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Further study on a moving-wall boundary-layer problem with mass transfer

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Summary. The boundary-layer problem of a semi-infinite flat plate moving in a free stream with mass transfer is discussed in this paper. The paper extends the work of previous researchers to the general situations including mass injection as well as suction on the wall and the case of wall moving in the same direction as the free stream velocity. The analysis is concentrated on the wall drag. The solutions are obtained by numerical techniques. Under certain conditions, current results will reduce to those obtained by other researchers.

1 Introduction

The similarity differential equation of the boundary-layer problem for a moving semi-infinite flat plate in a constant velocity free stream flow was first derived by Klemp and Acrivos [1] as follows:

$$f'''(\eta) + f(\eta)f''(\eta) = 0, \tag{1}$$

with the non-homogeneous boundary conditions

$$\eta = 0,\ f = 0,\ f' = -\lambda,\ \eta = \infty,\ f' = 1, \tag{2}$$

where f is the nondimensional stream function $f = \Psi(2U_\infty vx)^{-1/2}$, Ψ is the stream function, U_∞ is the free stream fluid velocity, v is the fluid kinematic viscosity, η is the similarity variable defined as $\eta = y(U_\infty/2vx)^{1/2}$, and λ is the ratio of the wall velocity to the free stream fluid velocity defined as $\lambda = U_w/U_\infty$, and it is assumed that the flat plate moves opposite to the free stream. The solution of Eq. (1) has been discussed by many researchers [2]–[5]. Vajravelu and Mohapatra [6] extended the problem by including mass injection on the wall surface. The drag reduction was discussed in their paper. In the above-mentioned paper, the discussions are focused on mass injection and wall moving in the opposite direction to the free stream. Mass transfer including injection and suction for a fixed flat plate was discussed by Schlichting and Bussmann [7]. In the general case of a moving-wall problem, the wall may move in the same direction as the free stream velocity, and mass suction can also be applied on the wall. In the present paper, general results of a moving-wall boundary layer problem in a constant-velocity free stream with mass transfer will be discussed.

2 Mathematical formulation

The laminar incompressible flow with mass transfer over a moving flat plate can be described by the following equations [6]:

$$u\frac{\partial u}{\partial x}+v\frac{\partial u}{\partial y}=v\frac{\partial^2 u}{\partial y^2}, \tag{3}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \tag{4}$$

subject to boundary conditions

$$u(x,0)=U_w,\quad v(x,0)=V_w(x),\quad \text{and}\quad u(x,\infty)=U_\infty, \tag{5}$$

where U_w is the plate moving velocity and $V_w(x)$ is the mass transfer velocity at the plate surface. The similarity equation can be obtained by defining a similarity variable as $\eta = y(U_\infty/2vx)^{1/2}$ and a stream function $\Psi = f(\eta)(2U_\infty vx)^{1/2}$. Plugging them into (3) and (4) yields

$$f''' + ff'' = 0 \tag{6}$$

with the associated boundary conditions

$$f(0) = f_0,\quad f'(0) = \lambda \quad \text{and} \quad f'(\infty) = 1, \tag{7}$$

where $\lambda = U_w/U_\infty$, which can be a positive number for $U_w > 0$ with the same direction as the free stream velocity and a negative number for $U_w < 0$ opposite to the free stream velocity, and $f_0 = V_w(2x/vU_\infty)^{1/2}$, which is positive for mass suction and negative for mass injection. The general analytical solution of Eq. (6) is not available. A numerical method has to be used to solve such a nonlinear ordinary differential equation. In the following section, a shooting Runge-Kutta method [8] is adopted to solve Eq. (6) with boundary conditions (7).

3 Results and discussions

3.1 Results for $\lambda_C \le \lambda \le 1$

The solution of Eq. (6) with boundary conditions (7) has been discussed for $f_0 < 0$ and $\lambda < 0$ [6]. It is shown that the solutions of Eq. (3) exist only for a certain range of wall moving velocity, $\lambda_C \le \lambda$, and mass injection, $f_{0,C} \le f_0$, where the subscript "C" denotes the critical parameter. The discussion here will be focused on the influences of f_0 and λ on the wall dynamic drag. Before solving the equation numerically, it is well known that the wall drag will reduce with the increase of mass injection and will rise with the increase of mass suction. However, the influence of the wall moving parameter, say λ, is not as clear as that of mass transfer. Figure 1 shows the results of $f''(0)$, which is directly relevant to the dynamic local wall stress, for different mass transfer and wall moving velocities. From Fig. 1, it is seen that, for $\lambda < 0$ and $f_0 < 0$, the solution will reduce to that of Vajravelu and Mohapatra's. In the figure, the blow-off mass injection parameter, say $f_0 = -0.8757$, for a fixed wall boundary-layer flow is denoted by a superscript "*". It is observed from Fig. 1 that, if there is mass suction on the wall surface, the solution domain will be enlarged by the increase of mass suction, and the wall shear will also increase with mass suction. Although mass suction will increase the wall drag, the heat transfer will also be enhanced by mass suction in many applications. The critical inverse wall velocity for flow separation will increase with the increase of mass suction. For a fixed wall boundary layer, the flow will be blown off if the mass-transfer parameter $f_0 < -0.8757$ [7].

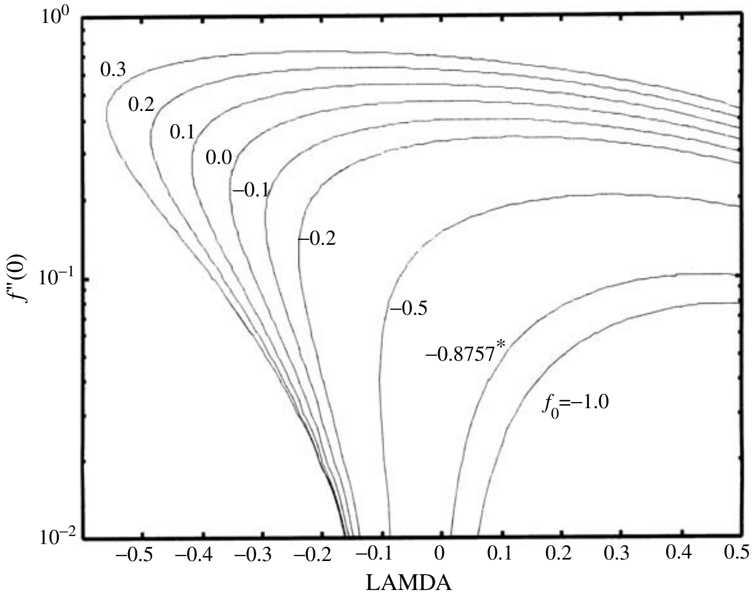

Fig. 1. Plots of $f''(0)$ versus λ for different mass transfer f_0

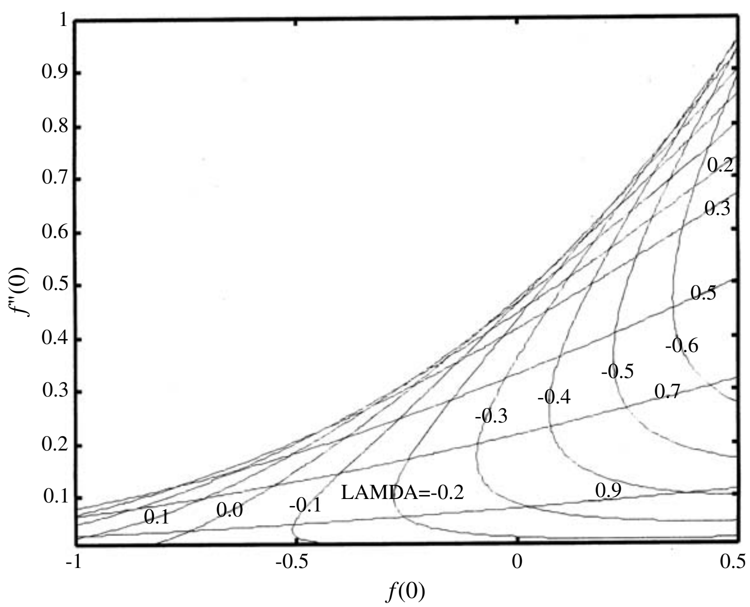

Fig. 2. Plots of $f''(0)$ versus f_0 for different λ

However, when the wall is moving in the same direction as the free stream, the flow will not be blown off even for very large mass injection. Figure 2 shows the same results of Fig. 1 from another perspective. The plots of $f''(0)$ versa f_0 for different λ are illustrated in Fig. 2. It is found that there exist two solutions for $\lambda < 0$ no matter whether f_0 is positive or negative, which is also shown in Fig. 1. The interesting observation is that the movement of the wall in the same direction as the free stream velocity has a similar influence to wall suction, which will restrain flow separation. Figure 3 depicts the same results as function of f_0 and λ for different $f''(0)$,

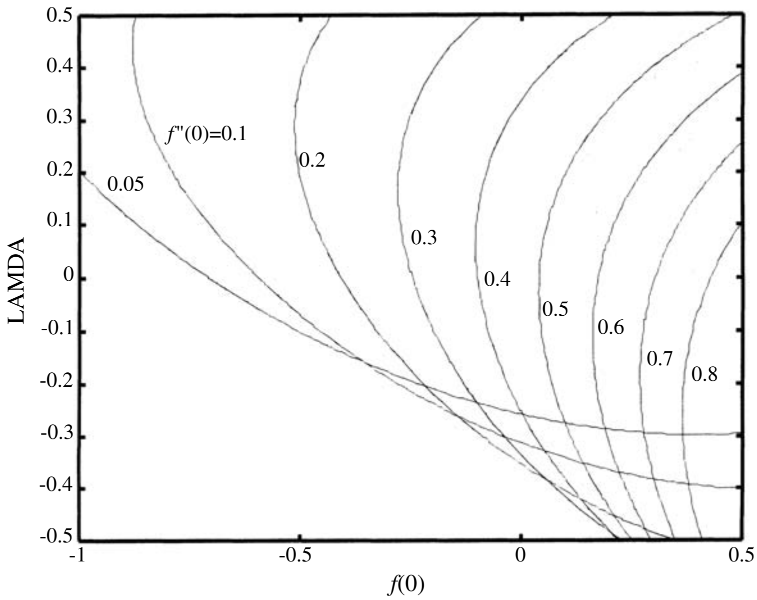

Fig. 3. Plots of λ versus f_0 for different $f''(0)$

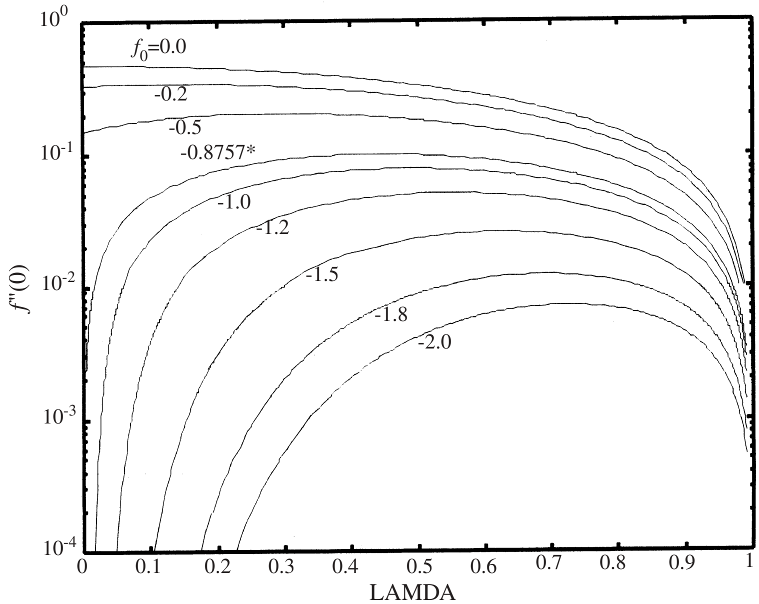

Fig. 4. Plots of $f''(0)$ versus λ under different mass transfer f_0 for $0 < \lambda < 1$

which also proves Vajravelu and Mohapatra's results that the solution of Eq. (4) only exists for a special domain of f_0 and λ.

Figure 4 shows the details for $0 \leq \lambda < 1$. It is found from Fig. 4 that when $0 < \lambda < 1$, there exists a unique solution for $-\infty < f_0 < +\infty$, which was proved by Hartman [9]. The wall local drag will decrease with increasing mass injection. However, the influence of λ on the wall drag is much different from that of mass transfer. It is found from Fig. 4 that there is a maximum wall drag for a constant mass injection in the domain of $0 < \lambda < 1$. Under certain mass injection,

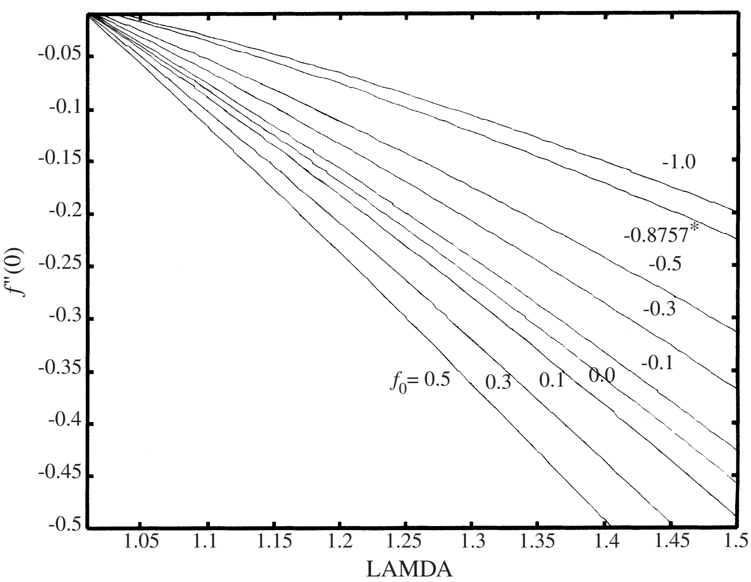

Fig. 5. Plots of $f''(0)$ versus λ under different mass transfer f_0 for $\lambda > 1$

there exists a pair of λ that will result in the same local wall drag. The physical meaning of this is that when the mass injection of the wall is constant, the wall drag will increase with raising wall velocity at first, then, after the wall drag reaches the peak point, it will decrease with increasing wall velocity. Observation also shows that the maximum wall drag will shift to a higher wall moving velocity with the increase of mass injection. A combination of Fig. 1 and Fig. 4 can also give the same trend of maximum local wall drag under specific mass transfer in the domain of $\lambda_C < \lambda < 1$. When $\lambda = 1$, there is a trivial solution for Eqs. (6) and (7), which is $f(\eta) = f_0 + \eta, f'(\eta) = 1$, and $f''(0) = 0$. In this case, the wall is moving at the same velocity as the fluid. There is no shear stress in the fluid.

3.2 Results for $\lambda > 1$

The above discussions are focused on $\lambda_C \leq \lambda \leq 1$. Solutions of Eqs. (6) and (7) also exist for $\lambda > 1$. In this case, the wall moving velocity is greater than the free stream velocity, and the wall drag force is opposite to the wall moving direction. The boundary layer is somehow similar to the boundary layer over a continuously stretching surface [10]. The nondimensional wall drag force is illustrated in Fig. 5 for $\lambda > 1$ under different mass transfer parameters. It is seen from Fig. 5 that the wall drag will increase with the increase of λ and mass suction, and it will decrease with increasing mass injection. It is also expected that the solution will exist for all $\lambda > 1$ and f_0.

4 Conclusions

(i) This paper extends the boundary-layer problem of a semi-infinite moving wall in a free stream with constant velocity to the general situations. Under some special conditions, the problem will reduce to the results obtained by previous researchers.

(ii) The critical reverse wall velocity will increase with the increase of mass suction without flow separation.

(iii) When $0 < \lambda < 1$, the wall drag will be in the same direction as the free stream velocity, there is no flow separation for any arbitrary mass transfer parameter, and there is a maximum wall drag under a certain mass injection. However, when $\lambda > 1$, the wall drag is opposite to the free stream velocity direction, and the wall drag will increase with increasing wall moving velocity ratio λ and mass transfer parameter f_0.

(iv) The analysis illustrates a general concept of interaction between mass transfer and wall movement, which can give us a deep insight into the boundary-layer problem.

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