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TECHNICAL NOTE

Determination of Poisson's Ratio of Rock Material by Changing Axial Stress and Unloading Lateral Stress Test

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Keywords Rock · Poisson's Ratio · Changing axial stress and unloading lateral stress test

List of Symbols

Variables

E Elastic modulus ($ML^{-1}T^{-2}$)
E_0 Deformation modulus ($ML^{-1}T^{-2}$)
v_1 Speed of unloading axial stress (LT^{-1})
v_3 Speed of unloading lateral stress (LT^{-1})
R^2 Coefficient of determination
t Time (T)
u_1 Axial displacement (L)

Greek Letters

ε_1 Axial strain
σ_1 Axial stress ($ML^{-1}T^{-2}$)
σ_3 Lateral stress ($ML^{-1}T^{-2}$)
σ_3^0 Initial lateral stress ($ML^{-1}T^{-2}$)
v Poisson's ratio

1 Introduction

Poisson's ratio is defined as the negative value of the ratio of the radial strain and the corresponding axial strain, caused by uniformly distributed axial stress, within the scope of the material's proportional limits. It reflects the lateral deformation of the material, and is widely used in elastic theory. Previous studies have shown that the upper and lower limits of Poisson's ratio are 0.5 and −1, respectively, and Poisson's ratio for the vast majority of materials is 1/3 (Lakes 1987; Shan et al. 2006). In the field of numerical analysis of geotechnical engineering and engineering design, Poisson's ratio is an important parameter reflecting the lateral deformation of the rock and soil mass and it directly affects the strength and deformation characteristics of the rock and soil mass.

According to Code for rock tests of hydroelectric and water conservancy engineering (Code DL/T 5368-2007), the Poisson's ratio of rock material should be determined by uniaxial compression tests. Adopting this method, resistance strain gauges should be pasted onto the surface of standard samples to obtain the lateral and axial strains, and then the Poisson's ratio can be obtained by the related formula. However, there are shortcomings to this method (Lau et al. 2004; You 2007; Zhou et al. 2007): (1) strain gauges can only measure the partial deformation of the rock sample and cannot represent the overall deformation properties of the sample; furthermore, the quality and direction of pasting and other factors will affect the measurement accuracy. (2) Under a uniaxial compression condition, the lateral strain of the rock accrues earlier, and then deviates faster from the linear relationship with the axial stress than the axial strain, which leads to a nonlinear relationship compared with the definition of Poisson's ratio. (3) Most engineering rock mass experiences mechanical excavation and unloading processes, which are different from the loading process required by the specification.

Based on the generalized Hooke's law, we propose a method for determining the Poisson's ratio of rock samples by changing the axial stress and unloading the lateral stress. By unloading axial and lateral stress triaxial tests (UAUL) and loading axial stress and unloading lateral

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stress triaxial tests (LAUL), the Poisson's ratio of the rock material can be obtained by measuring only three mechanical indexes: the axial stress σ_1, the lateral stress σ_3, and the axial strain ε_1. We conducted tests on cryptocrystalline amygdaloidal basalt using this method. The results were relatively consistent with the uniaxial compression tests, and their values were slightly larger but better adapted to practical engineering stress states.

2 The Theory of the Changing Axial Stress and Unloading Lateral Stress Test

The Poisson's ration of rock material is generally determined at the elastic stage. The expression for the Poisson's ratio [v, Eq. (2)] can be obtained from the derivative of the generalized Hooke's law [Eq. (1)]. Equation (2) shows that if the rock material's elastic modulus E is known, through UAUL and LAUL tests, v of the rock material can be obtained by measuring the three mechanical indexes mentioned above: σ_1, σ_3, and ε_1. This method can avoid the influence of unstable axial strain and does not require to paste resistance strain gauges on the specimen; moreover, it does not need to obtain the radial strain of the rock samples, making the method convenient and practical.

$$E\varepsilon_1 = \sigma_1 - 2v\sigma_3 \tag{1}$$

$$v = \frac{\mathrm{d}\sigma_1 - E\mathrm{d}\varepsilon_1}{2\mathrm{d}\sigma_3} = \frac{\mathrm{d}\sigma_1}{2\mathrm{d}\sigma_3} - \frac{E}{2} \cdot \frac{\mathrm{d}\varepsilon_1}{\mathrm{d}\sigma_3} \tag{2}$$

3 Changing Axial Stress and Unloading Lateral Stress Test

The cryptocrystalline amygdaloidal basalt used in the experiment was taken from the foothills of the Baihetan hydropower station project area of the Jinsha River. The diameter and height of the rock samples are 50 and 100 mm, respectively, and the parallelism and perpendicularity of those strictly conform to the requirements of the test specification. The instrument used in the test was the Rock High Pressure Tester, developed by the State Key Laboratory of Geohazard Prevention and Geoenvironment Protection. During the experiment, the test was controlled by a system control program according to predetermined requirements, and the program automatically recorded the axial stress σ_1, lateral stress σ_3, axial displacement u_1, and time t as well as other parameters.

In this test, the stress path has two patterns: the unloading axial and lateral stress mode and the loading axial stress and unloading lateral stress mode; the control mode is stress control. The stress levels considered in the tests have been chosen starting from the definition of the range of variation of the principal stresses in the case under study. The unloading and loading parameters are shown in Table 1, and the test steps are as follows:

Table 1 Parameters of the triaxial loading and unloading test

Sample no.	Stress path	Initial lateral stress σ_3^0 (MPa)	Speed of unloading axial stress v_1 (MPa/min)	Speed of unloading lateral stress v_3 (MPa/min)
A0–A4	UAUL	0, 3, 6, 9, 12	+0.25	0.5
B0–B4	LAUL	0, 3, 6, 9, 12	−0.25	0.5

1. Using hydrostatic pressure, the axial stress σ_1 is increased to the initial lateral stress σ_3^0, maintaining $\sigma_1 = \sigma_3$.
2. The axial stress σ_1 is increased to 80 % of the peak intensity, maintaining a constant σ_3.
3. The axial stress and lateral stress are varied at different rates until the lateral stress is 0 or the sample would be destroyed.

4 Process of Obtaining the Rock Material's Poisson's Ratio

In terms of rock material, the typical stress–strain curve has many obvious stages. At the initial compaction stage, the amount of lateral expansion is little or zero, resulting in inaccurate values of Poisson's ratio v. During the elastic stage, v is constant and the specification recommend that v should be determined at this stage. At the plastic reinforced stage, the rock sample volume expands and v presents nonlinear characteristics. Thus, based on the generalized Hooke's law, v can be obtained during the elastic stage by the changing axial stress and unloading lateral stress test.

Cryptocrystalline amygdaloidal basalt was used in the experiment, and Poisson's ratio was obtained by UAUL and LAUL; the results are shown in Fig. 1, Table 2, and Fig. 2, Table 3. During the unloading lateral stress tests, we ensured that the rock samples were always in the elastic deformation state. Figures 1 and 2 show that in the UAUL and LAUL, the axial stress σ_1, lateral stress σ_3, and axial strain ε_1 maintain a very good linear relationship (the coefficient of determination $R^2 = 0.8176$–0.9998, mean = 0.9500), indicating that the rock samples were in the elastic stage throughout the testing process. Therefore, Eq. 2 gives v under different lateral stress values.

For the cryptocrystalline amygdaloidal basalt, when the lateral stress was 3–12 MPa, the range of v was from 0.3333 to 0.4155 by UAUL and from 0.3223 to 0.4076 by LAUL. The v values obtained by the two test methods are

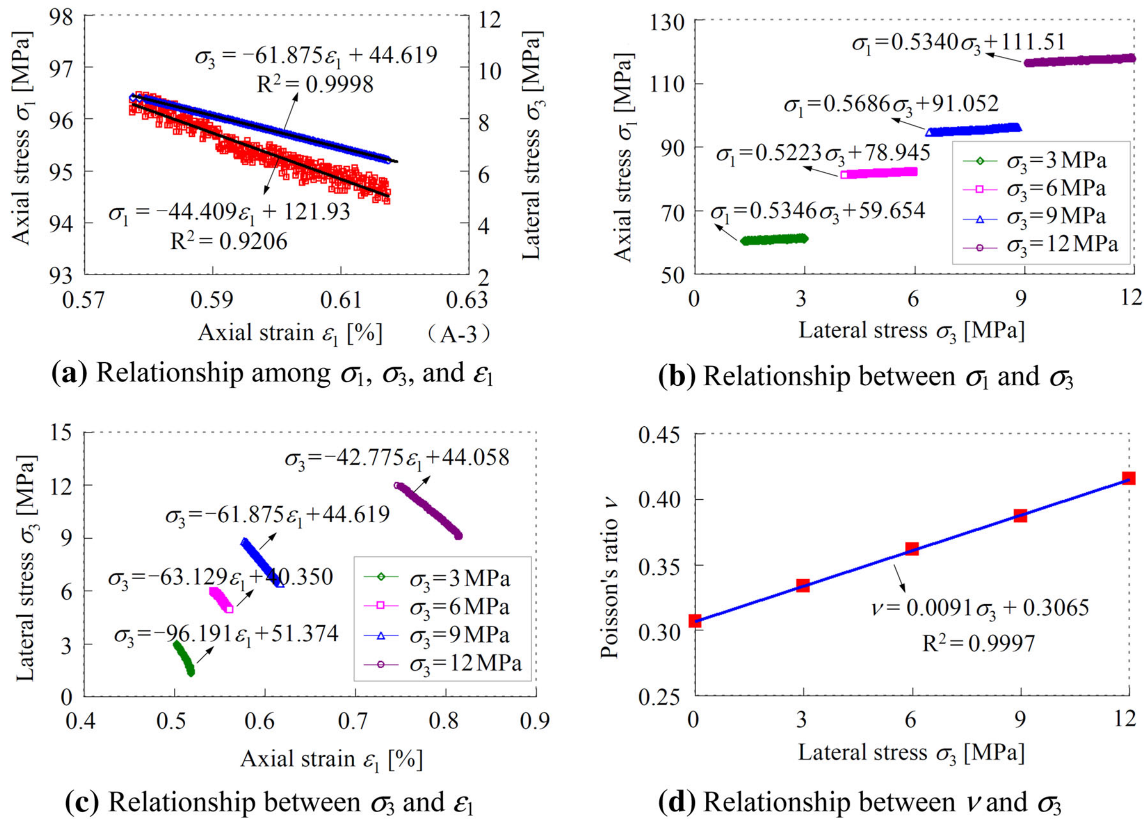


Fig. 1 Relationship between stress and strain of unloading axial and lateral stress triaxial tests

Table 2 Parameters of unloading axial and lateral stress triaxial tests

Sample no.	E (GPa)	$d\sigma_1/d\sigma_3$, R^2	$d\sigma_3/d\varepsilon_1$, R^2	v
A-0	12.7	–	–	0.3065
A-1		0.5346, 0.8176	−96.191, 0.9756	0.3333
A-2		0.5223, 0.8818	−63.129, 0.9908	0.3617
A-3		0.5686, 0.9841	−61.875, 0.9998	0.3869
A-4		0.5340, 0.9907	−42.775, 0.9981	0.4155

similar, with the values obtained by the LAUL method slightly lower than those of the UAUL method.

The results show that v and the lateral stress σ_3 are positively correlated. The relation between Poisson's ratio and the lateral stress obtained by UAUL and LAUL are presented in Eqs. 3 and 4, respectively. Thus, we can see that Poisson's ratio increases linearly with lateral stress, and that the lateral stress loading on the rock samples accounts for the softening effects.

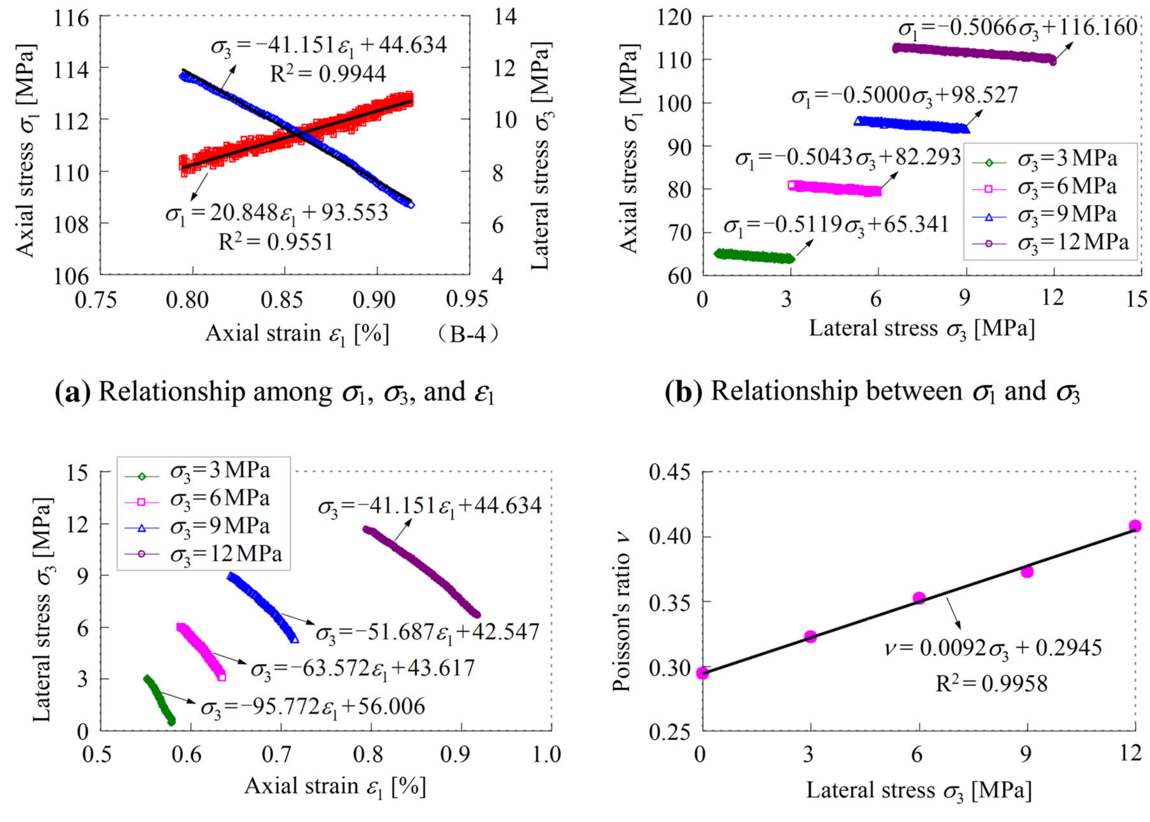
$$v = 0.0091 \cdot \sigma_3 + 0.3065 \tag{3}$$

$$v = 0.0092 \cdot \sigma_3 + 0.2945 \tag{4}$$

The existing specification recommend that the rock material's Poisson's ratio should be determined by uniaxial compression tests, in other words, Poisson's ratio can be defined only when the lateral stress $\sigma_3 = 0$. From the theoretical point of view, using the above two methods, v should be equal when the lateral stress is 0. The Poisson's ratio of cryptocrystalline amygdaloidal basalt obtained by UAUL and LAUL is 0.3065 and 0.2945, respectively, and the D value is 0.012. The two results are very close, and the D value is related to factors such as the experimental error and the sample difference.

5 Evaluation of the Changing Axial Stress and Unloading Lateral Stress Test

In rock mass engineering calculations, choosing the exact value of Poisson's ratio can be a problem. The application of laboratory test results in practice has many flaws as rock samples are strongly affected by the complexity of engineering rocks. In practice, a general range of Poisson's ratio values is given and a specific v is chosen according to the particular situation. Tang et al. (2001) reported, based on years of laboratory and field test results and on rock mass engineering experience, that a certain correlation exists between the deformation modulus and the rock's Poisson's ratio. They developed an empirical formula to calculate the Poisson's ratio of isotropic homogeneous rock



(a) Relationship among σ_1, σ_3, and ε_1

(b) Relationship between σ_1 and σ_3

(c) Relationship between σ_3 and ε_1

(d) Relationship between v and σ_3

Fig. 2 Relationship between stress and strain of the loading axial stress and unloading lateral stress triaxial tests

Table 3 Parameters of loading axial stress and unloading lateral stress triaxial tests

Sample no.	E (GPa)	$d\sigma d_1/d\sigma_3$, R^2	$d\sigma_3/d\varepsilon_1$, R^2	v
B-0	12.7	–	–	0.2945
B-1		−0.5119, 0.8400	−95.772, 0.9932	0.3223
B-2		−0.5043, 0.8654	−63.572, 0.9929	0.3520
B-3		−0.5000, 0.9248	−51.681, 0.9899	0.3729
B-4		−0.5066, 0.9603	−41.151, 0.9944	0.4076

mass (Eq. 5) and recommended that the value could be adjusted within ±0.05 in terms of anisotropic rock mass.

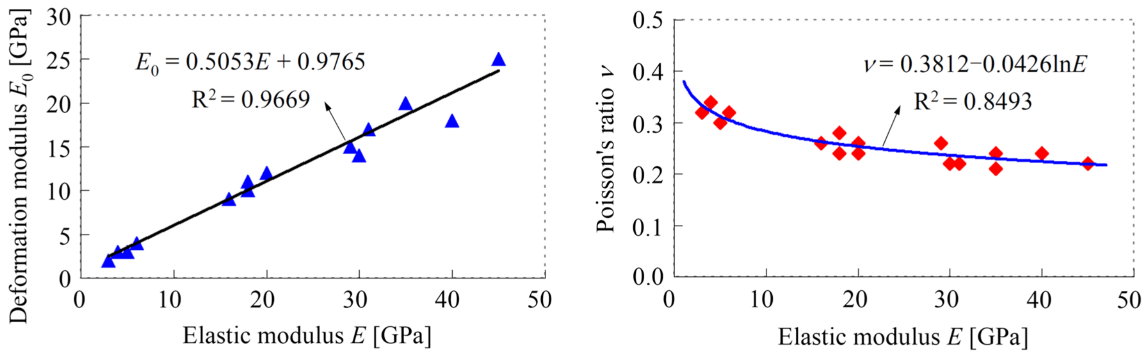
$$v = 0.40 - 0.05\lg\frac{E_0}{2} \tag{5}$$

To obtain the physical and mechanical parameters of basalt, the Hydrochina Huadong Engineering Corporation carried out a series of laboratory uniaxial compression tests. From this experimental data, we found that the basalt's deformation modulus E_0 and elastic modulus E had a high degree of linear relationship, while a certain correlation was found between v and E (Fig. 3). Through curve fitting, the corresponding empirical formula can be obtained (Eq. 6). Using Eq. (6) the Poisson's ratio of cryptocrystalline amygdaloidal basalt was calculated as 0.27.

$$v = 0.3812 - 0.0426\ln E \tag{6}$$

Thus, under the changing axial stress and unloading lateral stress test, Poisson's ratio v increased with lateral stress, showing a direct linear relationship, and Cryptocrystalline amygdaloidal basalt had a v of about 0.3, which is very close to the value obtained by Eq. 6 (0.27).

In rock mass engineering, rock does not appear only as pure material, and can often be in a complex stress and strain environment. Rock mass excavation is a mechanical process of unloading that affects the rock in a completely different way than the loading of rock samples under uniaxial loading tests. Therefore, Poisson's ratio obtained by the changing axial stress and unloading lateral stress test is better adapted to practical engineering stress states than that obtained by the uniaxial compression tests.



(a) Relationship between E_0 and E **(b)** Relationship between ν and E

Fig. 3 Diagrams of the mechanical parameters of Baihetan hydropower station basalt

6 Conclusions

Based on the generalized Hooke's law, a method for determining Poisson's ratio of rock samples was found through changing axial stress and unloading lateral stress test. With UAUL and LAUL, Poisson's ratio of rock material can be obtained by measuring only three mechanical indexes: the axial stress σ_1, the lateral stress σ_3, and the axial strain ε_1.

For brittle rock samples, Poisson's ratio v and the lateral stress σ_3 are positively correlated. Poisson's ratio increases with lateral stress and the lateral stress loading on the rock samples has some softening effects.

According to the changing axial stress and unloading lateral stress test, Poisson's ratio v of cryptocrystalline amygdaloidal basalt is relatively consistent with the uniaxial compression tests and better adapted to the stress states found in practical engineering.

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