RESEARCH

Copula-based multivariate analysis of hydro-meteorological drought

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Abstract
Droughts have far-reaching detrimental impacts on the environment, society, and economy, ranging from regional to national levels. As the drought characteristics are interrelated, multivariate analysis of those is necessary to understand the actual drought situation in a region. However, such studies are limited. Hence, this study aimed to develop a framework to investigate the meteorological and hydrological droughts using a multivariate analysis of drought characteristics in the Pennar River basin (a semi-arid region) of India, dominated by agricultural activity. Long-term observations (1980–2015) of precipitation, temperature, and streamflow were used to calculate the Standardized Precipitation Evapotranspiration Index (SPEI) and Streamflow Drought Index (SDI) at a 3-month timescale. Based on these indices, drought duration, severity, and peak were abstracted using the Run Theory. The best-fit marginal distribution was determined for every drought characteristic to establish the bivariate joint probability distribution (the copula function). Based on the best-fit copula function, the joint probabilities and the joint return periods were estimated. Results revealed that, for meteorological drought, Frank Copula and Survival Clayton copula were the best-fit copula function for the joint risk of drought duration and severity and the combination of drought duration and peak as well as drought severity and peak, respectively. The joint return period of a drought event with characteristics above the 25th percentile of the same varies between 1 and 2 years, whereas it spans between 1 and 45 years for the 50th percentile. This analysis will provide vital information for water management and planning in a region to mitigate droughts.

1 Introduction

Hydrological extreme events in today's world draw the attention of the public and scientific community because of their increase in severity and frequency, human impacts, and catastrophic physical damage. Droughts are one of the most challenging climate hazards which can adversely impact agriculture as well as social and economic sectors (Chandrasekara et al. 2021) (Spinoni et al. 2019). As per the sixth assessment report of the Intergovernmental Panel on Climate Change (IPCC), the temperature has already escalated globally by almost 1.1 °C since the pre-industrial period, and the 1.5 °C threshold is expected to be surpassed before 2040 (Allan et al. 2021). In recent decades, different places in India have witnessed frequent and severe droughts, leading it to become a drought-prone country with high vulnerability (Poonia et al. 2021b). Most Indian states are

badly impacted by recurring and long-term droughts, which have a wide range of negative consequences for water supplies, ecosystems, and socioeconomic development (Das et al. 2020) (Swain et al. 2021). Droughts are commonly classified into four: meteorological (Spinoni et al. 2019), hydrological, agricultural (lack of moisture content of soil and growth of plants), and socio-economic (Ayantobo et al. 2017); based on the nature of water shortages and the affected hydrological consequences. Meteorological drought is described as a prolonged lack of rainfall from its normal pattern, sometimes exacerbated by high evapotranspiration rates due to high temperatures. Meteorological droughts must be studied both for their own sake and as precursors to more severe hydrological and agricultural droughts (Ayantobo et al. 2019; Swain et al. 2020; Haied et al. 2017). Water shortages in hydrological storage systems, such as streamflow, groundwater, stored water supplies, power generation, and recreation, are known as hydrological droughts (Hong et al. 2015; Kwak et al. 2016). The most important variable to express the quantity of water available in surface water resources is the streamflow. Thus, hydrological drought is linked to the streamflow shortfall under normal conditions

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(Hasan et al. 2019; Kwak et al. 2016). In this study, the analysis of meteorological and hydrological drought has been carried out. The study of the relationship between meteorological and hydrological drought is particularly relevant for the management of water resources, early warning, and mitigation of drought for a particular watershed. Droughts caused by meteorological and hydrological factors are essentially linked, although with a temporal lag (Achite et al. 2022). Changes in hydrological patterns, such as a lack of precipitation and increased evaporation and transpiration, signify the onset of a drought. The meteorological drought subsequently becomes a hydrological drought, which leads to agricultural drought. Drought indices are the most effective way to assess the occurrence of droughts because there is no common definition of drought (Poonia et al. 2021a). Drought indices are more appropriate for assessing and monitoring drought than utilizing hydro-meteorological variables directly. Drought indices are calculated numerically utilizing hydro-meteorological inputs and drought indicators (Chandrasekara et al. 2021). Around the world, two types of drought indices are commonly used. One type of mechanism-based indicator considers the laws of hydrological circulation and spatiotemporal water balance. The other type is the index based on statistics, which focuses on the probability distributions of hydrometeorological variables, such as the Standardized Precipitation Index (SPI) (McKee et al. 1993). The latter has gained popularity in recent years due to its simplicity, consideration of geographical and topographical factors, changeable time scale, and cheap data requirements (Hong et al. 2015).

Many researchers used a variety of indices based on their applicability, such as SPI, Standardized Precipitation Evapotranspiration Index, SPEI (Vicente-Serrano et al. 2012), Palmer Drought Severity Index, PDSI (Palmer 1965), Surface Water Supply Index, SWSI (Shafer 1982), Standardized Runoff Index, SRI (Shukla and Wood 2008), Standardized Hydrological Index, SHI (Panu and Sharma 2009), Streamflow Drought Index, SDI (Hong et al. 2015; Pathak and Dodamani 2016; Udayar et al. 2019), Vegetation Condition Index, VCI (Kogan 1995), Standardized Soil Moisture Index, SSI (Hao and AghaKouchak 2013), Multivariate Drought Index, MDI (Rajsekhar et al. 2015), improved Multivariate Standardized Reliability and resilience Index, MSRRI (Guo et al. 2019), Reconnaissance Trivariate Drought Index, RTDI (Dixit and Jayakumar 2021), etc. to define droughts. Still, each drought indices have its unique advantages and disadvantages (Poonia et al. 2021a). The SPEI utilizes the benefits of the PDSI in terms of changes in evaporation demand (induced by temperature swings and trends) along with the SPI's easiness of operation and applicability of multi-scale analysis (Pandey and Dhama 2000; Rose and Chithra 2020). In this study, the Standardized Precipitation Evapotranspiration Index (SPEI) and Streamflow Drought

Index (SDI) were selected for meteorological and hydrological drought characterization, respectively.

Drought characteristics such as drought duration, severity, and peak can easily be extracted using the drought index values. These drought features can be analyzed in a multivariate or univariate manner. Traditional drought frequency analysis methods use the univariate technique (Poonia et al. 2021a). As previously stated, drought characteristics are interdependent and hence have a strong correlation; thus, univariate analysis of drought based on any of these characteristics cannot accurately explain the interdependence structure among them. Due to the multivariate nature of drought characteristics, simultaneous assessment is required, and the outcomes from the multivariate analysis of drought characteristics are important for estimating its potential risks (Mesbahzadeh et al. 2020). Since drought is a multivariate phenomenon, the drought condition can be described more thoroughly using multivariate analysis. Even though numerous probabilistic approaches for examining drought features have been developed in the past, a significant correlation cannot be shown using univariate analysis. Therefore, using a multivariate method and developing the joint dependence structure is preferable to characterize the interconnection between drought characteristics. Most of the multivariate distributions are generated from the univariate distributions of each variable, which have a few drawbacks, including the requirement that the marginal distribution is identical. Furthermore, for parameter estimations, complicated mathematical derivations are required (Poonia et al. 2021a). The Copula function is a great way to figure out the joint probability distribution of multiple variables. The most significant benefit of Copula theory is that it does not require the assumption that a univariate's marginal distribution is independent. The copula probability distribution function is a very effective correlation analysis and multivariate modelling tool. Hence it is suitable for constructing the joint distribution of the marginal distributions of different variables. It can deal with non-linear and non-symmetrical data and describe the correlation between the variables. Since the beginning of this century, copula functions have been widely used in the sectors of finance, meteorology, hydrology, and risk analysis of natural disasters (Wu et al. 2019; Madadgar and Moradkhani 2016; Evkaya et al. 2019). In the hydrological field, the copula function is now utilized in studying drought, flood, joint change of precipitation and flood, and other phenomena to represent the multivariate joint distribution (Dixit and Jayakumar 2022) (Daneshkhah et al. 2016) (Villarini et al. 2009) (Graler et al. 2013; Qin et al. 2021) (Hou et al. 2021). Archimedean class, elliptical or meta-elliptical class, extreme value class, and miscellaneous class are the four copula classes commonly used for hydrological analysis (Song and Singh 2010a) (Song and Singh 2010b) (Shiau and Modarres 2009) (Xu et al. 2015)

(Poonia et al. 2021a) (Poonia et al. 2021b) (Mirabbasi et al. 2012) (Lee et al. 2013).

Copula-based multivariate analysis will aid policymakers in developing better strategies for drought risk management and its mitigation. Several research using the copula function to analyze meteorological drought characteristics have been conducted; however, copula-based hydrological drought analysis in conjunction with meteorological drought is limited. The present study used the multi-variate technique to examine joint dependence structures among three drought characteristics of the meteorological and hydrological droughts for the study area. In this regard, the main objective of this study is thus to investigate both meteorological and hydrological drought in the Pennar River basin by using two standard drought indices, SPEI and SDI, at a 3-month timescale, its characterization using run theory and multivariate analysis of the drought characteristics using Copula function.

The paper starts with a brief introduction about the importance of drought analysis studies along with the objectives of the present study in Section 1, followed by data collection, study area, and methodology details in Section 2. Section 3 deals with results and discussions and in Section 4, a summary and conclusions from the work are presented.

2 Data and method

2.1 Study area

The study region for this work is the Pennar Basin of India. Drought has been a typical occurrence in the Pennar River Basin in recent decades. Between 1982 and 2013, the Pennar River basin in India witnessed 61 continuous droughts out of 107 total droughts in 29 years (Poonia et al. 2021b). The Pennar River is one of India's major east-flowing rivers, rising in the Thenanahesava peak of the Nandidurg range in Karnataka and draining into the Bay of Bengal near Nellore, Andhra Pradesh. It runs for 597 km and covers a total area of 55,213 sq. km., with 6,937 sq. km. in Karnataka and 48,276 sq. km. in Andhra Pradesh (Fig. 1).
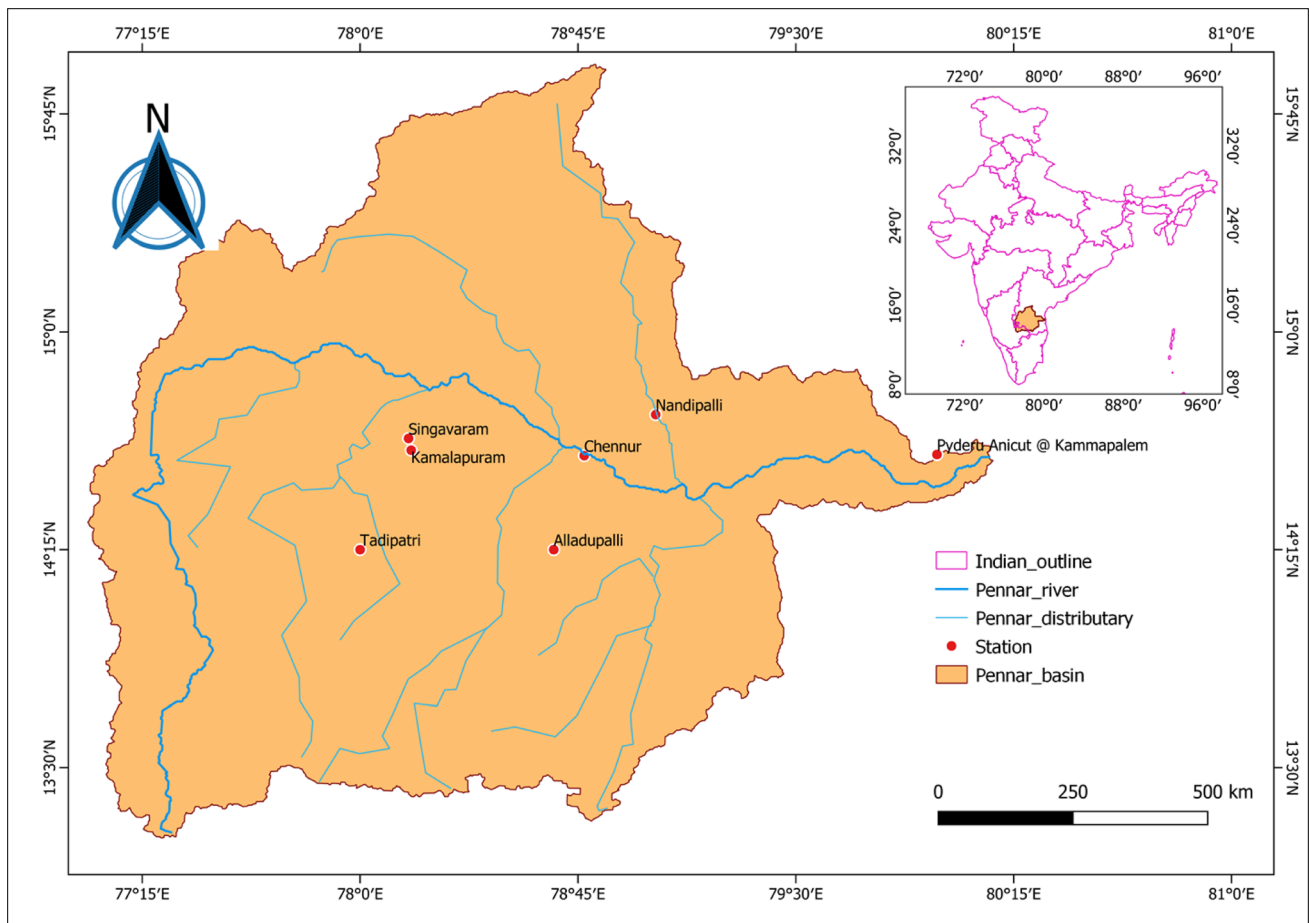

Fig. 1 Study area: Pennar River basin

This fan-shaped basin are lying within 13° 16′–15° 52′ N latitude and 77° 04′–80° 10′ E longitude and is bordered on the north by the Erramala range, on the east by the Eastern Ghats' Nallamala and Velikonda ranges, on the south by the Nandidurg hills, and on the west by the narrow ridge separating it from the Krishna basin's Vedavati valley. In the basin, there are seven hydrological observation sites, namely Chennur, Singavaram, Tadipatri, Kamallapuram, Alladupalli, Nandipalli, and Pyderu Anicut at Kammapalem. The basin's elevation ranges from less than 5 to 1250 m. Both the (southwest and northeast) monsoons bring rain to the basin. However, because the basin is in a semi-arid area, it receives little rainfall. The amount of rainfall in the basin varies spatially, with the coastal regions receiving more rainfall than the western parts. The average annual rainfall in the drainage basin ranges from 550 at Anantapur to 900 mm near Nellore. The mean maximum and minimum daily temperature varies from 40.3 to 34.7 °C, and from 20 C to 15.3 °C, according to the temperature data. Rivers in semi-arid environments are characterized by large yearly flow changes and poor water quality. Agriculture occupies much of the basin (approximately 53%), while forest covers the remainder (around 20%). The basin comprises fine to medium-textured clayey soils (Garg et al. 2017).

2.2 Data

Data was collected for this study from Indian Meteorological Department (IMD) and India WRIS (Water Resources Information System). 0.25-degree gridded data of monthly precipitation, 1-degree gridded monthly temperature data, and monthly flow data during the period 1980–2015 were used for the analysis. The flow data are extracted for the following stations as shown in Fig. 1: Alladupalli, Chennur, Kamallapuram, Nandipalli, Pyderu Anicut—Kammapalem, Singavaram, and Tadipatri. The monthly temperature data were regridded to 0.25-degree resolution using Inverse Distance Weighting method. Many researchers (Dixit et al. 2022; Poonia et al. 2021a) successfully used these high-resolution IMD datasets and India WRIS observed streamflow data. The meteorological data (rainfall, temperature) were interpolated to the seven hydrological stations using the bilinear interpolation technique.

2.3 Methodology

The most widely used drought indices, such as the standardized precipitation evapotranspiration index (SPEI) and the streamflow drought index (SDI), were used to define meteorological and hydrological drought. Drought indices used in this study are on a three-month time scale. A drought index with a shorter time scale is more capable of responding to drought events with low intensity and is more discrete. As a result, it can more accurately portray agricultural drought scenarios (if clubbed with the vegetation response) and is also ideal for seasonal drought monitoring (Kulkarni et al. 2020). With the help of obtained drought index value, the drought characteristics values were calculated using Run Theory. The interdependence structure of hydrological and meteorological characteristics is often investigated using multivariate analysis. Drought is a multivariate and complicated phenomenon that necessitates the use of effective tools (e.g., copula) for modelling its dependent characteristics. As a result, to execute bivariate distribution (Mirakbari et al. 2010), a copula-based strategy is used. Several copulas were employed to develop a joint dependence structure across drought characteristics. Then, using the maximum log-likelihood method, the parameters of the copula function were estimated. In addition, maximum log-likelihood (ML), Akaike Information Criteria (AIC), and Bayesian Information Criteria (BIC) values were used to pick the best-fit copula. Finally, bivariate probabilistic approaches, such as exceedance probabilities and joint return periods, were examined based on the optimal copula functions.

2.3.1 Drought indices

Drought events are characterized by an index based on the indicators of the drought. Several meteorological and hydrological parameters, such as precipitation, temperature, evapotranspiration, streamflow, and other water supply indicators, are combined into a single numerical value called an index to provide a complete picture for decision-making. To give planners and policymakers additional decision-making authority, such an index is frequently presented as a numerical value. Using these indices, the government or public and private groups assess and respond to drought (Eslamian et al. 2017). In 1992, the World Meteorological Organization defined the drought index as "an index related to some of the cumulative repercussions of a prolonged and abnormal moisture deficiency." Indices based solely on precipitation ignore the complexities of processes taking place on the land surface and are unable to justify the effects of evapotranspiration on soil moisture directly. Under warmer circumstances or any other changes in regional hydrology, this could be a major disadvantage. The SPEI (Vicente-Serrano et al. 2012) combines the benefits of both the SPI and the PDSI. Hence, the SPEI is an improvement over the SPI because it considers both precipitation and potential evapotranspiration components (Rose and Chithra 2020). The SPEI was often used to calculate the number of months and days in a drought on a monthly and daily scale. The SPEI index is a well-known drought monitoring index that is used all around the world. A detailed explanation of SPEI calculation is mentioned in (Eslamian et al. 2017). To calculate the SPEI-3 timescale, this study employs the SPEI package in the R language.

According to Nalbantis (Nalbantis and Tsakiris 2009), the cumulative streamflow volume $V_{i,k}$ can be obtained based on Eq. (1):

$$V_{i,k} = \sum_{j=1}^{3k} Q_{i,j}, k = 1, 2, 3, 4 \tag{1}$$

where $Q_{i,j}$ is the monthly streamflow volumes for the given time series, i (1, 2, ……….) is the hydrological year, and j (1, 2, ……,12) is the month within that hydrological year. The cumulative streamflow volume is calculated for the ith hydrological year and the kth reference period. In Eq. (1), $k = 1, 2, 3,$ and 4 are for the periods of October-December, January-March, April-June, and July–September, respectively. The Streamflow Drought Index (SDI) is defined based on $V_{i,k}$ for each reference period k of the ith hydrological year as follows:

$$SDI_{i,k} = \frac{V_{i,k} - \overline{V}_k}{s_k} \tag{2}$$

where $\overline{V}_k$ and s_k are the mean and the standard deviation of $V_{i,k}$ respectively. The truncation level in the definition of SDI is set to V_k; however, other values based on rational criteria could be used as well. (Nalbantis and Tsakiris 2009; Mishra et al. 2009), in their papers, have considered five

Table 1 Definition of drought states based on drought index values

State	Description of drought	Criterion	
0	Non-drought	$SPEI \geq 0.0$	$SDI \geq 0.0$
1	Mild	$-1.0 \leq SPEI < 0.0$	$-1.0 \leq SDI < 0.0$
2	Moderate	$-1.5 \leq SPEI < -1.0$	$-1.5 \leq SDI < -1.0$
3	Severe	$-2.0 \leq SPEI < -1.5$	$-2.0 \leq SDI < -1.5$
4	Extreme	$SPEI < -2.0$	$SDI < -2.0$

states (classes) of drought, each designated by an integer value ranging from 0 (non-drought) to 4 (extreme drought) and described by the criteria in Table 1. To calculate the SDI, this study used DrinC software (Tigkas et al. 2015).
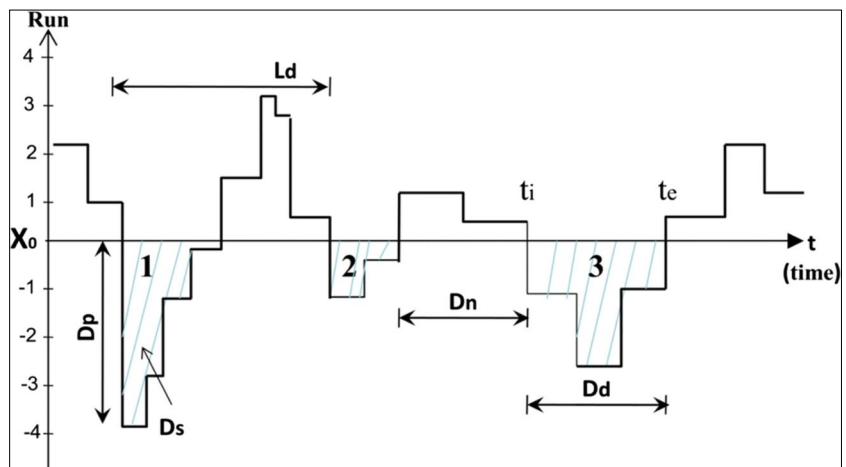
2.3.2 Drought characteristics

Several methods are utilized to determine drought features, including the Run Theory, discrete Markov process, percentile method, and so on. In identifying drought processes, the Theory of Runs is a commonly utilized time series analysis tool (Poonia et al. 2021a). The Theory of Runs was presented by Yevjevich (Yevjevich 1967; Wang et al. 2020) as a framework for defining and studying drought. The term "run" refers to a sequence of the same symbol that meets a set of requirements. The foundation of this idea is to select an appropriate threshold. That is, to determine whether a drought is beginning, continuing, or ending based on the relationship between the value of the drought index and the threshold. Some examples of Run Theory events are rainy days, droughts, continuous rain-free days, and alternating natural waters. In this work, drought characteristics such as drought duration, severity, and peak were extracted using the Theory of Runs from the drought index series (SPEI & SDI).

As shown in Fig. 2, each drought event is characterized by drought duration (D_d), drought severity (D_s) (Mishra et al. 2009), and drought peak (D_p) (Song and Singh 2010a). Drought duration (D_d) is defined by the number of consecutive intervals (months) where values remain below the truncation level X_0, while D_s and D_p are defined by the cumulative total of values throughout a drought period and the minimum value during a drought period, respectively (Qin et al. 2021; Ayantobo et al. 2017). The inter-arrival time (L_d) is defined as the sum of drought and non-drought duration. The three characteristics can be defined as follows:

Fig. 2 Sketch for the definition of drought characteristics showing three drought events, based on Run Theory (Ayantobo et al. 2019)

1. Drought duration (D_d): the duration for which the drought index value is less than zero.

$$D_d = t_e - t_i \tag{3}$$

where t_e = termination time and t_i = initiation time

2. Drought severity (D_s): the absolute value of the accumulated drought index value over the duration of the drought (Chen et al. 2013);

$$D_s = - \sum_{j=1}^{D_d} SPEI_j \tag{4}$$

SPEI will be replaced by other respective drought indices, during its calculation.

3. Drought peak (D_p): the absolute value of the minimum drought index value during the duration of the drought.

2.3.3 Marginal distribution

To investigate the joint distribution of the three variables, the marginal distribution of each drought characteristic should first be obtained using time-series data to establish a binary probability distribution for drought duration, severity, and peak. Different type of distributions was considered for the present analysis, and the best one was selected. The probability distribution functions considered in this study are Normal, Lognormal, Weibull, Exponential, Gamma, Generalized Gamma, Log-gamma, Generalized Extreme Value, and Generalized Pareto distributions. The maximum-likelihood estimation (MLE) method was employed to estimate the distribution parameters for each station. Kolmogorov–Smirnov (K-S) test, chi-squared test, and Anderson–Darling (A-D) test were used for selecting the optimal marginal distribution, with the notion that the threshold value should be as minimal as possible to preserve the largest sample. Eventually, the minimum statistics of Goodness of fit tests are subjected to assess the suitable marginal distribution. The Spearman (ρ) and Kendall (τ) correlation coefficient values were estimated to understand the correlation among different drought characteristics. The closer the correlation coefficients to 1, the stronger the correlation. The copula function can be used to simulate the joint probability distribution between the drought characteristics because of the positive correlation between the drought characteristics and the good fitting effect of each characteristic through different distribution functions.

2.3.4 Copula-based modelling

It is vital to remember that drought characteristics are intertwined; thus, a univariate analysis will only be able to provide a partial picture of the drought situation. Sklar presented copulas to connect univariate distribution functions to multivariate distribution functions. Copulas are functions that join marginal CDFs to generate a multivariate CDF, according to their definition. By isolating the impacts of dependence from the effects of margins, this function makes it easier to characterize the dependence properties. Let $F_{1, 2, ..., n}(x_1, x_2, ..., x_n)$ be the joint CDF of n associated arbitrary variables of $X_1, X_2, ..., X_n$ with the corresponding marginal CDF $F_1(x_1), F_2(x_2), ..., F_n(x_n)$. The n-dimensional CDF along with the univariate distributions of $F_1(x_1), F_2(x_2), ..., F_n(x_n)$ can be written as shown below in Eq. (5) (Nelsen 2007; Shiau 2006; Vazifehkhah et al. 2019) (Genest and Favre 2007; Tootoonchi et al. 2021):

$$H(x_1, x_2, ...x_n) = C\{F_1(x_1), F_2(x_2), ...F_n(x_n)\} = C(u_1, u_2, ...u_n) \tag{5}$$

where C is a d-dimensional copula; $F_k(x_k) = u_k$ for $k = 1,..., n$. The copula function is in the form: [0, 1] $d \rightarrow$ [0, 1], where any pair in the d-dimensional square of the univariate distributions corresponds to the copula function in [0, 1] of joint CDF.

Archimedean copulas and Elliptical copulas are the two most commonly utilized copula classes (Huang et al. 2014; Qin et al. 2021; Ayantobo et al. 2019; Wang et al. 2020; Poonia et al. 2021a) (Tsakiris et al. 2016) (Song and Singh 2010b). In this study, nine copula functions, such as Clayton, Frank, Gumbel, Joe, Student t copula (t-copula), Gaussian, Survival Clayton, Survival Joe, and Survival Gumbel, are considered for the analysis of drought.

Table 2 presents these copula functions, where θ, r, and ϑ represent the parameters of the respective copula functions, and u and v are the variables. In Eq. (6), Ĉ denotes the survival copula function and C represents the respective copula function. The best-fit copula can be selected based on either of K-S test, root mean square error (RMSE) value, ML value, Nash-Sutcliff efficiency (NSE) value, AIC, or BIC values. The parameters of the copula can be estimated using either inversion of scale-free measure of association, MLE, inference from margins (IFM), and canonical maximum likelihood (CFM) methods. In this study, to calculate the copula parameter(s), the IFM approach was applied and for selecting the best copula function ML, AIC, and BIC values were used. The 'copula' package in R programming was used to estimate bivariate joint distributions (Yan 2007; Azam et al. 2018; Hofert et al. 2014).

$$\widehat{C} = u + v - 1 + C(1 - u, 1 - v) \tag{6}$$

2.3.5 Estimation of parameters of copula

In this study, the inference from margins (IFM) approach has been employed to estimate the copula parameters.

Table 2 List of copulas used in this study and their parameter ranges

Copulas	Bivariate copula $C_\theta(u, v)$	Parameters θ
Archimedean copulas		
Clayton	$(u^{-\theta} + v^{-\theta} - 1)^{-\frac{1}{\theta}}$	$\theta\epsilon[-1, \infty)$
Frank	$-\frac{1}{\theta}\log\left[1 + \frac{(e^{-\theta u}-1)(e^{-\theta v}-1)}{(e^{-\theta}-1)}\right]$	$\theta\epsilon[-\infty, \infty)$
Gumbel	$\exp\left[-\left(u^{-\theta} + v^{-\theta}\right)^{\frac{1}{\theta}}\right]$	$\theta\epsilon[1, \infty)$
Joe	$1 - \left[(1-u)^\theta + (1-v)^\theta - (1-u)^\theta(1-v)^\theta\right]^{1/\theta}$	$\theta\epsilon[1, \infty)$
Elliptical or meta-elliptical copulas		
Student's t (or t)	$\int_{-\infty}^{t_\theta^{-1}(u)} \int_{-\infty}^{t_\theta^{-1}(v)} \frac{1}{2\pi\sqrt{(1-r^2)}}\left\{1 + \frac{x^2-2xy+y^2}{\theta(1-r^2)}\right\}^{-\frac{\theta+2}{2}} dxdy$	$\vartheta > 2, r\epsilon(0, 1]$
	$t_\vartheta(x) = \int_\infty^x \frac{\Gamma\left(\frac{\theta+1}{2}\right)}{\sqrt{\pi\theta}\Gamma\left(\frac{\theta}{2}\right)}(1 + \frac{y^2}{\theta})^{-\frac{\theta+1}{2}} dy, \vartheta \neq 0$	
Gaussian	$,\int_{-\infty}^{\varphi^{-1}(u)} \int_{-\infty}^{\varphi^{-1}(v)} \frac{1}{2\pi\sqrt{(1-\theta^2)}}\exp\left(\frac{2\theta xy-x^2-y^2}{2(1-\theta^2)}\right)dxdy$	$\theta\epsilon[-1, 1]$

In IFM, the log-likelihood function disintegrates the maximum log-likelihood into two segments: one from marginals (L_M) and the other from copula dependence (L_C), where L, C, and M denote the log-likelihood, copula dependence, and marginals, respectively. The parameters for each marginal distribution function are computed individually in the first stage of IFM. In the second step, using the computed value of the marginal distribution parameter, L_C is maximized to get an estimate for the copula parameter (θ) (Maity 2018). Hence, in this method, Eq. (7) was used for getting the estimates of parameters.

$$\left(\frac{\partial L_{M1}}{\partial\alpha_1}, \frac{\partial L_{M2}}{\partial\alpha_2}, \frac{\partial L_C}{\partial\theta}\right) = 0 \tag{7}$$

2.3.6 Selection of appropriate copula function

It is critical to select the appropriate copula family when developing the joint probability distribution of drought variables. The performance of different copula functions was evaluated using the ML value, AIC, and BIC values in this study. The residuals between model simulations and observations are reduced when a parameter set gives the maximum likelihood. As a result, it provides the best match to the observed data in this sense. Higher model complexity results in better model flexibility, which in turn leads to the best fit to the actual data (Menna et al. 2022; Fang et al. 2014), but can lead to overfitting. AIC is a cost-effective measure of statistical model quality that penalizes overfitting (Pontes Filho et al. 2020). The Kullback–Leibler distance is the basis of the AIC, while

the integrated neighborhood is the basis of the BIC in Bayesian theory (Achite et al. 2022). The smaller the value of AIC or BIC, the higher the fitting efficiency. AIC and BIC are calculated (Mesbahzadeh et al. 2020) using the following method:

$$AIC = -2\ln(MLE) + 2k \tag{8}$$

$$BIC = -2n\ln(MLE) + k\ln(MLE) \tag{9}$$

where k is the number of fitting parameters, MLE is the maximum likelihood of the copula function and n is the number of observations.

2.3.7 Probabilistic analysis

The drought risk probability can be defined as the joint probability of D_d, D_s, and D_p (Zhang et al. 2020). Drought properties have a copula-based joint dependence structure that can be used to obtain some significant information required for the management and mitigation of drought. For instance, it is deemed a crucial situation for a water-supply system if both drought characteristics (severity and duration) surpass specific thresholds at the same time. In this study, the exceedance probability is computed using Eq. (10), which is defined as the probability when all the considered drought variables exceed a specific threshold. Analysis of drought characteristics separately will not determine the exceedance probability; however, copulas can provide this information quickly (Poonia et al. 2021a).

$$P(D \geq d, S \geq s) = 1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s)) \tag{10}$$
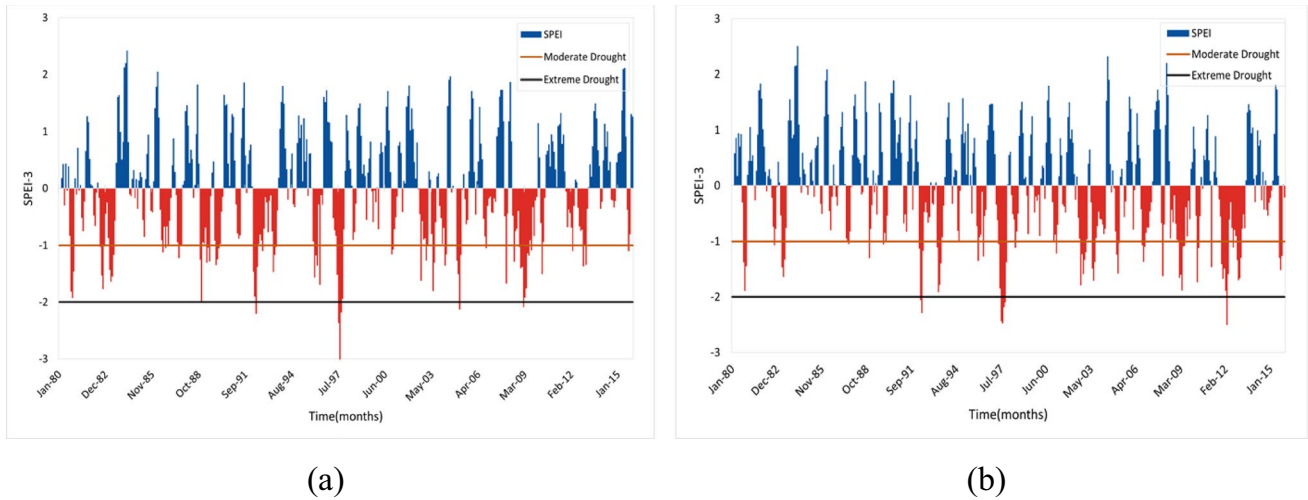
(a)

(b)

Fig. 3 SPEI-3 time series at **a** Alladupalli station, **b** Tadipatri station

2.3.8 Return period analysis

In drought assessment and water management, analysis of the return periods of drought characteristics renders a vital role. The univariate return period of drought duration (Shiau and Shen 2001) is calculated by Eq. (11):

$$T_D = \frac{E(L_d)}{1 - F_D(d)} \tag{11}$$

where $E(L_d)$ is the expected drought interarrival time, and $F_D(d)$ is the marginal CDF of drought duration. Similarly, in the case of drought severity and drought peak, $F_D(d)$ will be replaced by $F_S(s)$ and $F_P(p)$ for the expected return period of severity and peak, respectively.

The bivariate return period is calculated in this study using two criteria: (i) AND return period when both the drought variables exceed a particular value, and (ii) OR return period when either of the two drought variables exceeds a specific value. Both are expressed below:

$$T_{DS} = \frac{E(L)}{P(D \geq d, S \geq s)} = \frac{E(L)}{1 - F_D(d) - F_S(s) + C(F_D(d), F_S(s))} \tag{12}$$

$$T'_{DS} = \frac{E(L)}{P(D \geq d, or S \geq s)} = \frac{E(L)}{1 - C(F_D(d), F_S(s))} \tag{13}$$

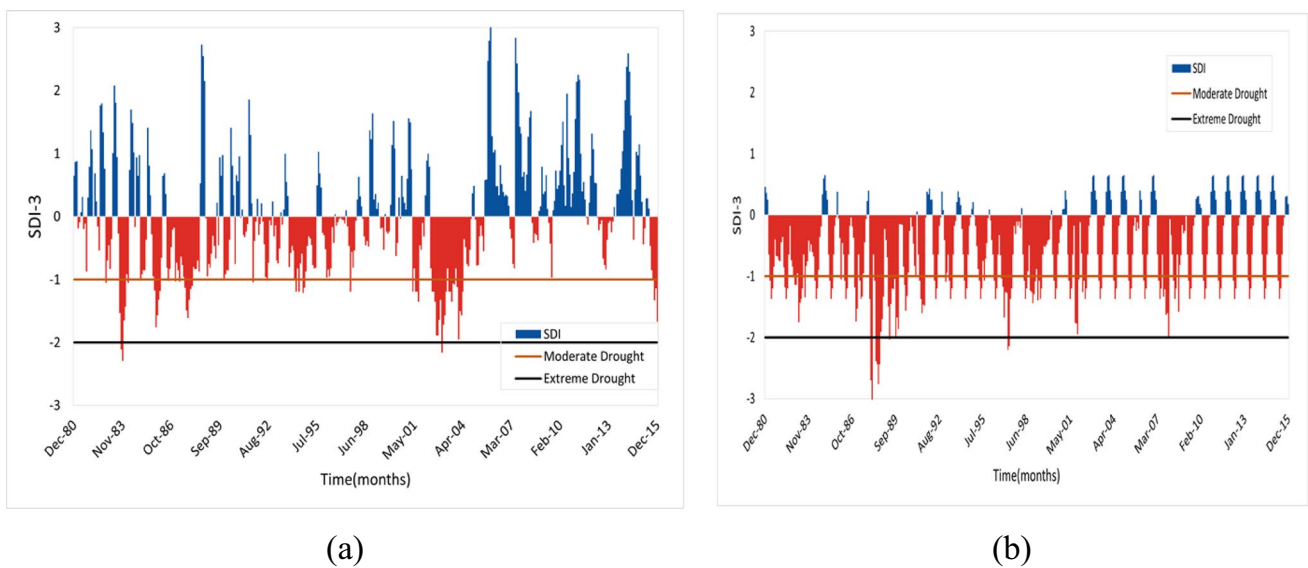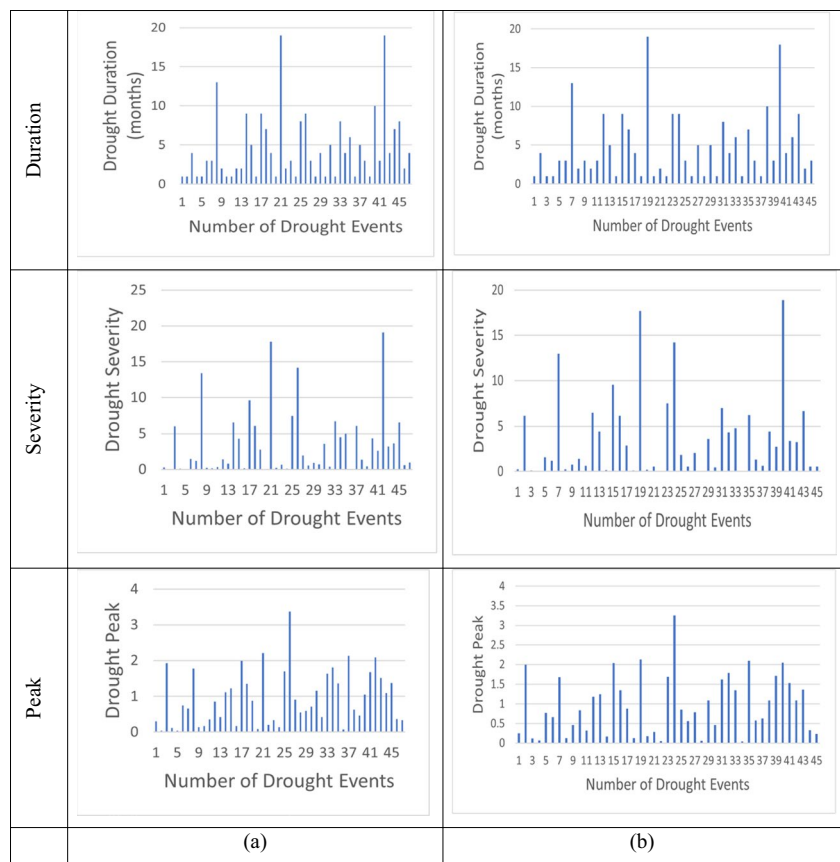where T_{DS} and T_{DS}' denote "AND" and "OR" joint return period respectively.

(a)

(b)

Fig. 4 SDI-3 time series at **a** Alladupalli station, **b** Tadipatri station

Fig. 5 Showing drought characteristics at (**a**) Alladupalli and (**b**) Chennur stations for the period of 1980–2015

(a) (b)

3 Results and discussion

3.1 Drought index

This research estimates SPEI and SDI on a 3-month time scale to replicate the drought characteristics for frequent droughts. As drought events occur more frequently, the drought index with a shorter time scale is better suited to adapt even to smaller drought occurrences. The SPEI of various time scales reflects the cumulative effect of drought. Equation (2) was used to obtain the SDI-3 value for each year. SPEI-3 and SDI-3 series were derived for the period of 1980 to 2015 at each of the seven hydrological stations, allowing the drought events to be recognized easily. SPEI-3 and SDI-3 series at Alladupalli and Tadipatri stations are presented in Figs. 3 and 4, respectively, in which positive values are shown in blue to represent non-drought periods and negative values are shown in red to indicate drought conditions. From the figures, it can be seen that the drought index has reached moderate and extreme drought levels in the range of 69 to 70 times and 6 to 9 times in case of meteorological drought. Whereas, in the case of hydrological drought, moderate drought has exceeded 9 to 180 times, and extreme drought has exceeded about 13 times. The major drought events observed in this study are consistent

with the findings of (Kumar et al. 2018), which took into account the GRACE data (available from 2002), and the drought event in 2003–2005 may therefore be compared to this study. Throughout the research period, the Pennar River basin showed a pattern of wetting and drying consecutively. Drought occurrence frequencies and seasonal fluctuations can be estimated at each level to comprehend the pattern of droughts better in the Pennar River basin.

3.2 Identification of drought characteristics

Before proceeding to the probabilistic analysis, the drought characteristics values (i.e., drought duration, severity, and peak) were computed using Run Theory from observed meteorological and hydrological droughts defined by SPEI-3 and SDI-3, respectively. These three drought characteristic values for each station were obtained using MS Excel and Python programming based on the drought index values. This paper seeks to examine the degree of drought risk based on the computed SPEI-3 and SDI-3; thus, the Run Theory threshold was set to zero. The drought duration, severity, and peak using the SPEI-3 at the Alladupalli and Chennur stations are shown in Fig.5 a and b, respectively. The average drought duration and severity for the study area coincide with the values in Poonia et al. (2021a) and Kumar et al.
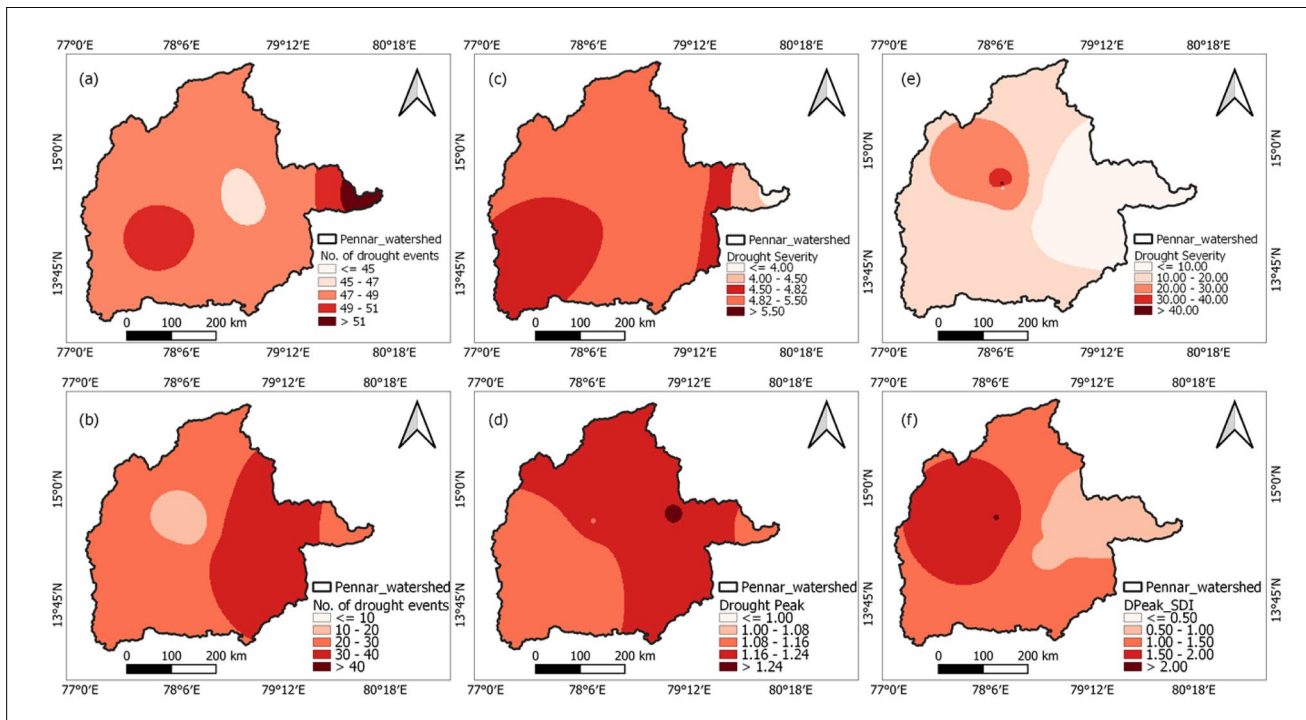
Fig. 6 Spatial plots of **a** the total number of meteorological drought events, **b** the total number of hydrological drought events, **c** Drought severity using SPEI, **d** Drought peak using SPEI, **e** Drought severity using SDI, and **f** Drought peak using SDI

(2018). Sajeev et al. (2021) studied the bivariate drought characteristics of two contrasting climatic regions of India (Rajasthan and Kerala), and the number of drought events for this study lies between the values obtained for those two regions in their study.

Figure 6 presents the spatial variation of the total number of drought events and various drought characteristics over the study area. There were 48 meteorological and 28 hydrological drought events on average during the period considered in this study (Fig. 6a and b). But just the spatial plots of the total number of drought events will not give proper judgement to the severity of the drought events, i.e., even if there is a smaller number of drought events, there may be a chance of having drought events with more severity and

larger peak. The drought events having a duration greater than 3 months, severity greater than 0.5, and peak greater than 0.25 have been considered for understanding the effects of drought severity over the basin, as droughts with values less than contemplated will not have much impact on the analysis. Results showed that the drought duration is almost equal in all stations. A spatial plot of drought severity (Fig. 6c and e) and peak (Fig. 6d and f) for both the drought types shows that Alladupalli station is having the most severe ($D_s = 5.04$) meteorological drought event, even if the total number of drought events are not the highest there. This result is in line with Sinha et al. (2018). Similarly, from the spatial plot, it can be inferred that Kamallapuram and Singavaram station, experienced more severe hydrological
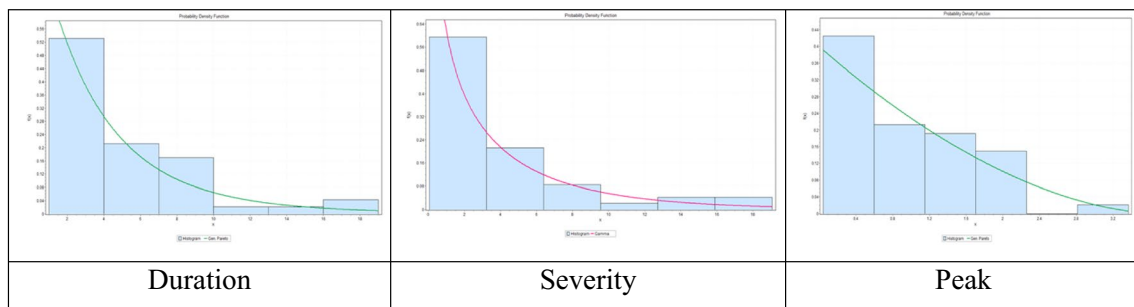

Fig. 7 PDF of best-fit distribution for various drought characteristics based on SPEI-3 at the Alladupalli station
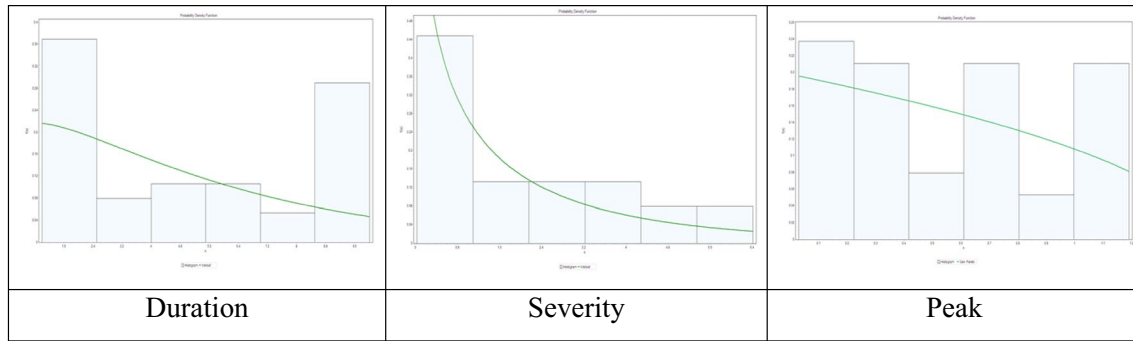
| Duration | Severity | Peak |

Fig. 8 PDF of best-fit distribution for various drought characteristics based on SDI-3 at the Nandipalli station

drought with larger peak, even though its total number of drought events is less than that of other stations.

According to preliminary research, analyzing drought duration, severity, and peak separately provides a finite assessment of drought characteristics; thus, it is preferable to use a multivariate approach and build a joint dependence structure to portray the interconnection between the drought characteristics, which is discussed in the next section.

3.3 Marginal distribution

The marginal distribution of each drought variable should be found first before exploring the joint distribution of the three variables. Different types of distributions were investigated in this study, and the best one was selected. The curve for the best-fit probability distribution function at Alladupalli station for the drought characteristics based on SPEI-3 is

Table 3 Best fit marginal distribution and its parameters for meteorological drought analysis

Region	Meteorological drought			Hydrological drought		
	Duration	Severity	Peak	Duration	Severity	Peak
Alladupalli	Gen. Pareto	Gamma	Gen. Pareto	Log − normal	Weibull	GEV
	$k=0.103$, $\sigma=3.666$, $\mu=0.443$	$\alpha=0.600$, $\beta=5.997$	$k=0.360$, $\sigma=1.353$, $\mu=-0.053$	$\sigma=0.920$, $\mu=1.314$	$\alpha=0.668$, $\beta=2.525$	$k=0.003$, $\sigma=0.444$, $\mu=0.494$
Chennur	Gen. Pareto	Weibull	Gen. Pareto	Gen. Pareto	Log − normal	Gen. Pareto
	$k=0.041$, $\sigma=4.394$, $\mu=0.441$	$\alpha=0.626$, $\beta=3.086$, $\gamma=0.04$	$k=-0.449$, $\sigma=1.508$, $\mu=-0.080$	$k=0.155$, $\sigma=4.729$, $\mu=0.292$	$\alpha=1.581$, $\beta=0.421$	$k=-0.549$, $\sigma=1.169$, $\mu=0.026$
Kamallapuram	GEV	Gamma	Gen. Pareto	GEV	GEV	Log − gamma
	$k=0.257$, $\sigma=2.216$, $\mu=2.633$	$\alpha=0.640$, $\beta=5.629$	$k=-0.398$, $\sigma=1.407$, $\mu=-0.057$	$k=0.851$, $\sigma=1.563$, $\mu=8.433$	$k=0.860$, $\sigma=1.485$, $\mu=7.708$	$\alpha=2.622$, $\beta=0.160$
Nandipalli	Gen. Pareto	Weibull	Gen. Pareto	Weibull	Weibull	Gen. Pareto
	$k=0.134$, $\sigma=4.861$, $\mu=0.243$	$\alpha=0.645$, $\beta=2.573$	$k=0.418$, $\sigma=1.458$, $\mu=-0.104$	$\alpha=1.139$, $\beta=5.417$	$\alpha=0.753$, $\beta=1.859$	$k=-0.659$, $\sigma=0.970$, $\mu=-0.025$
Pyderu Anicut	GEV	Log − normal	Gen. Pareto	Log − normal	Gen. Pareto	GEV
	$k=0.418$, $\sigma=1.458$, $\mu=-0.104$	$\sigma=1.301$, $\mu=0.485$	$k=-0.540$, $\sigma=1.470$, $\mu=0.086$	$\sigma=1.049$, $\mu=1.806$	$k=0.298$, $\sigma=4.521$, $\mu=-0.293$	$k=0.065$, $\sigma=0.402$, $\mu=0.569$
Singavaram	GEV	Gen. Pareto	Gen. Gamma	GEV	GEV	Normal
	$k=0.285$, $\sigma=2.162$, $\mu=2.502$	$k=0.216$, $\sigma=3.084$, $\mu=-0.409$	$k=0.908$, $\alpha=1.476$, $\beta=0.584$	$k=0.417$, $\sigma=16.688$, $\mu=13.339$	$k=0.320$, $\sigma=20.139$, $\mu=14.678$	$\sigma=0.907$, $\mu=1.733$
Tadipatri	Weibull	Log − Normal	Gen. Pareto	Gen. Pareto	Gen. Pareto	Log − gamma
	$\alpha=1.268$, $\beta=4.151$	$\sigma=1.510$, $\mu=0.225$	$k=-0.381$, $\sigma=1.284$, $\mu=-0.014$	$k=0.376$, $\sigma=5.489$, $\mu=5.422$	$k=0.462$, $\sigma=4.598$, $\mu=3.795$	$\alpha=3.990$, $\beta=0.108$

Table 4 Values of Kendall and Spearman's correlation coefficients for meteorological drought variables at different stations

Region	Kendall Tau			Spearman Rho		
	DD-DS	DD-DP	DS-DP	DD-DS	DD-DP	DS-DP
Alladupalli	0.821	0.668	0.85	0.938	0.84	0.958
Chennur	0.841	0.698	0.856	0.95	0.856	0.956
Kamallapuram	0.833	0.699	0.865	0.945	0.858	0.967
Nandipalli	0.818	0.698	0.874	0.934	0.859	0.965
Pyderu Anicut	0.827	0.634	0.782	0.938	0.786	0.929
Singavaram	0.838	0.709	0.865	0.945	0.865	0.967
Tadipatri	0.777	0.656	0.87	0.908	0.809	0.968

shown in Fig. 7. Even though streamflow changes could be the result of complicated interactions between climate change and land use land cover change, the variability in hydrological drought characteristics in the Pennar basin is more probably due to the change in meteorological factors such as precipitation and temperature in this study.

The curve for the best-fit probability distribution function at Nandipalli station for the drought characteristics based on SDI-3 is shown in Fig. 8. The list of the obtained best-fit marginal distributions and the estimated parameter values of the best-fit distributions for the meteorological and hydrological drought analysis from the drought characteristics are provided in Table 3.

3.4 Joint distribution of drought characteristics

In order to assess the correlation between drought characteristics, correlation analysis was performed in this study. Tables 4 and 5 show the results of correlation analysis among the three characteristics of meteorological and hydrological drought, respectively. The correlation coefficients between the three sets of variables at each of the seven stations are all near 1, which indicates that the variables in each group are strongly correlated.

Different copula functions have been used in the current work to build the joint distribution of the drought characteristics based on the drought index values. The ML, AIC, and BIC values were used to evaluate the best-fit copula function. For every station, the parameters and the ML, AIC & BIC values for the best-fit copula function for the meteorological and hydrological drought characteristics are tabulated in Tables 6 and 7, respectively. For the overall region of the study area, for meteorological drought, Frank Copula is the best-fit copula function for the joint risk of drought duration and severity, whereas Survival Clayton copula is the best-fit copula for the combination of both drought duration and peak as well as drought severity and peak (Table 6).

The bivariate probability distribution based on SPEI-3 and SDI-3 [including $P(D \leq d, S \leq s)$, $P(D \leq d, P \leq p)$, and $P(S \leq s, P \leq p)$], is obtained for each station using the best-fit copula function of the drought characteristics chosen. The resulting plot of joint CDF at Alladupalli and Chennur stations for each bivariate analysis (duration vs. severity, duration vs. peak & severity vs. peak) of meteorological and hydrological drought is presented in Figs. 9 and 10, respectively.

From Fig. 5a, the 50th percentile of drought duration, severity, and peak can be seen as 3 months, 1.415, and 0.749, respectively for meteorological drought at Alladupalli station. For example, at the 50th percentile of drought duration, drought severity, and drought peak, at Alladupalli station for meteorological drought, it can be read from Fig. 9a, that the joint probability $P(D \leq 3, S \leq 1.415)$ is 0.55, and $P(D \leq 3, P \leq 0.749)$ is 0.88, and $P(S \leq 1.415, P \leq 0.749)$ value is 0.832. Figure 9 shows that the drought occurrences at Alladupalli and Chennur stations were generally short-term high-severity and long-term moderate-severity events. As the duration and severity of a drought event increase, the value of joint cumulative probability also increases.

Table 5 Values of Kendall and Spearman's correlation coefficients for hydrological drought variables at different stations

Region	Kendall Tau			Spearman Rho		
	DD-DS	DD-DP	DS-DP	DD-DS	DD-DP	DS-DP
Alladupalli	0.793	0.592	0.797	0.911	0.757	0.932
Chennur	0.852	0.658	0.793	0.952	0.826	0.935
Kamallapuram	0.889	0.379	0.561	0.924	0.420	0.611
Nandipalli	0.779	0.578	0.796	0.915	0.739	0.924
Pyderu Anicut	0.774	0.456	0.676	0.922	0.642	0.844
Singavaram	0.926	0.821	0.862	0.982	0.915	0.944
Tadipatri	0.909	0.653	0.741	0.956	0.767	0.849

Table 6 Best fit copula, parameters, and ML, AIC & BIC values for the respective copula function at each station for meteorological drought analysis

Name of the Station	Characteristics	Copula Function	Estimated parameter	ML	AIC	BIC
Alladupalli	DD vs DS	Frank	19.412	148.52	− 295.04	− 292.44
	DD vs DP	T	0.878	108.45	− 212.90	− 207.69
	DS vs DP	Survival Clayton	1.191	112.50	− 222.99	− 220.39
Chennur	DD vs DS	Frank	− 3.341	22.94	− 43.87	− 41.27
	DD vs DP	Survival Clayton	2.629	124.99	− 247.97	− 245.37
	DS vs DP	Frank	− 1.717	6.68	− 11.35	− 8.75
Kamallapuram	DD vs DS	Frank	19.886	146.59	− 291.18	− 288.58
	DD vs DP	T	0.846	92.15	− 180.30	− 175.09
	DS vs DP	Survival Clayton	1.164	99.77	− 197.53	− 194.93
Nandipalli	DD vs DS	Frank	− 3.091	20.82	− 39.64	− 37.04
	DD vs DP	T	0.935	148.33	− 292.65	− 287.44
	DS vs DP	Frank	− 1.866	8.22	− 14.45	− 11.84
Pyderu Anicut	DD vs DS	T	0.977	202.71	− 401.43	− 396.22
	DD vs DP	Survival Clayton	1.033	45.73	− 89.47	− 86.86
	DS vs DP	Survival Clayton	0.733	37.21	− 72.42	− 69.82
Singavaram	DD vs DS	T	0.989	219.06	− 434.13	− 428.92
	DD vs DP	Survival Gumbel	10.826	193.04	− 384.07	− 381.47
	DS vs DP	Gumbel	5.198	187.62	− 373.24	− 370.64
Tadipatri	DD vs DS	Frank	− 4.720	41.67	− 81.33	− 78.73
	DD vs DP	Frank	− 0.719	1.22	− 0.44	2.17
	DS vs DP	Survival Clayton	1.065	48.90	− 95.81	− 93.20

Table 7 Best fit copula, parameters, and ML, AIC & BIC values for the respective copula function at each station for hydrological drought analysis

| Name of the station	Characteristics	Copula function	Estimated parameter	ML	AIC	BIC
Alladupalli	DD vs DS	T	− 0.911	94.56	− 185.12	− 179.91
	DD vs DP	Survival Clayton	1.027	35.13	− 68.25	− 65.65
	DS vs DP	Frank	− 1.617	6.41	− 10.81	− 8.20
Chennur	DD vs DS	Survival Clayton	11.538	279.61	− 557.21	− 554.61
	DD vs DP	Survival Clayton	0.943	38.81	− 75.62	− 73.01
	DS vs DP	Survival Clayton	0.796	31.94	− 61.88	− 59.28
Kamallapuram	DD vs DS	Survival Joe	2.154	23.16	− 44.31	− 41.70
	DD vs DP	Survival Joe	3.900	37.47	− 72.94	− 70.34
	DS vs DP	Gaussian	0.185	69.20	− 136.41	− 133.80
Nandipalli	DD vs DS	Frank	22.503	119.06	− 236.12	− 233.51
	DD vs DP	Clayton	1.888	33.72	− 65.44	− 62.83
	DS vs DP	Clayton	2.035	49.81	− 97.62	− 95.01
Pyderu Anicut	DD vs DS	Gaussian	0.955	138.93	− 275.85	− 273.25
	DD vs DP	Survival Clayton	3.121	105.43	− 208.86	− 206.25
	DS vs DP	Survival Clayton	2.206	97.36	− 192.72	− 190.11
Singavaram	DD vs DS	Gaussian	0.361	1.03	− 0.05	2.55
	DD vs DP	Survival Clayton	1.443	11.92	− 21.84	− 19.24
	DS vs DP	Gaussian	0.364	6.89	− 11.77	− 9.17
Tadipatri	DD vs DS	Survival Joe	22.484	187.51	− 373.01	− 370.41
	DD vs DP	Gaussian	0.631	88.27	− 174.54	− 171.93
	DS vs DP	Gaussian	0.608	105.61	− 209.22	− 206.61
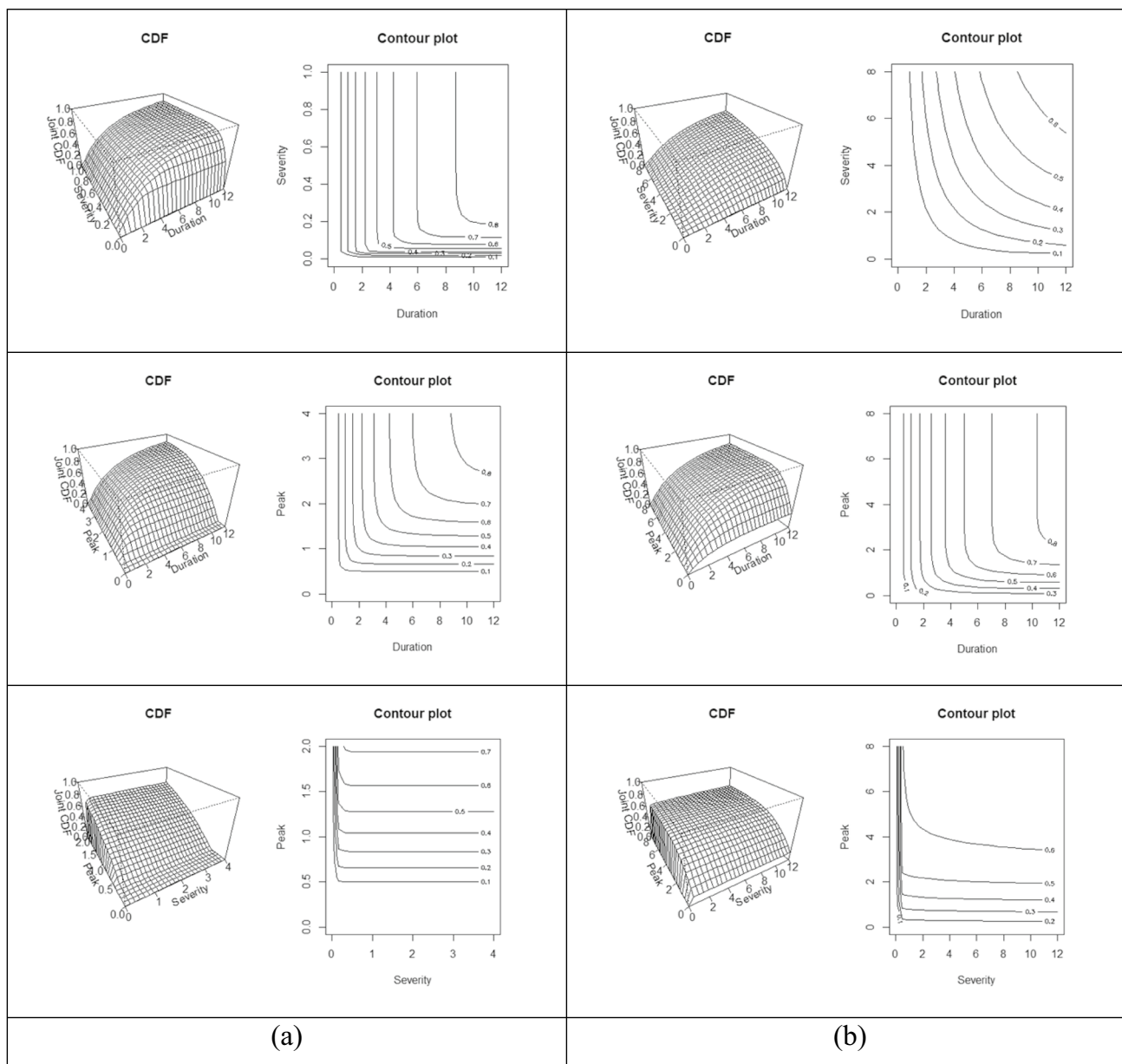
Fig. 9 Showing joint cdf of DD vs DS, DD vs DP and DS vs DP at (**a**) Alladupalli and (**b**) Chennur stations for meteorological drought

The joint exceedance probability values at the 25th percentile of drought duration vs severity as obtained in this study align with the results of Poonia et al. (2021a).

Equivalent results can be seen for different combinations, such as drought duration and peak, as well as drought severity and peak. In addition, the results of joint cdf for other stations are similar to the Alladupalli and Chennur stations. The results indicate that a significant percentage of the study area is vulnerable to meteorological drought conditions. Lack of precipitation poses a major risk to the basin, as there is a high probability of meteorological drought. This might be a response to the significant upward trend of mean temperature (Pörtner et al. 2022) and

the significant decreasing trend in seasonal and annual rainfall. Kumar et al. (2013) also hypothesize a general rise in moderate meteorological droughts during the past few decades in India. These findings will aid in quantifying the frequency of various levels of drought events.

3.5 Joint return period analysis of drought characteristics

Drought assessment requires a thorough understanding of the return period. In order to compute the joint return period of drought events, the expected value
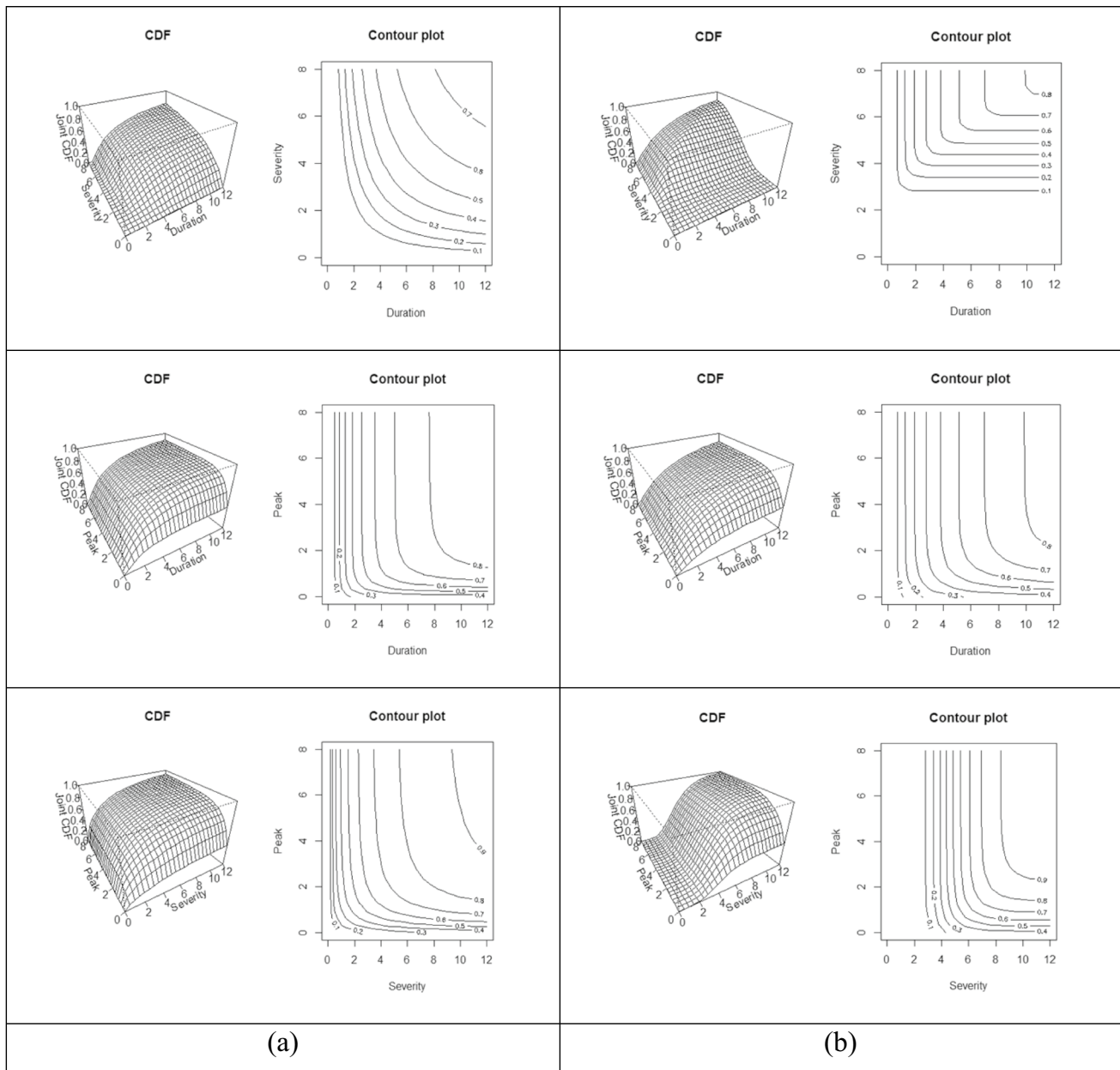
Fig. 10 Showing joint cdf of DD vs DS, DD vs DP and DS vs DP at (**a**) Alladupalli and (**b**) Chennur stations for hydrological drought

of the drought interarrival time must be calculated first. The drought interarrival time L_d for the stations Alladupalli and Chennur, obtained from the analysis of drought characteristics using the Run Theory, is 13.9 and 13.03 months, corresponding to 1.1 and 1.08 years, respectively, which are almost equal to 1 year. The expected interarrival time for other stations is also remarkably close to 1 year. The return period is calculated using the 'AND' and 'OR' criteria in this section. The joint return periods were computed using Eqs. (12) and (13) at the 25th and 50th percentile of each drought variable at every station for both meteorological and

hydrological droughts. Figures 11 and 12 show the joint return period for the "AND" and "OR" cases of meteorological and hydrological droughts for the 25th and 50th percentiles of the drought events, respectively. For the overall region of the study area, an average joint return period of 1.63 years and 9.3 years is obtained for the AND case of the 25th and 50th percentile, whereas 1.06 years and 1.23 years is obtained for the OR case of the 25th and 50th percentile for the meteorological drought. Similarly, for hydrological drought, an average joint return period of 1.66 years and 8.89 years is obtained for the AND case of the 25th and 50th percentile, whereas 1.16 years
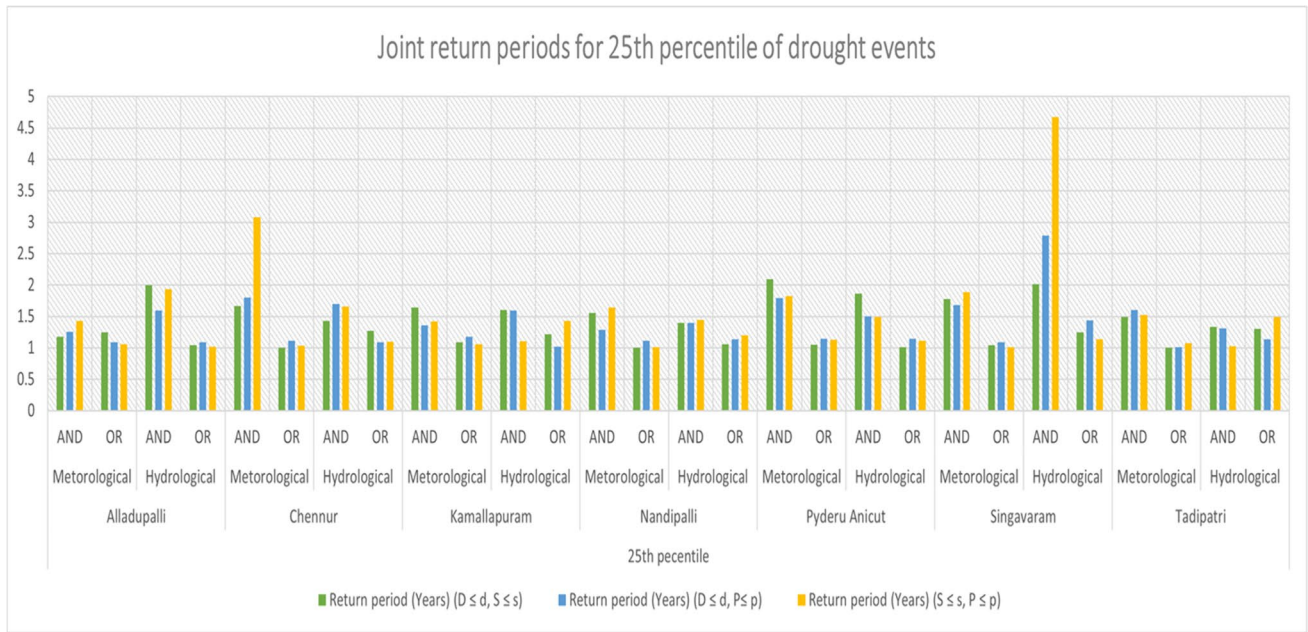
Fig. 11 Joint return periods for 25th percentile of drought events

and 1.34 years are obtained for the OR case of the 25th and 50th percentile. This shows that the drought events in the study area have a shorter return period. Amrit et al. (2018) found that droughts occur once every 5–6 years in a major section of Southern India, indicating a shorter return period. The joint return period values at the 25th and 50th percentile of drought duration vs severity for both the AND and OR case in this work and the fact that a shorter return period in the drought events prevailing in Southern India is also validated by Poonia et al. (2021a). In previous studies (Brunner et al. 2016; Vandenberghe et al. 2012), working with joint return periods (which is

not as straightforward as in the case of univariate analysis) one must choose whether to work with joint (OR / AND) or conditional probabilities, presents an extra challenge in the multivariate case. Hence, in our study, both the AND and OR cases are employed.

The joint return period of the "AND" case is greater than the "OR" case because the computation of the return period of the "AND" event is more restrained than that of the "OR" event. Since the best-fit marginal distribution function of the drought variables and best-fit copula function are determined in the preceding section, and the parameters corresponding to that have been computed, the return period which
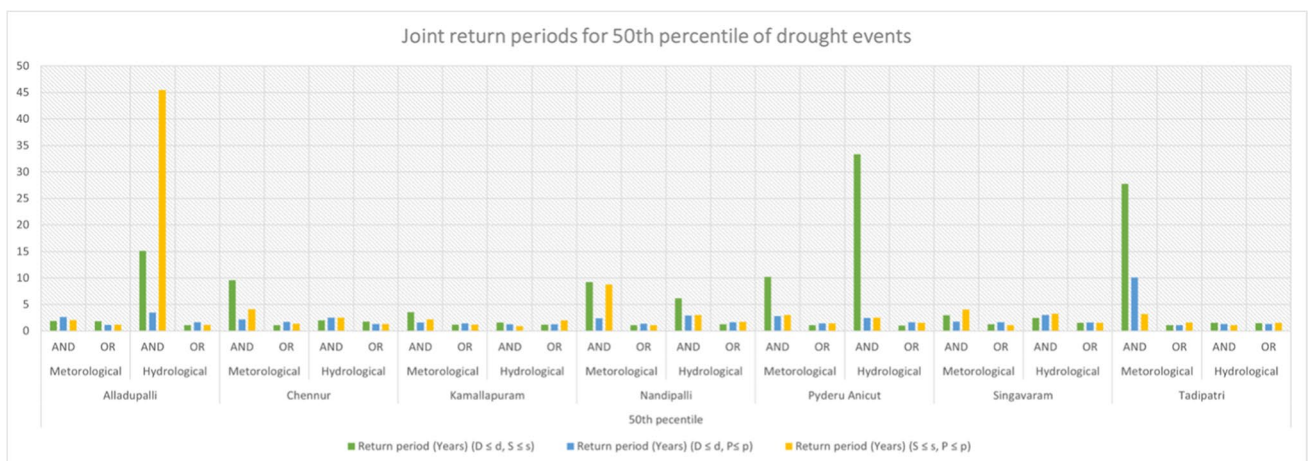

Fig. 12 Joint return periods for 50th percentile of drought events

corresponds to any particular drought duration, severity, or peak can also be calculated according to the requirement.

The return period notion employed in this study is well-known among water resources professionals. As a result of our findings, it is appropriate to raise concerns for drought control, particularly in locations with extremely high joint exceedance probability and low joint return period. If the hydraulic structures are designed based on the results of univariate frequency analysis, then they may under/overestimate drought characteristics, resulting in the failure of the structure or an increase in the structure's cost. These findings could be helpful in assessing the risk of water resource operations in severe and extreme drought scenarios. In short, the findings of this work can help in providing useful information to predict drought risk, optimize water resource allocation, and lessen the effects of drought on the study area in the future.

4 Summary and conclusion

Drought assessment is essential for water resource management and planning. Drought characteristics such as drought duration, severity, and peak were abstracted for the study area from 1980 to 2015 based on the Run Theory. To summarize, this research uses Copulas to conduct a bivariate frequency analysis of meteorological and hydrological droughts at seven stations in the study area. Compared to prior definitions of bivariate frequency analysis, this method is flexible, thorough, and offers several other benefits. Drought analysis based on copulas considers comprehensive interdependencies between drought characteristics throughout a geographic region. Meteorological and Hydrological drought characteristics in the Pennar River basin were studied from two perspectives: joint cumulative probability and joint return period. The notable outcomes of this study are summarized as follows:

1) In the case of meteorological drought, GEV, Gamma, and Generalized Pareto distributions were found to be the best marginal distributions for drought duration, severity, and peak, respectively in the study area.

2) In the case of hydrological drought, GEV was found to be the best marginal distribution for drought duration, and Generalized Pareto distribution was found to be the best marginal distribution for drought severity and peak in the study area. The selection of a particular marginal distribution for the whole study area is very difficult and hence the analysis has been done for the stations.

3) From the spatial plots, it can be comprehended that Alladupalli station is having the most severe meteorological drought and the whole study area is susceptible to severe meteorological drought. Kamallapuram and Singavaram station have experienced more severe hydrological drought with larger peak, even though the

total number of drought events are less than that of other stations. Hence, it should be noted that the total number of drought events does not always justify the severity of the drought for any given area.

4) The ML, AIC, and BIC values were utilized to pick out the optimal copula function. The drought variables showed an excellent correlation among themselves. Out of the nine copula functions, for the overall region of Pennar River basin, for meteorological drought, Frank Copula is found to be the best-fit copula function for the joint risk of drought duration and severity, whereas Survival Clayton copula is found to be the best-fit copula for the combination of both drought duration and peak as well as drought severity and peak.

5) Using joint probability values and joint return periods, the drought risk can be calculated, which provides significant information for the analysis of drought. The joint return period for the study area can vary from as least as 1 year to 2 years or more for the 25th percentile whereas it can vary from 1 to 45 years for the 50th percentile of the combination of drought characteristics considered in the current work.

For instance, using simply the univariate information offered by either of the drought variables may result in under or overestimation of the real drought situation, and thus the corresponding risk. Hence, a multivariate analysis should be adopted.

According to our findings, the whole Pennar River basin is susceptible to frequent droughts. It can be observed that the study area has a high exceedance probability and a smaller joint return period. The research used a zero-threshold value of the drought index to identify the drought events at a three-month timescale. A more thorough analysis will emerge from the modification of the drought index threshold value. Moreover, this can enhance the values for joint return period and be useful in comparing the probabilities of various classes of drought (Poonia et al. 2021a). Additionally, the analysis only for seven selected stations, due to the data unavailability, might not provide a proper understanding of the drought conditions for the whole basin. But with the limited dataset, copula-based methodology produces impressive results.

Lastly, in this article, no account has been made for the impact of climate change and land use land cover change on the analysis, despite the fact that climate change and land use land cover change will affect the calculation of drought return periods. Furthermore, the groundwater level has not been considered in the hydrological analysis, which will have a minor impact on the hydrological drought. In this regard, systematic and rigorous attempts should be made to estimate the expected variations in meteorological and hydrological parameters that affect drought variables and, ultimately, the frequency of drought events.

Author contribution Both authors contributed to the study conception and design. The analysis was done by Balaram Shaw under the supervision of Dr. Chithra NR. The first draft of the manuscript was written by Balaram Shaw, and it was revised by Dr. Chithra NR. Both authors have read and approved the final manuscript.

Data availability Data is available upon request to the corresponding author.

Code availability Not applicable.

Declarations

Ethics approval Not applicable.

Consent to participate Not applicable.

Consent for publication Not applicable.

Conflict of interest The authors declare no competing interests.

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