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ORIGINAL PAPER

Stability Analysis of Slopes with Spatially Variable Strength Properties

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Abstract

Natural variability of rock properties can significantly affect the strength of rock masses and factor of safety of slopes. The results of a comprehensive point load testing program showed that coefficient of variation of intact rock strength can reach unity in highly heterogeneous formations. Probabilistic numerical analysis was carried out to explore the effect of strength variability on uniaxial compressive strength of large heterogeneous samples. It was shown that mean large-scale strength decreases with increasing small-scale variability. The effect of spatial variability of strength properties on slope stability was examined using limit equilibrium and shear strength reduction methods. Both approaches gave similar results indicating that for stable slopes, increasing strength variability leads to a reduction in mean factor of safety and increase in the probability of failure. In addition, ignoring spatial variability in probabilistic slope analysis can lead to erroneous estimates of the probability of failure. Based on the results of probabilistic analyses on large heterogeneous samples and slopes, an equivalent uniaxial compressive strength can be obtained by reducing the mean strength by one-third of its standard deviation. This relationship was validated using a dataset of back-analyzed strength values in heterogeneous open pit slopes.

Keywords Probabilistic analysis · Strength variability · Heterogeneous sample · Equivalent strength · Limit equilibrium · Strength reduction

1 Introduction

Slope stability analysis has long been a subject of interest in geotechnical engineering and numerous methods of analysis have been introduced over the years. Yet, design of technically operational and economically optimal slopes remains a challenge in many civil and mining projects. A significant contributing factor is the inherent complexity of geological materials and structures (e.g., Andriani and Parise 2015). Such complexity makes it difficult to develop a representative model which accurately captures the important details such as location and geometry of different geotechnical units and behavior of materials under different hydro-mechanical conditions. Furthermore, properties of a given geotechnical unit are not constant and vary by location. This is evident by the fact that results of a series of standard tests on samples obtained from different locations inside a given geological unit are different indicating spatial variability of properties. Extreme variability can be observed in large copper and gold-rich porphyry deposits formed through alteration and mineralization processes (Sillitoe 1997). This poses a major challenge in determining representative properties for open pit slope stability analysis in such deposits.

Conventional slope stability analysis is based on the factor of safety defined as the factor by which the shear strength must be divided to bring the slope to the verge of failure. To account for strength variability and model uncertainty, a conservative deterministic factor of safety is used to provide a sufficient margin of safety in case actual slope condition (geology, shear strength, ground water, loading, support, etc.) is less favorable than that assumed in the design. The main problem with this approach is that there is no clear scientific basis for choosing acceptable levels of factor of safety in different projects. Hence, deterministic approach is vulnerable to subjectivity and the design is often too conservative and occasionally unsafe.

Probabilistic analysis offers a more effective approach by explicitly taking variability into account. In a probabilistic analysis, material properties are treated as random variables

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with statistical distributions estimated from characterization and testing programs. By analyzing a sufficient number of models with random properties following the established distributions, it is possible to generate a distribution of factor of safety instead of a single deterministic value. This allows the designer to explicitly evaluate the chances of factor of safety falling below an acceptable level and provides a strong basis for design of optimal slopes using risk analysis.

Early attempts in probabilistic slope stability analysis were made by Alonso (1976), Tang et al. (1976) and Harr (1977). Priest and Brown (1983) presented a practical approach for probabilistic stability analysis of rock slopes along with design acceptance criteria. The benefits of probabilistic analysis were further illustrated by Whitman (1984), Christian et al. (1994) and Wolff (1996). El-Ramly et al. (2002) presented an improved approach for probabilistic slope stability analysis in practice. All probabilistic analyses up to this point were based on the Limit Equilibrium (LE) method which requires assumptions regarding the geometry of slip surface and interslice forces. Dawson et al. (1999) and Griffiths and Lane (1999) used the Shear Strength Reduction (SSR) method proposed by Zienkiewicz et al. (1975) and applied it to slope stability analysis. Unlike LE method, the SSR method is based on a full stress–strain analysis which allows the natural formation of slip surface. Griffiths and Fenton (2004) carried out probabilistic analysis using the SSR method on an undrained clay slope. Hammah et al. (2009) used the SSR method to explore the effect of variability in geometry and orientation of joint network on stability of rock slopes. Chiwaye and Stacey (2010) applied a response surface to the results of the SSR method and subsequently used it for risk analysis of an open pit slope. The effect of spatial variability on slope stability was investigated by Jefferies et al. (2008), Griffiths et al. (2009), Srivastava (2012), Allahverdizadeh et al. (2015) and Javankhoshdel et al. (2017). Rafiei Renani et al. (2018) compared the results of probabilistic slope analysis using the LE and SSR methods.

In this study, a probabilistic approach is adopted to explore the effect of spatial variability of strength properties on large-scale strength of heterogeneous rock masses. A cohesive framework is presented for incorporating rock mass heterogeneity in slope stability analysis using the LE software, *SLIDE* (Rocscience Inc 2018) and the SSR method in the finite difference code, *FLAC3D* (Itasca Inc 2012). Using the results of deterministic and probabilistic analyses, a relationship is developed to estimate the equivalent strength of heterogeneous rock masses.

2 Statistical Description of Variability

To explicitly take variability into account in a probabilistic analysis, model parameters are treated as random variables with specified statistical distributions. For a random variable, the mean is the statistically expected value and standard deviation shows the extent of variability and dispersion around the mean value. Variability can also be expressed in terms of the dimensionless coefficient of variation, COV defined as the ratio of standard deviation to the mean.

Probability density function specifies how data are scattered around the mean. Normal distribution is perhaps the most commonly used distribution in probability theory. It is characterized with a bell shaped symmetrical distribution around the mean and closely captures the distribution of many random variables. Due to the symmetrical shape, however, it can produce negative values for the random variable, especially when the coefficient of variation is high.

Since many geotechnical properties cannot assume a negative value, an asymmetric distribution which can only produce non-negative values is preferred. Lognormal distribution, which is the distribution of a variable whose logarithm is normally distributed, only produces non-negative values. It closely captures the observed variability of many geotechnical properties and is extensively used in probabilistic slope stability analysis (e.g., Parkin and Robinson 1992; Nour et al. 2002; Griffiths and Fenton 2004). Weibull distribution is another asymmetric function producing non-negative values which is widely used in statistical analysis of rock failure (e.g., Wong et al. 2006; Amaral et al. 2008; Krumbholz et al. 2014).

In the absence of sufficient data to establish a full statistical distribution, a triangular density function may be adopted by specifying the minimum, maximum, and the most likely value of a property. Empirical guidelines, past local experience and expert opinion can be used to define triangular distributions of geotechnical properties (El-Ramly et al. 2002). Figure 1a shows the probability density functions for a random variable with a mean of 100 and COV of 0.5. While the normal distribution extends into negative values, the Weibull and triangular functions start at zero with similarly high slopes. It can be observed that the lognormal distribution is more concentrated in the intermediate range and is less likely to produce near zero values.

In addition to the statistical distribution of the values of a geotechnical property, it is important to consider how those values are distributed in space. In a given geotechnical unit, the values of a property obtained from samples in close proximity are expected to be closer than those obtained from samples farther apart. In statistical terms, as the distance between sampling locations increases the correlation between the obtained values of a property decreases. This phenomenon can be described using spatial correlation functions such as the Markovian function:

$$\rho = \exp\left(-2\frac{\delta}{L}\right), \tag{1}$$

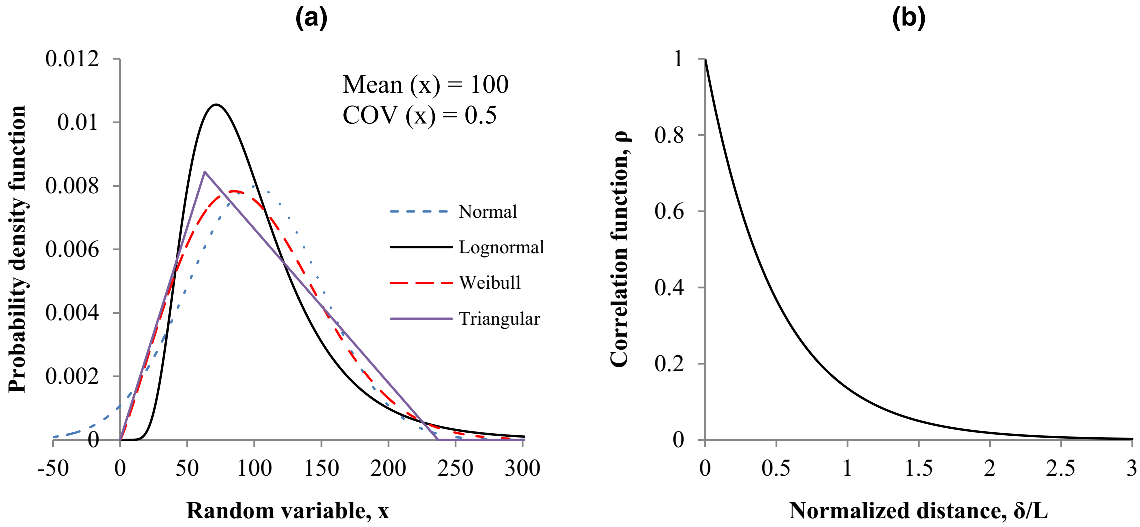


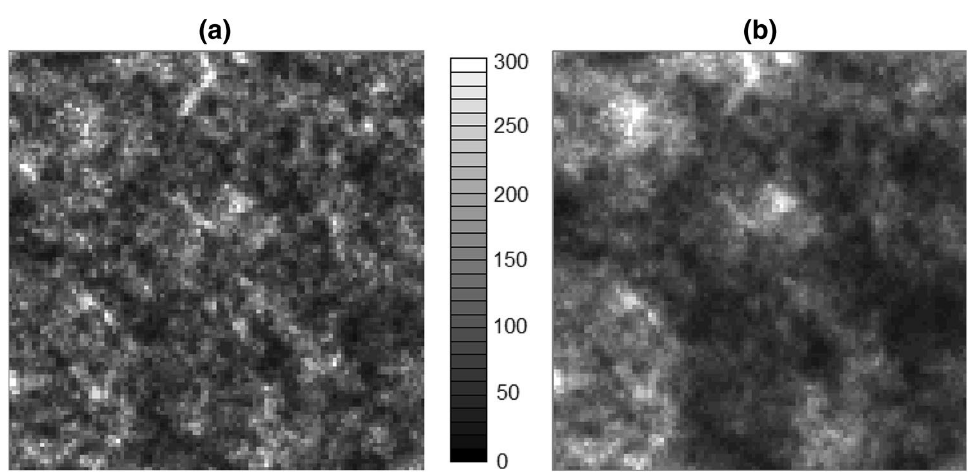
Fig. 1 Common functions for describing: **a** probability distribution and **b** spatial correlation structure

where ρ represents the correlation between the values of a property sampled at a distance δ in a random field with spatial correlation length of L. As shown in Fig. 1b, the spatial correlation length may be interpreted as the distance beyond which the correlation is weak and the values are almost independent. Random fields following the Markovian spatial correlation structure are commonly generated using the local average subdivision method (Fenton and Vanmarcke 1990) and used in probabilistic analysis of geostructures (e.g., Griffiths and Fenton 2001, 2004, 2007; Allahverdizadeh et al. 2015; Javankhoshdel et al. 2017). The same approach is adopted in the analyses presented in this study. A more detailed discussion on statistical characteristics and methods of generation of spatially correlated random fields is beyond the scope of this study and is given by Vanmarcke (1980, 1983), Fenton and Vanmarcke (1990), El-Ramly et al. (2002), Griffiths and Fenton (2001, 2004, 2007), Griffiths et al. (2009) and Srivastava (2012).

Figure 2 shows the spatial distribution of a random variable with identical lognormal statistics (mean of 100 and COV of 0.5) over a 100 × 100 m area using different correlation lengths of 5 m and 20 m. While the smaller correlation length corresponds to a highly erratic spatial distribution with significant changes of variable over short distances, the higher correlation length leads to a higher degree of spatial continuity.

Quantifying the spatial correlation length requires sampling and testing at close spacing, seldom feasible in practice (El-Ramly et al. 2005). In the absence of site specific data, the values reported for similar geological units may be used as a first estimate and the influence of correlation length on the design can be investigated through sensitivity analysis. The spatial correlation length for geomaterials reported in

Fig. 2 Spatial distribution of a variable with a lognormal distribution, mean of 100 and COV of 0.5 over a 100 × 100 m area using: **a** correlation length of 5 m and **b** correlation length of 20 m



the literature is typically a few meters and can reach tens of meters (Phoon et al. 1995; Phoon and Kulhawy 1999; El-Ramly et al. 2005).

3 Rock Strength Variability at Small Scale

Geomaterials are heterogeneous in microscopic scale. Even the most visibly uniform rocks are composed of numerous minerals, grains, cements, and voids of varying shapes and sizes. As a result, carrying out a series of standard test on carefully prepared samples of a macroscopically uniform rock provides a range of outcomes. While more pronounced in natural materials, strength variability is also observed in manufactured materials. Ellingwood et al. (1980) indicated that strength variability with coefficients of variation of up to 0.14, 0.15, and 0.21 can be observed in aluminum, steel, and concrete, respectively. Phoon et al. (1995) showed that coefficients of variation of effective friction angle and undrained shear strength of clay can reach up to 0.50 and 0.55, respectively.

In massive uniform geological units, strength variability of rocks may be modest making the determination of representative strength a relatively straightforward task. On the other hand, considerable variability may be observed in rock masses located in complex geological settings experiencing alteration, mineralization, and fault activity. For example, El-Ramly et al. (2005) reported the results of tests on a highly decomposed granite in which coefficient of variation of effective cohesion was as high as 1.0. Such conditions can also be found in large copper and gold-rich porphyry deposits due to alteration and mineralization events (Sillitoe 1997). Figure 3 shows the cores obtained from a gold-rich porphyry deposit. Determining a representative value of intact rock strength for slope stability analysis in such highly variable rock masses poses a major design challenge.

Quantifying the extent of strength variability is the first step in establishing a representative strength. Small-scale variability of rock strength may be explored using a series of standard uniaxial and triaxial compression tests. However, it should be noted that due to strict sample preparation requirements, such standard tests may only be carried out on larger and stronger pieces of rock obtained from a core. For example, significant portions of the cores shown in Fig. 3 do not meet the size and shape requirements of a standard uniaxial compression test. Typical exclusion of the weaker portions of rock from standard testing introduces a bias in statistical analysis leading to overestimating the mean strength and underestimating the coefficient of variation.

In an attempt to investigate the true distribution of rock strength, unaffected by sampling bias, Kostak and Bielenstein (1971) carried out an extensive testing program on 420 samples of Matinenda sandstone obtained from uniformly

Fig. 3 Cores from a gold-rich porphyry deposit with significant variability

distributed locations over the investigated volume of rock. Through careful drilling and sampling at predetermined regular intervals, they were able to obtain an unbiased sample including original defects in correct proportion to sound core. Statistical analysis of their test results showed that the mean strength obtained by typical exclusion of defective samples is overestimated by 10%, but more importantly, the obtained coefficient of variation is almost half the true value.

Since performing such an extensive direct testing program is not practical in most projects, carrying out indirect tests such as the point load test on core samples at regular intervals may provide an unbiased estimate of the strength distribution. Not only the point load test is quicker and easier to perform than standard uniaxial compression test, but it can also be carried out on small irregular shaped pieces of rock which would otherwise be excluded from testing.

In this study, a comprehensive point load testing program was carried out to quantify strength variability in highly heterogeneous porphyry deposits. Test samples were obtained from 54 mm NX cores and tested according to the ISRM suggested method (ISRM 1985). The tests were carried out on 446 samples of altered porphyry chlorite, 364 samples of altered primary sulfide, 95 samples of altered secondary sulfide and 51 samples of altered granodiorite. The values of point load strength index were used to estimate uniaxial compressive strength using the correlation recommended by Bieniawski (1975) and ISRM (1985). Figure 4 shows the strength histograms obtained from the point load tests on porphyry samples. It can be observed that lognormal distribution can provide a reasonable fit to the histograms of rock strength in porphyry deposits.

The results of tests on a wide range of rocks published in literature (Kostak and Bielenstein 1971; Martin 1993; Medhurst and Brown 1998; Ruffolo and Shakoor 2009; Glamheden et al. 2010; Azimian 2017; Cui et al. 2017) were compiled to establish a possible range for the coefficient

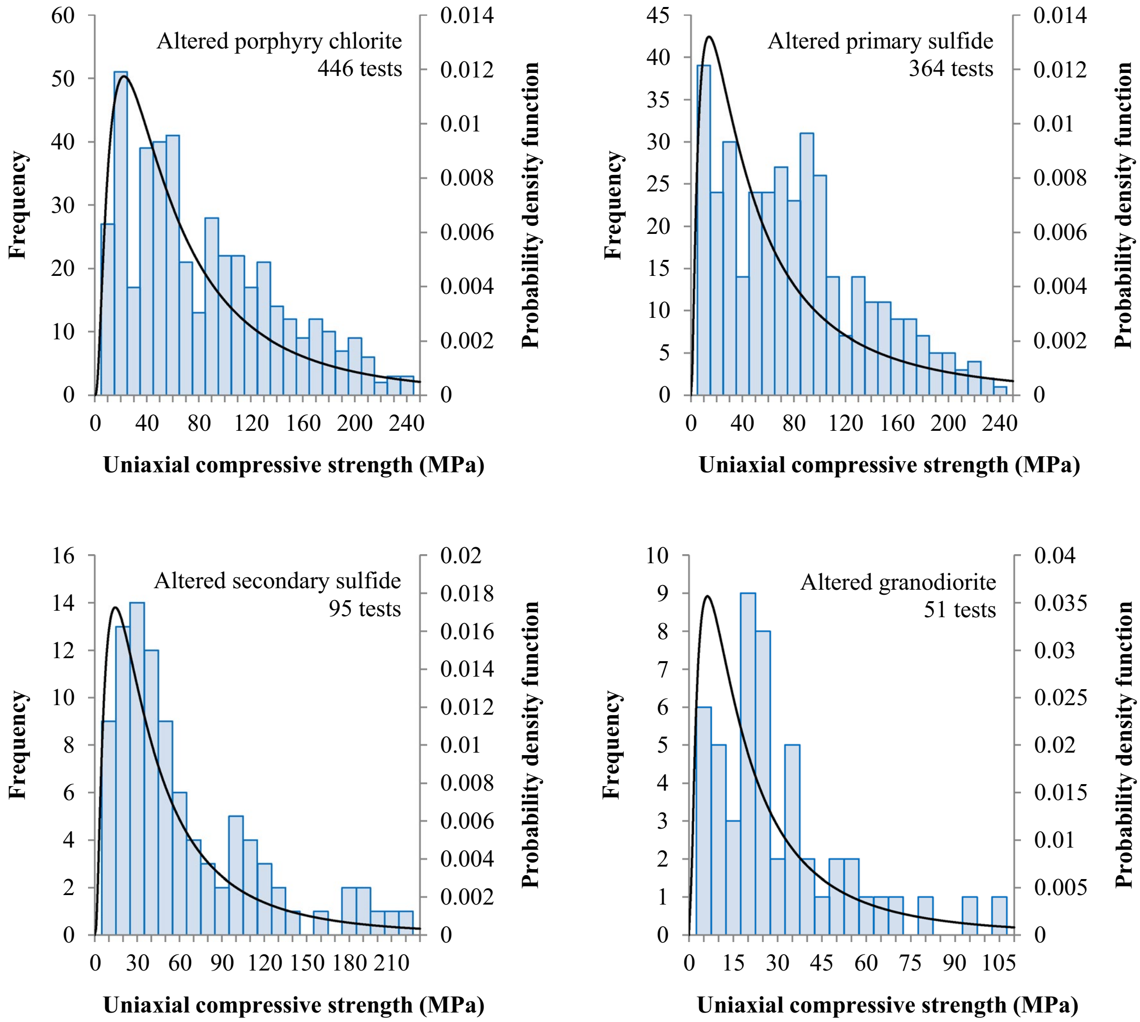


Fig. 4 Distribution of intact rock strength in a porphyry deposit obtained from point load tests

of variation of intact rock uniaxial compressive strength. Figure 5 shows that the coefficient of variation in massive uniform rocks such as Lac du Bonnet granite and Aspo diorite is typically below 0.20. On the other end of the spectrum, highly heterogeneous rocks in porphyry deposits can show coefficients of variation of nearly 1.0.

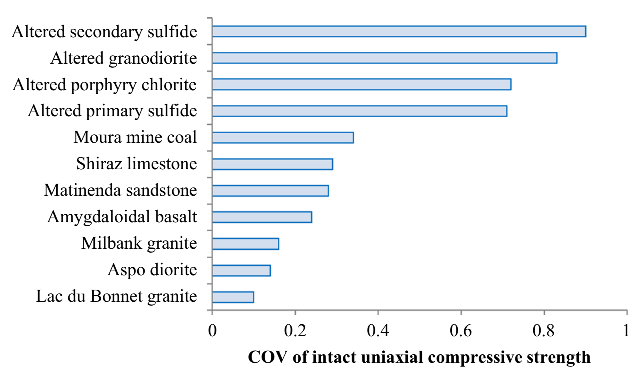


Fig. 5 Coefficient of variation of intact uniaxial compressive strength for various rocks

4 Framework of Probabilistic Analysis

Probabilistic analysis can be carried out once the variability of model parameters is defined in terms of statistical distributions. By incorporating the distributions of input parameters in the analysis, it is possible to estimate the distributions of desired output parameters such as overall strength, maximum displacement or factor of safety. There are a number of ways in which probabilistic analysis can be conducted.

In terms of the approach used to estimate the statistics of output parameters, there are two general categories of methods. The first category includes methods such as the point estimate method (Rosenblueth 1975) and first-order second moment method (Wong 1985) which estimate the mean and standard deviation of model output by carrying out a limited number of analyses with certain input parameters. Once the mean and standard deviation of the output parameter are estimated, a statistical distribution is assumed to calculate the probability of the output parameter exceeding an acceptable threshold. These methods are relatively quick but the results may not be very accurate due to the estimation of mean and standard deviation from relatively limited number of data and also the assumptions made regarding the distribution of the output parameters.

The second category of probabilistic approaches includes methods such as the Monte Carlo and Latin Hypercube methods (Olsson and Sandberg 2002). These methods require running numerous simulations with random values of input parameters to obtain a representative sample of the output parameter. While computationally expensive, these methods can provide more accurate estimates of the mean and standard deviation of the output parameter. In addition, no assumption is required regarding the distribution of the output parameter as it is directly obtained in the process. Finally, probability of unacceptable outputs can be readily estimated by dividing the number of simulations with unacceptable outputs by the total number of simulations.

In probabilistic analysis of slopes, the objective is typically estimating the distribution of factor of safety, FOS. There are two general approaches for determining the slope FOS. The Limit Equilibrium (LE) method is based on dividing the sliding mass into slices, solving force and/or moment equilibrium equations, calculating FOS, and repeating the process using different slip surfaces to find the most critical slip surface with the lowest FOS. It is very common to use the LE method in practical slope stability analysis as it relatively quick and easy, and there is extensive experience in using the results of the LE method for slope design. On the other hand, the results depend on the force and/or equilibrium equations satisfied, assumed relationship between interslice normal and shear forces, and more importantly the assumptions and algorithms used in finding the most critical slip surface (Krahn 2003). Traditional probabilistic limit equilibrium analysis involves solving numerous random realizations with different parameters. However, in each realization, the values of parameters were constant throughout the slope. This corresponds to an unrealistic correlation length of infinity. In this study, recent developments in modeling spatial variability in limit equilibrium analysis have been utilized and the effect of finite correlation length is considered.

An alternative approach for calculating slope FOS is using the Shear Strength Reduction (SSR) method implemented in a full stress–strain analysis with finite element or finite difference method (Dawson et al. 1999; Griffiths and Lane 1999). In this approach, trial SSR factors are applied to shear strength parameters to bring the slope to the verge of failure. The SSR factor corresponding to a state of limiting equilibrium (transition from numerical convergence to divergence) is defined as the critical SSR factor. This is the same definition used for factor of safety in limit equilibrium analysis (Griffiths and Lane 1999). While computationally demanding, the SSR method involves a full stress–strain analysis in which not only full equilibrium but also displacement compatibility is satisfied. As a result, it provides useful information regarding the slope movement and deformation pattern making it possible to use displacement as a design factor. More importantly, no assumptions or secondary algorithms are required to find the critical slip surface as it is naturally formed during the analysis. The inherent domain discretization schemes in finite difference or finite element method readily accommodate the incorporation of spatial variability by assigning variable properties to elements in a single slope model.

5 Probabilistic Analysis of Large Heterogeneous Samples

In addition to heterogeneity in small scale, rock properties are also variable at large scale due to geological processes such as sedimentation, metamorphism, folding and faulting. It is crucially important to properly identify different geological and geotechnical units present in the domain of the project and treat them separately in analysis and design. The difference in properties of such units is a major contributor to overall rock mass heterogeneity. Inside each geological unit, however, there is another level of heterogeneity. Due to difficulties in carrying out a sufficient number of large-scale tests, it is typically not feasible to directly determine large-scale strength variability.

Probabilistic analysis offers an alternative approach for estimating large-scale strength variability in a given geological unit. It is possible to model spatial variability of rock properties in finite element or finite difference analysis by assigning variable mechanical properties to different elements. Large heterogeneous samples can be subjected to loading and their overall strength can be recorded. By generating and testing a sufficient number of large heterogeneous samples in the framework of probabilistic analysis, the distribution of large-scale strength may be obtained. This approach of generating and testing samples through numerical modeling is similar to that adopted in the framework of synthetic rock mass modeling (e.g., Mas Ivars et al. 2011) using discrete element methods.

In this study, a series of uniaxial compression tests on large heterogeneous samples were simulated in the finite difference code, *FLAC3D* (Itasca Inc 2012). The bottom of the samples was fixed and an increasing downward displacement with a slow rate of 10^{-8} m/s was applied to the top boundary to induce compression without undesirable dynamic effects. Vertical stresses at the midheight of the samples were monitored during loading and the maximum stress was recorded. The Hoek–Brown criterion (Hoek et al. 2002) was used to specify the strength of rock:

$$\sigma_1 = \sigma_3 + \sigma_{ci}\left(m_b \frac{\sigma_3}{\sigma_{ci}} + s\right)^a, \tag{2}$$

where σ_1 and σ_3 are the major and minor principal effective stresses at failure, respectively, σ_{ci} is the uniaxial compressive strength of intact rock and m_b, s, and a are dimensionless parameters related to the rock type and rock mass quality.

Lognormal and Weibull distributions with mean of 100 MPa and COV of up to 1 were used to represent the small-scale variability of uniaxial compressive strength with a correlation length of 2 m. To isolate the effect of variability in uniaxial compressive strength, typical parameters for a fracture-free rock were used with Young's modulus of 40 GPa, Poisson's ratio of 0.25 and m_b, s, and a values of 20, 1 and 0.5, respectively. To investigate the effect of sample size, numerical tests were carried out on samples with 6×6×15 m, 10×10×25 m, and 20×20×50 m dimensions. For each combination of coefficient of variation and sample size, 100 numerical tests were carried out to obtain a representative distribution of large-scale strength.

As an example, Fig. 6a shows the contours of small-scale uniaxial compressive strength in a 10×10×25 m sample following a lognormal distribution with a COV of 1.0. The cumulative distribution of small-scale strength within the correlation length (2×2×2 m) as well as large-scale strength for heterogeneous samples of various sizes obtained from numerical tests is shown in Fig. 6b. It can be observed that mean strength of all large-scale samples is about 65 MPa representing a 35% reduction from the mean small-scale strength of 100 MPa. In addition, large-scale strength varies over a much narrower range than small-scale strength. Figure 6b shows that increasing sample size leads to decreasing strength variability.

Figure 7 shows the summary of numerical test results on large-scale heterogeneous samples. It can be observed that increasing small-scale variability leads to a reduction in mean and increase in the COV of large-scale strength. For a given level of small-scale variability, the mean strength values of large-scale samples were similar, while the largest sample showed the least amount of strength variability. The results of lognormal and Weibull distributions were very similar for samples with the COV of small-scale strength below 0.5. At higher levels of strength variability, however, the Weibull distribution gave slightly lower mean and somewhat higher COV of large-scale strength.

6 Probabilistic Analysis of Heterogeneous Slopes

In this section, application of probabilistic slope stability analysis is illustrated using two examples. The first example involves a simple slope with a single geological unit with Mohr–Coulomb behavior while the second example is focused on a realistic slope with more complex geometry and nonlinear Hoek–Brown strength criterion. Limit

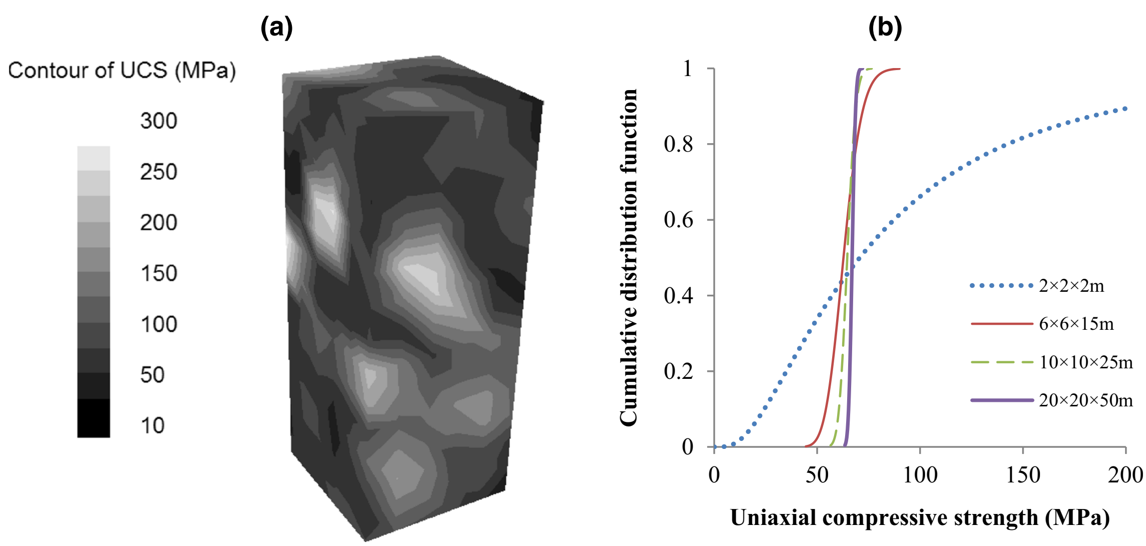


Fig. 6 Numerical testing of large heterogeneous samples: **a** spatial distribution of uniaxial compressive strength in elements of a numerical test sample and **b** cumulative distribution of uniaxial compressive strength for individual elements and the overall test samples

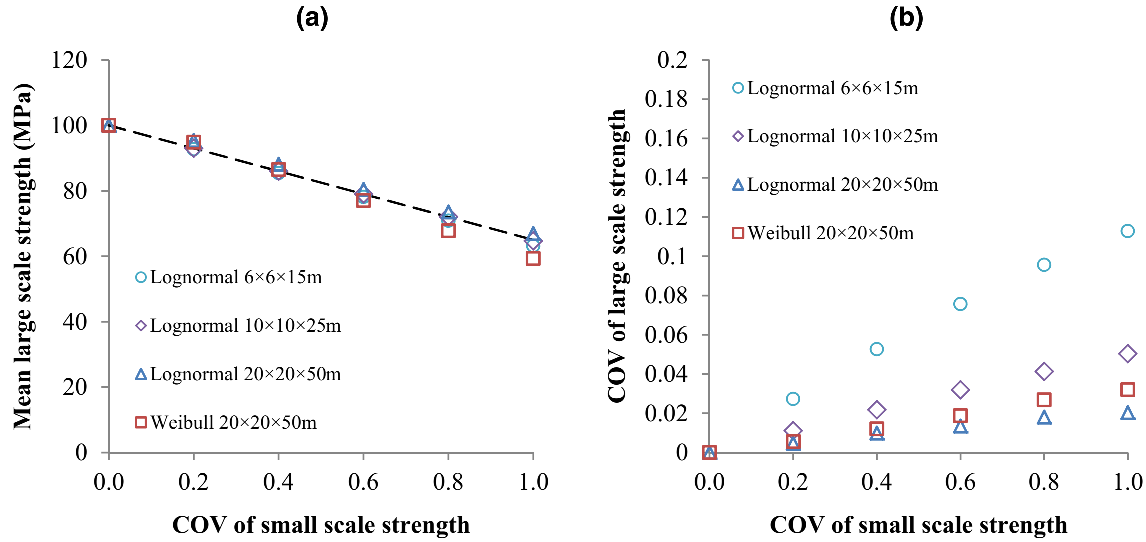


Fig. 7 Effect of small-scale strength variability on: **a** mean and **b** coefficient of variation of large-scale strength

equilibrium analysis and shear strength reduction method were used in stability analysis of the slopes. General limit equilibrium method (Morgenstern and Price 1965; Fredlund and Krahn 1977) satisfying both force and moment equilibrium along with non-circular slip surface search was used in the LE software, *SLIDE*. Slope stability analysis with the shear strength reduction method was carried out using the finite difference code, *FLAC3D*.

6.1 Slope #1 with Mohr–Coulomb Behavior

The first series of probabilistic analyses were carried out on a slope with simple geometry and Mohr–Coulomb material. The slope is 100 m high with a 60° slope angle, unit weight of 26.5 kN/m^3, cohesion of 426 kPa and friction angle of 35°. First, deterministic analyses were carried out using the LE and SSR methods and the results are shown in Fig. 8. It can be observed that the value of FOS and the predicted slip surfaces from both methods are quite similar.

A series of probabilistic analyses were followed with Mohr–Coulomb shear strength parameters, cohesion and friction angle as variables. It was assumed that cohesion and friction angle follow a lognormal distribution with similar coefficients of variation. In probabilistic analysis using the limit equilibrium method, Monte Carlo approach was adopted. In addition to the infinite spatial correlation length in traditional probabilistic limit equilibrium analysis, correlation lengths of 4, 8, 16 m were also modeled. The distribution of FOS obtained from Monte Carlo method was used to calculate mean FOS and probability of failure, POF.

In probabilistic analysis using the strength reduction method, a limited number of realizations with correlation length of 4 m were analyzed due to prohibitive computation

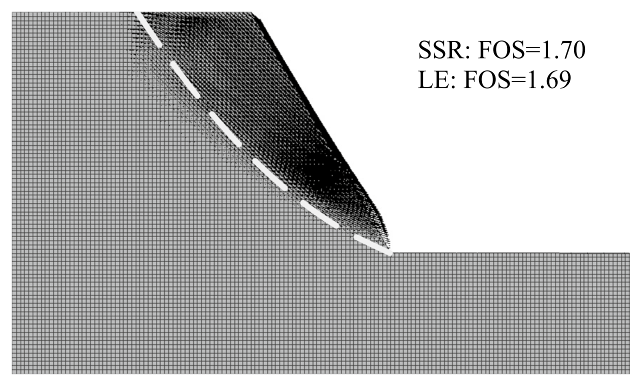


Fig. 8 Deterministic analysis of Slope#1, the dashed line indicates the slip surface using the LE method and the velocity vectors show the zone of failure using the SSR method

time. Subsequently, mean and standard deviation of FOS were determined and used to estimate POF. As an example, Fig. 9 shows the distribution of cohesion throughout the slope with COV of 0.5.

To evaluate the sufficient number of realizations in probabilistic analysis using the LE and SSR methods, convergence curves indicating the evolution of mean and standard deviation of FOS with increasing number of realizations were examined. Figure 10 shows the convergence curves for a slope with a COV of 0.5 for 1000 realizations using the LE method and 100 realizations using the SSR method. Although it is ideal for a consistent comparison to analyze the same number of realizations using the LE and SSR methods, it is currently impractical because analyzing one realization using the SSR method can take longer than 1000 trials using the LE method. Using finite correlation lengths in the

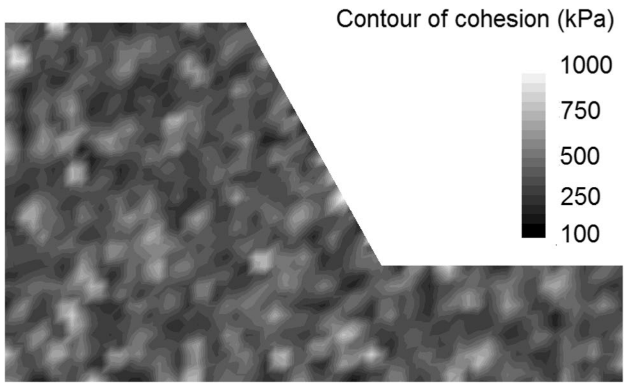


Fig. 9 Spatial distribution of cohesion with COV of 0.5 in Slope#1

LE method, mean and standard deviation of FOS become stable after about 300 realizations. Ignoring spatial correlation (using infinite correlation length) in the LE method causes the mean FOS to shows more volatility and standard deviation of FOS to significantly increase compared to finite correlation length scenarios. Fortunately, the results of the more computationally expensive SSR method converge with fewer realizations, taking about 20 trials for mean FOS and 50 trials for standard deviation of FOS to stabilize.

Probabilistic analyses were carried out with different values of spatial correlation lengths and COV of cohesion and friction angle. Using the large representative samples of FOS values obtained from the LE method, it was possible to investigate the type of distribution function produced for each scenario. As an example, Fig. 11 shows the best fit distributions for the slope with COV of 0.5 using different correlation lengths. It can be observed that increasing correlation length leads to a wider range of possible FOS values. Note that while the FOS distribution for finite correlation lengths was symmetric and could be represented with a normal distribution, ignoring spatial variability led to an asymmetric lognormal distribution for FOS. The combined effect of asymmetric distribution and higher spread of FOS using infinite correlation length resulted in a large probability of failure represented by the area under the probability density function with FOS < 1.

Figure 12a shows the effect of strength variability and spatial correlation length on the mean FOS in Slope #1. It can be observed that as material heterogeneity increases mean FOS obtained from LE and SSR methods decreases. The results of LE and SSR methods using finite correlation lengths are reasonably close. Ignoring spatial correlation, however, can lead to an overestimation of mean FOS, especially at higher levels of strength variability.

Having established the mean, standard deviation, and probability distribution of FOS, it is also possible to determine POF. In probabilistic analysis using LE method, POF was determined as the ratio of the number of Monte Carlo realizations with FOS < 1 to the total number of realizations. In probabilistic analysis with the SSR method, the mean and standard deviation of FOS along with the normal distribution obtained from the finite correlation lengths (Fig. 11) were used to estimate POF.

The relationship between probability of failure, strength variability and spatial correlation length is shown in Fig. 12b. Both LE and SSR methods predicted that for Slope#1, increasing strength variability causes an increase in POF. The results of LE and SSR methods using similar correlation lengths were in good agreement. Increasing spatial correlation length led to an increase in POF, especially at higher levels of strength variability. It can be observed that the values of POF obtained by ignoring spatial correlation (infinite correlation length) are considerably overestimated.

Cohesion and friction angle were assumed to be independent variables in most of the analyses. However, negative cross correlation between cohesion and friction angle has been reported for some geomaterials which can reach correlation coefficient of -0.5 in certain cases (e.g., Yucemen et al. 1973; Wolff 1985; Hata et al. 2012). To explore the effect of strong negative cross correlation, a correlation coefficient R of -0.5 between cohesion and friction angle was also incorporated in probabilistic analysis with spatial correlation length of 16 m. It can be observed from Fig. 12 that introducing the negative cross correlation causes an increase in mean FOS and decrease in POF. This is because negative cross correlation causes lower values of cohesion to be likely accompanied with higher friction angles and vice versa which effectively reduces the overall strength variability. This is consistent with previous findings (e.g., Griffiths et al. 2009; Chiwaye and Stacey 2010; Javankhoshdel and Bathurst 2016).

6.2 Slope #2 with Hoek–Brown Behavior

The framework of probabilistic analysis described previously can be extended to more realistic slopes with complex geology and nonlinear failure envelope. The second slope analyzed in this study has an overall slope angle of 46° with effective height of 75 m. The Hoek–Brown criterion (Hoek et al. 2002) was used to specify the strength of the two geological units encountered in the slope. Hoek et al. (2002) provided relationships to determine the cohesion and friction angle of an equivalent Mohr–Coulomb criterion fitted to the Hoek–Brown envelope. The Hoek–Brown parameters for the two rock mass units along with unit weight γ and equivalent cohesion c^* and friction angle φ^* are given in Table 1.

Deterministic slope stability analyses were carried out with the Hoek–Brown criterion using LE and SSR methods. In addition, the equivalent Mohr–Coulomb criterion was used in limit equilibrium analysis denoted as LE*. It can be observed from Fig. 13 that the critical slip surface is almost

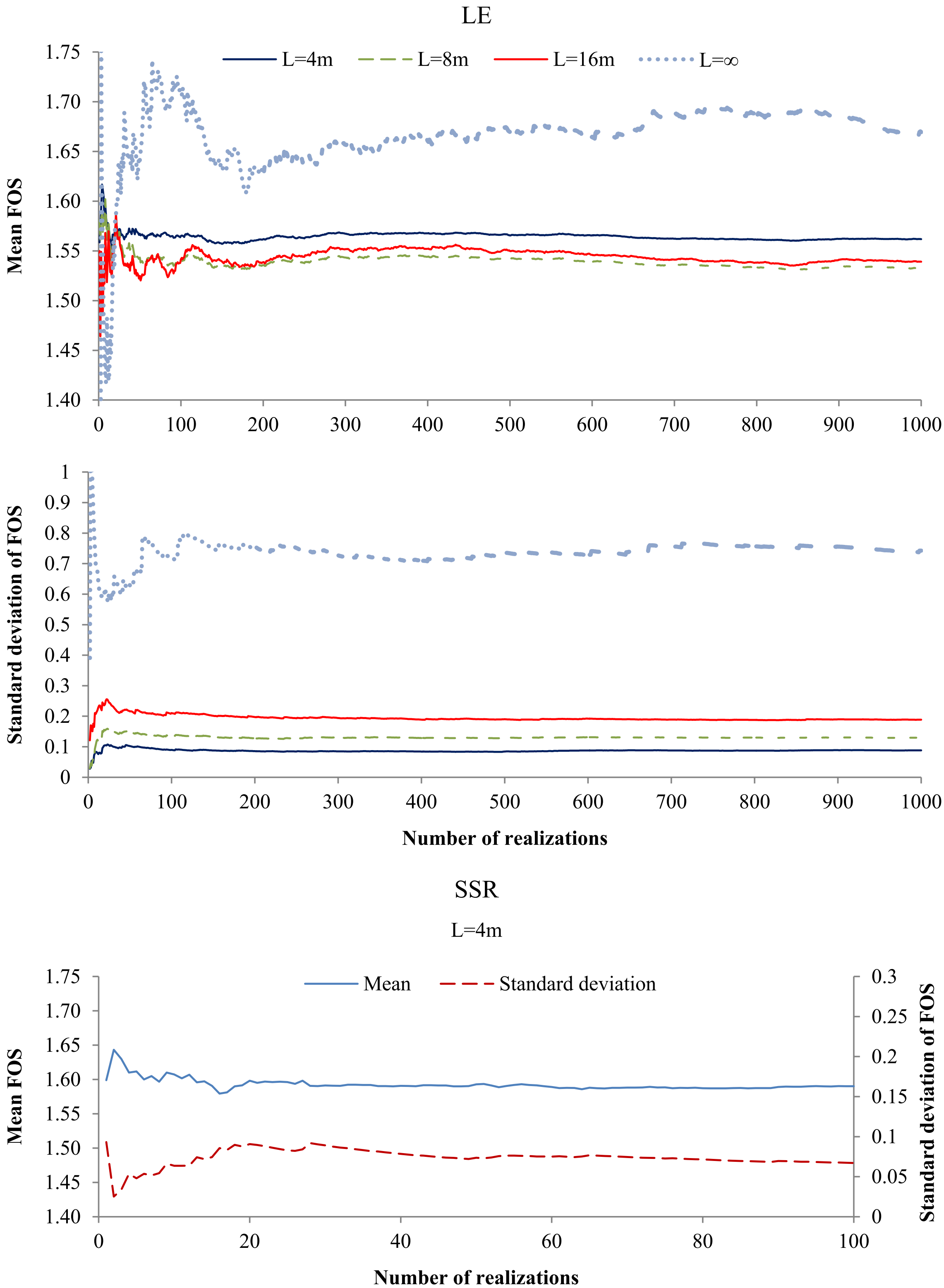


Fig. 10 Convergence curves for Slope #1 using COV of 0.5 for cohesion and friction angle

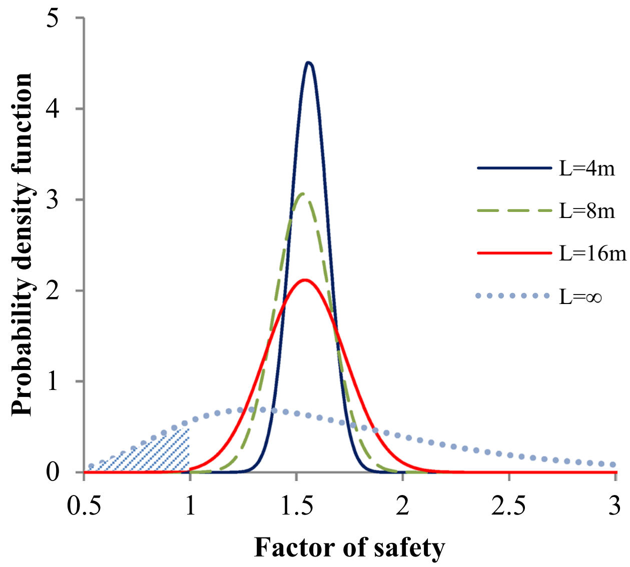


Fig. 11 Distribution of FOS for Slope #1 using COV of 0.5 for cohesion and friction angle

identical in all cases starting at the crest from the boundary of the two rock units, going through the weaker rock mass and daylighting at the slope face. The values of FOS obtained from LE and SSR methods with the Hoek–Brown criterion were in close agreement. Limit equilibrium analysis using the equivalent Mohr–Coulomb criterion led to 4% overestimation of the FOS for Slope#2.

In probabilistic stability analysis of this slope, uniaxial compressive strength of intact rock σ_{ci} was considered as the only variable with a lognormal distribution. The COV(σ_{ci}) was assumed to be the same for Unit 1 and Unit 2 rock masses. To investigate the effect of strength heterogeneity, different values of COV (σ_{ci}) were used in probabilistic analysis. Due to prohibitively long computation times, a single correlation length of 2.5 m was used in numerical modeling. Probabilistic analysis with SSR method was carried out using the nonlinear Hoek–Brown criterion. As an example, Fig. 14 shows the distribution of intact rock uniaxial compressive strength with COV of 0.5 in the weaker Unit 1 and stronger Unit 2.

In probabilistic limit equilibrium analysis, a wide range of correlation lengths were analyzed. For the case of infinite correlation length (ignoring spatial correlation), the nonlinear Hoek–Brown criterion was used. However, simultaneous implementation of spatial variability and the Hoek–Brown criterion was not supported in the current version of the *SLIDE* software and it was necessary to use the equivalent Mohr–Coulomb criterion in those cases. Hence, it was necessary to translate the distribution of σ_{ci} to those of equivalent cohesion and friction angle.

To this end, equations relating the Hoek–Brown and equivalent Mohr–Coulomb parameters (Hoek et al. 2002)

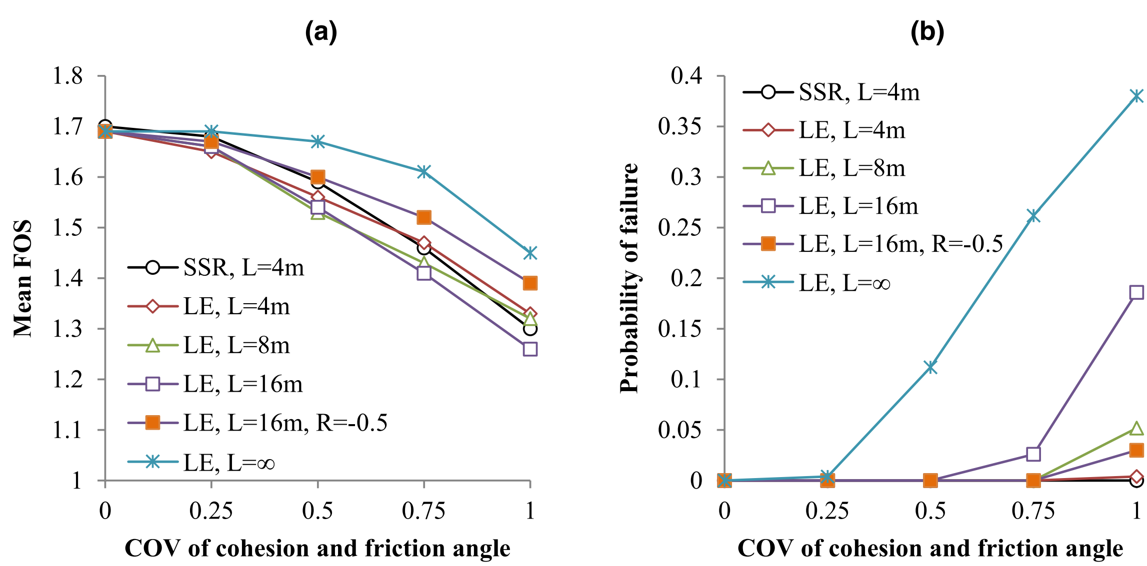


Fig. 12 Effect of strength variability and correlation length on: **a** mean factor of safety and **b** probability of failure for Slope#1

Table 1 Rock mass properties for Slope#2

Rock mass	γ (kN/m^3)	σ_{ci} (MPa)	m_b	s	a	c^* (kPa)	φ^* (°)
Unit 1	26.5	25	0.177	5.36×10^{-5}	0.511	201	24
Unit 2	26.5	75	0.683	5.53×10^{-4}	0.504	577	43

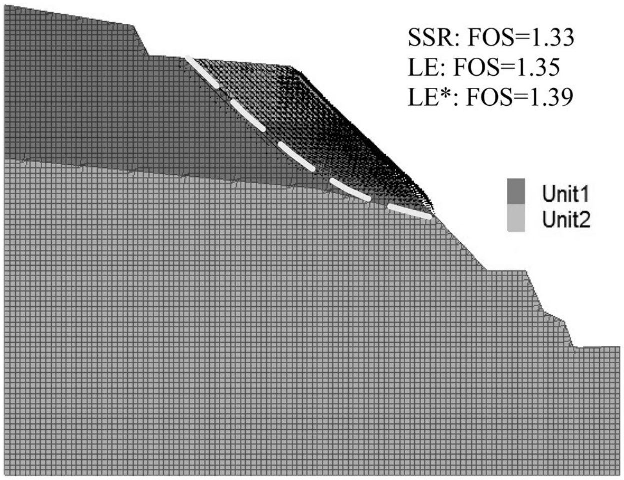


Fig. 13 Deterministic analysis of Slope #2, the dashed line indicates the slip surface using the LE method and the velocity vectors show the zone of failure using the SSR method

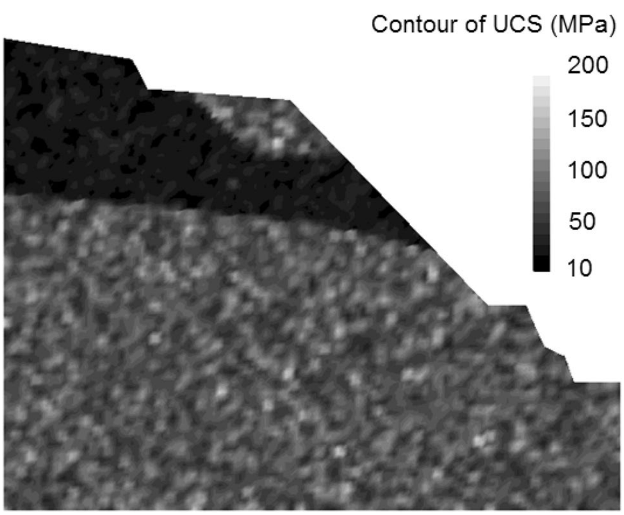


Fig. 14 Spatial distribution of intact rock uniaxial compressive strength with COV of 0.5in Slope #2

were utilized. Due to the complex forms of these equations, an analytical approach for calculating the statistics of equivalent cohesion and friction angle is cumbersome and therefore, a numerical approach was adopted. For each scenario, 10,000 random values were generated from the distribution of σ_{ci} and were used in the equations to generate representative distributions of equivalent cohesion and friction angle.

As an example, the distributions of equivalent cohesion and friction angle of Unit 1 corresponding to COV(σ_{ci}) of 0.5 are shown in Fig. 15. The relationship between mean and COV of equivalent Mohr–Coulomb parameters with COV(σ_{ci}) for Unit 1 is shown in Fig. 16. It can be observed that by increasing COV(σ_{ci}), the mean values of equivalent cohesion and friction angle slowly decrease while their COV values linearly increase. The relationships between COV(σ_{ci}), and COV of equivalent Mohr–Coulomb parameters for Unit 1 can be given by

$$\mathrm{COV}(c^*) = 0.38\mathrm{COV}\left(\sigma_{\mathrm{ci}}\right), \tag{3}$$

$$\mathrm{COV}(\varphi^*) = 0.25\mathrm{COV}\left(\sigma_{\mathrm{ci}}\right). \tag{4}$$

Having established the distribution of equivalent cohesion and friction angle, probabilistic analysis of Slope #2 was carried out using the LE* method with 1000 realizations and mean factor of safety and probability of failure were calculated. The results of probabilistic analysis of Slope #2 using the LE and SSR methods with the nonlinear Hoek–Brown criterion as well as the LE* method with the equivalent Mohr–Coulomb criterion are shown in Fig. 17.

Similar to the Slope #1, increasing strength variability led to a reduction in mean factor of safety and increase in probability of failure. The mean FOS obtained using finite spatial correlation lengths in the LE* and SSR methods followed a similar trend. However, there was a consistent gap of about 0.07 between mean FOS values obtained from the LE* and SSR methods due to approximation of the nonlinear Hoek–Brown criterion with an equivalent Mohr–Coulomb criterion. The values of probability of failure obtained from the LE* and SSR methods with finite correlation lengths were in close agreement. It can be observed that ignoring spatial correlation in Slope #2 leads to considerable overestimation of the probability of failure, especially at higher levels of strength variability. In cases where spatial correlation is ignored, the POF from the LE method with the Hoek–Brown criterion is higher than that from the LE* method with the equivalent Mohr–Coulomb criterion due to overestimation of mean FOS in the LE* method.

While equivalent cohesion and friction angle were independent in most of the analyses, a cross-correlation coefficient R of -0.5 was also used in probabilistic analysis along with spatial correlation length of 25 m. It can be observed from Fig. 17 that considering the negative cross correlation leads to an increase in mean FOS and decrease in POF. The negative cross correlation causes the effect of lower cohesions to be likely compensated with higher friction angles and vice versa which improves the overall slope stability. This is consistent with the results for Slope#1 and previous studies (e.g., Griffiths et al. 2009; Chiwaye and Stacey 2010; Javankhoshdel and Bathurst 2016).

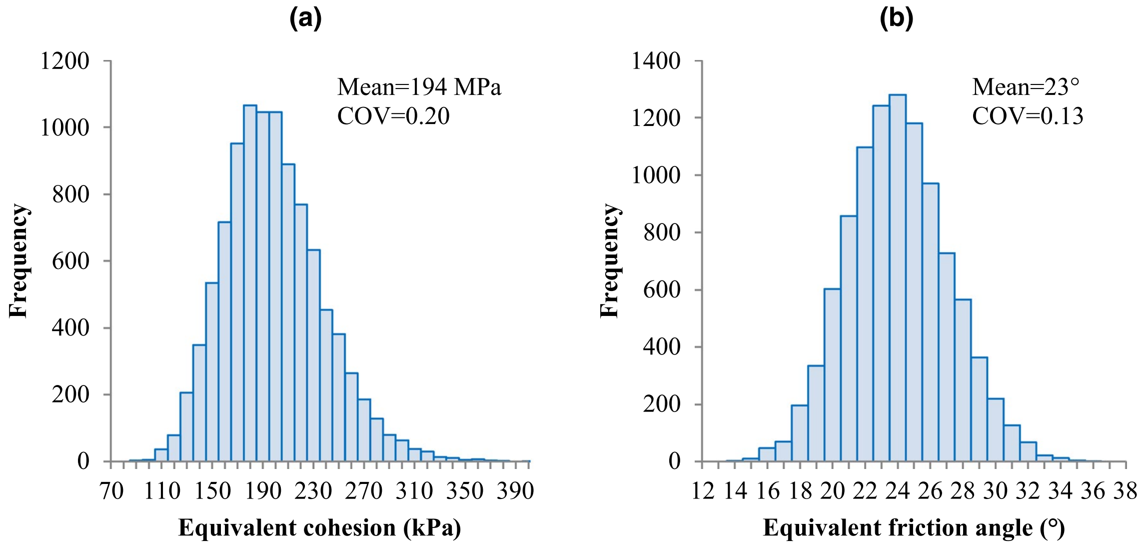


Fig. 15 Distribution of: **a** equivalent cohesion and **b** equivalent friction angle for Unit 1 with COV(σ_{ci}) of 0.5

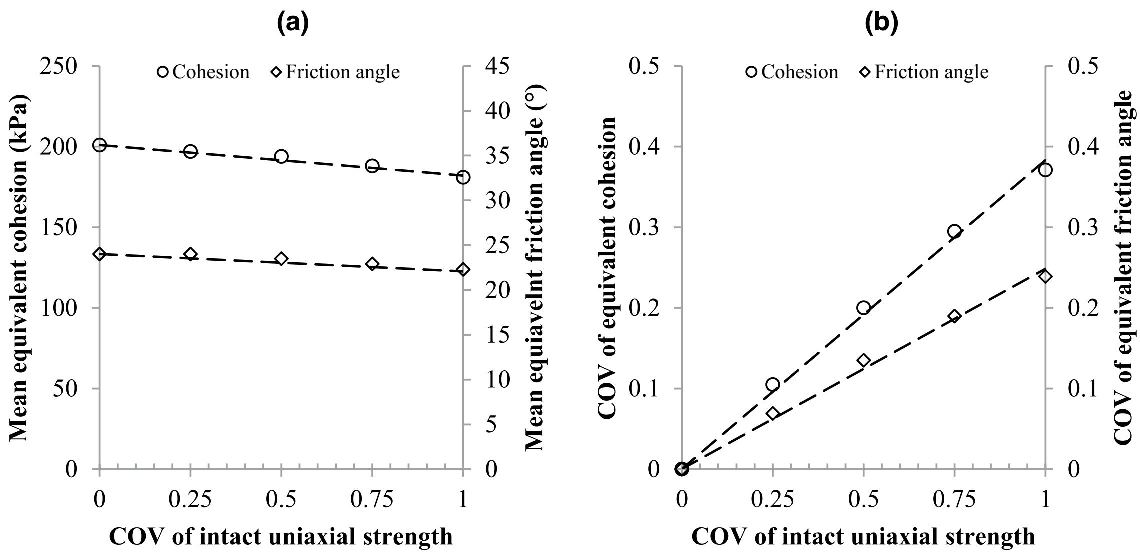


Fig. 16 Relationship between COV(σ_{ci}) and: **a** mean and **b** COV of equivalent cohesion and friction angle for Unit 1

7 Discussion

The deterministic factors of safety and slip surfaces obtained from LE and SSR methods were in reasonable agreement for the slopes analyzed in this study. The results of probabilistic analysis using both approaches showed that increasing strength variability leads to a reduction in mean FOS and increase in POF. This is the case for slopes with deterministic FOS > 1 such as those analyzed in this work. A reverse trend may be observed for slopes with deterministic FOS < 1 (Griffiths and Fenton 2004).

Traditional probabilistic analysis in which spatial variability is ignored consistently gave the highest mean factors of safety. This is because properties are constant within the slope in each realization which does not capture the formation of failure surface through weaker regions. The standard deviation of factor of safety is also significantly higher when spatial variability is ignored implying a wider spread of FOS (Fig. 10). This is again due to using constant properties within the slope in each realization where occurrence of extreme strength properties directly leads to extreme values of FOS. In reality; however, regions with extreme

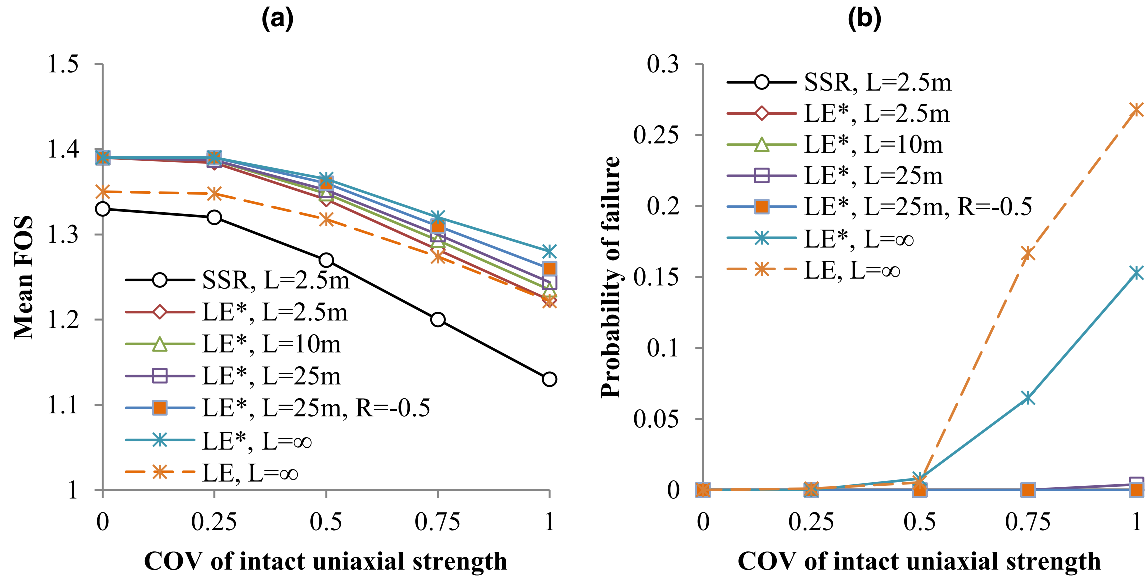


Fig. 17 Effect of strength variability and correlation length on: **a** mean factor of safety and **b** probability of failure for Slope#2

properties are surrounded by zones with intermediate properties and the effect of extreme properties is significantly moderated. The distribution of FOS is also quite asymmetric when spatial variability is ignored making lower values of FOS more likely to occur (Fig. 11). The combined effect of wide spread and asymmetric distribution of FOS leads to significant overestimation of probability of failure. This is in keeping with previous findings of El-Ramly et al. (2002), Cho (2007), Hong and Roh (2008) and Allahverdizadeh et al. (2015). Ignoring spatial variability in probabilistic slope analysis may only be justified if the spatial correlation length is significantly greater than the length of the critical slip surface. Preliminary findings of Allahverdizadeh et al. (2015) and Javankhoshdel et al. (2017) suggest that there is minimal change in probability of failure once the spatial correlation length exceeds about five times the slope height.

Natural variability of properties can be modeled more realistically when properties are allowed to vary spatially within the slope. Incorporating spatial variability in probabilistic analysis of Slope#1 with Mohr–Coulomb behavior provided similar results using LE and SSR methods. This suggests that mean factor of safety and probability of failure of simple slopes with Mohr–Coulomb behavior may be obtained using the more computationally efficient limit equilibrium analysis. However, for slopes with more complex geometry, loading and mechanical behavior or in cases where information about displacements is required, numerical modeling and SSR method may be adopted. Increasing correlation length from 4 m to 16 m led to minor reduction in mean FOS and increase in POF only at very high levels of strength variability. This is consistent with the findings of El-Ramly et al. (2006) who reported negligible change in FOS and POF by increasing spatial correlation length over a reasonable range. This suggests that using reasonable estimates of correlation length may give sufficiently accurate results in stable slopes with average variability.

For Slope #2 with Hoek–Brown behavior, the results of LE and SSR methods followed a similar trend. However, using the equivalent Mohr–Coulomb criterion with empirically estimated parameters led to a consistent gap between mean FOS values obtained from the LE* and SSR methods. This is in keeping with the findings of Li et al. (2008) who reported slight overestimation of FOS due to approximation of the nonlinear Hoek–Brown criterion. One way to reduce the discrepancy is to apply a correction factor to the FOS values obtained using the equivalent Mohr–Coulomb criterion to match the FOS from the Hoek–Brown criterion in a deterministic analysis. In Slope #2 for example, applying a correction factor of 1.35/1.39 causes the deterministic FOS obtained from the Hoek–Brown and equivalent Mohr–Coulomb criteria to match (Fig. 13).

In most of the analyses presented here, cohesion and friction angle were considered as independent variables. However, it was shown that incorporating a negative cross correlation between cohesion and friction angle can lead to an increase in mean FOS and decrease in POF. Therefore, for slopes with deterministic FOS > 1, probabilistic analysis with independent cohesion and friction angle provides conservative results (e.g., Chiwaye and Stacey 2010; Javankhoshdel and Bathurst 2016). The negative cross correlation, though reported for certain geomaterials, is not always significant (e.g., El-Ramly et al. 2005, 2006) and may be relied on only if confirmed by the results of experiments on a given material.

While spatial variability of strength properties was discussed in this study and incorporated in slope stability analysis, it is recognized that such analysis can be prohibitively time consuming, especially using the SSR method. Hence, it is valuable to establish relationships to approximate the behavior of a heterogeneous material with that of an equivalent homogeneous material. Once the parameters of the equivalent material are obtained, a single deterministic analysis can be carried out to estimate the results of a computationally expensive probabilistic analysis with numerous realizations.

Eurocode 7 (CEN 2004) suggests the use of a characteristic value defined as a cautious estimate of the value affecting the occurrence of design failure. As suggested by Schneider (1999) and confirmed by Orr (2000), the characteristic value of a strength property such as intact uniaxial compressive strength can be estimated using

$$\sigma_{\mathrm{ci}}^{\mathrm{EC7}} \approx \bar{\sigma}_{\mathrm{ci}}\left[1 - 0.5\mathrm{COV}\left(\sigma_{\mathrm{ci}}\right)\right], \tag{5}$$

where $\sigma_{\mathrm{ci}}^{\mathrm{EC7}}$ is the characteristic intact uniaxial compressive strength according to Eurocode 7 and $\bar{\sigma}_{\mathrm{ci}}$ and $\mathrm{COV}\left(\sigma_{\mathrm{ci}}\right)$ are mean and coefficient of variation of intact uniaxial compressive strength, respectively.

It is useful to examine how the regulatory-based guidelines of Eurocode 7 compare with the results of modeling-based probabilistic analysis in this study. Based on the results of numerical tests on large-scale heterogeneous samples shown in Fig. 7, the equivalent strength σ_{ci}^{*} can be estimated using

$$\sigma_{\mathrm{ci}}^{*} = \bar{\sigma}_{\mathrm{ci}}\left[1 - 0.35\mathrm{COV}\left(\sigma_{\mathrm{ci}}\right)\right]. \tag{6}$$

For heterogeneous slopes, it is constructive to establish relationships between the model parameters in a deterministic analysis which gives the FOS equal to the mean FOS from a probabilistic analysis. A series of deterministic analyses were carried out on Slope#2 using the Hoek–Brown criterion with gradually reducing the intact uniaxial compressive strength down to 20% of the mean value. Figure 18a shows that as expected, reducing strength leads to reduction of factor of safety. Once the relationship between strength and FOS in a deterministic analysis is established, it is possible to match the deterministic FOS with mean probabilistic FOS in Fig. 17 and find corresponding levels of COV of intact uniaxial compressive strength. Figure 18b shows the relationship between the equivalent intact strength used in deterministic analysis which gave the FOS identical to mean FOS from a probabilistic analysis for any given level of strength variability.Adopting a linear function similar to that used in Eurocode 7, the relationship between equivalent strength and strength variability for slope stability analysis can be given by

$$\sigma_{\mathrm{ci}}^{*} = \bar{\sigma}_{\mathrm{ci}}\left[1 - 0.3\mathrm{COV}\left(\sigma_{\mathrm{ci}}\right)\right]. \tag{7}$$

The slight nonlinearity of the trend can be captured using the following equation for median strength assuming lognormal distribution:

$$\sigma_{\mathrm{ci}}^{*} = \frac{\bar{\sigma}_{\mathrm{ci}}}{\sqrt{1 + \mathrm{COV}^{2}\left(\sigma_{\mathrm{ci}}\right)}}. \tag{8}$$

Note that Eqs. (7) and (8) are developed based on the results of slope stability analysis with $\mathrm{COV}\left(\sigma_{ci}\right) < 1$ and caution should be exercised in extrapolating beyond this range. The equivalent strength from these equations can be

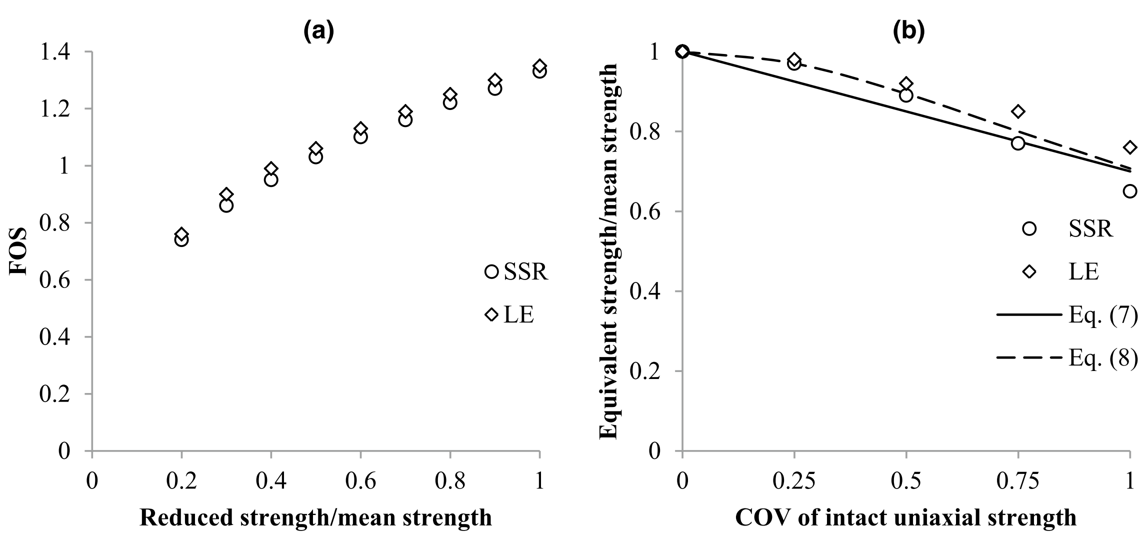


Fig. 18 Relationship between equivalent intact uniaxial compressive strength and: **a** deterministic FOS and **b** corresponding COV of intact strength with the same probabilistic mean FOS for Slope#2

readily used in a single deterministic slope analysis to estimate the mean FOS. This is useful in early stages of design where data are limited and estimating the factor of safety of a heterogeneous slope is the primary objective. Probability of failure, however, may only be obtained from a complete probabilistic analysis such as those presented in Sect. 6.

It can be observed that the results of probabilistic analysis of large heterogeneous samples and slopes suggest relationships for equivalent strength similar to the Eurocode 7 guidelines. In Eurocode 7, however, the characteristic strength is conservatively chosen whereas the equivalent strength in this study provides an unbiased estimate of the strength of a heterogeneous material. Considering the completely different geometry and loading path of the numerically tested heterogeneous samples and heterogeneous slopes, similarity of the relationships obtained for equivalent strength is worth noting.

Over decades of consulting at the Rosario open pit mine in northern Chile, a dataset of back-analyzed intact rock strength values which reproduced the observed behavior of highly heterogeneous slopes has been compiled (Silva-Mandiola 2018). The independently developed dataset of back-analyzed strength values was used to evaluate the equivalent strength relationships proposed in this study. Figure 19 shows that there is a surprisingly close agreement between the equivalent strength calculated from Eq. (7) and the back-analyzed strength values which predicted slope behavior.

For Slope#1 in this study, the Mohr–Coulomb criterion was used and rock mass cohesion and friction angle were considered as variables. In probabilistic analysis of Slope#2, however, the uniaxial compressive strength of intact rock was the only variable while other Hoek–Brown parameters were kept constant. This was done intentionally to isolate the effect of variability in intact rock strength and develop relationships for an equivalent strength. In reality, there is also some level of variability in other strength parameters which add to the overall strength variability of rock mass (Hoek 1998). The framework of probabilistic analysis described in this study can be readily applied to include any number of strength properties as variables with no additional complexity.

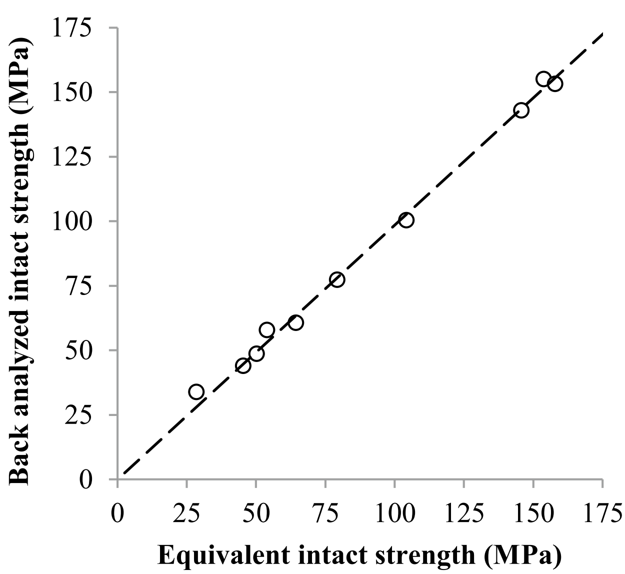


Fig. 19 Comparison of the proposed equivalent strength with back-analyzed strength of heterogeneous slopes

8 Conclusions

Variability of properties is an inherent characteristic of geomaterials which can affect the performance of engineering structures. Strength variability of rocks was discussed and the results of systematic point load testing program in porphyry deposits were presented indicating that coefficient of variation of intact rock uniaxial compressive strength can reach unity.

The effect of small-scale strength variability on the strength of large samples was explored using numerical modeling. Heterogeneous samples following specific strength distributions were generated and tested under uniaxial compression to determine large-scale strength. Following the framework of probabilistic analysis, numerous realizations were analyzed and strength distribution of large-scale heterogeneous samples was established.

Strength heterogeneity was also introduced in slope stability analysis using limit equilibrium and shear strength reduction methods. The results of deterministic analysis with both approaches were similar. It was observed that for the stable slopes analyzed in this study, increasing strength variability leads to a reduction in mean factor of safety and increase in probability of failure. Traditional probabilistic analysis in which spatial variability is ignored consistently overestimated mean factor of safety. Due to high spread and asymmetric distribution of factor of safety in this case, probability of failure was significantly overestimated. More realistic modeling of rock heterogeneity was achieved by incorporating spatial variability and allowing properties to vary within the slope. The results of limit equilibrium and shear strength reduction methods were in good agreement for Mohr–Coulomb slopes with finite spatial correlation lengths. Approximation of the nonlinear Hoek–Brown criterion with an equivalent Mohr–Coulomb criterion caused minor overestimation of factor of safety.

The results of the comprehensive probabilistic analysis of large-scale heterogeneous samples and slopes were utilized to develop relationships for equivalent strength. Despite considerable difference in geometry and loading path involved in the analysis of heterogeneous samples and slopes, a similar linear relationship was found between equivalent strength and coefficient of variation of intact strength. Based on the results of this study, reducing mean uniaxial compressive strength of intact rock by one-third of its standard deviation can provide an estimate of the equivalent strength of intact rock. This relationship was further validated using the values of back-analyzed strength which reproduced the observed behavior of heterogeneous slopes.

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