Acta Mechanica 171, 41–57 (2004)
DOI 10.1007/s00707-004-0128-8

Acta Mechanica
Printed in Austria

# Love waves propagation in a piezoelectric layered structure with initial stresses

**Z. Qian, F. Jin** and **Z. Wang,** Xi'an, China, and **K. Kishimoto,** Tokyo, Japan

Received February 5, 2004; revised April 19, 2004
Published online: July 26, 2004 

**Summary.** The propagation behavior of Love waves in a piezoelectric layered structure with inhomogeneous initial stress is studied. Solutions of the mechanical displacement and electrical potential function are obtained for the isotropic elastic layer and transversely isotropic piezoelectric substrate, respectively, by solving the coupled electromechanical field equations. Firstly, effects of the inhomogeneous initial stress on the dispersion relations and phase velocity of Love wave propagation are discussed. Then the influence of the initial stress gradient coefficient on the stress, mechanical displacement and electrical potential distribution in the layer and the substrate is investigated in detail. The results reported in this paper are not only meaningful for the design of surface acoustic wave (SAW) devices with high performance, but also effective for evaluating the residual stress distribution in the layered structures.

## 1 Introduction

Since White [1] invented the interdigital transducers utilized for transmitting and receiving surface acoustic waves (SAWs) in 1965, SAWs have been introduced into electrics formally. In less than forty years, nearly one hundred kinds of surface acoustic wave devices/sensors have been presented in the world. Surface acoustic wave devices (such as filters, delay lines, oscillators and amplifier etc.) are widely used in practical engineering based on the observation and investigation on the properties of surface waves (such as Rayleigh waves, Love waves, etc.). Love wave sensors are highly sensitive micro-acoustic devices, due to the acoustic energy concentration within a few wavelengths near the surface. Such a kind of device is particularly useful for the measurement of mass density, viscosity and acoustic-electric properties of liquids. Jakoby and Vellekoop [2] presented a review on the properties of Love waves and their applications to sensor devices.

Usually, to achieve high performance, for such a kind of device a typical layer/substrate configuration form is adopted, which means that this kind of device is a layered structure with a thin layer deposited on the substrate. However, due to the nonuniform material properties, coefficients of thermal expansion and chemical/nucleation shrinkage/growth during the processing and cool down to operating or room temperature, the presence of initial stress is unavoidable. On the other hand, to prevent the piezoelectric material from brittle fracture, the layered piezoelectric structure is usually pre-stressed during the manufacture process, where the initial stress is negative with the magnitude of 100 MPa, even up to 1000 MPa [3]. The initial stress in the layered structures can lead to delamination, microcracking, debonding and degradation of the layer, it also can lead to a dramatical change of the dispersion relation corresponding to the wave propagation in the above-mentioned structures. The initial stress has great influence on the performance and reliability of surface acoustic wave devices.

Jin and Wang [4] have studied the propagation behavior of Love waves in a piezoelectric layered structure with a pre-stressed elastic layer and a piezoelectric substrate. The effect of the initial stress on the phase velocity of the Love surface wave is considered and numerical examples are given. Liu and Wang [5] have investigated the influence of the initial stress on the propagation behavior of Love waves in a piezoelectric layered structure with a pre-stressed piezoelectric layer and an elastic substrate, and meaningful theoretical results are obtained. Jin, Wang and Kishimoto [6] have taken into account the propagation behavior of Bleustein-Gulyaev (B-G) waves in a pre-stressed piezoelectric layered structure, and significant results for the engineering application of B-G wave are obtained. However, in all of the above-mentioned research work, the initial stresses in the layer are regarded as uniform and constantly distributed for the convenience of mathematical treatment. Actually, the residual stresses in the layer are inhomogeneous. To the authors' best knowledge, no work has been carried out so far to discuss the effect of inhomogeneous initial stress on the propagation behavior of a Love wave in a piezoelectric layered structure. However, this is significant for the design of high quality surface acoustic wave devices.

In this contribution, we will analytically investigate the propagation behavior of Love waves in a piezoelectric layered structure with inhomogeneous initial stress. Firstly, the effect of the inhomogeneous initial stress on the dispersion relations is discussed. Then the influence of the initial stress gradient coefficient on the phase velocity of Love wave propagation is discussed. Finally, the influences of the gradient coefficient of the initial stress on the distribution of stress, mechanical displacement and electrical potential are investigated in detail. The results obtained in this paper will be meaningful for both theoretical research and engineering application of Love waves.

## 2 Statement of the problem

The basic configuration supporting the propagation of Love waves consists of a layer which is deposited on a substrate, as shown in Fig. 1. In the simplest case, both layer and substrate are isotropic media. In order to enable the electric excitation of Love waves, a piezoelectric material is chosen as substrate. Here, Love wave propagation in such a kind of layered piezoelectric structure will be taken into account. The structure consists of a homogeneous isotropic elastic layer with uniform thickness of $h$ deposited perfectly on a transversely isotropic piezoelectric substrate. The poling direction of the piezoelectric substrate is along the $z$-axis, perpendicular to the $xy$-plane. It is assumed that there exist inhomogeneous initial stresses in the layered structure, and the upper surface of the layer is mechanically traction free. Usually, the thickness of the substrate is much greater than that of the layer for surface acoustic wave devices, such that the structure can be treated as a layered piezoelectric half-space problem, and the initial stress in the substrate is negligible.

From above description, it is clear that we treat the wave propagation in above-mentioned layered structure to be a two-dimensional plain-strain problem. The main object of our present research will be focused on the exploration of suitable analytical solutions of the wave propagation in the layered structure with nonuniform initial stress.

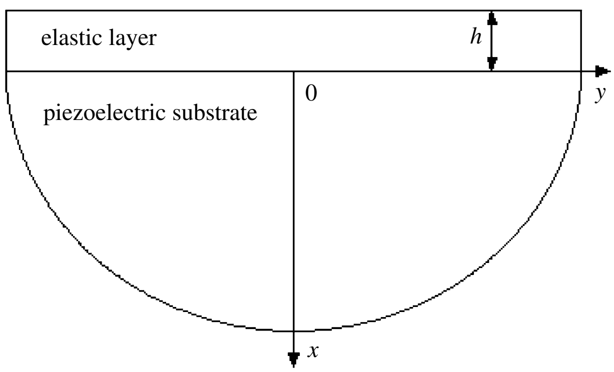


**Fig. 1.** Configuration of layered piezoelectric structure and coordinate system

For the wave motion of small perturbation, the field equations for the elastic layer with initial stresses and the piezoelectric substrate can be expressed respectively as follows [5], [7], [8]:

$$\begin{aligned} &\sigma_{ij,j} + (u_{i,k}\sigma^0_{kj})_{,j} = \rho \ddot{u}_i, \\ &D_{i,i} + (u_{i,j}D^0_j)_{,i} = 0, \end{aligned} \tag{1.1}$$

and

$$\begin{aligned} &\sigma_{ij,j} = \rho' \ddot{u}_i, \\ &D_{i,i} = 0, \end{aligned} \tag{1.2}$$

where $i, j, k = 1, 2, 3$, $\rho$ and $\rho'$ are the mass density of elastic and piezoelectric media, and $u_i$ and $D_i$ denote the mechanical and electrical displacements in the $i$-th direction, respectively. $\sigma_{ij}$ is the stress tensor, $\sigma^0_{kj}$ the initial stress tensor and $D^0_j$ the initial electrical displacement. The dot denotes time differentiation, the comma followed by the subscript $i$ indicates space coordinate differentiation with respect to the corresponding coordinate $x_i$, and the repeated subscript index implies summation with respect to that index.

For the isotropic elastic medium and transversely isotropic piezoelectric medium (such as crystal class of 6mm and piezoelectric ceramics) with the $z$-axis being the symmetric axis of the material, the constitutive equations can be read in the following forms in terms of components

$$\begin{aligned} &\sigma_x = (\lambda + 2\mu)s_x + \lambda s_y + \lambda s_z, \\ &\sigma_y = \lambda s_x + (\lambda + 2\mu)s_y + \lambda s_z, \\ &\sigma_z = \lambda s_x + \lambda s_y + (\lambda + 2\mu)s_z, \\ &\tau_{yz} = \mu s_{yz}, \\ &\tau_{zx} = \mu s_{zx}, \\ &\tau_{xy} = \mu s_{xy}, \\ &D_x = \varepsilon E_x, \\ &D_y = \varepsilon E_y, \\ &D_z = \varepsilon E_z, \end{aligned} \tag{2.1}$$

and

$$
\begin{aligned}
&\sigma_x = c_{11}s_x + c_{12}s_y + c_{13}s_z - e_{31}E_z,\\
&\sigma_y = c_{12}s_x + c_{11}s_y + c_{13}s_z - e_{31}E_z,\\
&\sigma_z = c_{13}s_x + c_{13}s_y + c_{33}s_z - e_{33}E_z,\\
&\tau_{yz} = c_{44}s_{yz} - e_{15}E_y,\\
&\tau_{zx} = c_{44}s_{zx} - e_{15}E_x,\\
&\tau_{xy} = \tfrac{1}{2}(c_{11} - c_{12})s_{xy},\\
&D_x = e_{15}s_{zx} + \varepsilon_{11}E_x,\\
&D_y = e_{15}s_{yz} + \varepsilon_{11}E_y,\\
&D_z = e_{31}s_x + e_{31}s_y + e_{33}s_z + \varepsilon_{33}E_z \ ,
\end{aligned}
\tag{2.2}
$$

where $\lambda$ and $\mu$ are Lamé constants, and $\varepsilon$ is dielectric constant of the isotropic elastic medium. $c_{11}, c_{12}, c_{13}$ and $c_{44}$ are the elastic constants, $e_{15}$, $e_{31}$ and $e_{33}$ are the piezoelectric constants, and $\varepsilon_{11}$ and $\varepsilon_{33}$ are the dielectric constants of the piezoelectric medium. The strain components $s_{ij}$ and the electrical field intensity $E_{\mathrm{k}}$ in Eqs. (2.1) and (2.2) can be calculated by the following formulae:

$$
\begin{aligned}
&s_x = \frac{\partial u}{\partial x}, s_y = \frac{\partial v}{\partial y}, s_z = \frac{\partial w}{\partial z},\\
&s_{yz} = \frac{\partial w}{\partial y} + \frac{\partial v}{\partial z}, s_{zx} = \frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}, s_{xy} = \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x},\\
&E_x = -\frac{\partial \varphi}{\partial x}, E_y = -\frac{\partial \varphi}{\partial y}, E_z = -\frac{\partial \varphi}{\partial z}.
\end{aligned}
\tag{3}
$$

Here, it can be assumed that Love wave propagation is along the positive direction of the $y$-axis without loss of generality. And there exists only one inhomogeneous initial stress component $\sigma_y^0$, which is a function of the space coordinate $x$ in the layer, such that the mechanical displacement components and the scalar electrical potential function $\varphi$ are as follows:

$$
\begin{aligned}
&u = v = 0,\\
&w = w(x, y, t),\\
&\varphi = \varphi(x, y, t).
\end{aligned}
\tag{4}
$$

Substituting Eq. (4) into Eqs. (1.1), (1.2) and (3), we have

$$
\begin{aligned}
&\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} + \sigma_y^0(x)\frac{\partial^2 w}{\partial y^2} = \rho\frac{\partial^2 w}{\partial t^2},\\
&\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0,
\end{aligned}
\tag{5.1}
$$

$$
\begin{aligned}
&\frac{\partial \tau_{zx}}{\partial x} + \frac{\partial \tau_{yz}}{\partial y} = \rho'\frac{\partial^2 w}{\partial t^2},\\
&\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} = 0
\end{aligned}
\tag{5.2}
$$

and

$$
\begin{aligned}
&s_x = s_y = s_z = 0,\\
&s_{yz} = \frac{\partial w}{\partial y}, s_{zx} = \frac{\partial w}{\partial x}, s_{xy} = 0,\\
&E_x = -\frac{\partial \varphi}{\partial x}, E_y = -\frac{\partial \varphi}{\partial y}, E_z = 0.
\end{aligned}
\tag{6}
$$

Substitution of Eqs. (6) into Eqs. (2.1) yields

$$
\begin{aligned}
&\sigma_x = \sigma_y = \sigma_z = 0,\\
&\tau_{yz} = \mu\frac{\partial w}{\partial y}, \quad \tau_{zx} = \mu\frac{\partial w}{\partial x}, \quad \tau_{xy} = 0,\\
&D_x = -\varepsilon\frac{\partial \varphi}{\partial x}, \quad D_y = -\varepsilon\frac{\partial \varphi}{\partial y}, \quad D_z = 0.
\end{aligned}
\tag{7}
$$

Let $w_1$ and $\varphi_1$ denote the mechanical displacement and electrical potential function in the region $-h < x < 0$, then from Eqs. (5.1) and (7), we have the following field equations for the isotropic elastic layer:

$$
\begin{aligned}
&\mu\frac{\partial^2 w_1}{\partial x^2} + [\sigma_y^0(x) + \mu]\frac{\partial^2 w_1}{\partial y^2} = \rho\frac{\partial^2 w_1}{\partial t^2},\\
&\frac{\partial^2 \varphi_1}{\partial x^2} + \frac{\partial^2 \varphi_1}{\partial y^2} = 0.
\end{aligned}
\tag{8}
$$

Here, we can see that for the motion described by Eq. (4), the field equation (8) contains only one material constant, i.e., $\mu$ as the particles are making transversely horizontal polarizing movement in the isotropic elastic layer. Moreover, $w_1$ and $\varphi_1$ are not coupled.

Substitution of Eqs. (6) into Eqs. (2.2) produces

$$
\begin{aligned}
&\sigma_x = \sigma_y = \sigma_z = 0,\\
&\tau_{yz} = c_{44}\frac{\partial w}{\partial y} + e_{15}\frac{\partial \varphi}{\partial y}, \quad \tau_{zx} = c_{44}\frac{\partial w}{\partial x} + e_{15}\frac{\partial \varphi}{\partial x}, \quad \tau_{xy} = 0,\\
&D_x = e_{15}\frac{\partial w}{\partial x} - \varepsilon_{11}\frac{\partial \varphi}{\partial x}, \quad D_y = e_{15}\frac{\partial w}{\partial y} - \varepsilon_{11}\frac{\partial \varphi}{\partial y}, \quad D_z = 0.
\end{aligned}
\tag{9}
$$

Let $w_2$ and $\varphi_2$ denote the mechanical displacement and electrical potential function in the region $x > 0$, then from Eqs. (5.2) and Eqs. (9), we have the following coupled electro-mechanical field equations for the piezoelectric substrate:

$$
\begin{aligned}
&c_{44}(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}) + e_{15}(\frac{\partial^2 \varphi_2}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial y^2}) = \rho'\frac{\partial^2 w_2}{\partial t^2},\\
&e_{15}(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial y^2}) - \varepsilon_{11}(\frac{\partial^2 \varphi_2}{\partial x^2} + \frac{\partial^2 \varphi_2}{\partial y^2}) = 0.
\end{aligned}
\tag{10}
$$

Usually, the space above the upper surface of the layer is air, for the dielectric constant $\varepsilon_0$ of air is much smaller than that of the elastic medium and is negligible, thus the space above the layer can be treated as vacuum. Let $\varphi_0(x, y, t)$ denote the electrical potential function in the air ($x < -h$), therefore $\varphi_0$ satisfies the following Laplace's equation:

$$
\frac{\partial^2 \varphi_0}{\partial x^2} + \frac{\partial^2 \varphi_0}{\partial y^2} = 0.
\tag{11}
$$

When the Love wave propagates in the layered structure, as shown in Fig. 1, the mechanical displacement components and the electrical potential function must satisfy Eqs. (8), (10) and (11). Moreover, the related mechanical and electrical variables must satisfy the boundary conditions and the continuity conditions along the interface, which are described as follows:

(i) The mechanical traction-free condition at $x = -h$

$$\tau_{zx}^{(1)}(-h, y) = 0.$$

(ii) The electrical boundary conditions at $x = -h$, for the electrically open case,

$$\varphi_0(-h, y) = \varphi_1(-h, y),$$
$$D_x^{(0)}(-h, y) = D_x^{(1)}(-h, y)$$

and $\varphi_1(-h, y) = 0$ for the electrically shorted case.

(iii) The continuity conditions at $x = 0$: the normal components of mechanical displacement, stress, electrical potential function and electrical displacement are continuous,

$$w_1(0, y) = w_2(0, y),$$
$$\tau_{zx}^{(1)}(0, y) = \tau_{zx}^{(2)}(0, y),$$
$$\varphi_1(0, y) = \varphi_2(0, y),$$
$$D_x^{(1)}(0, y) = D_x^{(2)}(0, y).$$

(iv) For $x \to +\infty$, $w_2 \to 0$, $\varphi_2 \to 0$. For $x \to -\infty$, $\varphi_0 \to 0$.

## 3 Solutions of the mechanical displacements and electrical potential functions

The solutions of the mechanical displacement and electrical potential function in Eqs. (8) can be assumed as follows:

$$\begin{aligned} w_1(x, y, t) &= W_1(x)\exp[ik(y - ct)], \\ \varphi_1(x, y, t) &= \Phi_1(x)\exp[ik(x - ct)], \end{aligned} \tag{12}$$

where $k$ $(= 2\pi/\lambda)$ is the wave number with $\lambda$ being the wavelength, $i = \sqrt{-1}$, $c$ is the phase velocity of wave propagation, and $W_1(x)$ and $\Phi_1(x)$ are the undetermined functions, respectively. Substitution of Eqs. (12) into Eqs. (8) yields

$$W_1''(x) + k^2 b_1^2(x) W_1(x) = 0, \tag{13.1}$$

$$\Phi_1'' - k^2 \Phi_1 = 0, \tag{13.2}$$

where $b_1(x) = \sqrt{\frac{c^2 - \sigma_y^0(x)/\rho}{c_{sh}^2} - 1}$ with $c_{sh} = \sqrt{\mu/\rho}$ being the bulk shear wave velocity in the layer.

It can be obtained directly from Eq. (13.2) that

$$\Phi_1(x) = A_2 e^{-kx} + B_2 e^{kx}. \tag{14}$$

Usually, it is difficult to obtain the exact solution of Eq. (13.1). But for high-frequency short waves, i.e., for the waves whose wave number possesses the character $k \gg 1$, the Wentzel-Kramers-Brillouin (WKB) asymptotic approximation technique [9] can be applied to obtain the asymptotic approximation of Eq. (13.1). The solution procedure can be described as follows:

Firstly, the following transformation is introduced in our analysis:

$$W_1(x) = \mathrm{e}^{\int \phi(x)\mathrm{d}x}. \tag{15}$$

Therefore, Eq. (13.1) can be transformed to the following form:

$$\phi^2(x) + \phi\prime(x) + b_1^2(x)k^2 = 0. \tag{16}$$

Expression (16) is a nonlinear differential equation regarding the variables $\phi$. To solve Eq. (16), we seek an expansion of $\phi$ in inverse powers of $k$, that is to say, we can write

$$\phi = \phi_0 k + \phi_1 + \frac{\phi_2}{k} + \frac{\phi_3}{k^2} + \ldots . \tag{17}$$

Substitution of Eq. (17) into Eq. (16) and expanding Eq. (16) in inverse powers of $k$ leads to

$$(\phi_0^2 + b_1^2)k^2 + (2\phi_0\phi_1 + \phi_0')k + (\phi_1^2 + \phi_1' + 2\phi_0\phi_2) + (2\phi_0\phi_3 + 2\phi_1\phi_2 + \phi_2')k^{-1} + \ldots = 0,$$

where the superscript $'$ denotes the derivative with respect to coordinate $x$.

Equating the coefficients of each power of $k$ to zero, we get an infinite number of equations:

$$\phi_0^2 + b_1^2 = 0,$$

$$2\phi_0\phi_1 + \phi_0' = 0,$$

$$\phi_1^2 + \phi_1' + 2\phi_0\phi_2 = 0,$$

$$\vdots$$

Then the solutions of $\phi_0, \phi_1, \phi_2, \ldots$ can be obtained from above expressions. If we only keep the first two solutions, i.e., solutions of $\phi_0$, $\phi_1$ and substitute them into Eq. (17), then the asymptotic solution of Eq. (16) can be easily obtained. Now considering the transformation (15), we can obtain the solution of Eq. (13.1) as follows:

$$W_1(x) = \frac{A_1}{\sqrt{b_1(x)}}\mathrm{e}^{-ikq(x)} + \frac{B_1}{\sqrt{b_1(x)}}\mathrm{e}^{ikq(x)}, \tag{18}$$

where $q(x) = \int b_1(x)\mathrm{d}x$.

Substitution of Eq. (14) and Eq. (18) into Eqs. (12) yields the solutions of the mechanical displacement and electrical potential functions as follows:

$$w_1(x,y,t) = \left[\frac{A_1}{\sqrt{b_1(x)}}\mathrm{e}^{-ikq(x)} + \frac{B_1}{\sqrt{b_1(x)}}\mathrm{e}^{ikq(x)}\right]\exp[ik(y-ct)],$$

$$\varphi_1(x,y,t) = (A_2\mathrm{e}^{-kx} + B_2\mathrm{e}^{kx})\exp[ik(y-ct)]. \tag{19}$$

It should be pointed out that in order to ensure the above asymptotic approximation solution valid enough, the initial stress function $\sigma_y^0(x)$ must be slowly varying in the layer, i.e., the initial stress distribution has to satisfy the following condition (for detailed derivative procedures, please see the Appendix):

$$\left|\sigma_y^0(0) - \sigma_y^0(-h)\right| << \rho c_{sh}^2 kh.$$

For Eqs. (10), its solution forms can be assumed as follows:

$$\begin{aligned} w_2(x,y,t) &= W_2(x)\exp[ik(y-ct)], \\ \varphi_2(x,y,t) &= \Phi_2(x)\exp[ik(x-ct)]. \end{aligned} \tag{20}$$

Substituting Eqs. (20) into Eqs. (10), we may have

$$W_2'' - k^2\left(1 - \frac{c^2\rho\prime}{(c_{44}\varepsilon_{11} + e_{15}^2)/\varepsilon_{11}}\right)W_2 = 0, \tag{21.1}$$

$$\Phi_2'' - k^2\Phi_2 = \frac{e_{15}}{\varepsilon_{11}}(W_2'' - k^2W_2). \tag{21.2}$$

It should be noted that if we define the velocity of the bulk shear wave in the substrate as

$$c'_{sh} = \sqrt{(c_{44}\varepsilon_{11} + e_{15}^2)/\rho'\varepsilon_{11}},$$

then the velocity $c$ of the Love wave should satisfy the following condition:

$$c_{sh} < c < c'_{sh}.$$

Considering condition (iv) in Sect. 2 and Eq. (21.2) as an inhomogeneous equation with respect to $\Phi_2$ $(x)$, the solutions of mechanical displacement and electrical potential function in the piezoelectric substrate can be obtained as follows:

$$\begin{aligned} w_2(x,y,t) &= A'_1\mathrm{e}^{-kb_2x}\exp[ik(y-ct)], \\ \varphi_2(x,y,t) &= (A'_2\mathrm{e}^{-kx} + \tfrac{e_{15}}{\varepsilon_{11}}A'_1\mathrm{e}^{-kb_2x})\exp[ik(y-ct)], \end{aligned} \tag{22}$$

where $b_2 = \sqrt{1 - \frac{c^2}{c\prime_{sh}^2}}$.

From condition (iv) in Sect. 2, the solution of the electrical potential function in the region $x < -h$ can be easily obtained from Eq. (11) as follows:

$$\varphi_0(x,y,t) = A_0\mathrm{e}^{kx}\exp[ik(y-ct)]. \tag{23}$$

## 4 Solutions of the phase velocity

### *4.1 Electrically open case*

Substituting Eqs. (19), (22) and (23) and their corresponding stress and electrical displacement components into boundary conditions (i) and (ii) and continuity conditions (iii), we can obtain the following algebraic equations with respect to the unknown constants $A_1, B_1, A_2, B_2, A'_1, A'_2$ and $A_0$:

$$\begin{aligned} &\mathrm{e}^{-ikq(-h)}A_1 - \mathrm{e}^{ikq(-h)}B_1 = 0, \\ &\mathrm{e}^{kh}A_2 + \mathrm{e}^{-kh}B_2 - \mathrm{e}^{-kh}A_0 = 0, \\ &\mathrm{e}^{kh}A_2 - \mathrm{e}^{-kh}B_2 + \frac{\varepsilon_0}{\varepsilon}\mathrm{e}^{-kh}A_0 = 0, \\ &\mathrm{e}^{-ikq(0)}A_1 + \mathrm{e}^{ikq(0)}B_1 - \sqrt{b_1(0)}A'_1 = 0, \\ &\mu i\sqrt{b_1(0)}\mathrm{e}^{-ikq(0)}A_1 - \mu i\sqrt{b_1(0)}\mathrm{e}^{ikq(0)}B_1 - \frac{c_{44}\varepsilon_{11} + e_{15}^2}{\varepsilon_{11}}b_2A'_1 - e_{15}A'_2 = 0, \\ &A_2 + B_2 - A'_2 - \frac{e_{15}}{\varepsilon_{11}}A'_1 = 0, \\ &A_2 - B_2 - \frac{\varepsilon_{11}}{\varepsilon}A'_2 = 0. \end{aligned} \tag{24}$$

The nontrivial solution of (24) exists if and only if the determinant of the coefficient matrix equals zero, and this procedure leads to the following expression:

$$\begin{aligned}&\left[\left(\frac{\varepsilon_{11}}{\varepsilon}+\frac{\varepsilon}{\varepsilon_0}\right)\tanh(kh)+\frac{\varepsilon_{11}}{\varepsilon_0}+1\right]\left\{\mu b_1(0)\tan\left[k(q(0)-q(-h))\right]-\frac{c_{44}\varepsilon_{11}+e_{15}^2}{\varepsilon_{11}}b_2\right\}\\&\quad+\frac{e_{15}^2}{\varepsilon_{11}}\left[\frac{\varepsilon}{\varepsilon_0}\tanh(kh)+1\right]=0.\end{aligned} \tag{25}$$

Equation (25) is the phase velocity equation for the propagation of a Love wave in the piezoelectric layered half-space for the electrically open case at the free surface. It can be seen from the equation that the wave velocity $c$ is related to the wave number $k$, so the Love wave in such structures is frequency dispersive.

### *4.2 Electrically shorted case*

For the electrically shorted case at the free surface of the layered piezoelectric half-space (the surface is plated with a very thin metal strip), the second and the third equations in Eqs. (24) should be replaced by the following expression which corresponds to the condition (ii) in Sect. 2:

$$\mathrm{e}^{kh}A_2+\mathrm{e}^{-kh}B_2=0.$$

Then, a homogeneous linear algebraic equation with respect to $A_1, B_1, A_2, B_2, A_1'$ and $A_2'$ can be obtained. By the similar procedure to the electrically open case, we can obtain the corresponding phase velocity equation for the electrically shorted case (here the equation will be given directly and the analysis procedures are omitted for conciseness and brevity):

$$\left[\frac{\varepsilon_{11}}{\varepsilon}\tanh(kh)+1\right]\left\{\mu b_1(0)\tan[k(q(0)-q(-h))]-\frac{c_{44}\varepsilon_{11}+e_{15}^2}{\varepsilon_{11}}b_2\right\}+\frac{e_{15}^2}{\varepsilon_{11}}=0. \tag{26}$$

As a matter of fact, the phase velocity equation for the electrically shorted case, which can be obtained through eliminating the terms related to $\varepsilon_0$, has the same form as that for the electrically open case.

Equations (25) and (26) are the phase velocity equations of Love wave propagation in the layered piezoelectric structure for the electrically open and shorted case, respectively. It is readily seen that the phase velocity $c$ is related to the initial stress, wavelength, layer thickness, elastic, dielectric and piezoelectric constants. The effect of the gradient coefficient of initial stress and $m$ (the ratio of layer thickness $h$ to wavelength $\lambda$) on the phase velocity will be discussed in Sect. 6.

## 5 Solutions of the stress and displacement fields

For the electrically open case, we can obtain from Eqs. (24) that

$$B_1=\beta_1A_1, A_2=\beta_2A_1, B_2=\beta_3A_1,$$

$$A_1'=\left[\frac{\varepsilon_{11}}{e_{15}}(\beta_2+\beta_3)+\frac{\varepsilon}{e_{15}}(\beta_3-\beta_2)\right]A_1, A_2'=\frac{\varepsilon}{\varepsilon_{11}}(\beta_2-\beta_3)A_1,$$

where

$$\beta_1 = -\frac{a_{31} - \dfrac{a_{34}a_{11}}{a_{14}} + \left(\dfrac{a_{34}a_{13}}{a_{14}} - a_{33}\right)\dfrac{a_{21}a_{14} - a_{11}a_{24}}{a_{23}a_{14} - a_{13}a_{24}}}{a_{32} - \dfrac{a_{34}a_{12}}{a_{14}} + \left(\dfrac{a_{34}a_{13}}{a_{14}} - a_{33}\right)\dfrac{a_{22}a_{14} - a_{12}a_{24}}{a_{23}a_{14} - a_{13}a_{24}}},$$

$$\beta_2 = -\frac{a_{21}a_{14} - a_{11}a_{24}}{a_{23}a_{14} - a_{13}a_{24}} + \frac{a_{22}a_{14} - a_{12}a_{24}}{a_{23}a_{14} - a_{13}a_{24}} \frac{a_{31} - \dfrac{a_{34}a_{11}}{a_{14}} + \left(\dfrac{a_{34}a_{13}}{a_{14}} - a_{33}\right)\dfrac{a_{21}a_{14} - a_{11}a_{24}}{a_{23}a_{14} - a_{13}a_{24}}}{a_{32} - \dfrac{a_{34}a_{12}}{a_{14}} + \left(\dfrac{a_{34}a_{13}}{a_{14}} - a_{33}\right)\dfrac{a_{22}a_{14} - a_{12}a_{24}}{a_{23}a_{14} - a_{13}a_{24}}},$$

$$\beta_3 = -\frac{a_{11}}{a_{14}} + \frac{a_{13}}{a_{14}}\frac{a_{21}a_{14} - a_{11}a_{24}}{a_{23}a_{14} - a_{13}a_{24}}$$

$$+ \left(\frac{a_{12}}{a_{14}} - \frac{a_{13}}{a_{14}}\frac{a_{22}a_{14} - a_{12}a_{24}}{a_{23}a_{14} - a_{13}a_{24}}\right) \frac{a_{31} - \dfrac{a_{34}a_{11}}{a_{14}} + \left(\dfrac{a_{34}a_{13}}{a_{14}} - a_{33}\right)\dfrac{a_{21}a_{14} - a_{11}a_{24}}{a_{23}a_{14} - a_{13}a_{24}}}{a_{32} - \dfrac{a_{34}a_{12}}{a_{14}} + \left(\dfrac{a_{34}a_{13}}{a_{14}} - a_{33}\right)\dfrac{a_{22}a_{14} - a_{12}a_{24}}{a_{23}a_{14} - a_{13}a_{24}}}$$

with

$$a_{11} = \mu i\sqrt{b_1(0)}\mathrm{e}^{-ikq(0)}, a_{12} = -\mu i\sqrt{b_1(0)}\mathrm{e}^{ikq(0)}, a_{13} = -\frac{c_{44}\varepsilon_{11} + e_{15}^2}{\varepsilon_{11}} b_2 \frac{\varepsilon_{11} - \varepsilon}{e_{15}} - \frac{e_{15}\varepsilon}{\varepsilon_{11}},$$

$$a_{14} = -\frac{c_{44}\varepsilon_{11} + e_{15}^2}{\varepsilon_{11}} b_2 \frac{\varepsilon_{11} + \varepsilon}{e_{15}} + \frac{e_{15}\varepsilon}{\varepsilon_{11}}, a_{21} = 0, a_{22} = 0, a_{23} = \left(1 + \frac{\varepsilon}{\varepsilon_0}\right)\mathrm{e}^{kh}, a_{24} = \left(1 - \frac{\varepsilon}{\varepsilon_0}\right)\mathrm{e}^{-kh},$$

$$a_{31} = \mathrm{e}^{-ikq(-h)}, a_{32} = -\mathrm{e}^{ikq(-h)}, a_{33} = 0, a_{34} = 0.$$

From Eqs. (19) and (22), one can obtain

$$\begin{aligned} w_1(x,y,t) &= \tfrac{A_1}{\sqrt{b_1(x)}}[\mathrm{e}^{-ikq(x)} + \beta_1 \mathrm{e}^{ikq(x)}]\exp[ik(y-ct)], \\ \varphi_1(x,y,t) &= A_1[\beta_2\mathrm{e}^{-kx} + \beta_3\mathrm{e}^{kx}]\exp[ik(y-ct)] \end{aligned} \tag{27}$$

and

$$\begin{aligned} w_2(x,y,t) &= A_1[\tfrac{\varepsilon_{11}}{e_{15}}(\beta_2+\beta_3) + \tfrac{\varepsilon}{e_{15}}(\beta_3-\beta_2)]\mathrm{e}^{-kb_2x}\exp[ik(y-ct)], \\ \varphi_2(x,y,t) &= A_1\left\{\tfrac{\varepsilon}{\varepsilon_{11}}(\beta_2-\beta_3)\mathrm{e}^{-kx} + [\tfrac{\varepsilon_{11}}{e_{15}}(\beta_2+\beta_3) + \tfrac{\varepsilon}{e_{15}}(\beta_3-\beta_2)]\mathrm{e}^{-kb_2x}\right\}\exp[ik(y-ct)]. \end{aligned} \tag{28}$$

Substitution of Eqs.(27) into Eqs.(7) yields the following solutions of the stress field in the layer, i.e.,

$$\begin{aligned} \tau_{zx}^{(1)} &= A_1\mu ik\sqrt{b_1(x)}[\beta_1\mathrm{e}^{ikq(x)} - \mathrm{e}^{-ikq(x)}]\exp[ik(y-ct)], \\ \tau_{yz}^{(1)} &= A_1\tfrac{\mu ik}{\sqrt{b_1(x)}}[\beta_1\mathrm{e}^{ikq(x)} + \mathrm{e}^{-ikq(x)}]\exp[ik(y-ct)]. \end{aligned} \tag{29}$$

Similarly, we have the following solutions of the stress field in the substrate from Eqs. (28) and (9), i.e.,

$$\begin{aligned} \tau_{zx}^{(2)} &= -A_1k\left\{\tfrac{c_{44}\varepsilon_{11}+e_{15}^2}{\varepsilon_{11}}b_2[\tfrac{\varepsilon_{11}}{e_{15}}(\beta_2+\beta_3) + \tfrac{\varepsilon}{e_{15}}(\beta_3-\beta_2)]\mathrm{e}^{-kb_2x} + \tfrac{e_{15}\varepsilon}{\varepsilon_{11}}(\beta_2-\beta_3)\mathrm{e}^{-kx}\right\}\exp[ik(y-ct)], \\ \tau_{yz}^{(2)} &= A_1ik\left\{\tfrac{e_{15}\varepsilon}{\varepsilon_{11}}(\beta_2-\beta_3)\mathrm{e}^{-kx} + \tfrac{c_{44}\varepsilon_{11}+e_{15}^2}{\varepsilon_{11}}[\tfrac{\varepsilon_{11}}{e_{15}}(\beta_2+\beta_3) + \tfrac{\varepsilon}{e_{15}}(\beta_3-\beta_2)]\mathrm{e}^{-kb_2x}\right\}\exp[ik(y-ct)]. \end{aligned} \tag{30}$$

## 6 Numerical simulation and discussions

Up to now, analytical solutions of the phase velocity and the stress field and mechanical displacement field for the piezoelectric layered structure have been obtained. Obviously, Eq. (25) and Eq. (26) are transcendental equations, which lead to the complexity of this problem. To study the propagation behavior of Love waves in this kind of structure and to graphically show the effect of the initial stress gradient coefficient on the dispersion relations and phase velocity, the following material system is considered: Pb glass layer on the ZnO substrate combination system. The material parameters used in computational analysis are taken from [4]. All the material properties used in the computation are summarized in Table 1. The dielectric constant of vacuum is $\varepsilon_0$=8.85 × 10$^{-12}$ F/m. As a numerical simulation of theoretical research, the variation pattern of initial stress can be taken as follows:

$$\sigma_y^0(x) = -A(1-\alpha x), \tag{31}$$

where $A$, $\alpha$ denote the magnitude and gradient coefficient of the inhomogeneous initial stress, respectively. According to Refs. [4], [5], it is known that the effect of the constant initial stress on the phase velocity is negligible on condition that $|\sigma_y^0| < 100$ MPa. Without loss of generality, it can be assumed that $A$=1000 MPa, $h$=0.005 mm in our following numerical analysis.

Then, $b_1(x)$ can be expressed as

$$b_1(x) = \sqrt{\frac{c^2 + A(1-\alpha x)/\rho}{c_{sh}^2} - 1},$$

and $q(x)$ can be obtained by integration of $b_1(x)$ as follows:

$$q(x) = \int b_1(x)\mathrm{d}x = -\frac{2}{3}\frac{\rho c_{sh}^2}{A\alpha} b_1^{5/2}(x).$$

### 6.1 *Effect of inhomogeneous initial stress on dispersion relations*

Firstly, the effect of initial stress on the dispersion relations of Love waves in a piezoelectric layered structure will be taken into account. For the given material system and initial stress, there exist only two variables, i.e., phase velocity $c$ and wave number $k$ in Eq. (25). The dispersion relations for two initial stress distribution cases, constant initial stress distribution and inhomogeneous initial stress distribution, are shown in Fig. 2 in comparison for a Pb glass layer/ZnO substrate combination system. From the fundamental mode to the higher third mode, the effect of the initial stress on the first four modes of the Love wave is calculated and shown in detail.

It can be seen from Fig. 2 that the inhomogeneous initial stress has no influence on the cut-off frequency for each mode of the Love wave. From computational results, a large difference of the initial stress effect on the dispersion relations for every Love wave mode can be found between the constant initial stress and inhomogeneous initial stress case. Due to the fact that

**Table 1.** Material properties used in computational analysis

| Materials | Elastic constant $c_{44}$ ($10^{10}$N/m$^2$) | Mass density $\rho$($10^3$kg/m$^3$) | Piezoelectric constant $e_{15}$ (C/m$^2$) | Dielectric constant $\varepsilon_{11}$ (F/m) |
|---|---|---|---|---|
| Pb glass | 2.18 | 3.879 | 0.0 | 5.1$\varepsilon_0$ |
| ZnO | 4.23 | 5.665 | −0.48 | 7.57$\varepsilon_0$ |

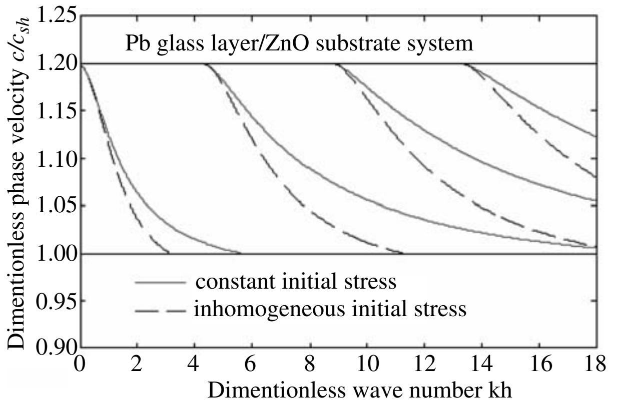


**Fig. 2.** The dispersion relations of Pb glass layer/ZnO substrate system with initial stress, for the electrically open case

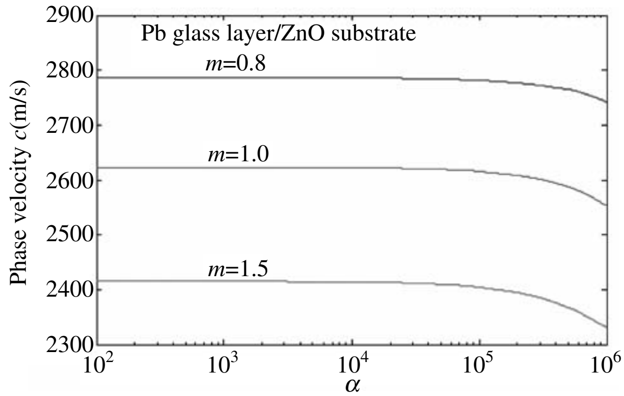


**Fig. 3.** Phase velocity $c$ vs gradient coefficient $\alpha$ of initial stress for different values of $m$, for the electrically open case

the initial stresses in the layered structures usually possess inhomogeneous character, the constant distribution assumption in Refs. [4], [5] is not exact enough, which also indicates that the analysis of the inhomogeneous initial stress effect in the paper is necessary. Actually, we calculate several other dispersion relations for an inhomogeneous initial stress for different gradient coefficients. However, due to their superposition together, they are not presented here.

### *6.2 Effect of inhomogeneous initial stress on the phase velocity*

The phase velocity $c$ can be calculated from Eq. (25) for different values of $m$ (the ratio of layer thickness $h$ to wavelength $\lambda$). The effect of the gradient coefficient of the initial stress on the phase velocity $c$ is shown in Fig. 3 for the first higher-order mode of Love wave in a Pb glass layer/ZnO substrate combination system.

It can be seen from Fig. 3 that the effect of the gradient coefficient of the initial stress on the phase velocity is negligible as $\alpha < 10^4$, but the phase velocity decreases with the increase of the gradient coefficient of the initial stress as $\alpha > 10^4$. The results in Fig. 3 mean that the more inhomogeneous the initial stress distribution, the smaller the phase velocity value. It is readily seen from Fig. 3 also that the values of the ratio of layer thickness to wavelength also have an important effect on the phase velocity $c$.

### 6.3 *Effects of inhomogeneous initial stress on the stress, mechanical displacement and electrical potential function*

Here, the stress, mechanical displacement and electrical potential function distribution in the Pb glass layer/ZnO substrate combination system will be taken into account for mode 1 of the Love wave. Without loss of generality, it is assumed that $A_1$=0.0001 mm and $m$=1.0. Variations of the stress components $\tau_{zx}$ and $\tau_{yz}$ with $x/h$, at $y$=0 are shown in Figs. 4 and 5 for different gradient coefficients $\alpha$, respectively. Also, variation of the mechanical displacement $w$ and electrical potential function $\varphi$ with $x/h$, at $y$=0 are shown in Figs. 6 and 7 for different gradient coefficients $\alpha$, respectively.

It is seen that the gradient coefficient of initial stress has an important influence on the stress and mechanical displacement distribution in a piezoelectric layered structure. Each distribution curve of the stress and mechanical displacement in Figs. 4–6 has a common node in the region $-1 < x/h < 0$ corresponding to mode 1 that we take into account, which also proves that the solutions we obtained in this paper are correct. It can be seen that the gradient coefficient changes not only the phase of the stress and mechanical displacement distribution but also the magnitude for the electrically open case.

For a layered structure, the small stress near the interface is expected to prevent the layered structure from debonding or fracture, while for surface acoustic wave devices/sensors that often

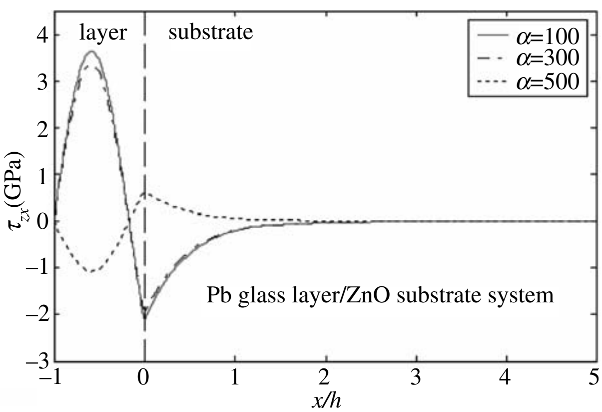


**Fig. 4.** Variation of $\tau_{zx}$ with $x/h$ for different gradient coefficients of the initial stress $\alpha$, for the electrically open case

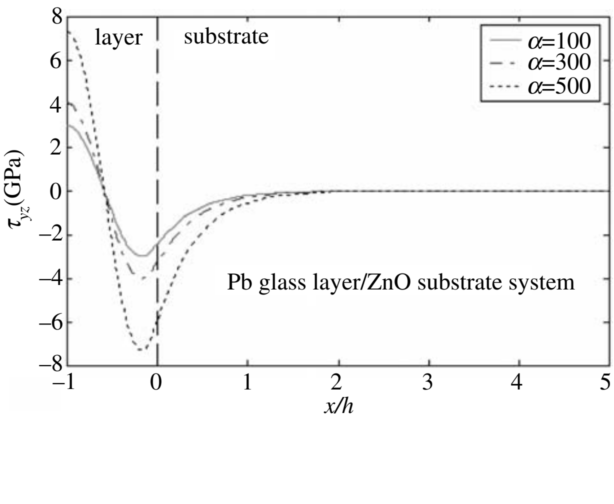


**Fig. 5.** Variation of $\tau_{yz}$ with $x/h$ for different gradient coefficients of the initial stress $\alpha$, for the electrically open case

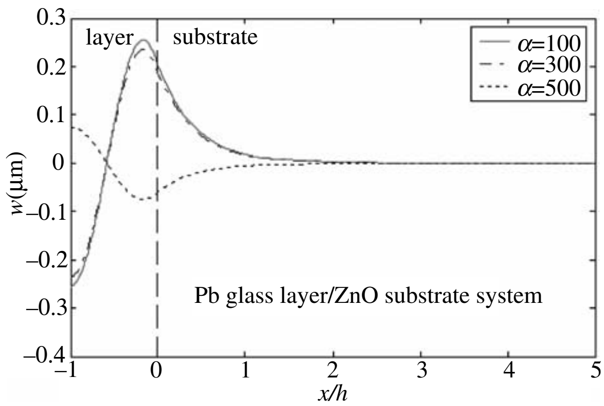


**Fig. 6.** Variation of $w$ with $x/h$ for different $\alpha$ for the electrically open case

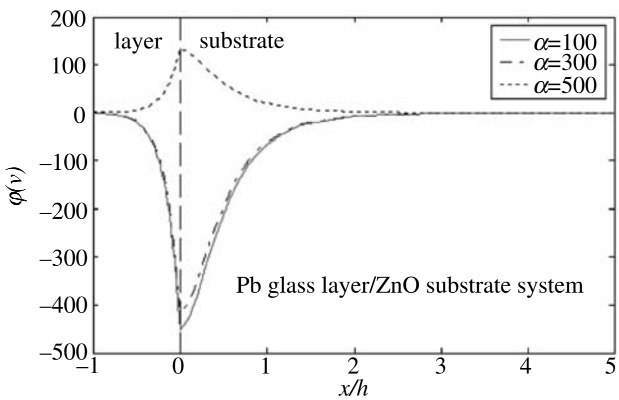


**Fig. 7.** Variation of electrical potential $\varphi$ with $x/h$ for different values of initial stress gradient $\alpha$, for the electrically open case

adopt piezoelectric layered structures, a large surface mechanical displacement is needed to increase sensibility. In order to obtain high performance of surface acoustic wave devices/sensors a small stress near the interface and large surface mechanical displacement can be obtained simultaneously through changing the gradient coefficient of the initial stress in the piezoelectric layer during the manufacture process of piezoelectric surface acoustic wave devices/sensors.

## 7 Conclusions

Usually, for the design of surface acoustic wave sensors, it is difficult to obtain high performance by using a single piezoelectric material. So, it is necessary to seek a combination of materials to fabricate sensors with a layered structure of electromagnetic media. However, due to the thermal mismatch of the film and the substrate materials and the intrinsic stress, there exist unavoidable initial stresses in the layered structure during the manufacturing process. Jin [4] and Liu [5] investigated the effect of constant initial stress on the propagation behavior of Love waves in the piezoelectric layered structure. Moreover, the initial stresses in the piezoelectric layered structures are often inhomogeneous according to Ref. [3]. Thus, it is significant to analyze the effect of the inhomogeneous initial stress on the propagation behavior of Love surface waves in the piezoelectric layered structure. Through the theoretical research and numerical simulation, some meaningful results are obtained in this paper:

(i) The results obtained in the analysis concerned with the effect of inhomogeneous initial stress on the dispersion relations indicate that inhomogeneous initial stress has no influence on the cut-off frequency for each mode of Love wave.

(ii) The effect of the gradient coefficient of the inhomogeneous initial stress on the phase velocity is negligible as $\alpha < 10^4$, but the phase velocity decreases with the increase of the gradient coefficient of the initial stress as $\alpha > 10^4$, which means that the more inhomogeneous the initial stress, the smaller the phase velocity.

(iii) The gradient coefficient changes not only the phase of the stress and mechanical displacement distribution but also the magnitude for the electrically open case. In order to obtain high performance of surface acoustic wave devices/sensors, small stress near the interface and large surface mechanical displacement can be obtained simultaneously by changing the gradient coefficient of initial stress in the piezoelectric layer during the manufacture process of piezoelectric surface acoustic wave devices/sensors.

(iv) Our research work also indicates that the WKB asymptotic approximation can provide a good alternative for the solution of high-frequency wave propagation in the structures with inhomogeneous initial stress. But what we should point out is that the initial stress should possess the slowly varying distribution property in the layer for the validity of the WKB technique to use.

On the other hand, it can be found in our analysis that for the treatment of the problem to be a two-dimensional plane-strain state, only the effect of the initial stress component $\sigma_y^0$ is taken into account. Actually, a practical SAW device is of finite length in the $z$-direction, and the initial stress component $\sigma_z^0$ affects the Love wave propagation to some degree. Both the effects of initial stress components $\sigma_y^0$ and $\sigma_z^0$ on the propagation of the Love wave need to be further investigated in the future.

## Appendix

The WKB solution (18) consists of the wave $u_1$ traveling in the $+x$-direction and the wave $u_2$ traveling in the direction $-x$, which means

$$u_1 = \frac{1}{b_1^{1/2}} \mathrm{e}^{-ikq(x)}, u_2 = \frac{1}{b_1^{1/2}} \mathrm{e}^{ikq(x)}. \tag{A1}$$

Substituting the expression of $u_1$ (or $u_2$) into Eqs. (13), we can get

$$(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + k^2 b_1^2) u_1 = f \neq 0. \tag{A2}$$

Therefore, the range of validity of the WKB solution is as follows:

$$|f| << \left| k^2 b_1^2 u_1 \right|, \tag{A3}$$

which means

$$\Delta = \left| \frac{(\frac{\mathrm{d}^2}{\mathrm{d}x^2} + k^2 b_1^2) u_1}{k^2 b_1^2 u_1} \right| << 1$$

due to the reason

$$\frac{du_1}{dx} = -\frac{1}{2}b_1^{-3/2}b_1'e^{-ikq(x)} + b_1^{-1/2}(-ikb_1)e^{-ikq(x)} = (-\frac{1}{2}b_1^{-3/2}b_1' - ikb_1^{1/2})e^{-ikq(x)},$$

$$\frac{d^2u_1}{dx^2} = \left[\frac{3}{4}b_1^{-5/2}(b_1')^2 - \frac{1}{2}b_1^{-3/2}b_1'' - \frac{1}{2}ikb_1^{-1/2}b_1'\right]e^{-ikq(x)} + (-\frac{1}{2}b_1^{-3/2}b_1' - ikb_1^{1/2})(-ikb_1)e^{ikq(x)}$$

$$= \left[\frac{3}{4}b_1^{-5/2}(b_1')^2 - \frac{1}{2}b_1^{-3/2}b_1'' - k^2b_1^{3/2}\right]e^{-ikq(x)}.$$

So

$$\Delta = \left|\frac{\left(\frac{d^2}{dx^2} + k^2b_1^2\right)u_1}{k^2b_1^2u_1}\right| = \left|\frac{\frac{3}{4}b_1^{-5/2}(b')^2 - \frac{1}{2}b_1^{-3/2}b_1''}{k^2b_1^{3/2}}\right| = \frac{1}{k^2}\left|\frac{3}{4}\frac{(b_1')^2}{b_1^4} - \frac{1}{2}\frac{b_1''}{b_1^3}\right| << 1.$$

Furthermore,

$$(b_1')^2 = \frac{1}{4\rho^2c_{sh}^4}b_1^{-1}[\sigma_y^{0\prime}(x)]^2,$$

$$b_1'' = -\frac{1}{4\rho^2c_{sh}^4}b_1^{-3/2}[\sigma_y^{0\prime}(x)]^2 - \frac{1}{2\rho c_{sh}^2}b_1^{-1/2}\sigma_y^{0\prime\prime}(x),$$

therefore

$$\Delta = \frac{1}{k^2}\left|\frac{3}{16}\frac{1}{\rho^2c_{sh}^4}\frac{1}{b_1^5}\left[\sigma_y^{0'}(x)\right]^2 + \frac{1}{8}\frac{1}{\rho^2c_{sh}^4}\frac{1}{b_1^{9/2}}\left[\sigma_y^{0'}(x)\right]^2 + \frac{1}{4}\frac{1}{\rho c_{sh}^2}\frac{1}{b_1^{7/2}}\sigma_y^{0''}(x)\right|.$$

In the analysis of our manuscript

$$b_1 = \sqrt{\frac{c^2 - \sigma_y^0/\rho}{c_{sh}^2} - 1} = \frac{c}{c_{sh}}\sqrt{1 - \frac{\sigma_y^0}{\rho c^2} - \frac{c_{sh}^2}{c^2}} = \frac{c}{c_{sh}}\sqrt{1 + (\frac{Af(x)}{\rho c^2} - \frac{c_{sh}^2}{c^2})},$$

in which A is the amplitude of initial stress and f(x) the distribution function of initial stress.

So we can know the fact that $b_1 \approx 1$ , and $\sigma_y^0(x)$ should be slowly varying in the range [0, $-h$]. Then, the following expression can be obtained:

$$\Delta = \frac{1}{k^2}\frac{5}{16}\frac{1}{\rho^2c_{sh}^4}\left[\sigma_y^{0'}(x)\right]^2 << 1. \tag{A4}$$

Finally, the following condition can be obtained:

$$\left|\sigma_y^0(0) - \sigma_y^0(-h)\right| << \rho c_{sh}^2 kh, \tag{A5}$$

which is the condition that the initial stress distribution function should be satisfied.

## Acknowledgements

The financial support of the work by the National Natural Science Foundation of China (No.10002013, 50135030) is gratefully acknowledged. F. Jin also greatly acknowledges the support by the Japan Society for the Promotion of Science (JSPS) to conduct research work at Tokyo Institute of Technology, Japan.

## References

[1] White, R. M. et al.: Direct piezoelectric coupling to surface elastic waves. Appl. Phys. Lett. **7**, 314 (1965).
[2] Jakoby, B., Vellekoop, M. J.: Properties of Love waves: Applications in sensors. Smart Mater. Struct.: **6**, 668–679 (1997).
[3] Ohring, M.: The materials science of thin films, Boston: Academic Press 1992, pp. 413–450.
[4] Jin, F., Wang, Z. K., Wang, T. J.: The propagation behavior of Love waves in a pre-stressed piezoelectric layered structure. Key Engng Mater. **183–187**, 755–760 (2000).
[5] Liu, H., Wang, Z. K., Wang, T. J.: Effect of initial stress on the propagation behavior of Love waves in a layered piezoelectric structure. Int. J. Solids Struct. **38**, 37–51 (2001).
[6] Jin, F., Wang, Z., Kishimoto, K.: The propagation behavior of B-G waves in a pre-stressed piezoelectric layered structure. Int. J. Nonlinear Sci. Num. Simul. **4**, 125–138 (2003).
[7] Wang, Z. K., Shang, F. L.: Cylindrical buckling of piezoelectric laminated plates. Acta Mech. Solida Sinica **18**, 101–108 (1997).
[8] Liu, H., Kuang, Z. B., Cai, Z. M.: Propagation of Bleustein-Gulyaev waves in a prestressed layered piezoelectric structure. Ultrasonics **41**, 397–404 (2003).
[9] Akira, I.: Electromagnetic wave propagation, radiation and scattering. New York: Prentice-Hall 1991.

**Author's address:** Z. Qian, F. Jin (Corresponding author: jinfengzhao@263.net) and Z. Wang, Department of Engineering Mechanics, Xi'an Jiaotong University, Xi'an 710049, Shaanxi Province, P. R. China; K. Kishimoto, Department of Mechanical and Control Engineering, Tokyo Institute of Technology, 2-12-1, O-okayama, Meguro-ku, Tokyo 152-8552, Japan