrm{eq}} = \frac{k}{m}\begin{pmatrix} \frac{5-3c_0}{2(1-c_0)\rho_0} & -\frac{1}{(1-c_0)\rho_0} & 0 & 0 & -\frac{3}{2\varepsilon_0} & 0 & 0 & 0 & 0 \\ -\frac{1}{(1-c_0)\rho_0} & \frac{1}{(1-c_0)\rho_0 c_0} & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{21}{2(1-c_0)\varepsilon_0} & -\frac{21}{2(1-c_0)\varepsilon_0} & 0 & 0 & 0 & 0 & -\frac{9\rho_0}{(1-c_0)\varepsilon_0^2} \\ 0 & 0 & -\frac{21}{2(1-c_0)\varepsilon_0} & \frac{21}{2c_0(1-c_0)\varepsilon_0} & 0 & 0 & 0 & -\frac{9\rho_0}{c_0\varepsilon_0^2} & \frac{9\rho_0}{(1-c_0)\varepsilon_0^2} \\ -\frac{3}{2\varepsilon_0} & 0 & 0 & 0 & \frac{3\rho_0}{2\varepsilon_0^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{9\rho_0}{2c_0\varepsilon_0^2} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{9\rho_0}{2(1-c_0)\varepsilon_0^2} & 0 & 0 \\ 0 & 0 & 0 & -\frac{9\rho_0}{c_0\varepsilon_0^2} & 0 & 0 & 0 & \frac{54\rho_0^2}{5c_0\varepsilon_0^3} & 0 \\ 0 & 0 & -\frac{9\rho_0}{(1-c_0)\varepsilon_0^2} & \frac{9\rho_0}{(1-c_0)\varepsilon_0^2} & 0 & 0 & 0 & 0 & \frac{54\rho_0^2}{5(1-c_0)\varepsilon_0^3} \end{pmatrix}, \tag{11}$$

where $2\varepsilon_0 = 3\frac{k}{m}\rho_0 T_0$ denotes the density of internal energy of the mixture at equilibrium. Then, it is easy to see that the matrix $\left.\frac{\partial^2 h}{\partial \boldsymbol{F}^{(0)} \partial \boldsymbol{F}^{(0)}}\right|_{\mathrm{eq}}$ is positive definite so that h is a convex function with respect to $\boldsymbol{F}^{(0)}$ at least in a neighborhood of the equilibrium state.

Then, in order to prove that system (3) is a strictly entropy dissipative system [1], we firstly introduce new variables, the so-called entropy variables defined as

$$\boldsymbol{W} = \frac{\partial h}{\partial \boldsymbol{F}^{(0)}}. \tag{12}$$

Since h is a convex function with respect to $\boldsymbol{F}^{(0)}$ in a neighborhood of the equilibrium state, hereafter we neglect the second order terms in the dissipative fluxes. Therefore, after some tedious manipulations, the entropy variables assume the form

$$\begin{aligned}
W_1 &= \frac{k}{m}\left[\frac{5}{2} + \ln\frac{\rho(1-c)}{\sqrt{T^3}}\right] + \frac{cu}{(1-c)T}v + \frac{q^{(1)}+q^{(2)}}{\rho\frac{k}{m}T^2}v - \frac{1}{2T}v^2 + \frac{\sigma^{(2)}}{3\rho(1-c)\frac{k}{m}T^2}v^2 \\
&\quad - \frac{1}{3\rho\left(\frac{k}{m}\right)^2 T^3}\left[q^{(1)} - \frac{c}{1-c}q^{(2)}\right]v^3 - \frac{2q^{(2)}}{15\rho(1-c)\left(\frac{k}{m}\right)^2T^3}v^3, \\
W_2 &= \frac{k}{m}\ln\frac{c}{1-c} - \frac{u}{(1-c)T}v + \frac{1}{3\rho c\frac{k}{m}T^2}\left[\sigma^{(1)} - \frac{c}{1-c}\sigma^{(2)}\right]v^2 - \frac{2}{15\rho c\left(\frac{k}{m}\right)^2T^3}\left[q^{(1)} - \frac{c}{1-c}q^{(2)}\right]v^3, \\
W_3 &= \frac{1}{T}\left(v - \frac{c}{1-c}u\right) - \frac{1}{\rho(1-c)\frac{k}{m}T^2}q^{(2)} - \frac{2\sigma^{(2)}}{3\rho(1-c)\frac{k}{m}T^2} - \frac{q^{(2)}}{15\rho(1-c)\left(\frac{k}{m}\right)^2T^3}v^2 \\
&\quad + \frac{2}{3\rho\left(\frac{k}{m}\right)^2T^3}\left[q^{(1)} + q^{(2)}\right]v^2, \\
W_4 &= \frac{u}{(1-c)T} - \frac{1}{\rho c\frac{k}{m}T^2}\left[q^{(1)} - \frac{c}{1-c}q^{(2)}\right] - \frac{2}{3\rho c\frac{k}{m}T^2}\left[\sigma^{(1)} - \frac{c}{1-c}\sigma^{(2)}\right]v \\
&\quad - \frac{1}{15\rho c\left(\frac{k}{m}\right)^2T^3}\left[q^{(1)} - \frac{c}{1-c}q^{(2)}\right]v^2, \\
W_5 &= -\frac{1}{2T} - \frac{q^{(1)}+q^{(2)}}{3\rho\left(\frac{k}{m}\right)^2T^3}v, \\
W_6 &= \frac{\sigma^{(1)}}{2\rho c\frac{k}{m}T^2} - \frac{2q^{(1)}}{5\rho c\left(\frac{k}{m}\right)^2T^3}v, \\
W_7 &= \frac{\sigma^{(2)}}{2\rho(1-c)\frac{k}{m}T^2} - \frac{2q^{(2)}}{5\rho(1-c)\left(\frac{k}{m}\right)^2T^3}v, \\
W_8 &= \frac{2q^{(1)}}{5\rho c\left(\frac{k}{m}\right)^2T^3}, \\
W_9 &= \frac{2q^{(2)}}{5\rho(1-c)\left(\frac{k}{m}\right)^2T^3}.
\end{aligned} \tag{13}$$

Now we can split $\boldsymbol{W}$ into two sets of variables $\boldsymbol{W} = (\hat{\boldsymbol{V}}, \boldsymbol{V})$ corresponding to the conservation and the balance laws, respectively, namely

$$\hat{\boldsymbol{V}} = \begin{pmatrix} W_1 \\ W_2 \\ W_3 \\ W_5 \end{pmatrix} \qquad \boldsymbol{V} = \begin{pmatrix} W_4 \\ W_6 \\ W_7 \\ W_8 \\ W_9 \end{pmatrix}. \tag{14}$$

Consequently it is possible to prove the existence of a positive definite matrix $B(\boldsymbol{W})$ such that

$$Q(\boldsymbol{W}) = -B(\boldsymbol{W})(\boldsymbol{V} - \boldsymbol{V}_0), \tag{15}$$

where $\boldsymbol{V}_0$ denotes $\boldsymbol{V}$ evaluated in the equilibrium state (10), $Q(\boldsymbol{W})$ represents the non-null productions, while B is given by

$$B(\boldsymbol{W}) = cT\rho^2 \begin{pmatrix} \tilde{Z} & \frac{4\tilde{Z}v}{3} & -\frac{4\tilde{Z}v}{3} & \frac{\tilde{Z}\Theta}{2} & -\frac{\tilde{Z}\Theta}{2} \\ \frac{4\tilde{Z}v}{3} & -2\left(\hat{H}-\frac{8\tilde{Z}v^2}{9}\right) & -2\left(\tilde{H}+\frac{8\tilde{Z}v^2}{9}\right) & -2\left(\hat{H}-\frac{\tilde{Z}\Theta}{3}\right)v & -2\left(\tilde{H}+\frac{\tilde{Z}\Theta}{3}\right)v \\ -\frac{4\tilde{Z}v}{3} & -2\left(\tilde{H}+\frac{8\tilde{Z}v^2}{9}\right) & -2\left(\bar{H}-\frac{8\tilde{Z}v^2}{9}\right) & -2\left(\tilde{H}+\frac{\tilde{Z}\Theta}{3}\right)v & -2\left(\bar{H}-\frac{\tilde{Z}\Theta}{3}\right)v \\ \frac{\tilde{Z}\Theta}{2} & -2\left(\hat{H}-\frac{\tilde{Z}\Theta}{3}\right)v & -2\left(\tilde{H}+\frac{\tilde{Z}\Theta}{3}\right)v & -\left(\frac{5\hat{K}}{2}+2\hat{H}v^2-\frac{\tilde{Z}\Theta^2}{4}\right) & -\left(\frac{5\tilde{K}}{2}+2\tilde{H}v^2+\frac{\tilde{Z}\Theta^2}{4}\right) \\ -\frac{\tilde{Z}\Theta}{2} & -2\left(\tilde{H}+\frac{\tilde{Z}\Theta}{3}\right)v & -2\left(\bar{H}-\frac{\tilde{Z}\Theta}{3}\right)v & -\left(\frac{5\tilde{K}}{2}+2\tilde{H}v^2+\frac{\tilde{Z}\Theta^2}{4}\right) & -\left(\frac{5\bar{K}}{2}+2\bar{H}v^2-\frac{\tilde{Z}\Theta^2}{4}\right) \end{pmatrix}, \tag{16}$$

with

$$\tilde{Z} = (1-c)Z, \qquad \Theta = 5\theta + 3v^2, \qquad \theta = \frac{k}{m}T,$$

$$\hat{H} = \frac{\theta}{m}\left[cH^{(1)} + (1-c)H^{(2)}\right], \quad \tilde{H} = \frac{\theta}{m}(1-c)H^{(3)}, \quad \bar{H} = \frac{1-c}{c}\frac{\theta}{m}\left[(1-c)H^{(4)} + cH^{(5)}\right],$$

$$\hat{K} = \frac{\theta^2}{m^2}\left[cK^{(1)} + (1-c)K^{(2)}\right], \quad \tilde{K} = \frac{\theta^2}{m^2}(1-c)K^{(3)}, \quad \bar{K} = \tfrac{1-c}{c}\frac{\theta^2}{m^2}\left[(1-c)K^{(4)} + cK^{(5)}\right]. \tag{17}$$

The positive definiteness of the matrix B is guaranteed by the sign of the coefficients (4), which in the case of the ‘‘equal masses assumption’’ reduce to

$$
\begin{aligned}
&Z = \tfrac{2\pi}{m}Y_1^{12} > 0, && H^{(1)} = 6\pi\left(Y_2^{11} - Y_1^{11}\right)<0,\\
&H^{(2)} = H^{(5)} = \pi\left(3Y_2^{12} - 5Y_1^{12}\right)<0, && H^{(3)} = \pi\left(3Y_2^{12} - Y_1^{12}\right),\\
&H^{(4)} = 6\pi\left(Y_2^{22} - Y_1^{22}\right)<0, && K^{(1)} = 4\pi m\left(Y_2^{11} - Y_1^{11}\right)<0,\\
&K^{(2)} = K^{(5)} = 2\pi m\left(Y_2^{12} - 2Y_1^{12}\right)<0, && K^{(3)} = 2\pi m Y_2^{12} > 0,\\
&K^{(4)} = 4\pi m\left(Y_2^{22} - Y_1^{22}\right)<0,
\end{aligned} \tag{18}
$$

so that the system under consideration is strictly entropy dissipative [1].

Finally, as a consequence of the positive definiteness of the matrix B, the entropy production (9.5), which can be expressed in terms of $\boldsymbol{V}$ as [1]

$$
\boldsymbol{\Sigma} = \boldsymbol{V} \cdot B\boldsymbol{V}, \tag{19}
$$

is a positive quantity.

4 Kawashima condition

As system (3) is compatible with a convex entropy law and it is strictly dissipative, we verify the Kawashima condition in order to investigate the existence of global smooth solutions.

In order to check the Kawashima condition (2), we consider the characteristic problem associated to system (6), given by

$$
\left(A - \lambda A^{(0)}\right)\boldsymbol{d} = 0, \tag{20}
$$

where the eigenvalues λ satisfy the following characteristic equation:

$$
\begin{aligned}
&\overline{\lambda}\left[15\overline{\lambda}^4 - 25v\overline{\lambda}^3 - 43\frac{k}{m}T\overline{\lambda}^2 + 45\frac{k}{m}Tv\overline{\lambda} + 8\left(\frac{k}{m}T\right)^2\right]\\
&\quad\times\left\{15\rho^2\left(5\overline{\lambda}^4 - 26\frac{k}{m}T\overline{\lambda}^2 + 15\left(\frac{k}{m}T\right)^2\right) - 2\rho\left[5\left(31\overline{\lambda}^2 - 45\frac{k}{m}T\right)\left(\sigma^{(1)} + \sigma^{(2)}\right)\right.\right.\\
&\quad\left.\left. + 144\overline{\lambda}\left(q^{(1)} + q^{(2)}\right)\right] + 315\left(\sigma^{(1)} + \sigma^{(2)}\right)^2\right\} = 0,
\end{aligned} \tag{21}
$$

with $\overline{\lambda} = \lambda - v$.

Equation (21) evaluated in the equilibrium state reduces to

$$
\lambda\left(\lambda^2 - \frac{8}{3}\frac{k}{m}T_0\right)\left(\lambda^2 - \frac{1}{5}\frac{k}{m}T_0\right)\left(\lambda^2 - \frac{13-\sqrt{94}}{5}\frac{k}{m}T_0\right)\left(\lambda^2 - \frac{13+\sqrt{94}}{5}\frac{k}{m}T_0\right) = 0, \tag{22}
$$

from which we derive the following expressions for the eigenvalues:

$$
\begin{aligned}
&\lambda_1 = 0, \quad \lambda_{2,3} = \pm 2\sqrt{\frac{2}{3}\frac{k}{m}T_0}, \quad \lambda_{4,5} = \pm\sqrt{\frac{1}{5}\frac{k}{m}T_0},\\
&\lambda_{6,7} = \pm\sqrt{\frac{13-\sqrt{94}}{5}\frac{k}{m}T_0}, \quad \lambda_{8,9} = \pm\sqrt{\frac{13+\sqrt{94}}{5}\frac{k}{m}T_0}.
\end{aligned} \tag{23}
$$

Consequently, by virtue of Eq. (20), the right eigenvectors associated to the eigenvalues (23) read

$$\boldsymbol{d}_1 = \left(1,0,0,0,\frac{2}{3}\frac{T_0}{\rho_0},-\frac{5}{3}c_0\frac{k}{m}T_0-\frac{5}{3}(1-c_0)\frac{k}{m}T_0,0,0\right)^T,$$

$$\boldsymbol{d}_{2,3,4,5} = \left(0,1,0,\frac{\lambda}{c_0},0,\rho_0\left(\lambda^2-\frac{k}{m}T_0\right),-\rho_0\left(\lambda^2-\frac{k}{m}T_0\right),\right. \tag{24}$$

$$\left.\frac{15}{4}\rho_0\lambda\left(\lambda^2-\frac{7}{3}\frac{k}{m}T_0\right),-\frac{15}{4}\rho_0\lambda\left(\lambda^2-\frac{7}{3}\frac{k}{m}T_0\right)\right)^T,$$

$$\boldsymbol{d}_{6,7,8,9} = \left(1,0,\frac{\lambda}{\rho_0},0,\frac{5}{9}\frac{1}{\rho_0}\frac{m}{k}\left(\lambda^2-\frac{9}{5}\frac{k}{m}T_0\right),\frac{4}{9}c_0\lambda^2,\frac{4}{9}(1-c_0)\lambda^2,\right.$$

$$\left.\frac{5}{3}c_0\lambda\left(\lambda^2-3\frac{k}{m}T_0\right),\frac{5}{3}(1-c_0)\lambda\left(\lambda^2-3\frac{k}{m}T_0\right)\right)^T.$$

Finally, from system (3), we deduce

$$\nabla_{\boldsymbol{u}}\boldsymbol{f}|_0 = \rho_0\begin{pmatrix}
0\;0\;0 & 0 & 0 & 0 & 0 & 0 & 0\\
0\;0\;0 & 0 & 0 & 0 & 0 & 0 & 0\\
0\;0\;0 & 0 & 0 & 0 & 0 & 0 & 0\\
0\;0\;0 & -c_0\rho_0 Z & 0 & 0 & 0 & 0 & 0\\
0\;0\;0 & 0 & 0 & 0 & 0 & 0 & 0\\
0\;0\;0 & 0 & 0 & \frac{c_0H^{(1)}+(1-c_0)H^{(2)}}{m} & \frac{c_0H^{(3)}}{m} & 0 & 0\\
0\;0\;0 & 0 & 0 & \frac{(1-c_0)H^{(3)}}{m} & \frac{(1-c_0)H^{(4)}+c_0H^{(5)}}{m} & 0 & 0\\
0\;0\;0 & -\frac{5c_0\rho_0kT_0Z}{m} & 0 & 0 & 0 & \frac{c_0K^{(1)}+(1-c_0)K^{(2)}}{m^2} & \frac{c_0K^{(3)}}{m^2}\\
0\;0\;0 & \frac{5c_0\rho_0kT_0Z}{m} & 0 & 0 & 0 & \frac{(1-c_0)K^{(3)}}{m^2} & \frac{(1-c_0)K^{(4)}+c_0K^{(5)}}{m^2}
\end{pmatrix} \tag{25}$$

and, taking into account Eq. (24), we easily obtain

$$
\nabla_{\boldsymbol{U}}\boldsymbol{f}\cdot\boldsymbol{d}_1\big|_0 = 10\pi\rho_0\frac{k}{m^2}T_0\begin{pmatrix} 0\\ 0\\ 0\\ 0\\ 0\\ c_0A^{11}\\ (1-c_0)A^{12}\\ 0\\ 0 \end{pmatrix},
$$

$$
\nabla_{\boldsymbol{U}}\boldsymbol{f}\cdot\boldsymbol{d}_{2,3,4,5}\big|_0 = \frac{\pi}{m}\rho_0^2\begin{pmatrix} 0\\ 0\\ 0\\ -2\lambda Y_1^{12}\\ 0\\ -\left(\lambda^2-\frac{k}{m}T_0\right)\left(6A^{11}-Y_1^{12}+3Y_2^{12}\right)\\ \left(\lambda^2-\frac{k}{m}T_0\right)\left(6A^{12}-Y_1^{12}+3Y_2^{12}\right)\\ -5\lambda\left[2\dfrac{k}{m}T_0Y_1^{12}+\dfrac{3}{2}\left(\lambda^2-\dfrac{7}{3}\dfrac{k}{m}T_0\right)\left(2A^{11}+Y_2^{12}\right)\right]\\ 5\lambda\left[2\dfrac{k}{m}T_0Y_1^{12}+\dfrac{3}{2}\left(\lambda^2-\dfrac{7}{3}\dfrac{k}{m}T_0\right)\left(2A^{12}+Y_2^{12}\right)\right] \end{pmatrix}, \qquad (26)
$$

$$
\nabla_{\boldsymbol{U}}\boldsymbol{f}\cdot\boldsymbol{d}_{6,7,8,9}\big|_0 = -\frac{4\pi}{3m}\lambda\rho_0\begin{pmatrix} 0\\ 0\\ 0\\ 0\\ 0\\ 2c_0\lambda A^{11}\\ 2(1-c_0)\lambda A^{12}\\ 5c_0\left(\lambda^2-3\frac{k}{m}T_0\right)A^{11}\\ 5(1-c_0)\left(\lambda^2-3\frac{k}{m}T_0\right)A^{12} \end{pmatrix},
$$

where

$$A^{11} = \left[c_0\left(Y_1^{11} - Y_2^{11}\right) + (1 - c_0)\left(Y_1^{12} - Y_2^{12}\right)\right],$$
$$A^{12} = \left[c_0\left(Y_1^{12} - Y_2^{12}\right) + (1 - c_0)\left(Y_1^{22} - Y_2^{22}\right)\right], \quad (27)$$

which are nonzero as a consequence of the definition of the Y_i.

Therefore, in the equilibrium manifold all right eigenvectors are not in the null space of $\nabla_{\boldsymbol{U}}\boldsymbol{f}$, namely the Kawashima condition is satisfied. Consequently, since system (3) is strictly dissipative, we may conclude that binary mixtures of nonreacting ideal gases of Grad's type admit global smooth solutions if the initial data are sufficiently small.

5 Acceleration waves

Let's consider a moving curve $\Xi(t)$, usually called wave front, across which the field variables are continuous whereas their first derivatives may be discontinuous [12]. As it is well known, the normal speed of propagation V is equal to the characteristic eigenvalue evaluated in the unperturbed field $\boldsymbol{U}_r$ whereas the jump of the normal derivative of the field vector Π is proportional to the right eigenvector $\boldsymbol{d}$ evaluated in $\boldsymbol{U}_r$, that is

$$V = \lambda(\boldsymbol{U}_r), \quad (28.1)$$

$$\Pi = \delta\boldsymbol{U} = \Pi\boldsymbol{d}(\boldsymbol{U}_r). \quad (28.2)$$

If we consider as unperturbed field $\boldsymbol{U}_r$ the equilibrium state defined in (10), it is possible to establish the following relation between the acceleration waves and the Kawashima condition (2) [9, 10], i.e.,

$$\delta\boldsymbol{f}|_0 = \nabla_{\boldsymbol{U}}\boldsymbol{f} \cdot \delta\boldsymbol{U}|_0 \propto \nabla_{\boldsymbol{U}}\boldsymbol{f} \cdot \boldsymbol{d}|_0 \neq \boldsymbol{0}, \quad (29)$$

which holds for all the waves. This relation implies that, even if the production vanishes in equilibrium $\left(\boldsymbol{f}|_0 = \boldsymbol{0}\right)$, all the acceleration waves transport the disturbance of normal derivative of the production $\left(\delta\boldsymbol{f}|_0 \neq \boldsymbol{0}\right)$.

As it is well known, the amplitude Π of the jump satisfies the Bernoulli equation [12]

$$\frac{d\Pi}{dt} + a(t)\Pi^2 + b(t)\Pi = 0, \quad (30)$$

where $\frac{d}{dt}$ denotes the time derivative along bicharacteristic while, owing to the choice $\boldsymbol{U}_r = \boldsymbol{U}_0$, the coefficients a and b become constants and they are given, respectively, by

$$a = (\nabla_{\boldsymbol{U}}\lambda \cdot \boldsymbol{d})_0,$$
$$b = -\boldsymbol{l}_0 \cdot \left(\nabla_{\boldsymbol{U}}\hat{\boldsymbol{f}}\right)_0 \cdot \boldsymbol{d}_0. \quad (31)$$

Therefore, the solution of the Bernoulli equation (30) reads

$$\Pi(t) = \frac{\Pi_0 b e^{-bt}}{b - \Pi_0 a(e^{-bt} - 1)}, \quad (32)$$

Π_0 being the initial value of the amplitude.

From Eq. (32) we observe that if $a \neq 0$ the discontinuity becomes unbounded so that the acceleration waves may evolve into shock waves at the critical time

$$t_c = \frac{1}{b} \ln \left(\frac{1}{1 + \frac{b}{a} \frac{1}{\Pi_0}} \right), \tag{33}$$

provided that the initial amplitude Π_0 satisfies the condition

$$-1 < \frac{b}{a} \frac{1}{\Pi_0} < 0. \tag{34}$$

In our case, by virtue of Eq. (28.1), the characteristic velocities coincide with the eigenvalues (23).

It is easy to ascertain that the characteristic velocities λ_1, λ_6, λ_7, λ_8 and λ_9 are the same obtained for one single fluid described by Extended Thermodynamics of 13 moments [11] or, equivalently, by the Grad's theory. On the contrary, the characteristic speeds λ_2, λ_3, λ_4, and λ_5 are peculiar of the whole mixture. This result seems a natural consequence of the "equal masses assumption". Therefore, if the difference between molecular masses is negligible, it is possible to distinguish the properties concerning the single fluid from the ones characteristic of the whole mixture. An analogous result was also obtained for mixtures of Euler fluids [13].

In order to derive the amplitude of the jump (32), firstly we calculate the left eigenvectors corresponding to the characteristic velocities (23), that are

$$\boldsymbol{l}_1 = \frac{1}{L} \left(1, 0, 0, 0, \frac{\rho_0}{T_0}, -\frac{5}{4} \frac{m}{k} \frac{1}{T_0}, -\frac{5}{4} \frac{m}{k} \frac{1}{T_0}, 0, 0 \right),$$

$$\boldsymbol{l}_{2,3,4,5} = \frac{1}{\tilde{L}} \left(0, 1, 0, \frac{m}{k} \frac{c_0}{T_0} \lambda, 0, \frac{3}{4} \frac{m^2}{k^2} \frac{(1 - c_0)\left(\lambda^2 - \frac{k}{m} T_0\right)}{\rho_0 T_0^2}, -\frac{3}{4} \frac{m^2}{k^2} \frac{c_0 \left(\lambda^2 - \frac{k}{m} T_0\right)}{\rho_0 T_0^2}, \right.$$
$$\left. \frac{3}{8} \frac{m^3}{k^3} \frac{(1 - c_0)\lambda\left(\lambda^2 - \frac{7}{3} \frac{k}{m} T_0\right)}{\rho_0 T_0^3}, -\frac{3}{8} \frac{m^3}{k^3} \frac{c_0 \lambda \left(\lambda^2 - \frac{7}{3} \frac{k}{m} T_0\right)}{\rho_0 T_0^3} \right), \tag{35}$$

$$\boldsymbol{l}_{6,7,8,9} = \frac{1}{\overline{L}} \left(1, 0, \frac{m}{k} \frac{\rho_0}{T_0} \lambda, 0, \frac{5}{6} \frac{m}{k} \frac{\rho_0 \left(\lambda^2 - \frac{9}{5} \frac{k}{m} T_0\right)}{T_0^2}, \frac{1}{3} \frac{m^2}{k^2} \frac{\lambda^2}{T_0^2}, \right.$$
$$\left. \frac{1}{3} \frac{m^2}{k^2} \frac{\lambda^2}{T_0^2}, \frac{1}{6} \frac{m^3}{k^3} \frac{\lambda\left(\lambda^2 - 3 \frac{k}{m} T_0\right)}{T_0^3}, \frac{1}{6} \frac{m^3}{k^3} \frac{\lambda\left(\lambda^2 - 3 \frac{k}{m} T_0\right)}{T_0^3} \right),$$

where the scalar factors L, $\tilde{L}$ and $\overline{L}$, obtained requiring the orthonormality condition, are given by

$$L = \frac{15}{4},$$
$$\tilde{L} = \frac{13}{10} \frac{m}{k} \frac{1}{T_0} \left(\lambda^2 + \frac{27}{13} \frac{k}{m} T_0 \right), \tag{36}$$
$$\overline{L} = \frac{136}{45} \frac{m}{k} \frac{1}{T_0} \left(\lambda^2 + \frac{15}{34} \frac{k}{m} T_0 \right).$$

Using the characteristic equation (21), the expressions of the right (24) and left eigenvectors (35) and the coefficients a and b, given by Eq. (31), assume the following form:

$$a_1 = 0, \quad a_{2,3,4,5} = 0, \quad a_{6,7,8,9} = \frac{\lambda}{27\rho_0}\left(\frac{702kT_0 - 335m\lambda^2}{13kT_0 - 5m\lambda^2}\right),$$

$$b_1 = \frac{10\pi\rho_0}{3m}\left\{c_0 A^{11} + (1-c_0)A^{12}\right\},$$

$$b_{2,3,4,5} = \frac{\pi\rho_0}{2m(13m\lambda^2 + 27kT_0)}\Big\{25\big(m\lambda^2 + 3kT_0\big) \times \left[2(1-c_0)A^{11} + 2c_0 A^{12} + Y_2^{12}\right] + \big(27m\lambda^2 - 7kT_0\big)Y_1^{12}\Big\}, \tag{37}$$

$$b_{6,7,8,9} = \frac{75\pi\lambda^2\rho_0}{34m\lambda^2 + 15kT_0}\left\{c_0 A^{11} + (1-c_0)A^{12}\right\}.$$

Therefore, we may observe that the contact wave and the ones related to the whole mixture propagating with normal speeds λ_2, λ_3, λ_4 and λ_5 are locally exceptional, namely

$$\nabla_{\boldsymbol{U}}\lambda \cdot \boldsymbol{d}|_0 = 0. \tag{38}$$

In these cases the amplitude equation (30) becomes linear so that its solution (32) reduces to

$$\Pi(t) = \Pi_0 e^{-bt}. \tag{39}$$

As it is well-known, despite of the nonlinearity of the system under consideration, shocks never occur across the wave front [12], [14], [15].

On the contrary, the acceleration waves propagating with speeds λ_6, λ_7, λ_8 and λ_9 satisfy the genuine nonlinearity ($a \neq 0$) so that they may evolve into shock waves provided that the condition on the initial amplitude (34) holds.

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