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TECHNICAL NOTE

Effective Stress Principle for Partially Saturated Rock Fractures

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1 Introduction

Soils and rocks are commonly characterized as a porous or fractured medium, with liquid and gaseous fluids occupying and moving in the void space. The presence of water in the void space remarkably influences the deformation behaviors, mechanical properties and stress states of soils and rocks. It has been well recognized that the induced volume change, deformation and shear strength decrease of soils and rocks do not depend on the total stress applied, but on the effective stress defined at the saturated state due to the difference between the total stress and the fluid pressure in the pore space. The deformation of soils and rocks further alters the pore or fracture network and induces a non-negligible variation in hydraulic properties (Kirby 1991; Chen et al. 2007; Li et al. 2014a). Therefore, the concept of effective stress plays a dominant role in understanding the coupled hydromechanical behaviors of soils and fractured rocks.

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Von Terzaghi (1923) pioneered the principle of effective stress for saturated soils, in which the effective stress was defined as the difference between the total stress and the pore water pressure:

$$\sigma' = \sigma - u_{\mathrm{w}} \tag{1}$$

where σ denotes the total stress, u_{w} the pore water pressure, and σ' the effective stress.

The pores and voids of an unsaturated soil, however, are only partially occupied by water, with the rest being occupied by air, which leads to a different stress state in the soils. A modification of Terzaghi's effective stress principle is therefore required for unsaturated soils. Bishop (1959) proposed the principle of effective stress for unsaturated soils by introducing an effective stress parameter into Eq. (1):

$$\sigma' = (\sigma - u_{\mathrm{a}}) - \chi(u_{\mathrm{w}} - u_{\mathrm{a}}) \tag{2}$$

where χ denotes the effective stress parameter under partially saturated conditions, and u_{a} the pore air pressure.

From then on, various effective stress principles were proposed for porous and fractured media (e.g. Tuncay and Corapcioglu 1995; Laloui and Nuth 2009; Ghabezloo et al. 2009). The development of the effective stress models enhances our understanding on the theory of consolidation and shear strength of soils. For partially saturated rock fractures, however, few efforts are made to examine the applicability of the effective stress principle originally developed for unsaturated soils, given the apparent differences of the void structure between soils and rock fractures. This results in a difficulty in properly characterizing the deformation, strength and permeability of rock fractures at partially saturated states (Zandarin et al. 2013; Li et al. 2014b). The theory of effective stress should therefore be improved for partially saturated rock fractures.

In this note, the partially saturated rock fractures are regarded as a three-phase system, with the void space bounded by two opposite walls (solid phase) and occupied by both water (liquid phase) and air (gas phase). The equation of force balance among the solid, liquid and gas phases in the partially saturated fractures is established based on the microstructures of the void space and the interactions among the three phases. An effective stress equation is then proposed for smooth parallel fractures under partially saturated conditions. By virtue of the capillary law stating that the local area with smaller apertures is preferentially occupied by the wetting phase (water), the effective stress equation is derived for partially saturated rough-walled fractures with any aperture distributions. The proposed effective stress model is found to share the same form with Bishop's effective equation at partially saturated states, with the effective stress parameter being equated to the surface saturation of fractures, and it reduces to Terzaghi's effective stress equation at fully saturated state. Illustrative examples show that the effective stress parameter nonlinearly depends on the degree of saturation for rough-walled fractures, and the nonlinearity increases as the surface roughness of the fractures increases. Also, discussed in this note is the effect of surface contact on the effective stress of partially saturated rock fractures.

2 Surface Morphology of Rock Fractures

The surface roughness and aperture distribution of a fracture have strong influences on its mechanical and hydraulic properties. Early fracture models commonly assumed that the fractures are composed of two smooth parallel rock surfaces, which is an extremely rough approximation of real fracture geometries. The aperture distribution of rough-walled fractures has been proved to mostly follow a Gaussian, lognormal, Gama or truncated-Gaussian distribution, with the truncated distribution function being capable of describing the aperture distribution of fractures under normal loads (Walsh et al. 2008; Weerakone et al. 2012; Bertels et al. 2001; Liu et al. 2013). Photoelectric techniques, such as 3-D laser scanning and CT (computed tomography) scanning, were used to obtain the aperture distribution of rock fractures with sufficiently high resolution (Lanaro 2000; Bertels et al. 2001). Besides the fitted continuous distributions, discrete distributions can also properly describe the aperture distribution of rough-walled fractures. As an advantage over the fitted continuous distributions, a discrete distribution preserves any details of local apertures experimentally measured. If a rough-walled rock fracture is assumed to be composed of numerous infinitesimal parallel plates, the discrete aperture distribution can be expressed as:

$$f(x) = n_i \tag{3}$$

where x is the aperture, and n_i the fraction of local plates with an aperture of x.

3 An Effective Stress Principle for Smooth Parallel Fractures

As mentioned above, the state of stress in a partially saturated fracture is fundamentally different from the state of stress at fully saturated state. The liquid or gas saturation of the fracture may significantly influence its mechanical properties. Under partially saturated conditions, suction is commonly introduced and defined as the net interface force acting on the fracture surfaces for representing the combined effects of negative pore water pressure and surface tension. The macroscopic consequence of suction is a bonding force that tends to pull the opposite fracture walls closer, similar to the effect induced by an extra normal compression.

To describe this effect, we assume for simplicity that a fracture is composed of two parallel rock blocks, as shown in Fig. 1a. Each rock block has the same size with a radius of b. Also, shown in Fig. 1a, is a typical distribution of the fluid phases (water and air) in the fracture at partially saturated states. The void space on the concave side is occupied with air and the water pressure is lower than the air pressure. The water meniscus formed between the fluid phases is described with two radii a and r, the fracture aperture e and the contact angle θ. The magnitude of the capillary force arising from the water meniscus in the fracture can be represented as a function of water content by considering the local geometry of the air–water–solid interface.

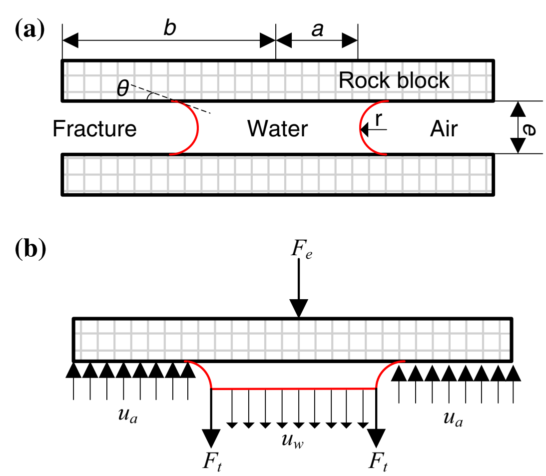

Fig. 1 Force analysis of a fracture under partially saturated condition

A free-body diagram for the forces acting on the fracture, including the actions of air pressure u_a, pore water pressure u_w, surface tension T_s and the applied external force F_e, is plotted in Fig. 1b. The positive, isotropic air pressure u_a exerts a compressive force on the fracture wall. The total force induced by the air pressure, F_a, is equal to the product of the magnitude of the air pressure and the area of the air–solid interface over which it acts:

$$F_a = u_a\left(\pi b^2 - \pi a^2\right). \tag{4}$$

The total force induced by the surface tension, F_t, acts along the perimeter of the water meniscus, and it reads:

$$F_t = -T_s \times 2\pi a. \tag{5}$$

The projection of the total force induced by the water pressure acting on the water–solid interface in the vertical direction, F_w, reads:

$$F_w = -u_w \times \pi a^2. \tag{6}$$

The resultant capillary force, F_r, is the sum of the above three forces:

$$F_r = u_a \pi b^2 - u_a \pi a^2 - T_s 2\pi a + u_w \pi a^2. \tag{7}$$

In the case of free of mechanical loading on the fracture, the air pressure is the unique contribution to the external force acting on the outer boundary of the rock block, which yields:

$$F_e = u_a \pi b^2 - (u_a - u_w)\pi a^2 - T_s 2\pi a \tag{8}$$

where F_e is the net interface force due to the interfacial interaction.

The stress resulting from the balance of the interfacial forces can be evaluated by accounting for the area over which it acts. Therefore, the magnitude of stress contributed by the capillary interface force over the area πb^2 is:

$$\sigma_w = u_a - (u_a - u_w)\frac{a^2}{b^2} - T_s \times 2\frac{a^2}{b^2}. \tag{9}$$

The capillary law states that an aperture of smaller opening is preferentially occupied by water in the wetting process, and it is adopted in this note to establish the relationship between the capillary pressure and the degree of saturation in a single smooth parallel fracture. The capillary pressure, P_c, defined as the difference between the air and water pressures is given by the Young–Laplace equation for a double curvature interface:

$$P_c = u_a - u_w = T_s\left(\frac{1}{R_1} + \frac{1}{R_2}\right) \tag{10}$$

where the difference $u_a - u_w$ is the capillary pressure, and R_1 and R_2, the two principal radii of the curvature of the interface. Specifically, R_1 is the radius of the in-plane curvature, and R_2 the one perpendicular to the fracture plane, which is determined by local aperture and contact angle.

As Kueper and Mcwhorter (1991) discussed, if a fracture is assumed to be composed of two parallel plates, the force balance yields the following entry pressure:

$$P_c = \frac{2T_s \cos\theta}{e}. \tag{11}$$

If the fracture opening is assumed to be circular in shape, then the entry pressure should be expressed as:

$$P_c = \frac{4T_s \cos\theta}{e} \tag{12}$$

In this note, Eq. (11) is adopted for smooth parallel fractures or rough-walled fractures assumed to be composed of numerous infinitesimal parallel plates, which represents the lower one of the above two extremes. Substituting Eq. (11) into Eq. (9) yields:

$$\sigma_w = u_a - (u_a - u_w)\frac{a^2}{b^2} - (u_a - u_w)\frac{ae}{b^2\cos\theta}. \tag{13}$$

The effective stress of a smooth parallel fracture subjected to an external total stress σ can then be expressed as:

$$\begin{aligned}\sigma' &= \sigma - \sigma_w = \sigma - u_a - \left(\frac{a^2}{b^2} + \frac{ae}{b^2\cos\theta}\right)(u_w - u_a)\\ &= (\sigma - u_a) - \chi(u_w - u_a)\end{aligned} \tag{14}$$

Equation (14) has the same form with Bishop (1959) effective stress equation for unsaturated soils [Eq. (2)], but with the effective stress parameter χ defined as:

$$\chi = \frac{a^2}{b^2} + \frac{ae}{b^2\cos\theta} \tag{15}$$

4 An Effective Stress Principle for Rough-Walled Fractures

The fluid distribution in partially saturated rough-walled fractures is different from that in smooth fractures. According to the capillary law, for any aperture distribution $f(x)$, when water starts to enter the fracture space with a local aperture X, the corresponding degree of water saturation of the fracture, S_r, can be expressed as:

$$S_r = \frac{\int_{x_{\min}}^{X} xf(x)\mathrm{d}x}{\int_{x_{\min}}^{x_{\max}} xf(x)\mathrm{d}x} \quad \text{(for continuous distributions)} \tag{16}$$

$$S_r = \frac{\sum_{x_{\min}}^{X} xn_i}{\sum_{x_{\min}}^{x_{\max}} xn_i} \quad \text{(for discrete distributions)} \tag{17}$$

where $x_{\min}$ and $x_{\max}$ are the minimum and maximum apertures of the fracture, respectively.

The liquid saturation can be alternatively expressed by the surface saturation, A_{S_r}, defined as the fraction of fracture surface wetted by water:

$$A_{S_r} = \frac{\int_{x_{\min}}^{X} f(x)\mathrm{d}x}{\int_{x_{\min}}^{x_{\max}} f(x)\mathrm{d}x} \quad \text{(for continuous distributions)} \tag{18}$$

$$A_{S_r} = \frac{\sum_{x_{\min}}^{X} n_i}{\sum_{x_{\min}}^{x_{\max}} n_i} \quad \text{(for discrete distributions)} \tag{19}$$

Obviously, the surface saturation of a partially saturated fracture varies from 0 to 1, which properly approximates the first term $\frac{a^2}{b^2}$ on the right-hand side of Eq. (15). Given that the aperture is commonly much smaller than the length of a fracture, the second term on the right-hand side of Eq. (15) is close to 0. Therefore, the surface saturation can be used to describe the effective stress parameter for smooth parallel fractures ($\chi = A_{S_r}$).

In a rough-walled fracture, the total force induced by the air pressure, F_a, is equal to the product of the magnitude of the air pressure and the area of the air–solid interface over which it acts:

$$F_a = u_a(1 - A_{S_r}) \tag{20}$$

The total force resulting from the surface tension, F_t, acts along the perimeter of the water menisci between the infinitesimal parallel plates:

$$F_t = -T_s \times B \tag{21}$$

where B is the total perimeter of water menisci between two fracture walls.

The projection of the total force induced by the water pressure acting on the water–solid interface in the vertical direction, F_w, is:

$$F_w = u_w A_{S_r} \tag{22}$$

The resultant capillary force, F_r, is the sum of the above three forces:

$$F_r = u_a(1 - A_{S_r}) - T_s \times B + u_w \times A_{S_r}. \tag{23}$$

At the stress-free state, the air pressure is the unique contribution to the external force acting on the outer boundary of the rock block, which results:

$$F_e = u_a(1 - A_{S_r}) - T_s \times B + u_w \times A_{S_r}. \tag{24}$$

The stress resulting from the capillary interface force over a unit area is:

$$\sigma_w = u_a(1 - A_{S_r}) - T_s \times B + u_w \times A_{S_r} \tag{25}$$

Substituting Eq. (11) into Eq. (25) yields:

$$\sigma_w = u_a - (u_a - u_w)A_{S_r} - (u_a - u_w)\frac{e^*B}{2\cos\theta} \tag{26}$$

where e^* is the maximum local aperture currently occupied by water.

The effective stress of a rough-walled fracture subjected to an external total stress σ is:

$$\begin{aligned}\sigma' &= \sigma - \sigma_w = \sigma - u_a + \left(A_{S_r} + \frac{e^*B}{2\cos\theta}\right)(u_a - u_w) \\ &= (\sigma - u_a) + \chi(u_a - u_w)\end{aligned} \tag{27}$$

Equation (27) is again reduced to the form of Bishop (1959) effective stress equation, with the effective stress parameter χ defined as:

$$\chi = A_{S_r} + \frac{e^*B}{2\cos\theta} \tag{28}$$

Similarly, if we ignore the second term on the right-hand side of Eq. (28), the effective stress parameter of rough-walled fractures can also be expressed as:

$$\chi = A_{S_r} \tag{29}$$

Equations (14) and (27), together with Eqs. (15) and (28), imply that both smooth parallel fractures and rough-walled fractures share the same form with Bishop's effective stress principle. Smooth parallel fractures are an idealized approximation of rough-walled fractures, and correspondingly, Eq. (15) can be regarded as a highly-simplified version of Eq. (28). It can be further observed that the surface saturation of a fracture attains 1 as the fracture is fully saturated with water, and at this state, Eqs. (14) and (27) reduces to Terzaghi's effective stress equation for saturated fractures (Eq. 1).

5 Illustrative Examples

As stated before, the effective stress of partially saturated rock fractures plays an important role in understanding the coupled hydromechanical behaviors of fractured rocks in various engineering practices, such as landslide mitigation, oil and gas exploitation, underground oil storage, and nuclear waste disposal. Existing studies commonly borrowed the relationship between the effective stress parameter and the degree of saturation developed for soils for convenient application in rock fractures (Lu and Likos 2004). The relationships presented in Eqs. (16), (18) and (29) provide a first approximation for estimating the effective stress developed in a partially saturated fracture. As an illustrative example, we consider a group of fractures of normal aperture distribution, with a uniform mean aperture being 0.1 mm and the standard deviation taking the values of 0, 0.01, 0.02, 0.04 and 0.06 mm, respectively. The relationship between the effective stress parameter (χ) and the degree of saturation [S_r, which is connected to the surface saturation A_{S_r} by Eqs. (16) and (18)] is plotted in Fig. 2. One observes that the effective stress parameter of the smooth parallel fracture (with standard deviation $s = 0$) is equal to its saturation degree, while the curves for rough-walled fractures are nonlinearly dependent on the degree of saturation. The nonlinearity increases with the increase of the standard deviation or surface roughness of the fractures. Figure 3 illustrates another example for rough-walled fractures, with the mean aperture being 0.1, 0.2 and 0.4 mm and the coefficient of variation (*s*/*e*) being 0.2 and 0.4, respectively. It is demonstrated from the plots that the nonlinearity of the curves is mainly determined by the coefficient of variation and less dependent on the mean aperture of fractures. The increase of fracture roughness results in an increase in nonlinearity of the relationship between the effective stress parameter and the degree of saturation.

Alterations of the surface contact area and aperture distribution of partially saturated rock fractures induced by normal or shear loading may significantly influence their mechanical and hydraulic properties (Li et al. 2014b). The influence of the surface contact on the effective stress can be represented using the ratio of surface contact area, α. In Eq. (20), if the ratio of the surface contact area increases from 0 to α, the total force resulting from the air pressure, F_a, reads:

$$F_a = u_a(1 - \alpha - A_{S_r}). \tag{30}$$

The resultant capillary force becomes:

$$F_r = u_a(1 - \alpha - A_{S_r}) - T_s \times B + u_w \times A_{S_r}. \tag{31}$$

Correspondingly, the effective stress is rewritten as:

$$\sigma' = \sigma - \sigma_w = \sigma - (1 - \alpha)u_a - A_{S_r}(u_w - u_a) \tag{32}$$

where $\sigma - (1 - \alpha)u_a$ is the net normal stress, and the effective stress parameter here, which is equal to the surface saturation, varies from 0 to 1–α.

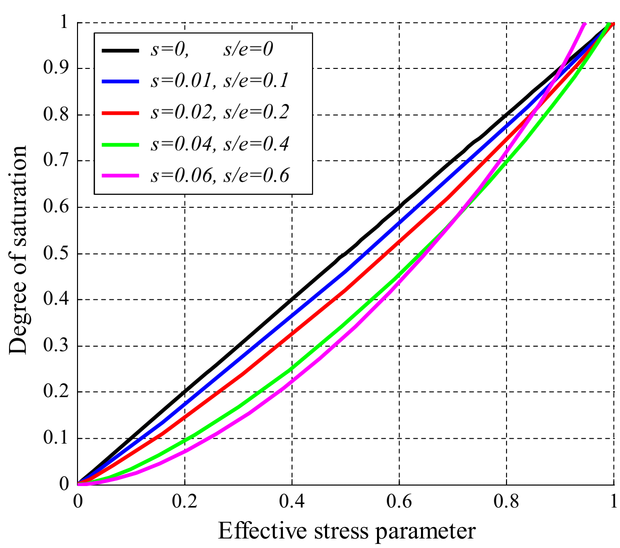

Fig. 2 Relationship between the degree of saturation (S_r) and the effective stress parameter (χ) for fractures with a uniform mean aperture but varying standard deviations

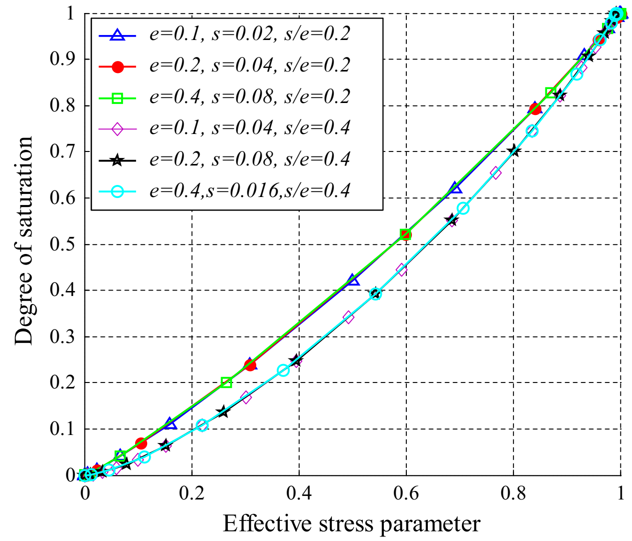

Fig. 3 Relationship between the degree of saturation (S_r) and the effective stress parameter (χ) for factures with different mean apertures and standard deviations

6 Conclusions

In this note, a new effective stress principle for partially saturated rock fractures was proposed. The proposed model shares the same form with Bishop's effective stress equation in unsaturated soil mechanics. However, the effective stress parameter is different from the traditional effective stress parameter for unsaturated soils. A simplified effective stress parameter equal to the surface saturation of fractures was suggested. At fully saturated state, the effective stress principle for partially saturated fractures reduces to Terzaghi's effective stress principle. The aperture distribution, especially the surface contact area, has a strong influence on the relationship between the effective stress parameter and the degree of saturation, because the surface contact reduces the areas of the air–solid and liquid–solid interfaces. The nonlinearity of the effective stress parameter versus the saturation degree curves is found to be mainly determined by the surface roughness (or the coefficient of variation), rather than the mean aperture of the fractures.

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