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ORIGINAL PAPER

Support Type Influence on Rock Fracture Toughness Measurement Using Semi-circular Bending Specimen

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Abstract
In the present manuscript, the importance of support type on accurate determination of rock fracture toughness in semi-circular bending (SCB) test is demonstrated. First, by employing the finite element (FE) method, a detailed study is conducted to assess the effect of support friction on the mode I stress intensity factor (SIF) in the SCB specimen. Then, three different rocks and three various support types are utilized to investigate the support type effect experimentally. Both the FE and experimental results show that utilization of different types of supports results in different friction coefficients between the SCB specimen and the bottom supports. This can generate significant errors in calculations of SIF and fracture toughness of rock samples. It is shown that using the fixed or roller-in-groove support types can generate a large amount of error in rock fracture toughness calculations, although, these types of supports are still commonly being used by researchers.

Keywords Support friction · Rock · Stress intensity factor · Semi-circular bending · Fracture toughness

Abbreviations

S	Support span
a	Crack length
R	Radius of the specimen
P	External load
t	Specimen thickness
r	Radius of the support rollers
K_{I}	Mode I stress intensity factor
K_{Ic}	Mode I fracture toughness
Y_{I}	Normalized mode I stress intensity factor

1 Introduction

Researchers often investigate rock fracture mechanics from two different points of view. The first one is the application of fracture mechanics for the prevention of crack growth in rock structures where the safety and stability should be guaranteed, including mines and tunnels. While the second one is the usage of fracture mechanics for those engineering purposes requiring fracture generation such as hydraulic fracturing or excavation. Mode I fracture toughness, defined as material resistance against crack extension under static loading, is among the most important parameters in engineering projects dealing with rock fracture mechanics. Hence, the precise determination of mode I fracture toughness in rock materials is of utmost importance.

Up to now, several specimens and testing procedures have been proposed by researchers to measure the mode I fracture toughness of rocks. These specimens and testing procedures can be divided into two groups. The first group is the testing methods which exert the mode I loading to the specimen through tensile or compressive loading and the second group is the methods providing the mode I situation by applying three- or four-point bending. Some examples in the first group are mentioned here. The first example is the short rod specimen proposed by Barker (1977) which was later approved by the international society of rock mechanics (ISRM) as an appropriate specimen for mode I fracture toughness determination in rock materials (Ouchterlony 1988). Cracked chevron notched Brazilian disk under diametrical compression is another specimen which has been suggested by ISRM (Fowell 1995). There are also other specimens for fracture toughness measurement of rocks by

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exerting compressive or tensile loadings, e.g., (Aliha et al. 2008; Tutluoglu and Keles 2012).

The second group belongs to the specimens which are subjected to pure bending. As a specimen recommended by ISRM for fracture toughness measurement of rocks (Ouchterlony 1988), the chevron bend provides mode I loading by applying three-point bending. Edge-cracked triangular specimen under three-point bending is another example which was proposed by Aliha et al. (2013). There are also other specimens in the literature for measuring the mode I fracture toughness by exerting bending loading (Tutluoglu and Keles 2011; Ayatollahi et al. 2016). The most recent method proposed by ISRM for fracture toughness measurement of rocks is the semi-circular bending (SCB) specimen (Kuruppu et al. 2014). Easy preparation, core-based specimen geometry, and simple fixture requirements are among the advantages of this specimen. The SCB specimen has been recognized as a favorite specimen for fracture toughness measurement, especially in recent years. Therefore, evaluating the errors that can affect the experimental results obtained from the SCB specimen is of paramount importance.

A brief review of literature shows that different types of supports have been used by researchers when the bending specimens are used for measuring the fracture toughness. For example, fixed supports have been utilized in Aliha and Bahmani (2017), roller-in-groove-type supports which cannot roll have been employed in Amaral et al. (2008) and Wei et al. (2018a) and roller-type supports with free rolling have been utilized by some other researchers (Wei et al. 2018b). The same groups of support types have been used for the SCB testing procedure too. For instance, there are some investigations on the SCB specimen which have utilized a roller-in-groove type of support that cannot roll freely (Dai et al. 2010, 2013; Funatsu et al. 2014; Yin et al. 2016; Yang et al. 2018; Yao et al. 2018; Li et al. 2018). Meanwhile, some other investigations have utilized freely rolling supports (Wei et al. 2016a, b; Gong et al. 2018) in their SCB experiments. The wide range of different support types used in various research studies makes it necessary to study the possible effects from the type of support on the fracture toughness measurement procedure.

In the present investigation, the influence of support type on the mode I stress intensity factor and fracture toughness of rock in the SCB specimen testing is studied using the finite element (FE) method and an experimental approach. The results show that the utilization of different support types leads to different friction coefficients between the SCB specimen and the supports which have a significant effect on the stress intensity factor and the value of fracture toughness measured for rocks. In the following, first, the FE method is used to investigate the influence of support friction on the mode I stress intensity factor in the SCB specimens. Then, the experimental procedure for rock fracture toughness determination is explained. Different rocks with various types of supports are considered in the experimental section. Finally, the results obtained from the numerical modeling and the experimental procedure are presented and discussed.

2 Finite element modeling

Figure 1 shows a schematics of the specimen and its geometrical parameters in which R is the radius of the specimen, a is the crack length, t is the specimen thickness, S is the distance between the bottom supports (span length), r is the radius of the support and P is the applied load. For all FE analyses, the values of R, t and P were constant and equal to $R = 75$ mm, $t = 30$ mm and $P = 1000$ N; while the other parameters varied in different sets of analyses. To model the boundary conditions of the SCB specimen, the two bottom supports were completely fixed. Also, the top support through which the external load is applied was considered to be solely moving in the vertical direction. Each of the three supports was modeled as a rigid part and the frictional contact was regarded between the supports and the sample. Then, arbitrary values of ($E = 16.5$ GPa) for elastic modulus and ($\nu = 0.25$) for Poisson's ratio were considered as the basic mechanical properties of modeled rocks (Wei et al. 2015). Meanwhile, to simulate the SCB samples, eight-node biquadratic quadrilateral elements with plane stress conditions were used. To determine the mode I stress intensity factors, the J-integral approach was employed and the results of path-independent J-integral converged after 3 or 4 contours. Moreover, to investigate the convergence of the numerical results against the mesh size, different mesh sizes around the crack tip were

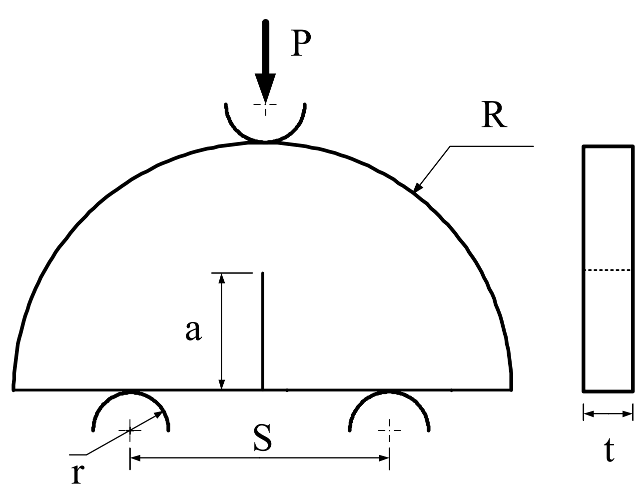


Fig. 1 Schematic view of the SCB specimen and its related parameters

considered and the results demonstrated very good convergence, with the variations of the obtained SIFs being below 0.1%. Figure 2 illustrates a sample of mesh pattern used for the cracked SCB specimens.

In the first set of FE analyses, the effect of support friction on the mode I SIF was investigated for different values of support span (S). To do so, four different normalized support spans ($S/2R = 0.5$, 0.6, 0.7 and 0.8) with 11 various friction coefficients ($\mu = 0, 0.1, 0.2, \ldots, 1$) were considered. Meanwhile, the crack length ratio was constant and equal to $a/R = 0.467$. Furthermore, for the second set of FE analyses, the support span was equal to $S/2R = 0.5$ and the influence of support friction for four different crack length ratios ($a/R = 0.4$, 0.467, 0.533 and 0.6) on the mode I SIF was assessed. Different values of parameters used in these two sets of FE simulations are summarized in Table 1. It should be noted that the parameters of Table 1 are within the range recommended by the ISRM standard for SCB testing (Kuruppu et al. 2014).

In this section, a detailed description of the SCB specimen dimensions and its FE simulation was presented. The results obtained from these simulations are presented later in sect. 4. In the next section, the experimental procedure is elaborated.

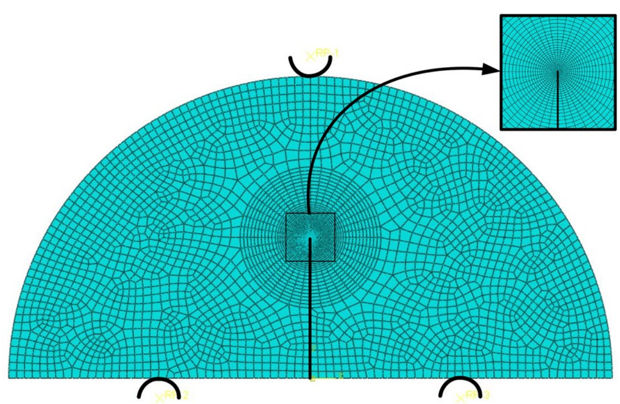


Fig. 2 The mesh pattern of the SCB specimen simulated in FE analysis

3 Experiments

In the first part of this section, the rock materials used for the fracture experiments are introduced. Then, the dimensions of SCB specimens and different types of supports which have been utilized in the experiments are presented. Finally, the experimental procedure for measuring the fracture toughness is elaborated.

3.1 Materials

Three different types of rocks were considered to perform a detailed experimental study on the effect of support type on rock fracture toughness measurement. The tested rocks were: green granite (extracted from rock mines in the northeast of Iran), the pink granite (excavated from northwest of Iran) and the white marble (extracted from a mine in the west of Iran). Figure 3 shows the SCB specimens prepared from these three types of rocks.

3.2 Specimen Dimensions

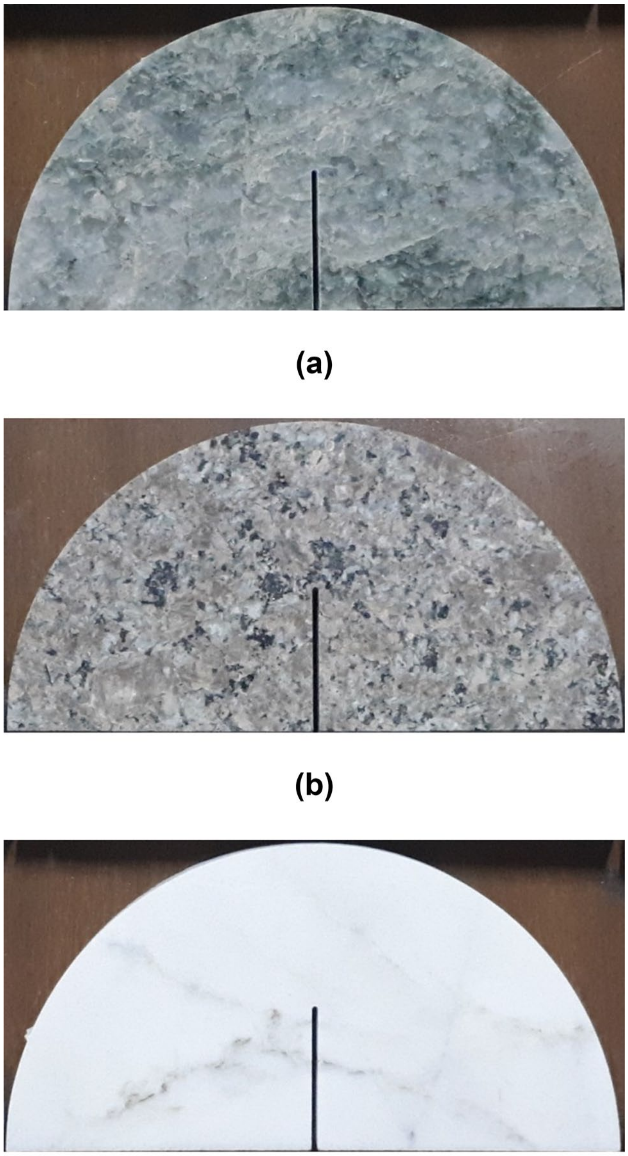
To prepare the SCB specimens, water jet machine was used. Similar to the dimensions considered for FE simulations, the following dimensions were used for preparing the rock samples: the specimen radius (R) was equal to 75 mm, the crack length ratio (a/R) was 0.467 and the thickness of specimen (t) was 18 mm.

3.3 Support Condition

To investigate the effect of support type on the measured value of fracture toughness, three support conditions were considered in the experiments. The first support type was two cylindrical rollers that could freely roll to simulate almost frictionless conditions, as shown in Fig. 4a. The second type of support consists of two cylindrical rollers trapped inside two grooves (see Fig. 4b) and the third one was the fixed-type supports made from steel as displayed in Fig. 4c. Figure 5 shows another example of the test setup corresponding to the roller-in-groove type of supports, as presented in

Table 1 Different values of parameters used for FE analysis

Parameters	Values for modeling different support lengths (set 1)	Values for different crack lengths modeling (set 2)
Normalized support span ($S/2R$)	{0.5, 0.6, 0.7, 0.8}	0.5
Normalized crack length (a/R) (mm)	0.467	{0.4, 0.467, 0.533, 0.6}
Radius of the specimen (R) (mm)	75	75
External load (P) (N)	1000	1000
Specimen thickness (t) (mm)	30	30
Radius of the support rollers (r) (mm)	5	5



(a)

(b)

(c)

Fig. 3 Different types of rocks. **a** green granite, **b** pink granite and **c** white marble

ISRM standard (Kuruppu et al. 2014) for fracture toughness testing on rocks. It is worth noting that the support span was equal to $S/2R = 0.5$ in all of the experiments.

3.4 Determination of Fracture Toughness

The tests were conducted using STM 150 universal testing machine (SANTAM®, Iran) with a fixed loading rate of 0.1 mm/min. To check the repeatability of the results, three tests were performed for each material and each type of support. Therefore, a total number of 27 fracture tests were conducted, the results of which are presented and discussed in Sect. 4.

4 Results and Discussion

Figure 6a shows the variations of normalized mode I SIF (K_I) for the SCB specimens in terms of the friction coefficient for different support spans ($S/2R = 0.5$, 0.6, 0.7 and 0.8) based on the FE results. As mentioned before, the crack length ratio in this set of FE analyses was the fixed value of $a/R = 0.467$. Moreover, Fig. 6b demonstrates the variations of normalized K_I for four different crack length ratios ($a/R = 0.4$, 0.467, 0.533 and 0.6) and a constant support span of $S/2R = 0.5$ under various values of friction coefficient. It should be mentioned that the SIF normalizing relation used in this study is similar to the one proposed by ISRM standard as (Kuruppu et al. 2014).

$$Y_I = \frac{2RT}{P\sqrt{\pi a}} K_I, \tag{1}$$

where Y_I is the normalized value of K_I. It is useful to remind that the Y_I values given along the vertical axis in Fig. 6 correspond to the cases of no friction for the presented curves. Then, by increasing the friction coefficient, Y_I decreases until it reaches a constant magnitude. The reason for this behavior is that the friction forces between the supports and the SCB specimen resist against the opening of the crack faces. Therefore, if the magnitude of such opposing forces increases, the mode I SIF decreases. Examining Fig. 6a more thoroughly, it can be said that as the support span increases, the error generated due to neglecting the friction effect decreases. To corroborate the mentioned conjecture, one can consider the below relation as the percentage error resulting from neglecting the effect of support friction in mode I SIF calculation.

$$\text{Error\%} = \frac{Y_{\text{I,non-frictional supports}} - Y_{\text{I,fricional supports}}}{Y_{\text{I,non-frictional supports}}} \tag{2}$$

In the above relation, $Y_{\text{I,non-frictional supports}}$ is the magnitude of Y_I when no friction exists between the supports and the SCB specimen and $Y_{\text{I,fricional supports}}$ is the magnitude of Y_I in the case where the support friction exists. According to Eq. (2), for the SCB specimen with $S/2R = 0.5$ and the friction coefficient of 0.4, neglecting the friction effect generates 83% error in SIF calculation; while for $S/2R = 0.8$ and friction coefficient of 0.4, the error is about 45% (see Fig. 6a). On the other hand, it can be concluded from Fig. 6b that the support friction results in almost the same amount of errors for different crack lengths. For instance, in the case where the crack length ratio is $a/R = 0.6$ and the friction coefficient

Fig. 4 Different support types, **a** roller support, **b** roller in groove support, **c** fixed support

is 0.4, neglecting the support friction effects leads to an error of 83% in SIF determination, while the same condition for *a/R* = 0.4 leads to 81% error.

As mentioned in the previous section, after performing a numerical study, some experiments were also conducted on the SCB specimens made from 3 different types of rocks. Table 2 demonstrates the fracture load obtained for each rock and support type.

It is seen from Table 2 that the support type has undeniable effects on the fracture loads obtained from the SCB tests on rocks. According to Table 2, the highest error due to the friction effect belongs to the case of pink granite when the fixed supports are used. It should be noted that the fracture loads obtained for the pink granite with roller-in-groove supports are close to the fracture loads in the case of fixed supports showing that the friction coefficient between the SCB and the supports in these two conditions is almost equal. Similar observations can be made for the green granite and white marble. In the following, the effect of support friction on the calculated values of fracture toughness is investigated.

To find the fracture toughness of SCB materials, typically two different methods have been used in the literature. The first one is using the analytical relation proposed by ISRM as (Kuruppu et al. 2014).

$$K_{IC} = Y_I \frac{P_{max}\sqrt{\pi a}}{2Rt}, \tag{3a}$$

$$Y_I = -1.297 + 9.516(s/2R) - (0.47 + 16.457(s/2R))(a/R) + (1.071 + 34.401(s/2R))(a/R)^2, \tag{3b}$$

where P_{max} is the fracture load determined from the tests. The second approach is to use the FE method to derive the critical SIF corresponding to the fracture load. The following procedure has been commonly used in different papers to derive the fracture toughness from the FE simulations. A node corresponding to one of the bottom supports is fixed in the both vertical and horizontal directions; while the node corresponding to the other bottom support is fixed vertically only in the loading direction. With this arrangement, by increasing the bending load, one of the supports can move freely in the horizontal direction. Then, the fracture load obtained from the experiment is applied to the simulated specimen and its corresponding SIF is considered as the material fracture toughness. It should be noted that in both analytical (Eq. 3a) and FE approaches described above, the fracture toughness is determined without considering the effect of friction between the SCB rock specimen and

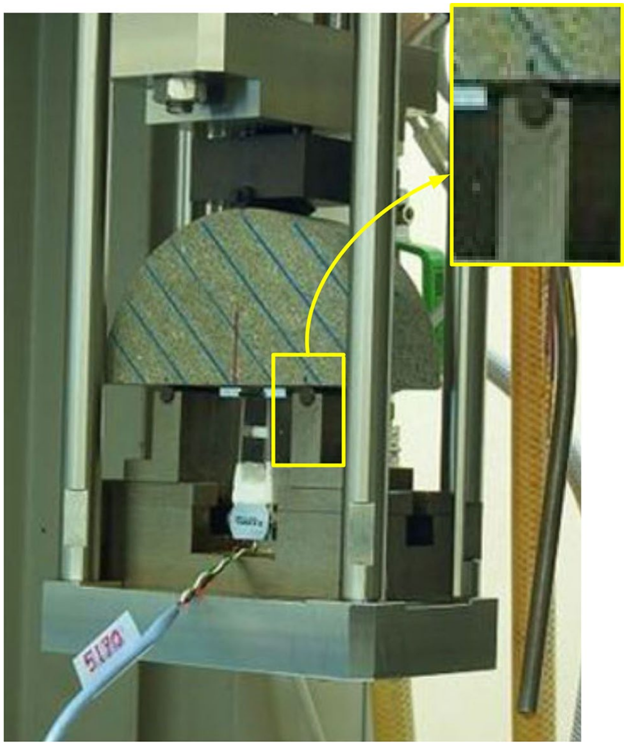


Fig. 5 A sample test setup corresponding to roller-in-groove type of support presented in Kuruppu et al. (2014)

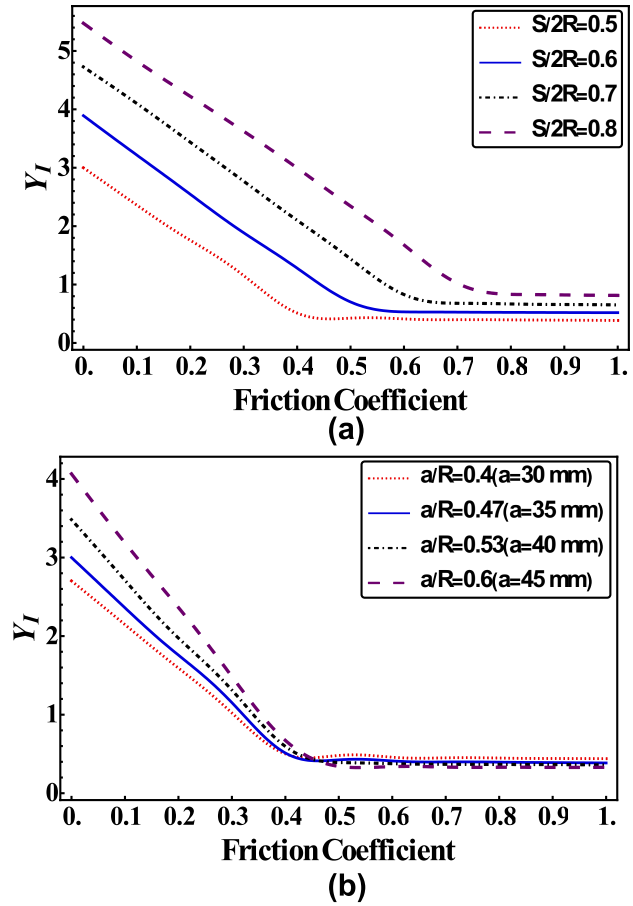


Fig. 6 The variation of normalized K_I versus the friction coefficient. **a** Different support span lengths (*S*) for $a = 35$ mm, **b** Different normalized crack lengths (*a/R*) for $S/2R = 0.5$

the supports. Table 3 demonstrates the values of fracture toughness calculated for the green granite, pink granite and white marble obtained for different types of supports using Eq. (3a) and FE method.

As shown in Table 3, both the FE simulations and Eq. 3a yield similar results for each material and support type which confirms the accuracy of both methods. Furthermore, it can be seen that the existence of friction between the supports and the specimen affects significantly the values of fracture toughness determined from the SCB testing. For example, in the case of pink granite, using the fixed supports generates 85.2% error in the fracture toughness measurements. Note that in Table 3, the condition corresponding to the roller type of support has been considered as the reference value in calculating the error percent for each type of rock. To avoid inaccurate results and to obtain a reliable value for fracture toughness of rocks using the SCB specimen, two options can be suggested. The first and the more convenient one is to use the roller supports which can easily and freely roll during the test procedure (see Fig. 3a). If the roller-type supports are not available, it is suggested to determine the magnitude of friction coefficient between the SCB specimen and the supports and then to consider the corresponding contact conditions in the FE simulation. It is clear that the second option is more complex and the reliability of its results is very much dependent on the accuracy of the measured friction coefficient between the supports and the test sample.

Table 4 presents the calculated values of friction coefficients between the rock specimens and the roller-in-groove and fixed types of supports in the conducted experiments. To obtain the friction coefficients reported in Table 4, the following procedure was adopted. First, the calculated value of fracture toughness presented in Table 3 for the roller-type support was considered as the reference fracture toughness for each type of rock. Then, the average values of fracture loads presented in Table 2 for the roller-in-groove and fixed-support types were exerted to the SCB specimen simulated in the FE code. Finally, the friction coefficient between the specimen and supports was varied until the mode I stress intensity factor in the simulated SCB specimen became equal to the reference fracture toughness.

As described above, a correct value of fracture toughness can be measured for rock samples when the roller-type supports are used, i.e., when the friction between the supports

Table 2 Fracture loads of SCB specimens for different support types and rocks

Material	Support type	P_1 (*N*)	P_2 (*N*)	P_3 (*N*)	P_{mean} (*N*)	Error %
Green granite	Roller	4262	4475	3921	4219	–
	Roller in groove	6133	6101	6439	6224	47.6
	Fix	6237	6861	5863	6320	49.8
Pink granite	Roller	2382	2386	2243	2337	–
	Roller in groove	4011	4131	3690	3944	68.8
	Fix	4156	4530	3990	4225	80.8
White marble	Roller	2036	2117	1914	2022	–
	Roller in groove	2440	2684	2318	2481	22.7
	Fix	2630	2709	2420	2586	27.9

Table 3 Fracture toughness values (MPa. mm$^{0.5}$) determined for different types of supports using FE simulation and also Eq. 3a

Material	Support type	FE simulation	Equation (3a)	Error%
Green granite	Roller	54.67	55.36	–
	Roller in groove	80.66	81.72	47.6
	Fix	81.88	82.98	49.8
Pink granite	Roller	30.28	30.69	–
	Roller in groove	51.09	51.79	68.7
	Fix	54.73	55.47	80.7
White marble	Roller	26.20	26.55	–
	Roller in groove	32.15	32.58	22.7
	Fix	33.51	33.95	27.9

Table 4 The values of friction coefficients estimated between different types of supports and the pink granite, green granite and white marble

Material	Support type	Friction coefficient
Green granite	Roller	0
	Roller in groove	0.13
	Fix	0.14
Pink granite	Roller	0
	Roller in groove	0.19
	Fix	0.22
White marble	Roller	0
	Roller in groove	0.085
	Fix	0.11

and the SCB specimen is almost zero. Therefore, for nonzero friction coefficients, there would be an error in the measured value of fracture toughness if the conventional calculation procedures which ignore the effects of support friction are employed. This error can be estimated for different values of friction coefficients by finite element simulation of each rock sample. To this end, first, the fracture load obtained experimentally from the roller-type support should be applied to the FE model of the specimen without considering friction. Then, the corresponding value of the SIF for each rock is considered as its fracture toughness. In the next step, for each value of friction coefficient in the FE simulation, the same load is applied and increased gradually until the SIF in that model reaches the material fracture toughness. Then, the below relation can be used to estimate the error due to neglecting the friction effects based on the FE results:

$$\text{Error\%} = \frac{P_{\text{fricional supports}} - P_{\text{non-frictional supports}}}{P_{\text{non-frictional supports}}}, \quad (4)$$

where $P_{\text{non-frictional supports}}$ is the load exerted to the SCB specimen to measure the material fracture toughness when no friction exists between the specimen and the supports and $P_{\text{frictional supports}}$ is the same parameter when friction is considered in the simulations.

As can be seen from Tables 3 and 4, the values of rock fracture toughness determined from SCB fracture tests using the roller-in-groove or fixed supports are always significantly higher than the correct value of fracture toughness obtained using the roller-type support. This is the same for all the three types of rocks tested in this study, although the highest increase occurs for the pink granite and the lowest increase for the white marble. There are a number of parameters that can affect the friction coefficient between the supports and the SCB specimen. The most important parameter can be the roughness of bottom surface in the SCB specimen. If the SCB specimen is cut by a technique which leaves very rough surface, the friction coefficient becomes larger and hence the value of fracture toughness measured with the fixed or roller-in-groove supports possesses larger errors.

In addition to the surface roughness due to the cutting method used, the microstructural features (e.g., the rock grain size, etc.) can also influence the friction coefficient in the bottom supports and give less accurate result for the calculated fracture toughness. Therefore, it is important to use an appropriate cutting method followed by a fine grinding to make the surface as smooth as possible. Although smooth surfaces can reduce the friction coefficient between the rock samples and the bottom supports, it is strongly recommended to make use of the roller-type supports instead of the roller-in-groove or fixed supports. Indeed, the roller-type supports generate very little friction forces because they can freely roll when the crack mouth gradually opens during the SCB test on rock samples. It is necessary to highlight that the values of fracture toughness which are measured in a SCB test using the roller-in-groove or fixed supports always overestimate the real fracture toughness of rock samples. For those rock engineering projects in which the stability and sustainability of the cracked rock structures are of paramount importance, the use of an overestimated fracture toughness would be dangerous and certainly unacceptable.

Finally, it is worth mentioning that the ISRM suggested method for measuring rock fracture toughness using the SCB specimen (Kuruppu et al. 2014) urges the use of roller-type supports in the SCB test configuration. However, as reviewed earlier in this paper, there are numerous articles published even in recent years where instead of the roller-type supports, the roller-in-groove or the fixed supports have been employed in the SCB testing, probably assuming that there is no significant difference between the values of fracture toughness obtained from any of these three options for the bottom supports. However, the results achieved in this paper underlined the importance of using roller-type supports in the SCB testing procedure.

5 Conclusions

The semi-circular bend (SCB) specimen was simulated by finite element method to study the effect of friction coefficient between the specimen and the bottom supports on the values of mode I stress intensity factor. It was found that there is a significant reduction in the mode I SIF of SCB specimen when the friction coefficient increases. Then, the SCB specimens prepared from three different rocks were tested and the values of fracture toughness were determined for each rock when three different types of supports were used in the test setup, i.e., roller, roller-in-groove and fixed supports. The experimental results revealed that the values of fracture toughness calculated for the cases of roller-in-groove and fixed supports overestimate the real fracture toughness of all three rocks significantly. It was finally shown that for achieving an accurate value for rock fracture toughness from the SCB testing procedure, the roller-type supports must be used.

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Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest.

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