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Reassessing the Joint Roughness Coefficient (JRC) Estimation Using Z_2

By

Z. Y. Yang, S. C. Lo, and **C. C. Di**

Department of Civil Engineering. Tamkang University, Tamsui, Taipei, Taiwan

1. Introduction

Rock joint roughness has attracted attention over the last 30 years because of its important influence on the shear strength of a rock joint. The most widely used formula for estimating the shear strengths of rock joints is Barton's empirical equation (Barton, 1973) given as:

$$\tau = \sigma_n \tan\left[\mathrm{JRC}\log_{10}\left(\frac{\mathrm{JCS}}{\sigma_n}\right) + \phi_b\right]. \tag{1}$$

In this equation, the JRC (joint roughness coefficient) value for a given joint profile can be estimated visibly by comparing it with the ten standard JRC profiles, whose JRC ranges from 0 to 20 (Barton and Choubey, 1977). The JRC is the commonly used measure of joint roughness in rock engineering practice and adopted by the ISRM commission on test methods (Brown, 1981). However, in practice it may be difficult to determine the proper JRC number, which is highly subjective because of the scale effect. If the profile is assigned to the wrong class, then the joint shear strength will introduce large estimation errors. To minimize subjectivity, alternate methods have been proposed for JRC estimation. Many researchers have thus attempted to calculate the JRC value from the profile geometry. At present, one commonly adopts Tse and Cruden's (1979) empirical statistical relationship between the JRC and Z_2 (the root mean square of the first derivative of the profile) to calculate typical JRC values (with a correlation coefficient $R = 0.986$):

$$\mathrm{JRC} = 32.2 + 32.47\log_{10} Z_2, \tag{2}$$

with Z_2 given as:

$$Z_2 = \mathrm{RMS}\left(\frac{\Delta y}{\Delta x}\right) = \frac{1}{L}\sqrt{\int_{x=0}^{x=L}\left(\frac{dy}{dx}\right)^2}. \tag{3}$$

In discrete form it may be expressed as:

$$Z_2 = \left[\frac{1}{m(\Delta x)^2}\sum_{i=1}^{m}(y_{i+1} - y_i)\right]^{1/2}. \tag{4}$$

In this equation, L is the total joint length, Δx is the sampling interval, $\Delta y = y_{i+1} - y_i$ is the difference between two adjacent sampling points; thus the $\Delta y/\Delta x$ is the asperity slope and m is the number of sampling intervals. However, the value of Z_2 is sensitive to sampling intervals (Yu and Vayssade, 1991). For estimating JRC, it cannot be employed without taking account the influence of sampling intervals. A relationship between the JRC and structure function (SF) was also proposed as (with a correlation coefficient $R = 0.984$):

$$\mathrm{JRC} = 37.28 + 16.5847 \log_{10} \mathrm{SF}, \tag{5}$$

where $\mathrm{SF} \cong (Z_2)^2$, so Eq. (2) and Eq. (5) are not independent. However, Eq. (5) is not well known to us. The increasing availability of image analysis hardware and low-cost digitizing pads could make Tse and Cruden's method a valuable objective alternative for the JRC assessment.

After examining the process for finding the JRC and Z_2 relationship given in Eq. (2), a possibly incorrect viewpoint becomes evident. During preparation of the discrete point data from the ten standard JRC profiles, the authors first enlarged the 10 cm long profiles by 2.5 times, both in the x- and y-coordinates. The new profile was then digitized along its total length ($L = 25$ cm) and 200 discrete data points taken at equal intervals ($\Delta x = 1.27$ mm) were analyzed. Thus the sampling ratio ($\Delta x/L$) is 1/200, the same as that used in our study. However, this process is a self-similarity transformation or so-called isotropic transformation according to the transformation law of fractal theory (Crownover, 1995; Yang and Chen, 1999). That is, the length and amplitude of the joint profiles are enlarged isotropically 2.5 times during this process. This changes the degree of the roughness of the joint profiles dramatically. The transformed profile could look much more erratic than the original profile, as the example displayed in Fig. 1 shows. In this figure, the roughness of the isotropic enlarged profiles is obviously exaggerated. The roughness of this enlarged profile is considerably different from the original profile. Thus, the roughness value (JRC) of the enlarged profiles has been changed. However, Tse and Cruden still used the former JRC value to correlate a new Z_2 for the enlarged profile data. Therefore, in this study we use copies of the 10 cm standard JRC profiles presented by Barton and Choubey (1977) to examine the new correlation formula between JRC and Z_2.

2. Reconstruction of the Standard JRC Profiles

The Fourier transform method has been used successfully in the characterization of joint surfaces (Raja and Radhakrishnan, 1977, Ueng and Chang, 1990; Adyan et al., 1996). The joint profile regarded as a periodic function in the whole length can be divided into several simple sine or cosine waves, each having a definite wavelength, amplitude and phase. Because it is not easy to digitize a standard JRC

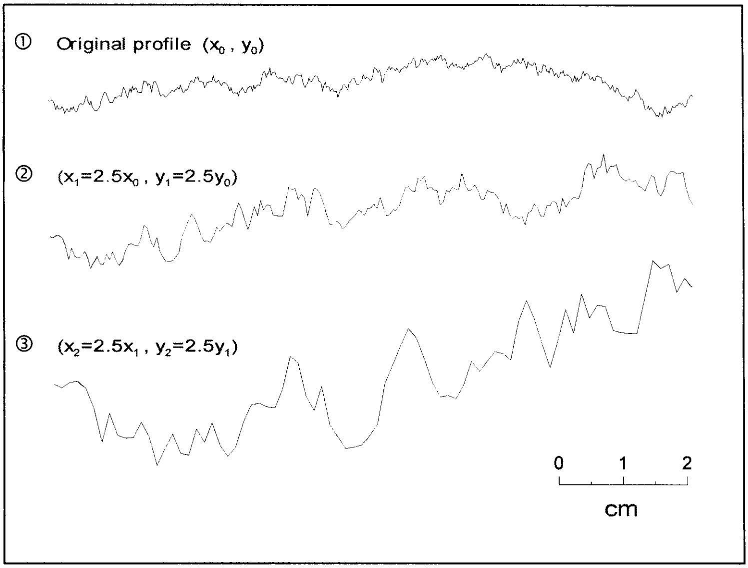

Fig. 1. Graphs of a profile ($H = 0.4$) after enlarging in x-direction by factors of 2.5 as well as in y-direction over two steps. (In each successive enlarged profile, right part of the data from the previous curve disappears due to clipping at the right boundary of the plot)

profile at equal intervals by hand, the Fourier series is adopted first to approximate the JRC profile. The profile geometry, represented by the function in the Fourier series, is then divided into many discrete data points at 0.05 cm intervals for calculating the Z_2 value.

2.1 Transformation of Fourier Series

When a function $y(x)$ varies periodically with distance x in a period L, it is possible to write the Fourier form of function $y(x)$ as:

$$y(x) = G_0 + \sum_{i=1}^{\infty} \left[G_i \cos\left(\frac{2\pi i}{L} x\right) + H_i \sin\left(\frac{2\pi i}{L} x\right) \right], \tag{6}$$

in which, the centerline average (CLA) equals G_0, and coefficients G_i and H_i can be expressed as:

$$G_0 = \left(\frac{1}{L}\right) \int_0^L y(x)\, dx, \tag{7a}$$

$$G_i = \left(\frac{2}{L}\right) \int_0^L y(x) \cos\left(\frac{2\pi i}{L}\right) dx, \tag{7b}$$

$$H_i = \left(\frac{2}{L}\right) \int_0^L y(x) \sin\left(\frac{2\pi i}{L}\right) dx. \tag{7c}$$

The above Fourier series can be adopted to describe any discrete function, such as the irregular joint profile. In order to define the roughness components, the irregular joint profile can be separated into sinusoidal components, which are the harmonics of a fundamental frequency within the entire profile length. The amplitude of the i-th term harmonic is given as $\sqrt[2]{G_i^2 + H_i^2}$.

2.2 Evaluation of the Reconstructed Profiles

In this study, the standard JRC profiles in 10 cm length were first digitized into about 500 data points by hand. In each profile, the 500 selected points were then used to back-calculate the coefficients G_0, G_i, H_i of the Fourier series. The first twenty harmonics were summed up to simulate the standard JRC profile and reconstructed as shown in Fig. 2. This shows that the reconstructed profiles are very similar to the original standard profiles.

2.3 The Fractal Property of JRC Standard Profiles

The ten standard JRC profiles, 10 cm long approximately, published in Barton and Choubey (1977), each of which were chosen by the authors as being the most typical of samples for 10 to 15 rock joint profiles. It is very important to consider whether the JRC profile is a fractal. Numerous researchers had regarded the JRC standard profile as a fractal curve. For example, Odling (1994) analyzed the fractal properties of the ten standard JRC profiles and found that most of the profiles were approximated fractal curves. Sakellariou et al. (1991) also stated that the JRC curves were essentially self-affine surfaces. The difference between the self-similar and self-affine surfaces is that a self-similar surface is statistically equivalent when scaled equally in both the axial and transverse directions, whereas a self-affine surface must be scaled differently in the perpendicular directions to maintain statistical similarity.

Mandelbrot (1983) suggested that the natural joint profile seems to behave in a self-affine manner (a fractal curve), appearing as traces of Brownian motion which is non-differentiable, so that its shape appears the same at any magnification, but its magnitude is not exactly the same for any sample length. Thus, to maintain the same roughness after enlarging the standard JRC profile, the self-affinity transformation concept must be obeyed (Mandelbrot, 1983; Crownover, 1995; Yang and Chen, 1999). That is, the joint profile can be stretched in the x-coordinate by a factor of r and in the amplitude by r^H. There will be no striking differences between the original and enlarged profiles (Yang and Chen, 1999). Here, the Hurst exponent H (Hurst et al., 1965) is an index to describe the roughness degree of a joint profile (Yang and Lo, 1997). This intrinsic value H, representing the roughness degree, which is invariant during the self-affinity enlargement transformation, can be obtained by trial-and-error according to the self-affinity properties (Crownover, 1995; Yang and Lo, 1997; Yang and Chen, 1999). The H value ranges from 0 to 1, and a rougher profile possesses a smaller H. For example, the

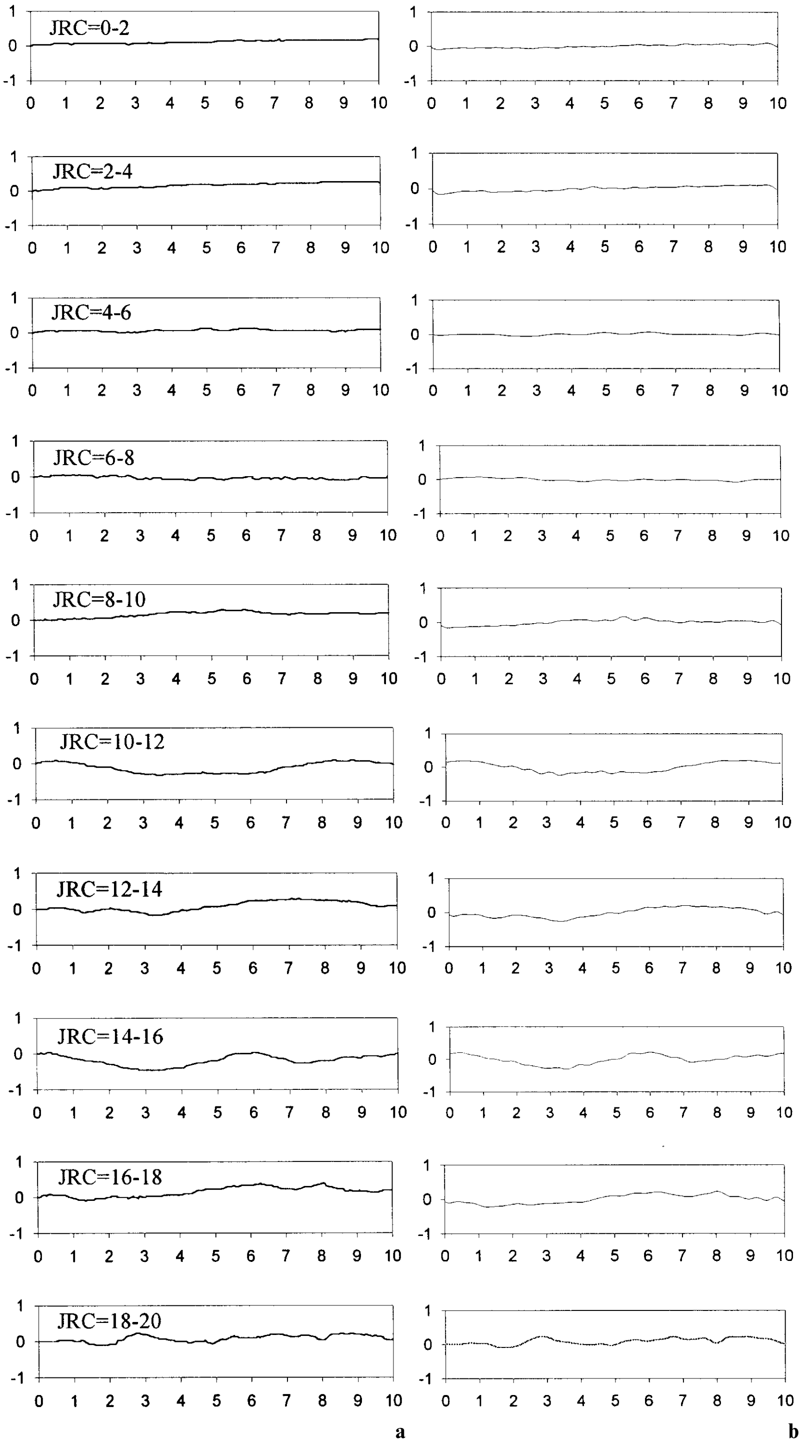

Fig. 2a,b. Graphs of the standard JRC profiles and reconstructed profiles. **a)** Standard JRC profile. **b)** Reconstructed JRC profile

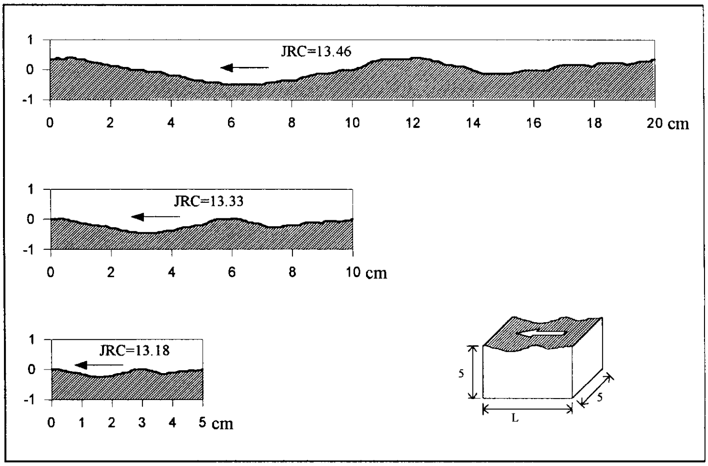

Fig. 3. The appearance and roughness change due to the self-affine transformation for a standard JRC profile ($H = 0.88$) (Yang and Chen, 1999)

JRC = 14–16 standard profile with $H \cong 0.88$ value (see also Yang and Chen, 1999) after self-affinity transformation is shown in Fig. 3. All of the profiles look about the same and will be perceived to have the same roughness. The new JRC value is 13.46 for a 20 cm profile and 13.18 for a 5 cm profile, calculated by Eqs. (2) and (4) using $\Delta x = 0.05$ cm (i.e., $\Delta x/L = 1/200$). They are indeed both very close to the original JRC value (JRC = 13.33) for the 10 cm length. The three profiles display a similar roughness and there is no scale effect in roughness between them.

3. Re-correlation Between JRC and Z_2

In order to examine and compare the JRC-Z_2 regression formula, Eq. (2), for each standard profile, we must keep the same JRC value as that used by Tse and Cruden. Thus we read the original JRC values in the JRC coordinate from their Fig. 4 (see Tse and Cruden, 1978) and re-draw as in Fig. 4(a) to examine our correct reading. Then, for each reconstructed JRC profile (see Fig. 2(b)), the discrete data points at equal intervals of 0.05 cm (keeping the sampling ratio as 1/200) used to calculate a new Z_2 using Eq. (4). The new Z_2 value is plotted with the corresponding original JRC value and shown in Fig. 4(b). A linear regression formula (with a higher correlation coefficient $R = 0.99326$ which is better than Tse-Cruden's) is obtained and given as:

$$\mathrm{JRC} = 32.69 + 32.98 \log_{10} Z_2. \tag{8}$$

From this new regression formula, we find that the regression result is somewhat different from the original Tse-Cruden's formula (see Eq. (2)). All of the new JRC values calculated using this formula are a little larger than those using Tse-Cruden's formula, but very close to one another (see Fig. 4(c)), even though some misconception exists in Tse and Cruden's previous work. A similar conclusion

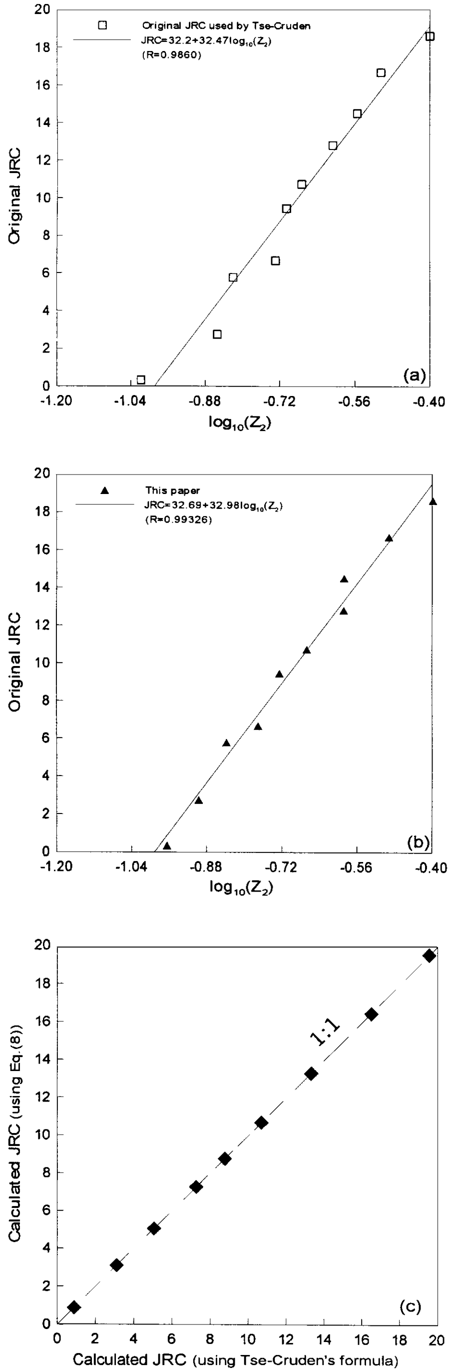

Fig. 4a–c. The correlation of JRC and $\log_{10} Z_2$: **a)** Re-plot of Tse and Cruden's Fig. 4. **b)** Correlation between the original JRC and new Z_2. **c)** Comparison of the calculated JRC using different formula

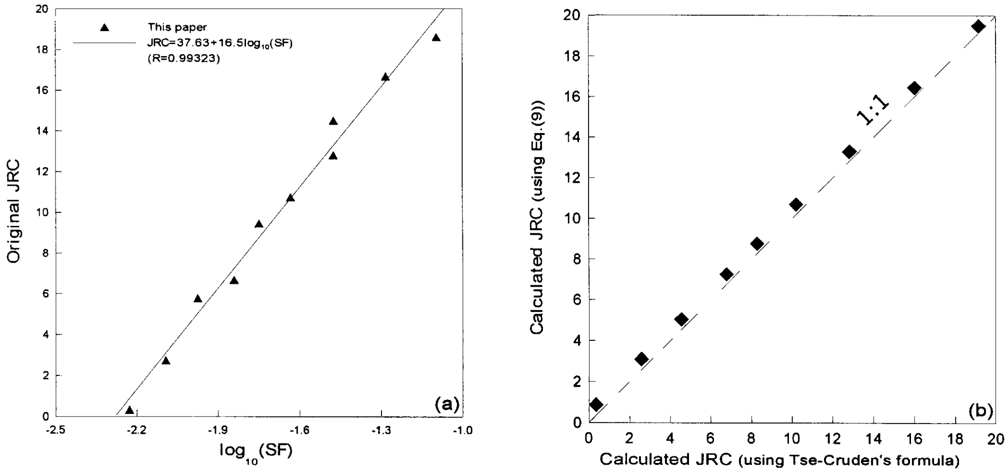

Fig. 5a,b. Plots of JRC values with SF. **a)** Correlation between the original JRC and new SF. **b)** Comparison of calculated JRC using different formula

between JRC and SF is shown as Fig. 5 and a regression formula (with a higher correlation coefficient $R = 0.99323$) is given as:

$$\mathrm{JRC} = 37.63 + 16.5\log_{10}(\mathrm{SF}). \tag{9}$$

After examining this consequence, we find that the results from the asperity slope for Tse-Cruden's isotropically enlarged profile (i.e., $2.5\Delta y/2.5\Delta x$) are identical to that for the original profile (i.e., $\Delta y/\Delta x$). That is, the new Z_2 calculated using Eq. (4) for the isotropically enlarged profiles is not different from the Z_2 value for the profiles in the original 10 cm scale. Because the parameter Z_2 is the root mean square of the tangents of the slope angles along the profile, it does not have a length dimension. This isotropic enlargement will not affect its value. This also holds in the relationship between JRC and SF because $\mathrm{SF} \cong (Z_2)^2$. However, in correlating the engineering coefficient with another surface parameter such as the Z_1, Z_3 and Z_4 (Tse and Cruden, 1979), the concept of the self-affinity transformation law should be considered.

In addition, Fig. 4(b) of this paper indicates that a surface with zero JRC still has a substantial value or an appreciable roughness, implying that Eq. (2) is only available for Z_2 in the range of 0.1 to 0.42. According to the explanation in Tse-Cruden's paper (see page 307),they suggested that the basic friction angle ϕ_b in Eq. (1) couldn't be regarded as a function solely of the rock substance. In fact, Barton and Choubey (1977) suggested that ϕ_b be replaced by ϕ_r (the residual friction angle). However, it has been demonstrated ϕ_r depends upon the roughness of the surfaces sheared (Tse and Cruden, 1979).

4. Conclusions

The formula relating the JRC with Z_2 proposed by Tse-Cruden has made the determination of a joint roughness objective and quantitative. However, according to the self-affinity transformation law, their elongating the standard JRC profiles was an incorrect concept. Yet, the parameter Z_2 corresponding to the asperity slope (also the structure function, SF) was unchanged due to this mistake. However, using the standard JRC profiles at the original scale, a similar formula with a higher correlation coefficient is obtained. In the future, if someone wants to scale up or scale down an original sample joint profile for preparing other artificial profiles that are different from its original length,this self-affinity transformation concept should be kept in mind.

References

Aydan, O., Shimizu, Y., Kawamoto, T. (1996): The anisotropy of surface morphology characteristics of rock discontinuities. Rock Mech. Rock Engng. 29, 47–59.

Barton, N. (1973): Review of a new shear strength criterion for rock joints. Engng. Geol. 7, 287–332.

Barton, N., Choubey, V. (1977): The shear strength of rock joints in theory and practice. Rock Mech. 10, 1–54.

Brown, E. T. (1981): Rock characterization testing and monitoring (ISRM suggested methods). Pergamon Press, Oxford.

Crownover, R. M. (1995): Introduction to fractals and chaos. John and Bartett, Boston.

Hurst, H. E., Black, R. P., Simaika, Y. M. (1965): Long-term storage: An experimental study. Constable, London.

Mandelbrot, B. B. (1983): The fractal geometry of nature. Freeman, New York.

Odling, N. E. (1994): Natural fracture profiles, fractal dimension and joint roughness coefficients. Rock Mech. Rock Engng. 27, 135–153.

Raja, J., Radhakrishnan, V. (1977): Analysis and synthesis of surface profile using Fourier series. Int. J. Mech. Tools. 245–251.

Sakellariou, M., Nakos, B., Mitsakaki, C. (1991): On the fractal character of rock surfaces. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 28, 527–533.

Tse, R., Cruden, D. M. (1979): Estimating joint roughness coefficients. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 16, 303–307.

Ueng, T. S., Chang, W. C. (1990): Shear strength of joint surfaces. Proc., 31st U.S. Symp. on Rock Mechanics, Balkema, Rotterdam, 245–251.

Yang, Z. Y., Chen, G. L. (1999): Application of the self-affinity concept to the scale effect of joint roughness. Rock Mech. Rock Engng. 32, 221–229.

Yang, Z. Y., Lo, S. C. (1997): An index for describing the anisotropy of joint surfaces. Int. J. Rock Mech. Min. Sci. 34, 1031–1044.

Yu, X., Vayssade, B. (1991): Joint profiles and their roughness parameters. Int. J. Rock Mech. Min. Sci. Geomech. Abstr. 28, 333–336.

Authors' address: Prof. Zon-Yee Yang, Department of Civil Engineering, Tamkang University, Tamsui, Taipei 25137, Taiwan.