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Unsteady MHD rotating flow over a rotating sphere near the equator

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Summary. The unsteady rotating flow of a laminar incompressible viscous electrically conducting fluid over a rotating sphere in the vicinity of the equator has been studied. The fluid and the body rotate either in the same direction or in opposite directions. The effects of surface suction and magnetic field have been included in the analysis. There is an initial steady state that is perturbed by a sudden change in the rotational velocity of the sphere, and this causes unsteadiness in the flow field. The nonlinear coupled parabolic partial differential equations governing the boundary-layer flow have been solved numerically by using an implicit finite-difference scheme. For large suction or magnetic field, analytical solutions have also been obtained. The magnitude of the radial, meridional and rotational velocity components is found to be higher when the fluid and the body rotate in opposite directions than when they rotate in the same direction. The surface shear stresses in the meridional and rotational directions change sign when the ratio of the angular velocities of the sphere and the fluid $\lambda \geq \lambda_0$. The final (new) steady state is reached rather quickly which implies that the spin-up time is small. The magnetic field and surface suction reduce the meridional shear stress, but increase the surface shear stress in the rotational direction.

1 Introduction

The rotating flows over a stationary or a rotating body have important and interesting applications in meteorology, in geophysical and cosmical fluid dynamics, in gaseous and nuclear reactors etc. The magnetohydrodynamics of rotating electrically conducting viscous fluids in the presence of a magnetic field are encountered in several important problems in geophysics and astrophysics. They can provide explanations for the observed maintenance and secular variations of the geomagnetic field [1]. It is also relevant in solar physics involved in sun spot development, the solar cycle and the structure of rotating magnetic stars [2].

The rotating flow over a stationary or a rotating sphere has received considerable attention in the literature. Banks [3] has theoretically and experimentally studied the rotating flow over a stationary sphere, whereas Singh [4] investigated this flow analytically for small values of the Reynolds number. At very large values of the Reynolds number the rotating flow near the pole of the rotating sphere reduces to the problem of a rotating flow over a rotating infinite disk. For the case of an axially symmetric flow, the Navier-Stokes equations can be reduced to a set of five first-order ordinary differential equations with the boundary conditions specified at two different locations. Numerical solutions to this problem have been obtained by several investigators [5]–[10] for various values of the ratio of the angular velocity of the disk and the fluid.

Rogers and Lance [5] found that in the absence of suction no solution exists for $-0.16054 > s > -1.4355$. Evans [6], Ockendon [7], Bodonyi [8], and Dykstra and Zandbergen [9], [10] have made a more detailed study of this problem. Hastings [11], Mc Leod [12]–[14], Bushell [15] and Hartman [16] have studied this problem theoretically by either taking $s > 0$ or allowing suction to be imposed.

Banks [3] obtained the similarity solution near the equator of a stationary sphere in a rotating fluid. For the case of a sphere rotating in an ambient fluid, no similarity solution was found in the vicinity of the equator, and the nature of the flow in this region has been discussed by Stewartson [17], Banks [18], [19], Singh [20], and Dennis et al. [21]. Ingham [22] has studied the rotating flow in the vicinity of the equator of a rotating sphere numerically and found that no unique solution exists.

In the above studies, the flow was assumed to be axisymmetric. The non-axisymmetric flow in rotating fluids has been considered by a few investigators [23]–[26]. Recently, the spin-up and spin-down problem over a rotating disk in a vertical plane in the presence of a magnetic field and buoyancy force has been investigated by Slaouti et al. [27]. In this problem the flow becomes non-axisymmetric due to the presence of the buoyancy force. Some further studies on the rotating flow due to the rotating disk in an ambient fluid were carried out by Vooren and Botta [28], [29], and Tarek et al. [30]. In all these analyses, except [27], steady flow was considered. However, the study of the unsteady MHD flow of a rotating fluid on a rotating body is important in the temporal evolution of rotating magnetic stars.

In this paper, we have investigated the unsteady MHD rotating flow of a viscous incompressible electrically conducting fluid in the vicinity of the equator of a rotating sphere. The magnetic field is applied in the radial direction. We have considered the case when there is an initial steady state which is perturbed by a sudden increase in the angular velocity of the sphere. This causes unsteadiness in the flow field. The fluid and the sphere either rotate in the same directions or in the opposite directions. The coupled nonlinear parabolic partial differential equations governing the boundary-layer flow of a rotating unbounded fluid in the vicinity of the equator of a rotating sphere have been solved numerically by using a finite-difference scheme. Analytical solutions have been obtained for large values of suction or the magnetic field. The computations have been carried out from the initial steady state to the final steady state. The results in the absence of a magnetic field and suction have been compared with those of Ingham [22].

2 Formulation and analysis

Let us consider the unsteady laminar viscous incompressible electrically conducting rotating unbounded fluid in the vicinity of the equator of a rotating sphere. The fluid as well as the body either rotate in the same directions or in the opposite directions with angular velocities Ω_f and Ω_b, respectively. There is an initial steady state which is perturbed by suddenly increasing the angular velocity of the sphere. This introduces unsteadiness in the flow field. We have taken spherical polar coordinates (r^*, θ, ϕ) with the origin at the centre of the sphere of radius a, and $\theta = 0$ is the axis of rotation. The motion is assumed to be independent of the azimuthal angle ϕ which implies that the flow is taken to be axisymmetric. It is possible to express the fluid motion in terms of the dimensionless velocity components (u, v, w) in the directions (r^*, θ, ϕ). The dimensionless velocity components (u, v, w) are obtained from the dimensional velocity components (u^*, v^*, w^*) by dividing them by $\alpha\Omega_f$. The dimensionless time τ is obtained from the dimensional time t^* by multiplying it by Ω_f (i.e. $t = \Omega_f\ t^*$). The dimensionless pressure p is obtained from the dimensional pressure p^* by dividing it by $\rho\, a^2\Omega_f^2$. The surface of the sphere is assumed to be electrically insulated. The magnetic field **B** is applied in the r-direction by placing a magnetic dipole at the centre of the sphere. The magnetic Reynolds number $\mathrm{Re}_m = \mu_0\sigma \bar{V}L \ll 1$, where μ_0 and σ are the magnetic permeability and electrical conductivity, respectively, and $\bar{V}$ and L are the characteristic velocity and length, respectively. Under this condition it is possible to neglect the induced magnetic field in comparison to the applied magnetic field. Since there is no applied or polarization voltage imposed on the flow field, the electric field $\bar{E} = 0$. Hence only the applied magnetic field contributes to the Lorentz force, and the components of the Lorentz force in the θ- and ϕ-directions are -Mv and -$M(w - r\sin\theta)$, respectively, where M is the magnetic parameter. Under the above assumptions, the Navier-Stokes equations based on the conservation of mass and momentum governing the unsteady rotating flow over a rotating sphere can expressed as

$$\frac{1}{r^2}\frac{\partial}{\partial r}(r^2 u) + \frac{1}{r\sin\theta}\frac{\partial}{\partial\theta}(v\sin\theta) = 0, \tag{1}$$

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial\theta} - \frac{(v^2+w^2)}{r} = -\frac{\partial p}{\partial r} + \mathrm{Re}^{-1}\left[\nabla^2 u - \frac{2u}{r^2} - \frac{2}{r^2}\frac{\partial v}{\partial\theta} - \frac{2v\cot\theta}{r^2}\right], \tag{2}$$

$$\begin{aligned}&\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial\theta} + \frac{uv - w^2\cot\theta}{r}\\ &\quad = -\frac{1}{r}\frac{\partial p}{\partial\theta} + \mathrm{Re}^{-1}\left[\nabla^2 v + \frac{2}{r^2}\frac{\partial u}{\partial\theta} - \frac{v}{r^2\sin^2\theta}\right] - Mv,\end{aligned} \tag{3}$$

$$\frac{\partial w}{\partial t} + u\frac{\partial w}{\partial r} + \frac{v}{r}\frac{\partial w}{\partial\theta} + \frac{uw + vw\cot\theta}{r} = \mathrm{Re}^{-1}\left[\nabla^2 w - \frac{w}{r^2\sin^2\theta}\right] - M(w - r\sin\theta), \tag{4}$$

where

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial}{\partial\theta}\right). \tag{5}$$

The boundary conditions are the no-slip conditions on the surface and the free stream conditions far away from the surface. These conditions can be expressed as

$$\begin{aligned}&u(1,\theta,t) = u_{0,}v(1,\theta,t) = 0, \quad w(1,\theta,t) = \lambda(1+\varepsilon)r\sin\theta,\\ &u(\infty,\theta,t) = v(\infty,\theta,t) = 0, \quad w(\infty,\theta,t) = r\sin\theta.\end{aligned} \tag{6}$$

Here $\mathrm{Re} = a^2\Omega_f/v$ is the Reynolds number, v is the kinematic viscosity, $\lambda = \Omega_b/\Omega_f$ is the ratio of the angular velocity of the sphere to the angular velocity of the distant fluid, $t = t^*\Omega_f$ is the dimensionless time, $r = r^*/a$ is the dimensionless radial distance, $M = \sigma B^2/\rho\Omega_f = \mathrm{Ha}/\mathrm{Re}$ is the magnetic parameter, $Ha = \sigma B^2 a^2/\mu$ is the Hartmann number, μ is the viscosity, a is the radius of the sphere, ε is a dimensionless constant, and u_0 is the velocity at the surface along r-direction and $u_0 < 0$ for suction.

In order to reduce the complexity of the Navier-Stokes equations (1)–(4), we use the boundary layer approximations. Hence, the radial coordinate is stretched such that

$$\eta = \mathrm{Re}^{1/2}(r-1), \tag{7}$$

and the velocity components and pressure are given by

$$u(r,\theta,t) = \mathrm{Re}^{-1/2}U(\eta,\theta,\tau), \quad v(r,\theta,t) = V(\eta,\theta,\tau),$$
$$w(r,\theta,t) = W(\eta,\theta,\tau), \ t=\tau, \ p=P. \tag{8}$$

Consequently, the boundary-layer equations reduce to

$$\frac{\partial U}{\partial \eta} + \frac{\partial V}{\partial \theta} + V\cot\theta = 0, \tag{9}$$

$$P = 2^{-1}\sin^2\theta, \tag{10}$$

$$\frac{\partial V}{\partial \tau} + U\frac{\partial V}{\partial \eta} + V\frac{\partial V}{\partial \theta} - W^2\cot\theta = -\sin\theta\,\cos\theta + \frac{\partial^2 V}{\partial \eta^2} - MV, \tag{11}$$

$$\frac{\partial W}{\partial \tau} + U\frac{\partial W}{\partial \eta} + V\frac{\partial W}{\partial \theta} + VW\cot\theta = \frac{\partial^2 W}{\partial \eta^2} - M(W - \sin\theta). \tag{12}$$

Here U, V, W are the transformed velocity components along the η-, θ-, ϕ- directions, respectively (i.e., U, V and W are the radial, meridional and rotational velocity components).

In the neighborhood of the pole, the above problem reduces to the unsteady counterpart of the well known Karman swirling flow. Here we are interested in the flow near the equator. For this flow, we write

$$U(\eta,\theta,\tau) = H(\eta,\tau),$$
$$V(\eta,\theta,\tau) = (\theta - \pi/2)F(\eta,\tau), \tag{13}$$
$$W(\eta,\theta,\tau) = G(\eta,\tau).$$

Substituting (13) in Eqs. (9), (11) and (12) and equating the coefficients of the lowest power in $(\theta - \pi/2)$ gives rise to partial differential equations with two independent variables η and τ, and these equations, after substituting $F = -H'$ from the continuity equations, can be expressed as

$$H''' - HH'' + H'^2 + G^2 - 1 - MH' - \partial H'/\partial\tau = 0, \tag{14}$$

$$G'' - HG' - M(G-1) - \partial G/\partial\tau = 0. \tag{15}$$

The boundary conditions are

$$H(0,\tau) = A, \quad H'(0,\tau) = 0, \quad G(0,\tau) = \lambda(1+\varepsilon),$$
$$H'(\infty,\tau) = 0, \quad G(\infty,\tau) = 1. \tag{16}$$

The initial conditions are given by the steady-state equations which are obtained from (14) and (15) by putting $\tau = \varepsilon = \partial H'/\partial\tau = \partial G/\partial\tau = 0$. The steady-state equations are given by

$$H''' - HH'' + H'^2 + G^2 - 1 - MH' = 0, \tag{17}$$

$$G'' - HG' - M(G-1) = 0 \tag{18}$$

with the boundary conditions

$$H(0) = A, \quad H'(0) = 0, \quad G(0) = \lambda, \quad H'(\infty) = 0, \quad G(\infty) = 1. \tag{19}$$

Here $H, H'(=-F)$ and G are the dimensionless velocity components along the radial, meridional and rotational directions, respectively, $A(=u_0\mathrm{Re}^{1/2})$ is the mass transfer parameter, and $A < 0$ for suction, and prime denotes derivative with respect to η.

It may be remarked that near $\lambda = 1$ it is rather difficult to obtain the numerical solution of Eqs. (14) and (15) under the boundary conditions (16), since the boundary layer grows. Therefore, the following scaling has been used near $\lambda = 1$ when $A = M = 0$:

$$\varepsilon_1 = 1 - \lambda(1+\varepsilon), \quad \eta_1 = \varepsilon_1^{1/4}\eta, \quad H(\eta,\tau) = \varepsilon_1^{1/4}h(\eta_1,\tau),$$

$$G(\eta,\tau) = 1 - \varepsilon_1 g(\eta_1,\tau). \tag{20}$$

Using (20) in (14) and (15), we obtain the following equations:

$$h''' - hh'' + h'^2 - 2g - \partial h'/\partial\tau = 0, \tag{21}$$

$$g'' - hg' - \partial g/\partial\tau = 0 \tag{22}$$

with the boundary conditions

$$h(0,\tau) = h'(0,\tau) = 0, \quad g(0,\tau) = 1, \quad h'(\infty,\tau) = g(\infty,\tau) = 0. \tag{23}$$

The corresponding steady-state equations along with the boundary conditions are given by

$$h''' - hh'' + h'^2 - 2g = 0, \tag{24}$$

$$g'' - hg' = 0, \tag{25}$$

$$h(0) = h'(0) = 0, \quad g(0) = 1, \quad h'(\infty) = g(\infty) = 0. \tag{26}$$

In Eqs. (21)–(26), prime denotes derivative with respect to η_1. The steady-state equations (24)–(26) are identical to those of Ingham [22].

3 Methods of solution

Equations (14) and (15) under the boundary conditions (16) and the initial conditions (17)–(19) have been solved by using an implicit, tri-diagonal, iterative finite-difference scheme similar to that of Blottner [32]. All the first order derivatives with respect to τ are replaced by two-point back-ward-difference formulae:

$$\partial S/\partial\tau = (S_{i,j} - S_{i-1,j})/\Delta\tau, \tag{27}$$

where S denotes any dependent variable H or G, and i and j are the node locations along the τ and η directions, respectively. First the third-order differential equation (14) is converted to a second-order one by substituting $H' = H_1$. Then these second-order equations are discretized by using three-point central difference formulae and all the first-order equations by employing the trapezoidal rule. The nonlinear terms are evaluated at the previous iterations. At each time-step of constant τ, a system of algebraic equations is solved iteratively by using the Thomas algorithm (see Blottner [32]). The same procedure is repeated for the next τ value, and the equations are solved line by line until the desired τ value is reached. A convergence criterion based on the relative difference between the current and previous iterations is used. When this difference reaches 10^{-5}, the solution is assumed to have converged and the iterative process is terminated.

A sensitivity analysis of the effect of the step sizes $\Delta\eta$ and $\Delta\tau$ and the location of the edge of the boundary layer η_∞ on the solutions was performed. Finally, the computations were carried out with $\Delta\eta = 0.02$, $\Delta\tau = 0.001$ for $0 \le \tau \le 0.1$, $\Delta\tau = 0.02$ for $\tau > 0.1$, and $\eta_\infty = 10$.

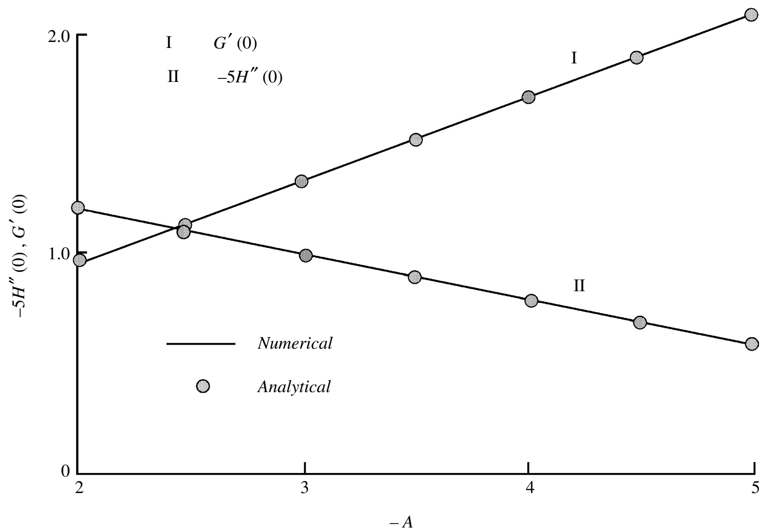

Fig. 1. Comparison of analytical and numerical results for $M = 1$, $\varepsilon = 0.2$, $\tau = 3$, $\lambda = 0.5$

Table 1. Comparison of surface shear stress in the meridional and rotational direction $(-H''(0), G'(0))$ for $A = M = \varepsilon = \tau = 0$

λ	Present results		Ingham [22]	
	$-H''(0)$	10 $G'(0)$	$-H''(0)$	10 $G'(0)$
0.995	0.02568	0.00665	0.025799	0.006661
0.99	0.04326	0.015834	0.04332	0.015837
0.975	0.08568	0.04968	0.08576	0.04973
0.95	0.14315	0.11802	0.14317	0.11808
0.90	0.23716	0.27979	0.23722	0.27985
0.85	0.31659	0.46281	0.31666	0.46287
0.80	0.38678	0.66068	0.38686	0.66076

4 Asymptotic solution for large suction

In this section, we have obtained approximate closed form solutions of the final steady-state equations obtained from Eqs. (14) and (15) by putting $\partial H'/\partial\tau = \partial G/\partial\tau = 0$, $\tau \to \infty$, for large values of the suction parameter $-A(-A \geq 2)$. Our numerical results show that for $-A \geq 2$ the radial velocity $H(\eta) = A = -A_0$, $A_0 > 0$ and the meridional velocity $H'(\eta) \ll 1$. Hence Eq. (15), after putting $\partial G/\partial\tau = 0$, reduces to

$$G'' + A_0 G' - M(G-1) = 0. \tag{28}$$

The solution of Eq. (28) under the conditions (16) can be written as

$$G = 1 + b_2 \exp(-b_1\eta), \tag{29}$$

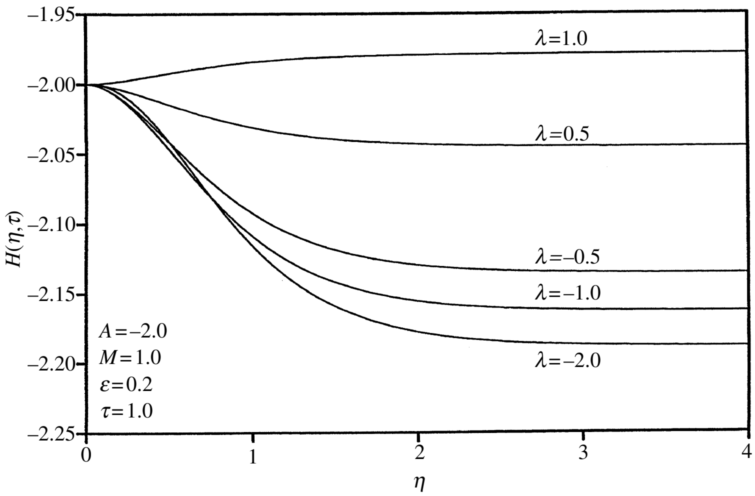

Fig. 2. Effects of λ on $H(\eta, \tau)$

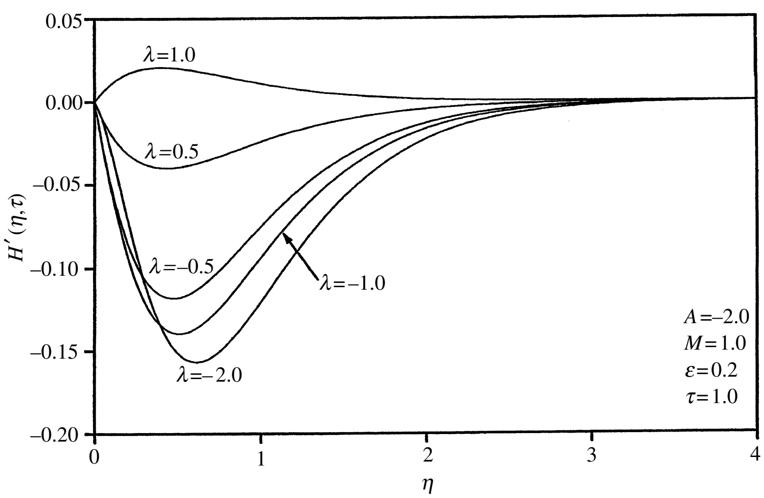

Fig. 3. Effects of λ on $H'(\eta, \tau)$

where

$$b_1 = 2^{-1}[A_0 + (A_0^2 + 4M)^{1/2}], \quad b_2 = \lambda(1 + \varepsilon) - 1. \tag{30}$$

The surface shear stress in the rotational direction is expressed in the form

$$G'(0) = -b_1 b_2. \tag{31}$$

Using relation (29) with $H = -A_0$ and $H' \ll 1$ on the steady-state equation corresponding to Eq. (14), we get

$$H''' + A_0 H'' - MH' = -[2b_2 \exp(-b_1\eta) + b_2^2 \exp(-2b_1\eta)]. \tag{32}$$

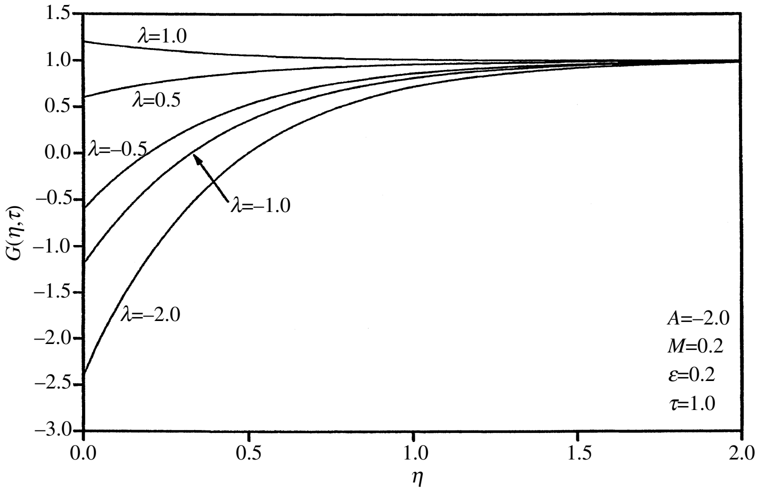

Fig. 4. Effects of λ on $G(\eta, \tau)$

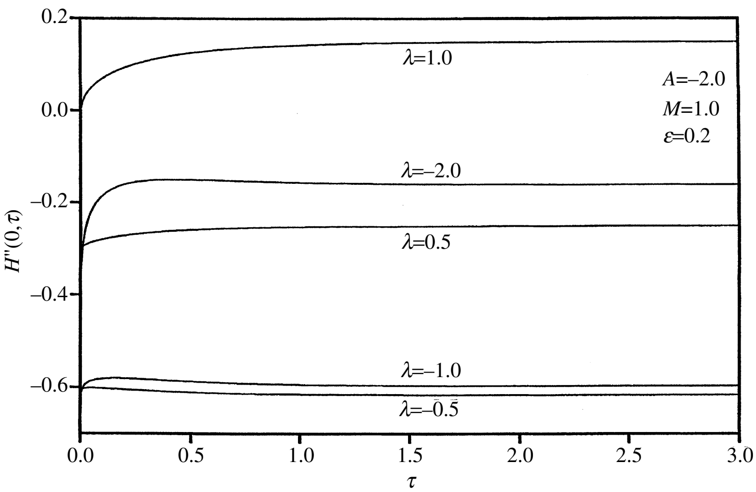

Fig. 5. Effects of λ on $H''(0, \tau)$

The solution of (32) under conditions (16) can be expressed as

$$H = -A_0 - (2b_1b_3)^{-1}b_2^2[2\exp(-b_1\eta) - \exp(-2b_1\eta) - 1]$$
$$- 2b_2(b_1^2b_4)^{-1}[(1 + b_1\eta)\exp(-b_1\eta) - 1], \tag{33}$$

where

$$b_3 = A_0^2 + A_0(A_0^2 + 4M)^{1/2} + 3M, \quad b_4 = (A_0^2 + 4M)^{1/2}. \tag{34}$$

The radial velocity far away from the surface is given by

$$H(\infty) = -A_0 - (2b_1b_3)^{-1}b_2^2 + 2(b_1^2b_4)^{-1}b_2. \tag{35}$$

The surface shear stress in the meridional direction is given by

$$H''(0) = b_2[(b_1b_2/b_3) + (2/b_4)]. \tag{36}$$

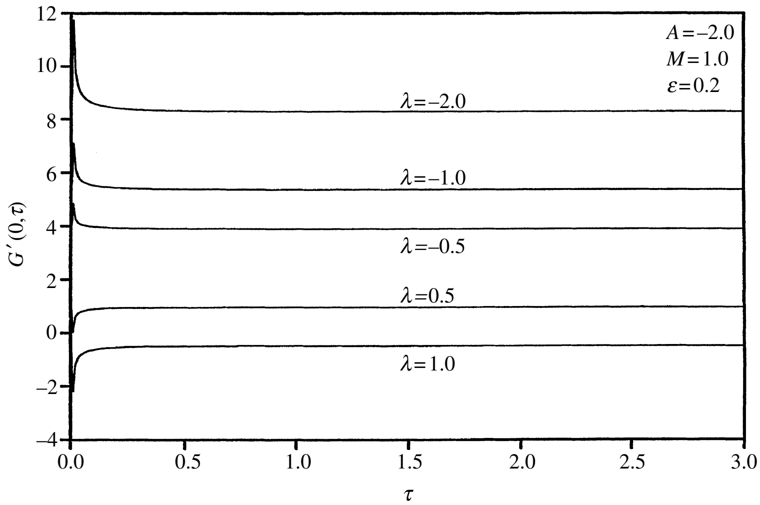

Fig. 6. Effects of λ on $G'(0,\tau)$

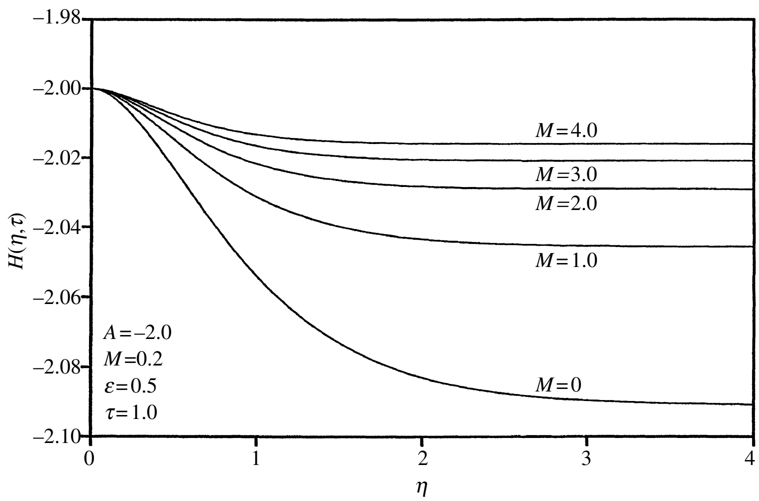

Fig. 7. Effects of M on $H(\eta,\tau)$

For $-A \geq 2(A_0 \geq 2)$, these analytical results are found to be in good agreement with the numerical results. The comparison is presented in Fig. 1. It may be remarked that the above analytical results are also valid for large $M(M \geq 4)$ when $A_0 \geq 1$. Further, these results also hold good for the initial steady-state case if we put $\varepsilon = 0$.

5 Results and discussion

Equations (14) and (15) under the boundary conditions (16) and initial conditions (17)–(19) have been solved by using the implicit finite-difference scheme as described earlier. In order to assess the accuracy of our method, we have compared the steady-state results ($H''(0)$, $G'(0)$) for

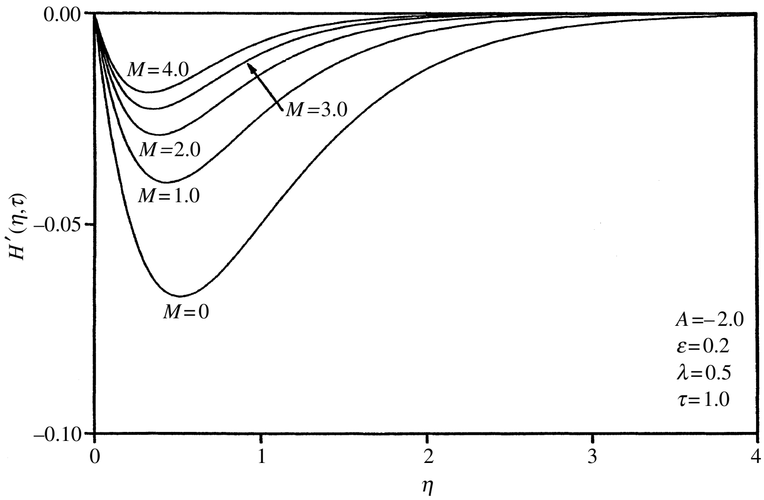

Fig. 8. Effects of M on $H'(\eta,\tau)$

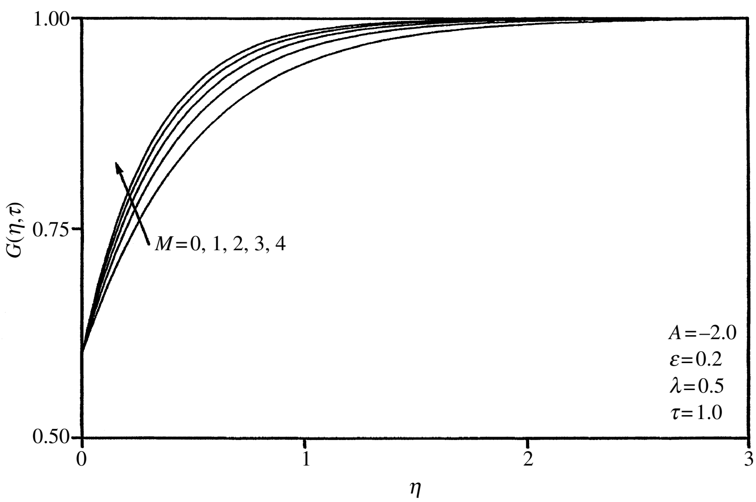

Fig. 9. Effects of M on $G(\eta,\tau)$

$0.8 \leq \lambda \leq 0.995$, when $A = M = 0$ with those of Ingham [22], and the results are found to be in good agreement. This comparison is given in Table 1.

Figure 1 presents the comparison of the surface shear stresses in the meridional and rotational directions ($H''(0)$, $G'(0)$) for the final steady-state case when $-A \geq 2$ ($A_0 \geq 2$), $M = 1$, $\lambda = 0.5$, $\tau = 3$, $\varepsilon = 0.2$, obtained by analytical and numerical methods. The results are in very good agreement for $-A > 2$.

Figures 2–4 show the effect of the ratio of the angular velocities of the body and fluid λ on the radial, meridional and rotational velocity components ($H(\eta,\tau)$, $H'(\eta,\tau)$, $G(\eta,\tau)$) for $A = -2$, $M = 1$, $\varepsilon = 0.2$, $\tau = 1$. The velocity profiles are found to change more when the body and the fluid rotate in opposite directions ($\lambda < 0$) than when they rotate in the same direction ($\lambda > 0$), because the relative angular velocity increases when they rotate in opposite directions. The velocity profiles, $H(\eta,\tau)$, $H'(\eta,\tau)$ and $G(\eta,\tau)$ are negative for $\lambda < \lambda_0$.

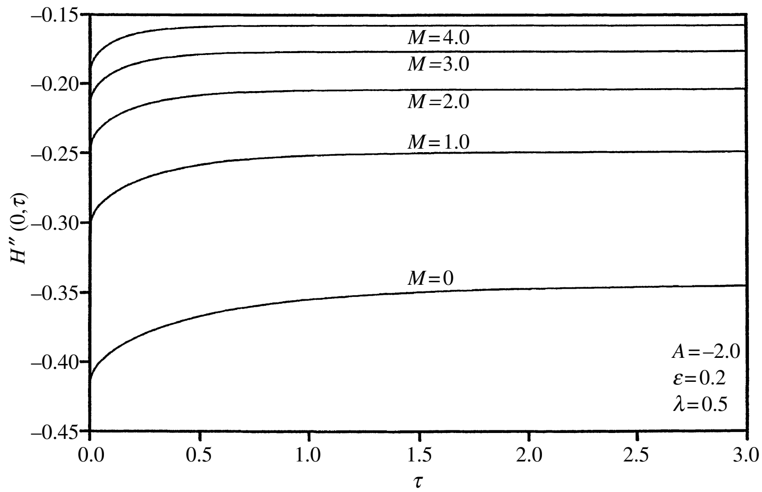

Fig. 10. Effects of M on $H''(0,\tau)$

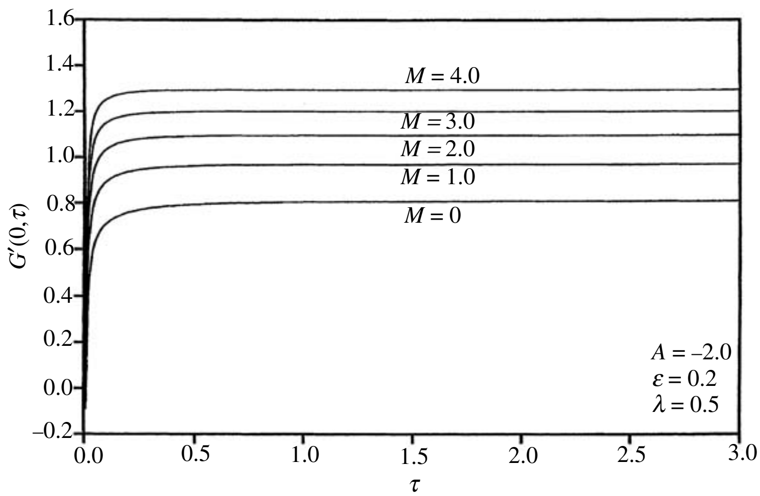

Fig. 11. Effects of M on $G'(0,\tau)$

Figures 5 and 6 present the effect of λ on the surface shear stresses in the meridional and rotational directions ($H''(0,\tau)$, $G'(0,\tau)$) for $A = -2$, $M = 1$, $\varepsilon = 0.2$. The surface shear stresses reach the new (final) steady state rather quickly (i.e., the spin-up time is small).

Figures 7–9 display the effect of the magnetic parameter M on the velocity components in the radial, meridional and rotational directions ($H(\eta,\tau)$, $H'(\eta,\tau)$, $G(\eta,\tau)$) for $A = -2$, $\varepsilon = 0.2$, $\lambda = 0.5$, $\tau = 1$. The velocity components in the radial and meridional directions ($H(\eta,\tau)$, $H'(\eta,\tau)$) decrease with increasing M, but the velocity component in the rotational direction increases. The reason for this trend is the reduction of the boundary-layer thickness with increasing M.

Figures 10 and 11 present the effect of the magnetic parameter M on the surface shear stresses in the meridional and rotational directions ($H''(0,\tau)$, $G'(0,\tau)$) for $A = -2$, $\varepsilon = 0.2$, $\lambda = 0.5$. The surface shear stress in the meridional direction ($-H''(0,\tau)$) decreases with increasing M, but the surface shear stress in the rotational direction ($G'(0,\tau)$) increases. Since the magnetic parameter

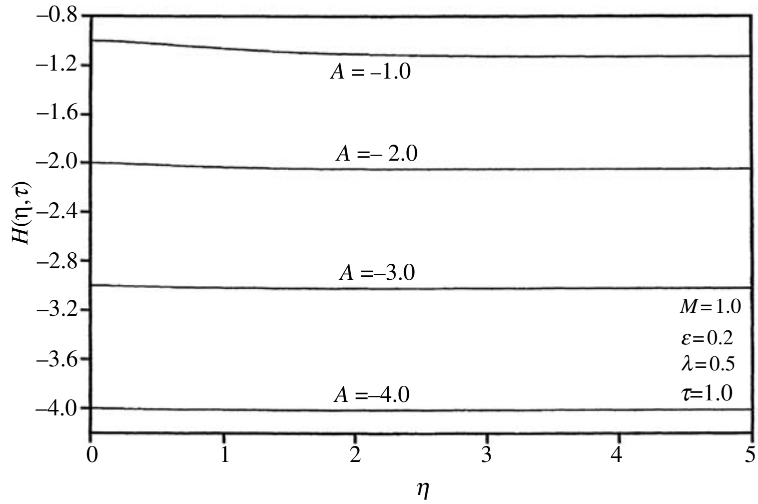

Fig. 12. Effects of A on $H(\eta, \tau)$

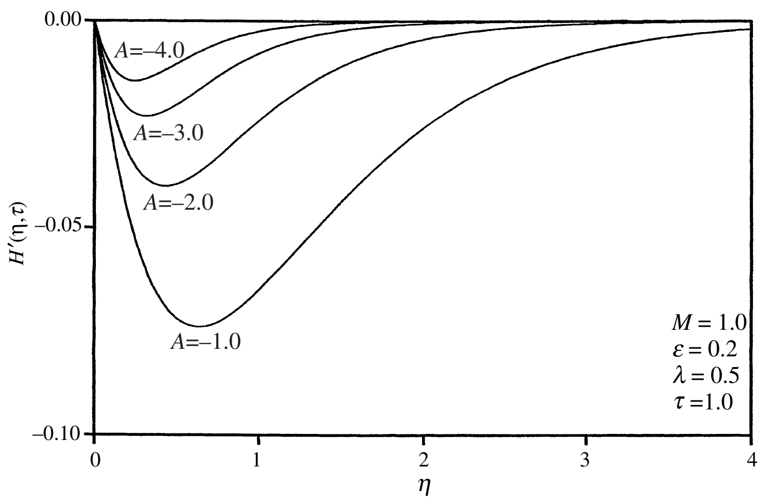

Fig. 13. Effects of A on $H'(\eta, \tau)$

M has stabilizing effect on the flow field, it retards the growth of the boundary layer. Consequently, the shear stress in the meridional direction decreases, but the shear stress in the rotational direction increases as M increases. For $A = -2$, $\lambda = 0.5$, $\tau = 3$, $\varepsilon = 0.2$, $-H''(0, \tau)$ decreases by about 112% as M increases from zero to 4, but $G^{/}(0, \tau)$ increases by about 62%. The spin-up time (time to reach the new steady state) reduces with increasing M.

In Figs. 12–14, the effect of the suction parameter ($-A$) on the radial, meridional and rotational directions ($H(\eta, \tau)$, $H'(\eta, \tau)$ and $G(\eta, \tau)$) for $M = 1$, $\lambda = 0.5$, $\varepsilon = 0.2$, $\tau = 1$ is given. For $-A \geq 2$, the velocity component in the radial direction ($H(\eta, \tau)$) becomes nearly a constant, i.e. $H(\eta, \tau) = -A$. The velocity component in the meridional direction ($H'(\eta, \tau)$) reduces with increasing suction, but the velocity component in the rotating direction ($G(\eta, \tau)$) increases. Like the magnetic field, suction also reduces the boundary-layer thickness. This results in a reduction

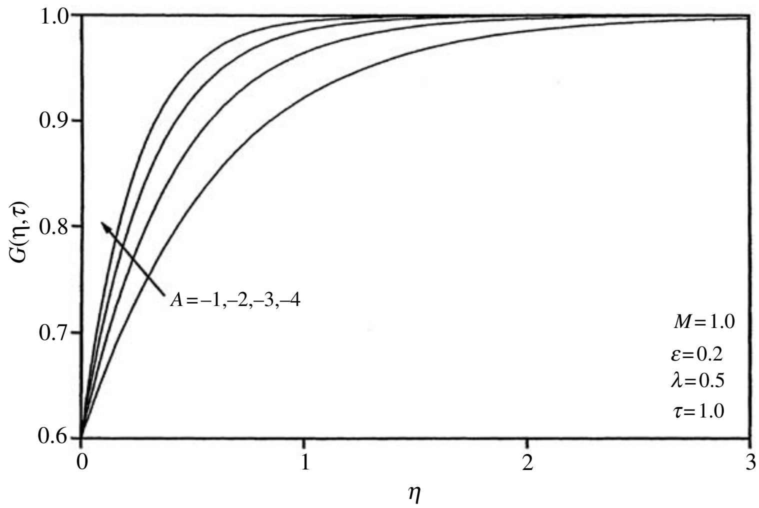

Fig. 14. Effects of A on $G(\eta, \tau)$

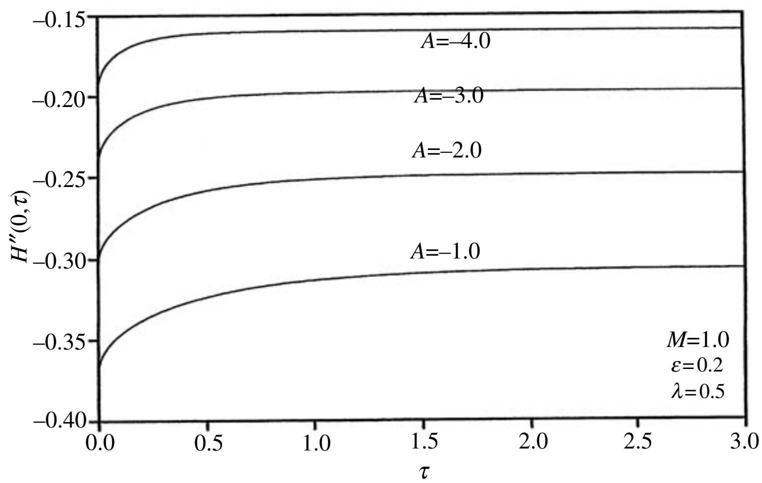

Fig. 15. Effects of A on $H''(0, \tau)$

in the radial and meridional velocity components ($H(\eta, \tau)$, $H'(\eta, \tau)$ and an increase in the rotational velocity component $G(\eta, \tau)$).

Figures 15 and 16 show the effect of the suction parameter ($A < 0$) on the surface shear stresses in the meridional and rotational directions ($H''(0, \tau)$, $G'(0, \tau)$) for $M = 1$, $\varepsilon = 0.2$, $\lambda = 0.5$. Like the magnetic field, the suction parameter reduces the surface shear stress in the meridional direction ($H''(0, \tau)$), but increases the surface shear stress in the rotational direction ($G'(0, \tau)$). This trend is due to the reduction of the boundary-layer thickness with increasing suction. The shear stress in the meridional direction ($-H''(0, \tau)$) for $M = 1$, $\varepsilon = 0.2$, $\lambda = 0.5$ decreases by about 47% as the suction ($-A$) increases from 1 to 4, whereas the shear stress in the rotational direction ($G'(0, \tau)$) increases by about 160%. The spin-up time reduces with increasing suction.

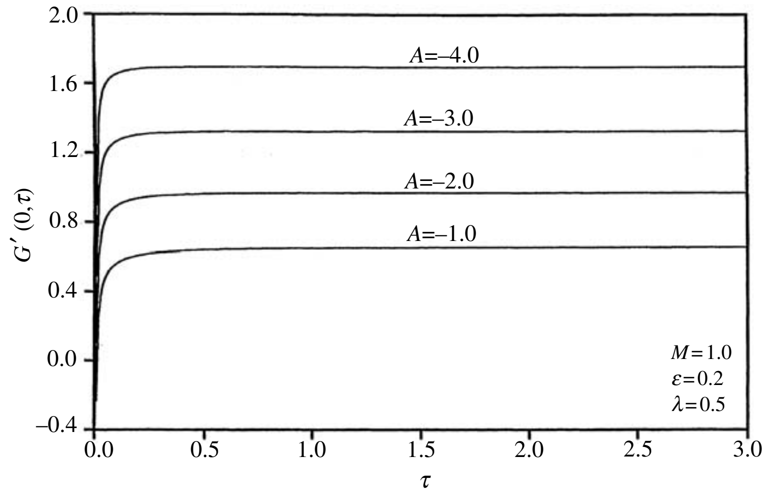

Fig. 16. Effects of A on $G'(0, \tau)$

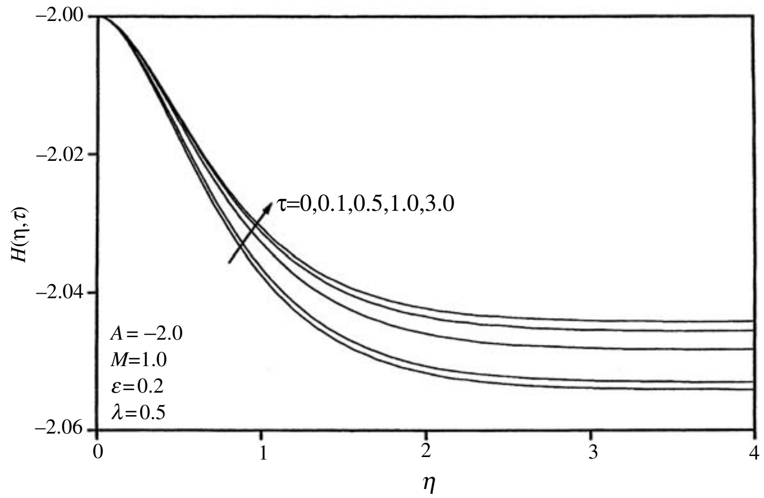

Fig. 17. Time development of $H(\eta, \tau)$

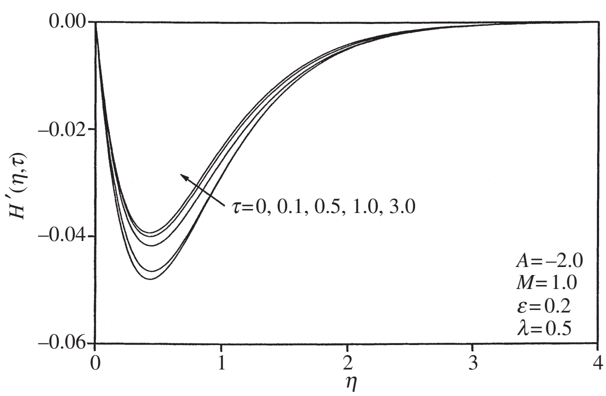

Fig. 18. Time development of $H'(\eta, \tau)$

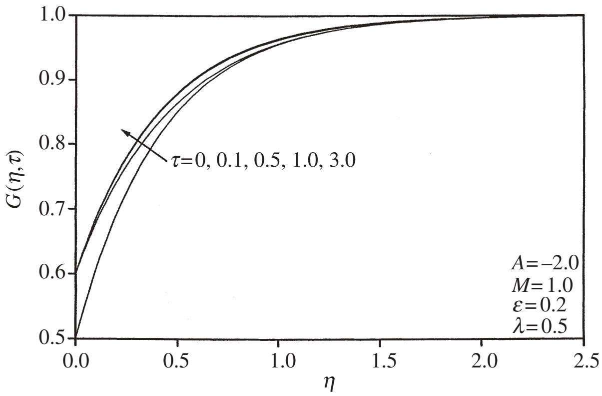

Fig. 19. Time development of $G(\eta, \tau)$

The temporal development of the velocity components in r, θ and ϕ directions ($H(\eta, \tau)$, $H^{/}(\eta, \tau)$, $G(\eta, \tau)$) for $A = -2, M = 1, \varepsilon = 0.2$ is shown in Figs. 17–19. The velocity profiles reach the new (final) steady state after time $\tau = 3$.

6 Conclusions

In the presence of suction and (or) a magnetic field, it is possible to obtain the solution of the flow problem near the equator when the fluid and the body either rotate in the same direction or in opposite directions. For large suction or magnetic field, analytical solutions have been obtained for the final and initial steady-state cases. Suction and (or) magnetic field reduce the surface shear stress in the meridional direction, but increase the surface shear stress in the rotational direction. The spin-up time reduces with increasing suction or magnetic field.

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