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Developments in the Assessment of In-situ Block Size Distributions of Rock Masses

By

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Summary

The block sizes in a rock mass play an important role in many rock engineering projects and therefore the assessment of in-situ block size distribution (IBSD) has been an increasing pursuit of researchers in mining, quarrying and highway cutting operations. This paper discusses further developments in the assessment of IBSD which build upon a broadly accessible approach for engineers published previously by the Geomaterials Unit. The original research provided look-up tables appropriate for field data, with theoretical joint set spacing distributions and an assumption that discontinuities extend indefinitely. The developments reported in the paper include: the prediction of IBSD with special reference to discontinuity sets with fractal spacing distributions; the influence of impersistence of discontinuities on the prediction of IBSD; and the use of grey correlation analysis when selecting a closely fitting theoretical distribution for discontinuity spacing data. Various approaches to IBSD assessment are discussed.

1. Introduction

The formation of individual blocks of intact rock of different sizes and shapes, i.e. in-situ blocks, results from the mutual intersection of discontinuity sets with different spacing and orientation characteristics. The ISRM (1978) has suggested that the in-situ block sizes are governed mainly by the spacing and the persistence of discontinuities, as well as the number of discontinuity sets.

In recognition of the fact that the IBSD is a major contributing factor in assessing the technical viability of a new quarry source of large blocks for armourstone production, a computer program was written (Wang et al., 1991; Wang, 1992). The program solves the problem of deriving the block sizes and shapes formed by pre-defined planar discontinuities which intersect each other. In fact, engineers have increasingly recognised that the IBSD plays an important role in many rock engineering projects. It has been examined in: mining and quarrying blast operations (Cunningham, 1983; Da Gama, 1983; Ord and Cheung, 1991; JKMRC, 1991; Wang et al., 1991), rock mass characterisation (Franklin, 1974;

ISRM, 1978; Hoek et al., 1992), stability analysis of excavations in jointed rock masses (Hoek and Bray, 1981; Goodman and Shi, 1986) and indirectly in fracture network flow modelling (Rives et al., 1992; Dershowitz, 1993). The prediction of IBSD (a better term would be 'assessment' since the true IBSD can rarely be evaluated) has been one of the main pursuits of mining and quarrying operations as it is believed to greatly influence blasting performance generally (Da Gama, 1983; Cunningham, 1983; Wang et al., 1990, 1991; JKMRC, 1991), and rock amour production for coastal defence in particular (CIRIA/CUR, 1991; Latham et al., 1994). In blasting for highway cuttings (Wang et al., 1992; Matheson, 1995) the IBSD is also important but, in all these applications, it remains notoriously difficult to assess. Certainly, the IBSD is becoming one of the main inputs to new blast design models.

Recent research (Lu and Latham, 1996; Lu, 1997) has built on an approach that Wang referred to as "the equation method". His approach provided the engineer with a practical formula and a series of look-up tables (Wang et al., 1990; Wang, 1992), an alternative to computer simulation requiring licensed software, in order to find appropriate coefficients to make up the cumulative curve for the in-situ block sizes. This paper outlines the average in-situ block volume and IBSD assessment methods of other researchers, draws together several recent refinements in the use of Wang's equation method and discusses the strengths and weaknesses of the various approaches to IBSD assessment.

2. Prediction of IBSD

2.1 Developments in IBSD Prediction

The earliest quantitative description to bear a relation to in-situ block sizes was the Rock Quality Designation (RQD), the proportion of borehole core that consists of 0.1 m or more of intact length of sound rock (Deere, 1964). Priest and Hudson (1976) extended RQD to scanline survey data and proposed an analytical relation between RQD and the discontinuity frequency from the scanline survey (Hudson and Priest 1979). The RQD value obtained from a borehole or a scanline is influenced by the measuring direction. To overcome this disadvantage, Kazi and Sen (1985) suggested the use of the Volumetric Rock Quality Designation (V. RQD). This parameter, which is similar to an average block volume, tells us little about the proportions of very small or massive blocks and the distribution of block volumes as a whole.

When a single representative measure or index of block size is all that is required, such as for rock mass ratings, there have been a number of proposals in addition to RQD. Franklin (1974) proposed a fracture spacing index I_f, to help describe block sizes. I_f is the diameter of a "typical block", estimated by visually selecting typical sizes of core or outcrop material and recording their average dimension. The ISRM (1978) suggested a Block Size Index I_b that is similar to I_f and estimated by selecting by eye several typical block sizes and taking their average dimensions. Obviously, both I_f and I_b are semi-quantitative and have more limited use in practice. It also suggested the Volumetric Discontinuity Count J_v, which is the sum of the number of discontinuities per meter for each discontinuity set present.

Palmstrom (1985) suggested empirical equations to link J_v, RQD and linear fracture frequency, and proposed a correlation between the in-situ block size and J_v. This method could roughly estimate upper and lower ranges of block sizes. Sen and Eissa (1992) derived analytical expressions and charts for different characteristic shapes, such as bars, plates or prisms, relating J_v, RQD, and block volumes. The block volume information of the rock mass was again given only in terms of average block volume. To assess the entire in-situ block volume distribution has been a more recent pursuit, benefiting from computer technology and modelling (for example, Da Gama, 1983; Hudson and Priest, 1979; Stewart, 1986; Xu and Cojean, 1990; Kleine and Villaescusa, 1990). While these modelling developments have greatly promoted our understanding of the IBSD of rock masses and advanced the assessment of IBSD, the restricted in-house or commercial nature of such computer modelling tools has discouraged many engineers from using IBSD information, which is now widely recognised to be of relevance to rock engineering projects.

Before looking at the recent developments that follow on from Wang's work, it is perhaps interesting to outline a general approach to deriving IBSD from simulated discontinuity networks, which has been adopted by several other teams of researchers. For example, Aler et al. (1996) have exploited the stochastic modelling methods of Xu and Cojean, (1990). The number of joint sets, requiring independent characterisation to create the network, is determined by analysing the discontinuity field data with both visual inspection of stereoplots and a discontinuity grouping program. For each set, statistical parameters that best describe the distributions of spacing, semi-trace length, dip angle and dip direction are considered. The objective is to obtain the optimum selection of sets and statistical parameters to generate a realistic discontinuity network for a statistically homogeneous region of the site in question. The simulated rock mass is then made up of intersecting discontinuities that are represented by flat discs, and their 3-D characteristics are derived from fitted distributions of the two-dimensional measures of the various geometrical parameters. Aler et al. (1996) reported that disc centers were generated with a uniform distribution, while size in 3-D was estimated from the fitted distribution of the semi-trace lengths observed in outcrop. Orientation of discs was generated from normal distributions calculated from observed dip and dip directions. Kolmogorov-Smirnov tests were then used to evaluate the quality of fit of different theoretical distribution parameters, particularly in the characterisation of discontinuity spacings and semi-trace lengths. Once the computer had simulated all the circular discs in space, the mutual intersections were examined and parts of planes that did not isolate complete blocks were eliminated. The computer program then calculated the block size distribution of all the completely formed blocks. The physical resemblance of the simulated geometry to that of the actual rock mass is probably one of the main criteria for assessing the degree of confidence in the results produced. For many potential users, the need for access to the simulation software remains a probable obstacle for this type of simulation. The FRACMAN program (Dershowitz et al., 1993) has a similar stochastic simulation capability designed originally for flow modelling.

2.2 Wang's Methods

The two techniques developed at QMWC by Wang and his co-workers (Wang et al., 1990, 1991; Wang, 1992) have been applied widely and differ from the above approach. The techniques use either orientation and location data from individual discontinuities or, only location data of the discontinuities mapped by either scanline or downhole CCTV techniques, combined with a knowledge of the main discontinuity set orientations derived from any suitable survey method. Both of them are incorporated in a computer program and these are called respectively the Dissection Method and the Equation Method (Wang, 1992). However, of great significance, the equation method can also be handled by simple manual methods and calculations.

Dissection Method

The Dissection Method uses a computer program to determine the exact IBSD produced by intersecting discontinuities within a boundary block formed by six persistent planes. The algorithm developing in this computer program was mainly based on the block theory by Goodman and Shi (1985). The algorithm and the associated program is briefly described below (see Wang et al., 1991 for details). The data set required to run the program contains discontinuity orientation parameters and intercepts with reference to an oriented scanline, all of which can be provided from detailed scanline surveying. Six discontinuities are chosen to form an executable six-sided block called the boundary block, for which the IBSD is to be computed. This boundary block is first dissected into two blocks of varying shape by a discontinuity, which is read from the discontinuity data file. These two are further dissected into three or four blocks by the next discontinuity. The dissection process is carried out until the last discontinuity in the working data file is executed, yielding an intermediate file of co-ordinates of corners of all natural blocks existing in the boundary block formed by the dissecting discontinuities. The sizes of these blocks are given in terms of volume, maximum length, and nominal diameter. Accordingly, the block size distribution is given. Block shape statistics are also available and may be of particular interest to producers of dimension stone. The geometrical pattern of discontinuities intersected with the boundary block can be viewed in three dimensions from the computer program. Their exact visual similarity with field exposure can be striking when introducing successive discontinuities deterministically, i.e. in their correct position relative to an origin in the field. An example of a 3-D view, created using the dissection method, is shown in Fig. 2. The authors are now aware that similar software has been developed elsewhere (e.g. MAKEBLK, see Maerz and Germain, 1996).

Equation Method

The Equation Method uses a set of empirical equations to estimate the IBSD. This set of empirical equations relates the IBSD to the principal mean spacings and the

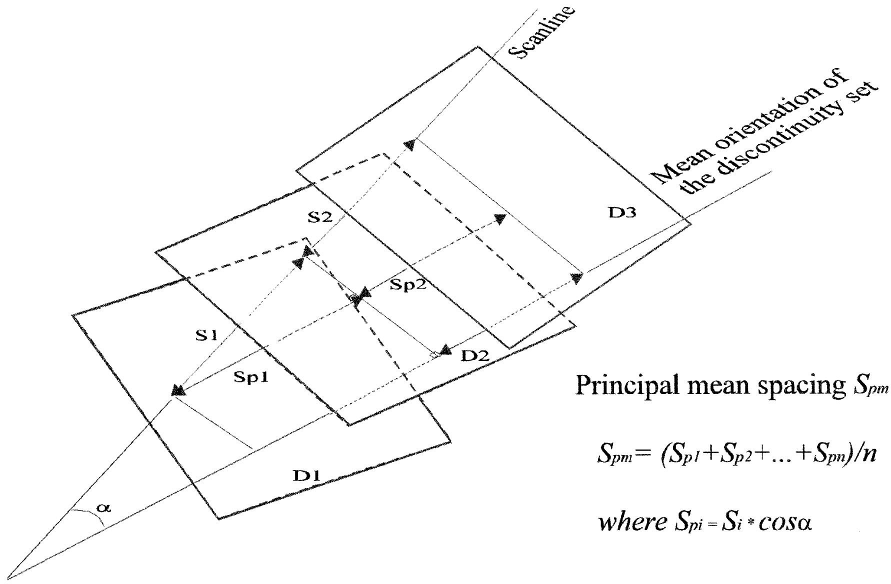

Fig. 1. Principal mean spacing: determined from the known spacing and orientation of the scanline and the mean orientation of the discontinuity set. ($D1$, $D2$ and $D3$ are three adjacent discontinuities)

mean orientations of the three principal sets of discontinuities, the latter being determined from a weighted grouping algorithm designed to best characterise the rock mass. Here, the principal mean spacing is the mean spacing in the direction parallel to the pole of mean orientation of a discontinuity set, as shown in Fig 1. The equations are obtained using the exact dissection method solutions described above and were derived from calibrating the results of extensive computer runs, which had different predefined mean spacings and spacing distributions. Each run utilises different random sequences to create the required theoretical distributions, and the results of each run provide the input data from which the best-fit equations yield the calibrated equations of interest – essentially a Monte Carlo simulation process. Depending upon which specific spacing distribution is chosen, different sets of empirical equations are offered which summarise the best fit for results from all the separate runs. The equations were all given by the general equation below:

$$V_{i,p} = \frac{C_{i,p} \times (S_{pm1} \times S_{pm2} \times S_{pm3})}{\cos\theta \cos\phi \cos\alpha}, \quad i = 10, 20, \ldots, 100, \tag{1}$$

where, $V_{i,p}$ and $C_{i,p}$ ($i = 10, 20, \ldots 100$) are respectively block sizes of percentage passing and empirical coefficients calibrated from the results of computer runs; i are percentages; S_{pm1}, S_{pm2} and S_{pm3} represent the three principal mean spacings; and θ, ϕ and α are the angles between the mean orientations of the three discontinuity sets. The $C_{i,p}$ in Eq. (1) for discontinuity sets with negative exponential and uniform distributions as well as with a certain lognormal distribution spacing law, obtained by Wang (1992), are summarised in Table 1. The possible error range introduced when using this method will be wider than implied by the confidence

Table 1. $C_{i,p}$ with 90% confidence intervals for the relationships in Eq. 1

	Uniform		Neg. exp.		Log-nor	
P (%)	$C_{i,p}$	Range	$C_{i,p}$	Range	$C_{i,p}$	Range
10	0.375	0.157	0.332	0.131	0.469	0.099
20	0.700	0.292	0.710	0.249	0.965	0.207
30	1.052	0.435	1.207	0.423	1.513	0.334
40	1.460	0.607	1.852	0.645	2.22	0.542
50	1.939	0.787	2.708	0.984	3.099	0.731
60	2.548	1.036	3.980	1.550	4.287	1.029
70	3.343	1.384	5.867	2.596	5.956	1.501
80	4.495	1.802	8.948	4.581	8.497	2.243
90	6.623	2.691	15.332	9.532	13.377	4.227
100	17.772	9.348	38.992	23.734	38.277	17.569

Range is the standard deviation multiplied by a constant of 1.64. The 90% confidence interval for $C_{i,p}$ can be derived from $C_{i,p} \pm$ Range.

limits given if the real data are only poorly fitted by the assumed theoretical distributions.

In the dissection method, it is crucial to select the six boundary planes carefully in order to form an "executable boundary block" capable of yielding meaningful and reliable block size distributions. In addition, the execution of the computer programme is often very time-consuming owing to problems with the dissection method in terms of user-friendliness and user access. The equation method evolved from the dissection method as a simpler, time-saving method that has often been used successfully by masters-level students without recourse to the in-house sophisticated simulation or dissection software. Thus, the equation method is preferable to the dissection method in terms of cost-effectiveness, accessibility and user-friendliness.

2.3 Problems with the Equation Method

First, the estimation of IBSD of rock with discontinuities with a fractal spacing distribution has not been achieved. A systematic study of research data reported in the literature (Lu, 1997) has indicated that for spacings, while negative exponential, lognormal and uniform distributions appear popular, the fractal spacing distribution is also one encountered in rock masses (Gillespie et al., 1993; Boadu and Long, 1994; Lu and Latham, 1996), which is being increasingly recognised in geological engineering (Turcotte, 1992; Xie, 1993; Hobbs, 1993). However, there have been no studies reporting the prediction of the IBSD of rock with discontinuities described by a fractal spacing distribution. The probability density function for negative exponential spacing distributions is the well known simple relation

$$f(x) = \lambda e^{-\lambda x}, \tag{2}$$

where λ is the mean frequency. For fractal spacings it can be written

$$f(x) = Ax^{-(1+D)}, \tag{3}$$

where D is the fractal dimension and the spacings, x, have an upper and lower cutoff at a and b such that

$$A = D/(a^{-D} - b^{-D}), \quad 0 < D < 1. \tag{4}$$

Second, with the possible exception of Aler et al. (1996), consideration of the influence of discontinuity persistence has not concentrated on the estimation of IBSD, although significant progress in characterising and representing the influence of discontinuity persistence on mechanical and hydraulic properties of rock masses has been seen in recent years (Dershowitz and Herda, 1992; Einstein, 1993; Mauldon, 1994). Apart from the stochastic simulations, such as those based on intersecting discs of finite size, as described above, almost all reported methods to estimate the IBSD, including the techniques developed by Wang, assume that all discontinuities to be included in the analysis of a rock mass are persistent. A conceptual device, which coded the degree of persistence in such a way that both all and a subgroup of mapped discontinuities are analysed, enabled Wang et al. (1991) to provide upper and lower bounds for the IBSD when using the dissection method. Certainly, a rock mass with impersistent discontinuities is the normal, indeed, the universal case. This suggests that it would be a useful refinement if the influence of impersistence on Wang's equation method prediction of IBSD could be incorporated.

Last, classical, statistical procedures for the identification of the distribution law for discontinuity spacing, the law being a necessary prerequisite for using the equation method, leave room for improvement. Obtaining a functional relationship, which closely describes or nearly fits the distribution of a field measurement, say discontinuity spacing, is helpful to understand the nature and the implications of the variation of this parameter. Identification of the best type of theoretical spacing distribution to fit the field discontinuity data is just as important when applying the equation method to predict IBSD as it is when selecting a theoretical discontinuity network for stochastic simulation. This identification might be made using a conventional goodness-of-fit test, say the χ^2 or Kolmogorov-Smirnov test; but the conventional statistical methods for evaluating goodness-of-fit and selecting a preferred fit have shortcomings (Benjamin and Cornell, 1972). To overcome these, the introduction of a new technique was considered to be a useful additional tool.

3. Refinements

3.1 Prediction of IBSD for Discontinuities with a Fractal Spacing Distribution

The IBSD of rock masses intersected by discontinuity sets with fractal spacing distributions has been investigated by employing Wang's dissection method algorithm and Monte Carlo calibration methods in the manner described above.

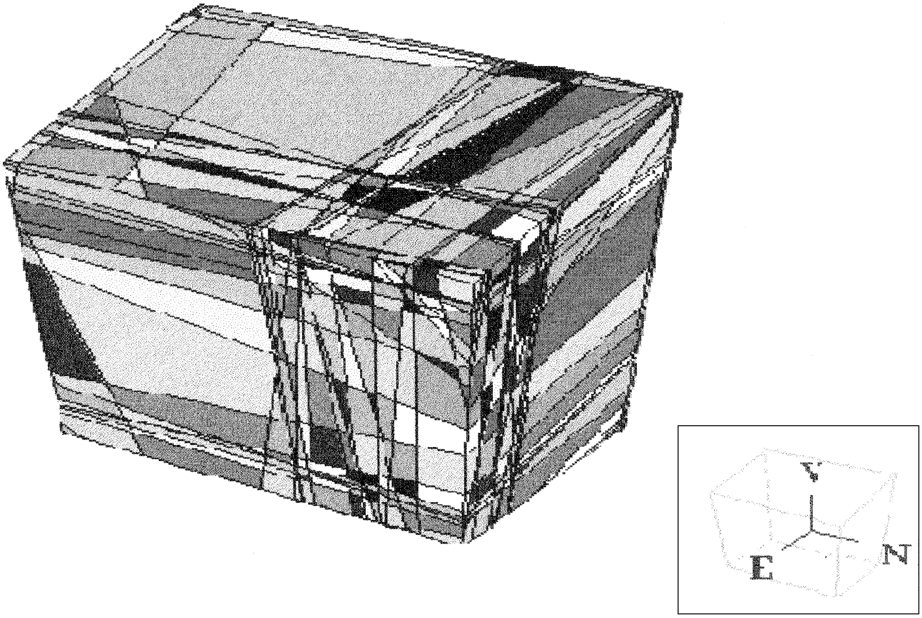

Fig. 2. 3-D view of a simulated rock mass consisting of discontinuities with fractal spacing distributions ($D = 0.36$, the rock mass volume $V = 2163\ \mathrm{m}^3$)

Figure 2 is an example of one of the 50 or so simulations used to calibrate Eqs. (5) and (6).

To make a valid simulation, appropriate cutoffs to exclude invalid spacing values have to be set for the fractal distribution. When taking field measurements, discontinuity spacing values below the resolution on a measuring tape will not be recorded. As such, there exists a lower cutoff. On other hand, both actual exposures and discontinuities are of finite size and spacing. Setting aside mechanistic reasons that may be the cause of a real scale dependence, the introduction of cutoffs acts like a long and short wavelength filter designed to remove sampling bias at the extremes, so that any underlying scale independence can better show itself. Discontinuity spacings in an actual engineering project will therefore have an upper cutoff. Thus, the spacing values measured will fall within a range defined by the lower and upper cutoffs. The lower cutoffs are often set out around 0.01–0.05 m (Priest and Hudson, 1976; Wang, 1992; Boadu and Long, 1994), and the upper cutoffs were usually reported below 10 m (Priest and Hudson, 1976; Wang, 1992; Gillespie et al., 1993). The lower and upper cutoffs were therefore chosen to be 0.05 m and 10 m in this simulation.

Two sets of empirical equations for predicting the IBSD of a rock mass with discontinuities of fractal spacing distributions have been derived from recent investigations (Lu and Latham, 1996; Lu, 1997) and are given by:

$$V_{i,p} = C_{i,p} \times (D_1 \times D_2 \times D_3)^{-b_{i,p}}, \tag{5}$$

Table 2. Coefficients $C_{i,p}$ and $b_{i,p}$ for the relationship between $V_{i,p}$ and the product of fractal dimensions of discontinuities with fractal distributions (Eq. 5)

Passing	Coefficient $C_{i,p}$	Standard error	90% confidence level Lower	Upper	Range	Error[a] (%)
10	0.0468	0.0033	0.0417	0.0526	0.0055	11.66
20	0.1440	0.0099	0.1284	0.1614	0.0165	11.47
30	0.3098	0.0218	0.2755	0.3485	0.0365	11.78
40	0.5642	0.046	0.4926	0.6462	0.0768	13.62
50	0.8453	0.066	0.7422	0.9628	0.1103	13.04
60	1.4447	0.1121	1.2695	1.6440	0.1872	12.96
70	2.5625	0.2379	2.1959	2.9904	0.3973	15.55
80	4.4490	0.5492	3.6254	5.4598	0.9172	20.62
90	9.2825	1.2878	7.3777	11.679	2.1507	23.17
100	31.3330	3.5021	26.025	37.722	5.8485	18.67
	$b_{i,p}$					
10	0.5949	0.025	0.5529	0.6368	0.0418	7.03
20	0.5423	0.0246	0.5010	0.5836	0.0411	7.59
30	0.5012	0.0253	0.4588	0.5436	0.0422	8.42
40	0.4711	0.0292	0.4220	0.5201	0.0488	10.36
50	0.4712	0.0280	0.4242	0.5181	0.0467	9.92
60	0.4404	0.0278	0.3938	0.4871	0.0465	10.55
70	0.4199	0.0332	0.3642	0.4756	0.0555	13.22
80	0.4072	0.0441	0.3333	0.4811	0.0736	18.07
90	0.3609	0.0494	0.2780	0.4438	0.0825	22.87
100	0.3054	0.0399	0.2384	0.3724	0.0667	21.84

[a] Error is the ratio of Range over the corresponding coefficient expressed in %.

and

$$V_{i,p} = C_{i,p} \times (S_{pm1} \times S_{pm2} \times S_{pm3})^{b_{i,p}}. \tag{6}$$

Where $V_{i,p}$ $(i = 10, 20, \ldots, 100)$ are block volumes of percentage passing (in m^3), and, $C_{i,p}$ and $b_{i,p}$ are empirical coefficients, see Tables 2 and 3; i are percentages; D_1, D_2 and D_3 are the fractal dimensions of the three sets of discontinuity spacing values, and S_{pm1}, S_{pm2} and S_{pm3} are the principal mean spacing values of the three sets of discontinuities.

Eqs. (5) and (6) provide us with a tool for predicting the IBSD of a rock mass for which the three sets of discontinuities have fractal spacing distributions.

Whereas Eq. (1) is a linear relationship between the IBSD and the principal mean spacings, Eq. (6) is non-linear. This indicates that the IBSDs of rock masses with fractal spacing distributions appear different from those with negative exponential, lognormal and uniform spacing distributions when identical mean spacings are considered. This might be explained as follows: discontinuities with a fractal spacing distribution tend to give a pattern in such a way that some discontinuities are closely clustered while others are sparsely distributed (see Fig. 2); in contrast, both the negative exponential and the uniform spacing distribution are fairly evenly distributed. It is therefore important to distinguish whether the type of spacing distribution of discontinuities is clearly of a fractal form in applications

Table 3. Coefficients $C_{i,p}$ and $b_{i,p}$ for the relationship of $V_{i,p}$ and the product of principal mean spacing values of discontinuities with fractal distribution (Eq. (6))

Passing	Coefficient $C_{i,p}$	Standard error	90% confidence level Lower	Upper	Range	Error (%)
10	0.4649	0.0077	0.4523	0.4779	0.0128	2.75
20	1.1685	0.0245	1.1283	1.2101	0.0409	3.50
30	2.1606	0.0448	2.0871	2.2367	0.0748	3.46
40	3.5458	0.0965	3.3882	3.7107	0.1612	4.55
50	5.3165	0.115	5.128	5.512	0.1920	3.61
60	8.0903	0.1855	7.7864	8.4061	0.3098	3.83
70	13.3920	0.5029	12.578	14.258	0.8398	6.27
80	22.6070	1.3562	20.455	24.985	2.2649	10.02
90	39.6660	3.0117	34.954	45.0130	5.0295	12.68
100	108.9700	5.7909	99.724	119.070	9.6708	8.88
	$b_{i,p}$					
10	0.7882	0.0109	0.770	0.806	0.018	2.30
20	0.7200	0.014	0.697	0.743	0.023	3.21
30	0.6719	0.0137	0.649	0.695	0.0228	3.40
40	0.6433	0.0179	0.613	0.673	0.03	4.66
50	0.6440	0.014	0.620	0.668	0.024	3.70
60	0.6053	0.0151	0.580	0.631	0.0252	4.17
70	0.5874	0.0247	0.5459	0.629	0.0413	7.03
80	0.5900	0.0395	0.5237	0.656	0.0659	11.18
90	0.5335	0.0499	0.4497	0.617	0.083	15.63
100	0.4675	0.035	0.4088	0.526	0.058	12.50

for which the in-situ block size distribution is likely to be of major significance. This will certainly be the case in quarrying for armourstone, aggregates and building stone, especially for operations of quarrying in which the proportion of big blocks is deemed critical (see Fig. 4).

3.2 Consideration of the Influence of Impersistence

Persistence is a very important property but difficult to characterise (ISRM, 1978). There are two definitions of persistence in current usage. One is suggested by the ISRM (1978), where the persistence is defined as the percentage of total area of a plane through the rock mass, which is formed by discontinuities co-planar with this reference plane. That is, the persistence P_l is a ratio defined by $P_l = \left(\sum_i a_{Di}\right)/A_D$ (where, A_D is the area of a sampled region of the plane, i.e. the reference plane, and a_{Di} is the area of the *ith* discontinuity in A_D). Another is suggested by Einstein et al. (1983), where the persistence is defined as the limit of the above ratio as the size of the reference plane, A_D approaches infinity. The persistence, according to the first definition, will be closely related to the value of A_D. The greater A_D, the smaller will be the value of P_l. According to the second definition, the persistence could approach zero as A_D approaches infinity. In other words, these two definitions would lead to a scale-dependent persistence value. The work on disconti-

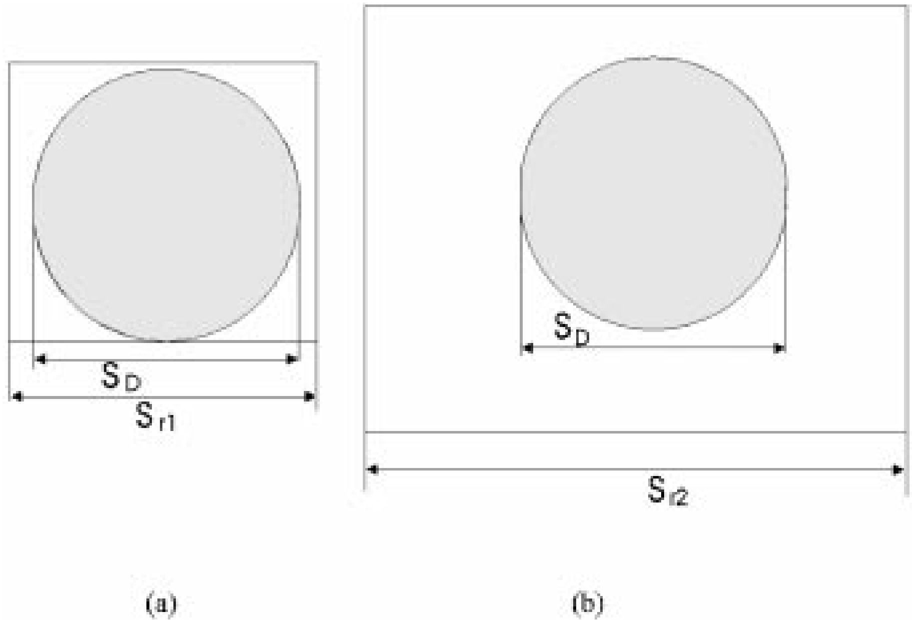

Fig. 3. Schematic illustration of the relative impersistence factor

nuity persistence by Dershowitz and Herda (1992) and Mauldon (1994) focussed on the description of discontinuity intensity and is helpful in understanding the persistence related to scale and non-scale dependence. However, the work appears not to address the issue of the influence of impersistence on the prediction of IBSD. Whether the assumption of all-persistent discontinuities (this will be referred to as the "all-persistent discontinuities" assumption) is good or not, in the context of prediction of IBSD, it is critically dependent upon two factors: one is the scale of the in-situ rock mass of interest, and another is the mean size of discontinuities. The greater the scale of the rock mass, the worse the approximation; the larger the mean discontinuity size, the better the approximation (Fig. 3). The assumption is probably acceptable for a small volume of a rock mass or for a rock mass with discontinuities having a large mean discontinuity size (Fig. 3a), but with the increase of the rock mass in question, the errors related to this assumption will increase (Fig. 3b). As such, a factor to characterise these properties has been introduced for the prediction of IBSD (Lu, 1997). The factor is referred to as the "relative impersistence factor", F_{imp}, as follows

$$F_{\text{imp}} = \begin{cases} \dfrac{S_D}{S_r} & S_D < S_r \\ 1 & S_D \geq S_r \end{cases}, \tag{7}$$

where S_D is mean discontinuity size, which can be estimated using the techniques developed by Lu (1997) as briefly shown below, and S_r represents the characteristic size of the rock mass under consideration, say, the cube root of the volume of the rock mass (see Fig. 2) of interest.

Provided that an estimation of mean discontinuity size can be made, it has been shown that the influence of impersistence on in-situ block size can be elucidated by comparing the mean discontinuity size to the scale of the domain of in-situ rock mass of interest (Lu, 1997).

Assuming that discontinuities are circular discs of negligible thickness and the centers of discontinuities hold a three-dimensional Poison process, Warburton (1980) has made a valuable derivation of the distribution of trace lengths formed by where the intersections of parallel circular planar discs, which is given by

$$f(l) = \frac{1}{m_d} \int_l^{\infty} \frac{l g(R)\, dR}{\sqrt{R^2 - l^2}}, \tag{8}$$

where l represents the trace length of a discontinuity, R is the diameter of the discontinuity with the circular disc shape, m_d represents the mean diameter of discontinuities, $f(l)$ is the probability density distribution of the discontinuity trace lengths, and $g(R)$ represents the probability density distribution of the discontinuity diameters.

Theoretically, the distribution of discontinuity diameter can be estimated for any continuous form of $g(R)$ by applying Warburton's derivation. However, difficulties in both mathematics and practical sampling make it hardly possible to determine $g(R)$ and its control parameters directly. Therefore, alternative techniques of determining the discontinuity diameter distribution and its governing parameters (Warburton, 1980; Villaescusa and Brown, 1992) were sought. It was found that there is a prerequisite that an analytical form of discontinuity diameter distribution has to be set up in advance when utilising the techniques used by Warburton and Villaescusa and Brown. Although it might be reasonable to assume that the diameters of discontinuities would take a particular form, the impossibility of dismantling a rock mass and the difficulties raised in sampling have so far made it impossible to prove the assumption.

By contrast, it is possible and easy to confirm assumptions about the distribution forms of trace lengths produced by discontinuities. Consequently, imposing a distribution form of trace lengths is more reasonable than doing so for discontinuity diameters. An algorithm and the associated computer program, DIATRACE, for determining the discontinuity diameter distribution, which is not based on the assumption of the distribution of discontinuity diameters, but on that of trace lengths, has been developed by Lu (1997). Note that Aler et al. (1996), in their work on block size assessment, reported use of the simplifying assumption that the diameter distribution is the same as the trace length distribution. Using the program DIATRACE to relate discontinuity diameter distribution to the distribution of measured trace lengths, one is able to estimate the mean diameter of a population of discontinuities. This estimation is further facilitated by the developments of techniques for the estimation of mean trace length of discontinuities with fractal and lognormal (Lu, 1997) and with negative exponential distributions (Priest and Hudson, 1981). The estimation of mean trace length can be obtained from a simple counting survey of an exposure, with discontinuities censored at different pre-defined levels. The method to estimate the discontinuity size is still subject to the commonly applied constraint of circular disc discontinuities. Nevertheless, it provides a tool to estimate discontinuity size. This is of great significance for considering the influence of impersistent discontinuities on the prediction of IBSD.

Introducing the above relative impersistence factor F_{imp}, a prediction of IBSD, which incorporates the influence of impersistent discontinuities, can be made very simply by extending the existing equation method as follows:

$$V_{i,p} = \frac{1}{(F_{\mathrm{imp}})^q}(V_{i,p})_0, \tag{9}$$

in which $(V_{i,p})_0$ represents the prediction result of IBSD from the all-persistent assumption, $V_{i,p}$ is the corrected result, incorporating the influence of impersistent discontinuities on the result, and q is a constant less than 1. It was tentatively suggested that it would take a value of between 1/5 and 1/2, which was later supported by a case study (Lu, 1997). It is also worth noting that the impersistence factor can be applied with equal relevance and simplicity to the block size output from the dissection method.

Now Eq. (1) for predicting IBSD of discontinuities can be updated to give

$$V_{i,p} = \frac{C_{i,p}}{(F_{\mathrm{imp}})^q}\frac{S_{pm1} \times S_{pm2} \times S_{pm3}}{\cos\theta\cos\phi\cos\alpha}, \quad i = 10, 20, \ldots, 100 \tag{10}$$

while Eqs. (5) and (6) for predicting IBSD of discontinuities with fractal spacing distributions can be updated to give

$$V_{i,p} = \frac{C_{i,p}}{(F_{\mathrm{imp}})^q}(D_1 \times D_2 \times D_3)^{-b_{i,p}} \tag{11}$$

$$V_{i,p} = \frac{C_{i,p}}{(F_{\mathrm{imp}})^q}(S_{pm1} \times S_{pm2} \times S_{pm3})^{b_{i,p}}, \tag{12}$$

The above technique has been successfully applied to a highway cutting site in which the impersistence factor was calculated from analysis of field data (Lu, 1997).

3.3 Comparison of IBSD for Discontinuities with Different Types of Spacing Distribution

Comparison of Eq. (1) and Eq. (6) indicates that the non-linear form is noted for the fractal spacing distribution, whereas a linear form was found for the rock mass with negative exponential, lognormal and uniform spacing distributions. Figure 4 gives a comparison of IBSD curves for the special case where the principal mean spacing value of each discontinuity set has been given the same value of 1.0 m. Clearly, there is a significant difference between the resulting IBSDs. The IBSD of a rock mass with discontinuities that have a fractal spacing distribution is much larger than that with the other three spacing distributions. The fractal IBSD curve is less steep, giving blocks that are more widely distributed. That is, more

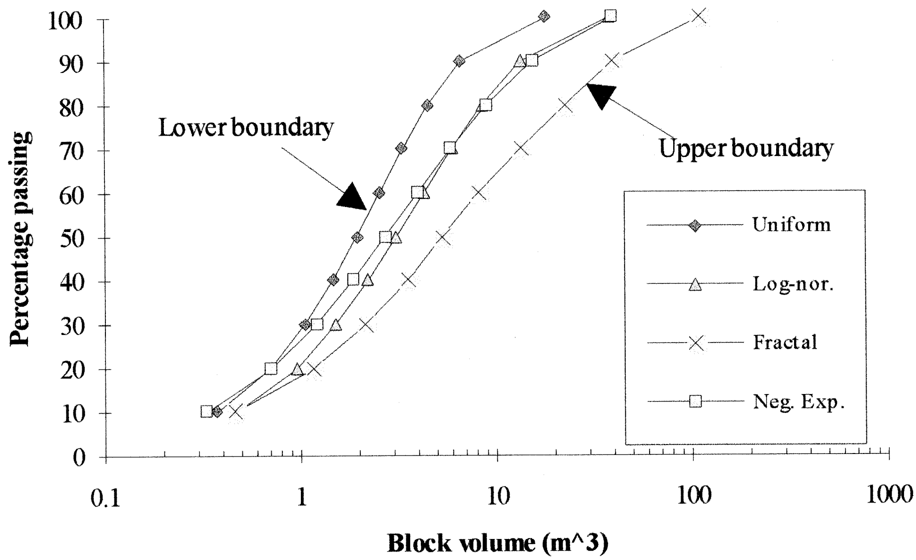

Fig. 4. Comparison among IBSDs with 4 different spacing distributions (all the mean spacings are 1.0 m)

small and more "mammoth" blocks will be produced from the rock mass with discontinuities with fractal distributions. Among the four different spacing distributions, the IBSD intersected by discontinuities with a uniform spacing distribution will form the lower boundary IBSD curve (see. Fig. 4).

In practice, the more typical situation is that between the various sets, a mix of two or more kinds of theoretical distributions are needed to best describe the discontinuity spacings. In such cases, we can reasonably predict that the IBSD will fall in the range formed by the uniform (lower boundary) IBSD curve and the fractal (upper boundary) IBSD curve (see Fig. 4).

This is supported by a preliminary examination made in a highway cutting case study, as shown in Fig. 5 (Lu, 1997). The discontinuity spacing data from each of three sets, which characterise the rock mass at the field site, was subject to statistical analysis to determine which distribution law would give the best fit to the spacing data. It was typically found that more than one type of distribution law was required to give the best fit for all three sets of spacing data. In Fig. 5, the dissection result using the raw scanline data is likely to be the most representative, not only because the other results introduce theoretical spacing distributions, but also because only one type of spacing distribution is imposed on all three sets of spacing data. Further research is therefore needed to address how to weigh the combined influence when two or more kinds of discontinuity spacing distribution laws are acting together in a rock mass. For example, Monte Carlo type procedures, similar to those reported in Wang et al. (1990) and in Lu and Latham (1996), could be used to include two or more different, discontinuity spacing distributions.

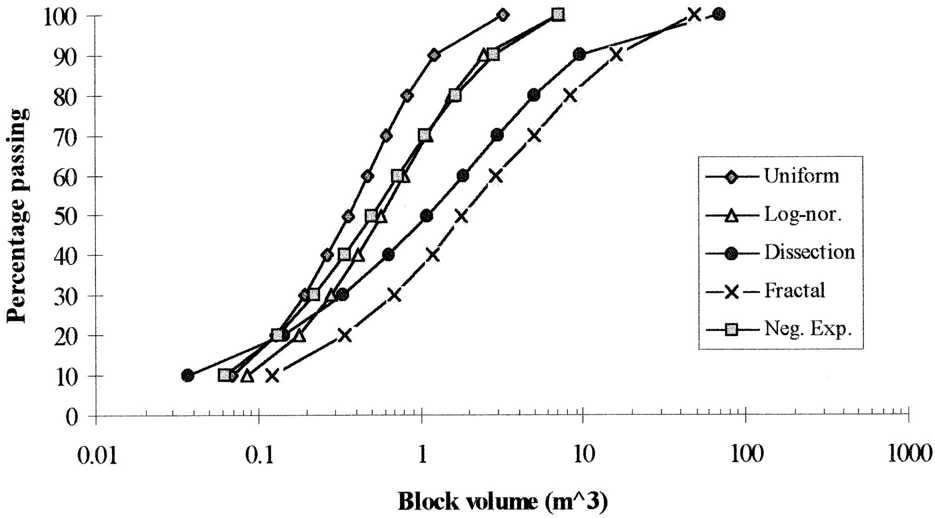

Fig. 5. Comparison of IBSD predictions between the dissection and the equation methods at a highway cutting field site

3.4 Goodness-of-Fit Using Grey Correlation Analysis

It is necessary, when using the equation method, to identify the spacing distribution law of discontinuities, and further confidence is required in obtaining these. The technique of Grey Correlation Analysis was introduced to help select a close fitting distribution for discontinuity spacings (Lu, 1997). A Grey Correlation Analysis is the analysis of correlation between various parameters influencing a system and the identification of which parameter relationships will be dominant. This is outlined as follows:

Let the main data set of interest be the parent array X_0, and the influencing data sets be sub-arrays, $X_i,\ i = 1, 2, \ldots, n$, n is the number of influencing data sets, then

$$\begin{aligned} X_0 &= \{X_0(1), X_0(2), \ldots, X_0(K)\}, \\ X_i &= \{X_i(1), X_i(2), \ldots, X_i(K)\}. \end{aligned} \tag{13}$$

The Correlation Coefficient of sub-array X_i to the parent array X_0 at time k, $r_i(k)$, is defined by the formula (Den, 1985) below:

$$r_i(k) = \frac{\delta_{\min} + \eta\delta_{\max}}{|x_0(k) - x_i(k)| + \eta\delta_{\max}}, \tag{14}$$

where η is the recognition coefficient of the range between 0 and 1, and usually takes on the value of 0.5; $\delta_{\min}$ and $\delta_{\max}$ are given by

$$\begin{aligned} \delta_{\min} &= \min_i\left(\min_k |x_0(k) - x_i(k)|\right) \\ \delta_{\max} &= \max_i\left(\max_k |x_0(k) - x_i(k)|\right) \end{aligned} \tag{15}$$

The above correlation coefficient $r_i(k)$, characterises the deviation degree between X_i and X_0 at time k. Summarising the deviation degrees between X_i and X_0 at all times, gives the correlation degree between X_i and X_0 as

$$R_i = \frac{1}{K}\sum_{k=1}^{K} r_i(k). \tag{16}$$

R_i in the above formula is referred to as the Correlation Measure (Den, 1985). The coefficient $r_i(k)$ and the correlation measure R_i satisfy the following relations:

$$0 \leq r_i(k) \leq 1 \quad \text{and} \quad 0 \leq R_i \leq 1. \tag{17}$$

The closer the relationship between X_i and X_0, or the more similar X_i is to X_0, the greater will be the associated Grey correlation measure. Only when both X_i and X_0 are completely superimposed will the correlation measure be equal to 1.

The main application of the Grey Correlation Analysis relevant to this paper is the provision of a comparative analysis between several proposed discontinuity spacing distribution, resulting in the identification of the best spacing distribution law fitting the measured discontinuity spacing data. In a simple clear-cut case, this analysis may be made visually. Situations in the real world are often more complicated. For example, when there are a large number of data points and the curve shapes are similar in some intervals, but different in others, the quantitative Grey Correlation Analysis can provide a useful statistical tool to obtain solutions to such problems as selecting best-fit discontinuity spacing distribution.

Compared to classical regression, the Grey Correlation Analysis has the following characteristics: there is no need for a large population, or a lot of sample data; sample data need not satisfy an explicit functional relation and calculation is simple and convenient.

To select a preferable distribution of discontinuity spacing from among several contenders, let the observed data of discontinuity spacings be represented by the parent array X_0 mentioned above, and let the values at corresponding observing points for the *ith* contending distributions be represented by X_i. The Grey Correlation Analysis can then compare and select a theoretical spacing distribution from among several contending distributions. Figure 6 is an example of spacing data from a highway cutting site that was analysed, using the Grey Correlation Analysis (Lu, 1997). The fitted fractal, negative exponential and Weibull distributions are compared with the actual measurements in Fig. 6. Obviously, it is not easy to visually isolate, from this figure, a preferred distribution among these three proposed distributions. Applying the Grey Correlation Analysis to this case, the Grey correlation measures of these three theoretical distributions, when correlated with the observed spacing data, are 0.810 for the fractal distribution, 0.782 for the Weibull distribution and 0.718 for the negative exponential distribution. The fractal distribution appears to be the most correlated with the observed data. Using the Kolmogorov-Smirnov test, both the fractal and Weibull distributions can be accepted at the level of significance of 0.15, while at this level the negative exponential distribution should be rejected. Thus, the Grey Correlation Analysis has helped us conclude that the fractal distribution is arguably the best choice for the spacing distribution of this set of discontinuities.

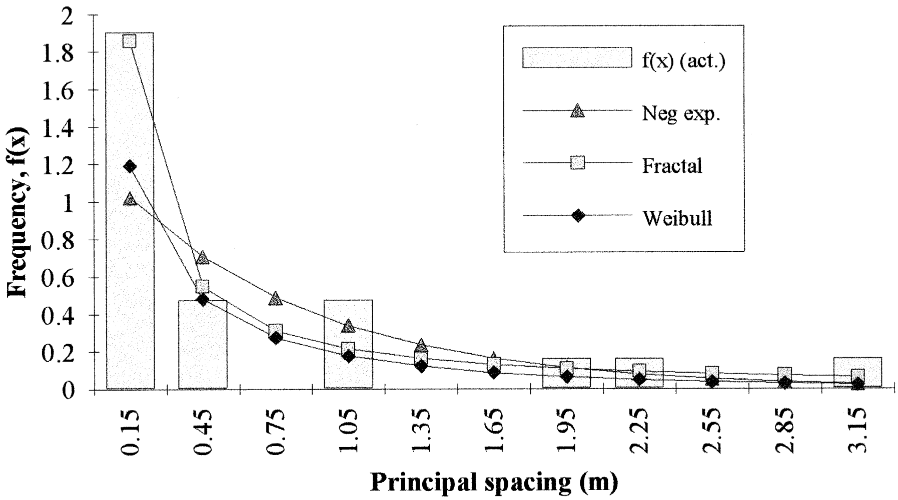

Fig. 6. Comparison between three proposed distributions and the measured distribution

4. Discussion

This paper has focused on discontinuity pattern assessment with application to IBSD prediction. Interest in IBSD is at the fringe of current work on discontinuity pattern modelling which, judging from the literature, are primarily concerned with flow studies and jointed rock stability studies (e.g. Dershowitz and Einstein, 1988; Itasca, 1992). The UDEC programs (Itasca, 1992), originating from the discrete element work of Cundall (e.g. Cundall and Strack, 1979), appear to provide the main source of rock block generator. However, as far as the authors are aware, the UDEC type block generators, though potentially suitable, have not been harnessed to measure the average block volumes and the IBSD of rock masses. To model the discontinuity network principally, so as to derive the IBSD and thereby provide the necessary input to predict the fragmentation of a blast more accurately, has been an aim of recent research (JKMRC, 1991; Wang et al., 1992; Lu and Latham, 1996; Aler et al., 1996). There follows a brief examination of the tools available to this group of blast fragmentation modellers, seeking estimation of the IBSD:

It seems to the authors that the stochastic network pattern (e.g., Aler et al., 1996) has the following features: any number of discontinuity sets with their specific geometric distributions can be superimposed to create the network of fractures and blocks for study, and the spacing and orientation distributions of each individual joint set are separately accounted for. However, the visual representation of the rock mass is a stochastic one so that joint locations bear no spatial relation to a fixed origin at a field site, although the general pattern may seem realistic. For blast modelling, this may be of little consequence. The accuracy will be poor if the theoretical best-fits cannot represent the measured spacing and trace length distributions. Expertise and a license to use sophisticated simulation software are required.

Alternatively, using the dissection method, the IBSD can be well determined for a blocky rock mass with the advantage that the visual representation of discontinuities bears the same positional relationships with respect to a chosen reference origin as would the discontinuities at the field site in question. The simulation will appear to be more realistic for a rock mass with planar persistent discontinuities. The drawbacks of the dissection method are that the influence of impersistence on IBSD is not satisfactorily included, and the implementation usually needs experience and may give long run times. The data acquisition from scanlines must be chosen optimally so as to best characterise the whole block volume. In addition, the rock mass simulation software is of restricted access.

Lastly, the updated equation method with the impersistence correction as presented in this paper is very simple to apply and there is no need for block generation software. However, a preliminary analysis of the raw geometric data is required. This preliminary analysis mainly involves selecting the best-fit laws of discontinuity spacing distributions, which can now include fractal distributions, and estimating the principal mean spacing. The Grey Correlation Analysis presented in this paper can help with the selection of the laws for discontinuity spacing distributions. Many types of scanline and area mapping surveys can be used for data acquisition.

However, questions with the updated equation method remain. Possible differences in dispersion of discontinuity orientation and type of spacing distribution within the three sets cannot be accounted for at present. Also, it is assumed that three discontinuity sets can adequately characterise the rock mass geometry. This approximation often holds in practice, and grouping techniques can be applied to achieve the best three-set description. It has been suggested that the index q in Eq. (9) take the value of 1/5–1/2. With IBSD results being sensitive to q, it is suggested that further calibration of q is needed.

Most approaches to block creation tend to fall down in regard to structural geological mechanisms. For example, it is the episodically evolving tectonic stress fields that create the networks of natural fractures and blocks in which conjugate shear fractures and extension joint systems, including terminations of fractures against other discontinuities, are commonplace. Stochastic models using data idealised from scanlines are not usually suited to modelling such features. The blocky rock mass generator of Heliot (1988) is one example where structural geological principles have been introduced.

5. Conclusions

Both background to IBSD assessment and a discussion of IBSD and blocky rock mass modelling have been presented. The very simple but relatively little known methods of providing look-up tables for plotting IBSD have been further documented here. These methods have been refined in two important ways: by including the fractal spacing distribution, which is an increasingly popular possible choice for the best-fit model; and by reporting the relative impersistence factor that compensates for the assumption, used in the formula calibration, that all discontinuities persist. Of more general interest, a novel approach to selecting the best-fit, when several theoretical discontinuity spacing distributions seem likely contenders, has been introduced.

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