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Comprehensive Stability Evaluation of Rock Slope Using the Cloud Model-Based Approach

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Abstract This article presents the cloud model-based approach for comprehensive stability evaluation of complicated rock slopes of hydroelectric stations in mountainous area. This approach is based on membership cloud models which can account for randomness and fuzziness in slope stability evaluation. The slope stability is affected by various factors and each of which is ranked into five grades. The ranking factors are sorted into four categories. The ranking system of slope stability is introduced and then the membership cloud models are applied to analyze each ranking factor for generating cloud memberships. Afterwards, the obtained cloud memberships are synthesized with the factor weights given by experts for comprehensive stability evaluation of rock slopes. The proposed approach is used for the stability evaluation of the left abutment slope in Jinping 1 Hydropower Station. It is shown that the cloud model-based strategy can well consider the effects of each ranking factor and therefore is feasible and reliable for comprehensive stability evaluation of rock slopes.

Keywords Rock slope · Stability evaluation · Cloud models · Membership cloud · Analytical hierarchy process · Jinping 1 Hydropower station

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1 Introduction

Stability acts as an important and permanent issue during the design and construction of slopes in hydropower stations, especially the dam abutment slopes in mountainous areas. The abutment slope over the dam is a geological structure having complex stability problems. The slope failure would result in high risk to the safety of the dam. For arch dams, the abutment slope is even an essential part of the resisting system to undertake the loads transferred from the dam. Thus, geological engineers and researchers have paid special attention to the stability of the abutment slopes in the dam area. During the last decades, substantial works have been approached for evaluation of slope stability: the empirical method (Hoek and Bray 1981; Bieniawski 1979; Goodman 1989; Aydan et al. 1989; Romana et al. 2003; Rodrigo and Hürlimann 2008; Pantelidis 2009; Taheri and Tani 2010; Jhanwar 2012), the analytical analysis (Hoek and Bray 1991; Lam and Fredlund 1993; Nawari et al. 1997; Bye and Bell 2001; Rodrigo and Hürlimann 2008; Liu et al. 2008; Latha and Garaga 2010; Saada et al. 2012) and the numerical modeling (Goodman and Shi 1985; Hoek and Bray 1991; Jeongi-gi and Kulatilake 2001; Wang et al. 2003; Hatzor et al. 2004; Stead et al. 2006; Kveldsvik et al. 2009; Alejano et al. 2011) just to mention a few. These methods generally propose a deterministic approach for slope stability analysis. In practice, these different kinds of methods are often used in a coupled way in order to perform a reliable stability evaluation of complex rock slopes.

However, it is recognized that uncertainty is the main characteristic of most engineering problems such as hydropower stations. Nothing is assured except for the uncertainty itself (Li and Du 2007). Slope engineering deals with natural data that are filled with uncertainties

(Liu et al. 2012). The fundamental features in uncertainties are the randomness and the fuzziness which are discussed in probability theory and fuzzy mathematics, respectively. As known, either of the two theories is capable of accounting for randomness or fuzziness but insufficient to handle both alone. Based on this point, in the past decade, some uncertainty methods have been proposed and applied in slope stability evaluation, e.g., the fuzzy method (Park and West 2001; Park et al. 2005, 2012; Jimenez-Rodriguez et al. 2006; Aksoy and Ercanoglu 2007; Duzgun and Bhasin 2009; Abbas et al. 2011), the matter-element method (Wang and Pan 2004). Generally, there are three steps for implementation of these methods: (1) collecting data related to slope stability; (2) ranking related factors; (3) realization of comprehensive evaluation of slope stability. In these evaluations, slope stability is often ranked as five grades which are shown in Table 1.

Most of the uncertainty methods are developed based on certain mathematical models which describe the uncertainty effects of the factors on slope stability. Also, the analytical hierarchy process (AHP) (Saaty 1980) is utilized in some methods to give comprehensive evaluations of all factors related to slope stability. For example, the fuzzy methods that deal with the fuzziness are applied to give fuzzy memberships which show the degrees of one evaluating factor belonging to the ranking grades. However, the fuzzy method is incapable of expressing the randomness in the slope data.

The concept of "cloud" is proposed on the basis of probability theory and fuzzy mathematics to account for the fuzziness and randomness simultaneously (Li et al. 1998a). The cloud model portrays the randomness and fuzziness as well as their relationships in linguistic terms (Li et al. 1998b). The cloud model synthesizes the characteristics of fuzzy and probability theory and thus can generate results with fuzzy and random signification (Li et al. 2009). In this study, we apply the cloud models to the evaluation of slope stability. A new analysis strategy is proposed. We first utilize the cloud models to generate cloud memberships which demonstrate the degree of each evaluated factor belonging to the five ranked grades. Also, we consider the varying contributions of different factors by introducing the weight matrix which is obtained by expert opinions. Then we apply the AHP, like in fuzzy methods, to obtain a comprehensive evaluation of slope stability.

2 Slope Stability Ranking System

The slope is commonly ranked into five grades in stability evaluations, which is shown in Table 1 (after Romana 1985; Tomás et al. 2007; Abbas et al. 2011) with some descriptive statements of rock slope behaviors. Based on this, the factors related to slope stability are also classified into five grades according to their value range. There are various factors that affect rock slope stability. These affections are often showing characteristics with randomness and fuzziness since they would change with time or circumstances and cannot be quantified easily. The impactions of the factors have been considered in many classification systems for rating rock mass quality.

A rearrangement of the characteristics of the existing rock mass classification systems (Bieniawski 1976; China tMoWRotPsRo 1995; Rodrigo and Hürlimann 2008; Pantelidis 2009) leads to the following findings: (1) the factors related to the general condition of rock slopes and the condition and geometric characteristics of discontinuities constitute the base of the existing classifications; (2) the factors commonly adopted are related to the geometric, excavation process and supports of the rock slope; (3) the factors relevant to the environmental changes such as the precipitation and the seismic characteristics are also listed in most rating systems. In a general situation, a large number of factors could have potential influences on the stability of rock slopes due to variability and divergence of engineering backgrounds. In this study, we try to give a comprehensive but not exhaustive list of parameters in relation with the rock slope stability analysis. However, our investigation is mainly limited to rock slopes involved in hydroelectric engineering in mountainous regions.

Deformation modulus and the UCS of rock are widely used to specify the basic mechanical behaviors of rock mass and thus listed in Table 2 for stability evaluation of rock slopes. Many parameters have been used to specifying the fracturing of rock mass, such as the rock quality

Table 1 Slope stability ranking and empirical description (after Romana 1985; Tomás et al. 2007; Abbas et al. 2011)

Ranking grade	V	IV	III	II	I
Description	Very unfavorable	Unfavorable	Fair	Favorable	Very favorable
Stability	Completely unstable	Unstable	Partially stable	Stable	Completely stable
Safety	Dangerous	Unsafe	Moderately safe	Safe	Completely safe
Failures	Big planar of soil-like	Planar or big wedges	Some joints or many wedges	Some blocks	None
Treatment	Re-excavation	Important/corrective	Systematic	Occasional	None

Table 2 Classification of evaluating factors [after (Bieniawski 1976; China tMoWRotPsRo 1995; Liu and Chen 2007; Xu et al. 2011)]

Rating factor	Ranking grade				
	V	IV	III	Π	Ι
Geological factors (X_1)					
Deformation modulus X_{11} (GPa)	0-1.3	1.3-6.0	6.0-20.0	20.0-33.0	33.0-50.0
Integrity index X_{12}	0-0.15	0.15-0.35	0.35-0.55	0.55-0.75	0.75-1
RQD X_{13}	0–25	25-50	50-75	75–90	90-100
UCS X_{14} (MPa)	0–25	25-50	50-100	100-250	>250
Initial geo-stress X_{15} (MPa)	20–25	14–20	8-14	2-8	0–2
Discontinuities (X_{16})					
Cohesion X_{16-1} (Mpa)	0-0.05	0.05 - 0.08	0.08-0.12	0.12-0.22	0.22-0.32
Friction angle X_{16-2} (°)	0-13	13–21	21–29	29-37	37–45
Volumetric count X_{16-3}	>35	20-35	10-20	3-10	0–3
Integrity description X_{16-4}	Very bad	Bad	Moderate	Good	Very good
Distance between dip direction of discontinuity and cutting face X_{16-5}	Very close	Close	Fair	Far	Very far
 Engineering factors (X_2)					
Slope height X_{21} (m)	80-100	60-80	45-60	30-45	0-30
Slope angle X_{22} (°)	60-80	45-60	35–45	20-35	0–20
Reinforcement X_{23}	Barely	Insufficient	Fair	Sufficient	Surplus
Drainage condition X_{24}	Very unfavorable	Unfavorable	Fair	Favorable	Very favorable
Excavation method X_{25}	Large explosion	Explosion	Mixed	Manual	No excavation
Excavation intensity and volume X_{26}	Very unfavorable	Unfavorable	Fair	Favorable	Very favorable
 Environmental factors (X_3)					
Mean annual rainfall X_{31} (mm)	1,500-2,000	1,100-1,500	800-1,100	600-800	0–600
Daily maximum rainfall X_{32} (mm)	100-150	70–100	50-70	25-50	0–25
Saturate water content X_{33} (%)	75-100	55–75	40–55	20-40	0–20
Seismic acceleration X_{34} (m/s ²)	0.20-0.40	0.15-0.20	0.1-0.15	0.05-0.1	0-0.05
Rainfall duration and intensity X_{35}	Very unfavorable	Unfavorable	Fair	Favorable	Very favorable
Water table X_{36}	Very high	High	Fair	Low	Very low
Catchment area X ₃₇	Very unfavorable	Unfavorable	Fair	Favorable	Very favorable
Vegetation type X_{38}	Very unfavorable	Unfavorable	Fair	Favorable	Very favorable
Monitoring factors (X_4)					
Surface deformation rate X_{41} (mm/month)	8-10	5-8	3–5	2–3	0–2
Internal deformation rate X_{42} (mm/month)	2–3	1.5–2	0.8–1.5	0.3–0.8	0–0.3
Pre-stressed anchorage force X_{43} (%)	25-30	20–25	15-20	8-15	0–8
Route inspection of slope condition X_{44}	Very bad	Bad	Moderate	Good	Very good

destination (RQD), the number of fractures per cubic meter rock (J_v) , the integrity index (K_v) , the cracking coefficient and the average spacing of joints (d_p) . And there are some empirical relations for connecting these parameters. But such relations are generally qualitative and there are no theoretical demonstrations. Moreover, each of the parameters has certain limitations in specifying the failure of rock mass (China tMoWRotPsRo 1995). For example, K_v cannot exactly quantify the joint number; J_v and d_p cannot account for the bonding degree and in particular the opening degree as well as the filled material features of joints. Based on these, we adopt RQD, J_v and K_v as the key parameters for the stability evaluation of rock slope in Table 2.

Discontinuity is another critical factor affecting stability of rock slopes and it is included in many rock mass rating systems (Bieniawski 1979; Hoek and Bray 1981, 1991; China tMoWRotPsRo 1995). Such parameters like the cohesion and the friction angle of filling material, the volumetric discontinuity count, the difference between dip directions of discontinuities and dip directions of cutting faces, etc., are often used to specify the effects of discontinuities on slope stability. In this study, due to limited data availability, we only listed some representative parameters in Table 2.

Water infiltration is another important factor affecting the stability of slopes. Especially for the natural slope or open pit mine slope, their failure is frequently induced by rainfalls. The effects of precipitation on the stability of rock slopes are directly undertaken by the water infiltration. The water table is often very low during the construction stage before water-impoundment and running of the reservoir for the high steep rock slope (e.g., the case slope is more than 500 m) of hydropower stations in steep mountainous areas. Thus, the water table is not included in Table 2. Due to difficulty in obtaining reliable pore water pressure during the construction of rock slopes, we adopt the precipitation factor in Table 2 to specify the effects of rainfalls. Also, the parameters related to rainfall duration, rainfall intensity, catchment area and vegetation type, could also have an influence on rock slope stability.

Another important factor contributing to the stability of slope is the reinforcement and treatment measures as well as other engineering activities such as drainage passage, excavation method, excavation intensity and volume. These parameters are also widely included in the stability evaluation of engineering rock slopes.

Besides, the monitored physical variables (like the displacement) are also considered in this study since nowadays most complicated slopes are installed with in situ monitoring instruments to guarantee the safety. Hence we consider four categories of factors: (a) the geological category; (b) the engineering category; (c) the environmental category and (d) the monitoring category. The factors counted in each category in this study are as follows.

- (a) The geological category: (1) deformation modulus X_{11} ; (2) intactness index of rock mass X_{12} ; (3) rock quality designation (RQD) X_{13} ; (4) uniaxial compressive strength (UCS) X_{14} ; (5) initial in situ stresses X_{15} ; (vi) condition of discontinuities X_{16} .
- (b) The engineering category: (1) slope height X_{21} ; (2) slope angle X_{22} ; (3) reinforcements X_{23} ; (4) drainage condition X_{24} ; (5) excavation method X_{25} ; (vi) excavation intensity and volume X_{26} .

- (c) The environmental category: (1) mean annual precipitation X_{31} ; (2) daily maximum rainfall X_{32} ; (3) saturated water content of slope X_{33} ; (4) seismic acceleration X_{34} ; (5) rainfall duration and intensity X_{35} ; (6) water table X_{36} ; (7) catchment area X_{37} ; (8) vegetation type X_{38} .
- (d) The monitoring category: (1) surface deformation rate X_{41} ; (2) internal deformation rate X_{42} ; (3) Pre-stressed anchorage force X_{43} ; (4) route inspection of slope condition X_{44} .

The factor "intactness index" in the geological category (also known as velocity index of rock mass) is the degree of intactness of the rock mass. It is calculated as

$$X_{12} = K_{\rm v} = (V_{\rm pm}/V_{\rm pt})^2 \tag{1}$$

where V_{pm} is the velocity of elastic P-wave in the rock mass; V_{pt} is the velocity of elastic P-wave in rock sampling of the corresponding rock mass. If discontinuities develop in the rock mass, then V_{pm} is smaller than V_{pt} . The more and wider the discontinuities are, the smaller V_{pm} is than V_{pt} . Thus, this factor can be recognized as an evaluation related to discontinuities in the rock mass.

The parameter "volumetric count" is used to evaluate rock mass integrity and calculated as

$$J_{v} = S_{1} + S_{2} + \dots + S_{n} + S_{k} \tag{2}$$

where J_v is number of joints per cubic meter; S_n is number of the *n*th group of joints counted in the length of one meter; S_k is number of ungrouped joints per cubic meter. This parameter is also a quantitative way to evaluate the discontinuities in the rock mass.

Some of the above factors have been ranked in Chinese Design Standard for Engineering Classification of Rock Masses (China tMoWRotPsRo 1995) and former research publications (Bieniawski 1976; Liu and Chen 2007; Xu et al. 2011). However, the monitoring factors have not been discussed yet. The factors of monitoring category are difficult to be classified since monitoring data vary tremendously for one slope to another. As recorded, the displacement rate of Huangnashi landslide is 50 mm/month for surface deformation and 10 mm/month for inner deformation; while that of the Xintan landslide is 85.9-399 mm/day (Li 2004). Deformations of rock slopes are much smaller than that of soil-like slopes. And they vary significantly as a result of diversity of the rock formations and conditions of the slope. The classification of monitoring factors of the left abutment slope of Jinping 1 Hydropower Station has been discussed preliminarily (Tan et al. 2009). The ranking results of the monitoring factors given in Table 2 are obtained from monitoring analysis on the longtime monitoring data from the year 2005-2011. The ranking results of all the factors are given in Table 2.

3 Comprehensive Evaluation Strategy

The comprehensive evaluation of slope stability is performed on the basis of cloud membership results which are produced by the cloud models and cloud generators. The process of production of cloud memberships are shown in Fig. 1.

3.1 Cloud Models and Cloud Generators

Cloud model is defined as the uncertain transformation model between the qualitative concept expressed by linguistic terms and the corresponding quantitative representation expressed by three numerical descriptors (Li and Du 2007). The normal cloud model is fundamental in cloud theory. The normal cloud model is quite different from a normal distribution and the universality of normal cloud has been proved theoretically. The normal cloud model conveys the numerical features of the qualitative concept with a group of independent parameters to represent the uncertainties.

Let *U* be the universe of discourse expressed by quantitative digits, and *C* be a qualitative concept in *U*. If the definite parameter $x \in U$ is a random occurrence in *C*, and the certainty $\mu(x) \in [0, 1]$ of *x* to *C* is the random number with steady tendency, i.e.

$$\mu(x) \to [0, 1] \ \forall x \in Ux \to \mu(x) \tag{3}$$

Fig. 1 Production of cloud

memberships

Then, the distribution of x in U is defined as a cloud, and each $(x, \mu(x))$ is named as a cloud drop.

Particularly, if $x \sim N(E_x, E_n^{\prime 2})$, $E_n^{\prime} \sim N(E_n, H_e^2)$ and the certainty $\mu(x) \in [0, 1]$ satisfies

$$\mu(x) = e^{-\frac{(x-E_x)^2}{2(E_n')^2}}$$
(4)

Then, the distribution of x in U is defined as a normal cloud.

The models used in this study are normal cloud models. The parameters (the numerical descriptors) of cloud model are the expectation E_x , the entropy E_n and the hyperentropy H_e (Liu et al. 2013). There are bell-shaped cloud as well as the half bell-sharp cloud and trapezium-sharp cloud which are called the half cloud and trapezoidal clouds, respectively.

Cloud models are implemented by cloud generators. As shown in Fig. 2, there are two basic cloud generators: the forward and the backward cloud generators. The forward cloud generator CG is used to generate the cloud drops $P(x_i, \mu_i)(i = 1, 2, \dots n)$ given the cloud descriptors $N(E_x, E_n, H_e)$, which is the transformation from the qualitative knowledge to its quantitative representation. And the backward generator CG⁻¹ is the transformation to derive the qualitative concept represented by three descriptors of cloud drops $P(x_i, \mu_i)(i = 1, 2, \dots n)$. Combinations of the two cloud generators can be used alternately to produce various kinds of cloud models which can bridge the gap





(b) X-condition and Y-condition cloud generators

Fig. 2 Cloud generators

between the qualitative concept and the quantitative knowledge (Li et al. 2009). Obviously, approximations should be made if only a few cloud drops are available for backward cloud generators. The more plentiful the cloud drops are, the more accurate the numerical descriptors (E_x, E_n, H_e) will be.

As it is shown in Fig. 2, the cloud drops $drop(x_0, \mu_i)$ can be generated by the *X* condition cloud generator given three numerical descriptors (E_x, E_n, H_e) and a specific $x = x_0$. Also, the cloud drops (x_i, μ_i) can be generated by the *Y* condition cloud generator given three numerical descriptors (E_x, E_n, H_e) and a specific $y = \mu_i$. Cloud generators are fundamental for uncertainty reasoning with cloud models. Combinations of *X* condition and *Y* condition cloud generators can be structured to obtain the reasoning generator for the rules like "If A Then B". In this study, we utilize the forward cloud generators and *X* condition cloud generators to produce cloud memberships as shown in Fig. 2a, b.

3.2 Cloud Transformation

As it is stated in Fig. 2, we need to implement cloud transformations before implementing cloud generators. Cloud transformation is the process which transforms data from the original data space to cloud model space, i.e., transferring the natural data into a form that can be accepted by the cloud generators. This process can be called "cloudification" in the similar way as fuzzification in fuzzy theory. There are different methods to implement cloud transformation. One possible way is as follows.

Assuming that non-negative data in the universe of discourse U after normalization can be denoted as $\{C_1, C_2, \ldots, C_n\} = [0, a_1], (a_1, a_2], \ldots, (a_{n-1}, 1]$, the cloud transformations can be implemented by the following equations.

$$\begin{cases} C_1 & E_{x_1} = 0 & En_1 = a_1/3 & H_{e_1} = k \\ C_2 & E_{x_2} = (a_1 + a_2)/2 & E_{n_2} = (E_{x_2} - \frac{3}{2}E_{n_1})/3 & H_{e_2} = k \\ C_3 & E_{x_3} = (a_2 + a_3)/2 & E_{n_3} = (E_{x_3} - \frac{3}{2}E_{n_2})/3 & H_{e_3} = k \\ \cdots & \cdots & \cdots \\ C_n & E_{x_n} = 1 & E_{n_n} = (E_{x_n} - E_{x_{n-1}})/3 & H_{e_n} = k \end{cases}$$
(5)

Data normalization, which is applied to remove the influence of units in different types of data, should be implemented before cloud transformation using Eq. (3) or (4). If the larger of a factor value the more favorable of the factor, data normalization can be implemented as

$$x'_{ij} = \frac{x_{ij} - x_{i\min}}{x_{i\max} - x_{i\min}}.$$
(6)

Otherwise, if the less of a factor value the more favorable of the factor effect, data normalization is implemented as

$$x'_{ij} = \frac{x_{i\max} - x_{ij}}{x_{i\max} - x_{i\min}}.$$
(7)

In the above expression, x'_{ij} is the normalized data of a factor; x_{ij} is the original data of this factor; x_{imax} and x_{imin} are the maximum value and the minimum value of the factor, respectively.

After data normalization, the factor values are transferred in the interval [0, 1] for the quantitative data. For the qualitative data, i.e., the descriptive factors, we assume that each rank of description has the same weight. Hence the values of each rank can be normalized in the form of subintervals in [0, 1]. The five ranking grades (V, IV, III, II, I) of the qualitative factors are thus quantitatively expressed by five data intervals [0.0, 0.2), [0.2, 0.4), [0.4, 0.6), [0.6, 0.8), [0.8, 1.0], respectively. And the ranking grades of quantitative factors are normalized as varying subintervals in [0, 1] according to their ranking values in Table 2.

3.3 Comprehensive Evaluation

Rock slope stability is affected by many factors. The cloud memberships of a factor can only represent the effect of one unique factor. However, slope stability evaluation needs to take into account the effects of all the related factors. Hence comprehensive evaluation should be proposed. Moreover, different factors have varying contributions to accounting for slope performance; this should also be considered in slope stability evaluation. The varying contributions of factors can be quantified by factor weights. As shown in Fig. 3, the weights of the factors are synthesized in the evaluation, which are obtained by inquiring experts who have been engaged in the evaluated slope. The implementation of the strategy of comprehensive evaluation is given in Fig. 3.



Fig. 3 Comprehensive evaluation process of rock slope stability

4 Stability Evaluation of Left Abutment Slope of Jinping 1 Hydropower Station

4.1 Slope Data Collection

The Jinping 1 Hydropower Station is the critical hydroelectric project in the midstream and downstream of Yalong River. It is located at the east part of Oinghai-Tibet plateau in Yanyuan and Muli District of Liangshan State, Sichuan Province, China, as shown in Fig. 4. This region is of complex topographical and geological conditions formed by the coupled effects of continuous uplifting of Qinghai-Tibet plateau and rapidly sapping of the steep valleys. The 305 m water retaining structure of the project is currently the top highest arch dam of double curvatures worldwide. The left abutment slope in the dam area is a reverse slope over 800 m high and presents the interphase micro-geomorphology of the ridge and the super fissure. The lower part under height 1,820-1,900 m of the slope is formed by marble with the slope ratio between 55° and 70°; the upper part is of sandy slate with slope ratio between 40° and 50° . There are faults, joints and disturbed belts extruded between rock layers growing intensively in the rock mass of left bank slope (Song et al. 2009). An overview of the slope is given in Fig. 5.

Most faults are in the direction NE–NNE and of large scale in the left abutment slope. The azimuth situates mostly in N30°–50°E, SE $\angle 60^\circ$ –80° for the faults with a crush width between 1 and 3 m, such as the faults f_5 , f_8 , f_2 and the spot dike (X). Faults in the direction NEE–EW

develop secondly, represented by f_{42-9} with attitude EW, S $\angle 40^{\circ}$ -60°. Joints are principally developing into three categories: (1) N15°-35°E, NW $\angle 30^{\circ}$ -45°, mainly fractures in the bedding plane; (2) SN–N30°E, SE $\angle 60^{\circ}$ -80°; (3) N50°-70°E, SE $\angle 50^{\circ}$ -80°, mostly rigid structure planes. The assembly of those joints would lead to potential unstable blocks.

Substantial investigations have been undertaken on this rock slope to identify its geotechnical and mechanical behaviors (Qi et al. 2004; Zhang et al. 2009; Liu et al. 2010; Jin 2011); a number of evaluation factors are summarized in Table 3. As indicated in Fig. 5, most part of the slope surface is strengthen by shotcrete and rock bolt or pre-stressed cables and designed with drainage passages. Thus, the rainfall infiltration is thought to have limited influence on the global stability of this slope due to these measures. Another point, the pore water pressure data are not reliable for this slope due to damage of slope construction activities and thus it is not taken into account in this study. Also, many other factors in Table 2 are not available for this slope and thus not included in Table 3. This does not affect the implementation of the cloud model-based evaluation strategy.

4.2 Cloud Memberships

Both the ranking and the collected factor data need to be transformed before implementation of evaluation. The ranking data are transformed by Eq. (3) while the collected factor data are transformed by Eq. (4) or (5). The results are shown in Tables 3 and 4.









Evaluation factor	Original	Cloudification $(X_{(i)0})$	Evaluation factor	Original	Cloudification $(X_{(ij)0})$
Deformation modulus X ₁₁	1.9 GPa	0.038	Reinforcements X ₂₃	0.70	0.700
Integrity index of rock mass X ₁₂	0.72	0.720	Drainage effect X ₂₄	0.75	0.750
Rock quality designation X ₁₃	85 %	0.850	Mean annual precipitation X ₃₁	607 mm	0.697
Wet UCS X ₁₄	105 Mpa	0.131	Daily maximum rainfall X ₃₂	49.2 mm	0.672
The initial geo-stress X ₁₅	21.49 Mpa	0.140	Water content X ₃₃	13 %	0.870
Cohesion of fault material X ₁₆₋₁	0.02 Mpa	0.063	Seismic acceleration X ₃₄	0.1 g	0.750
Friction angle of fault materialX ₁₆₋₂	16.7°	0.371	Surface deformation rate X ₄₁	1.89 mm/m	0.811
Volumetric count X ₁₆₋₃	17/m ³	0.283	Internal deformation rateX ₄₂	0.27 mm/m	0.910
Integrity description X ₁₆₋₄	0.60	0.600	Pre-stressed anchorage	8.73 %	0.709
Slope height X ₂₁	110 m	0.000	forceX ₄₃		
Slope angle X ₂₂	50°	0.375	Route inspection X ₄₄	0.75	0.750

Table 3 Original data and cloud transformed values of evaluation factors

To this end, the cloud memberships can be calculated as stated in Fig. 1. The cloud memberships of the mean annual precipitation X_{31} are calculated in detail hereby as an example. The cloud memberships of the other factors can be computed in the similar way and the results are given in Table 4.

According to the numerical descriptors of X_{31} given in Table 4, the cloud model graphs of factor X_{31} can be generated as shown in Fig. 6 by forward cloud generators. The subfigures (a), (b), (c), (d) and (e) represent the concept of ranking grades V, VI, III, II and I, respectively. The mean annual precipitation X_{31} is 607 mm and 0.697 after cloud transformation as given in Table 3. Hence the X condition cloud generator is applied here to produce cloud memberships in each ranking with $x_0 = 0.697$. The results are also shown in Fig. 6.

As it is shown in Fig. 6, no expected drops can be shown for ranking grade V and IV when $x_0 = 0.697$ since such as value is out of their counting ranges which depend on the numerical descriptors given in Table 4. However, due to randomness of cloud models, there may be several cloud drops generated, but they have few effects on the final evaluation. For each of the ranking grades III, II and I, there are some expected drops (in black) shown in Fig. 6. The expected drops in Fig. 6d are much more than those in Fig. 6c, e. But only one membership value should be used. Hence two ways are proposed to determine the final cloud membership in each ranking grade: $\mu = \text{Average}(\mu_i)$ or $\mu = \text{Maximum}(\mu_i)$. The former way is adopted in this study. Thus, the cloud membership vector of X_{31} is calculated originally as U = (0.0000, 0.0000, 0.0138,0.5277, 0.167). After normalization, the memberships can be obtained as $U_V(X_{31}) = 0$, $U_{II}(X_{31}) = 0.745$, $U_I(X_{31}) =$ 0.236.

The cloud graphs of all the other factors can be generated in the same way as that of the factor X_{31} according to the cloudification values in Table 3 and the CD values in Table 4 using the cloud generators in Fig. 2. We list the cloud membership values of all these factors in Table 4, but not in the form of cloud graphs for the reason of simplicity.

4.3 Evaluation Results

Due to complex geological conditions and insufficient discovery in pre-construction stage, the case slope is designed with a moderate complete system to detect and monitor the deformation and other specifications to guide the construction of this slope as well as to safeguard the safety of this slope. The weight matrix was obtained based on the evaluations of experts from various institutions and universities in China such as HydroChina, Chengdu Engineering Corporation (CHIDI), Yalong River Hydropower Development Company LTD, Institute of Geology and Geophysics of Chinese Academy of Sciences, which were involved in the consultation, design, construction and (or) research programs related to this slope. Some of the evaluations are obtained by formal inquiries; some are by face-to-face talk. We make a simple average of five evaluations as the final weight matrix (Table 5). Generally, the experts involved in geological investigations of this slope recommended higher weights for geological conditions, while those involved in its construction suggested higher weights for engineering and monitoring factors.

Due to multi-hierarchies of the rating factors in Fig. 3, evaluations should be performed first for four categories of factors and then for the final comprehensive ranking grade.

Table 4 (Cloud descriptors of r	ranking data	and calculated cloud merr.	nberships						
Factor	V		IV		Ш		Π		I	
	CD	CM	CD	CM	CD	CM	CD	CM	CD	CM
X_{11}	(0,0.009,0.003)	0.027	(0.073,0.02, 0.003)	0.973	(0.260,0.082,0.003)	0.000	(0.530, 0.136, 0.003)	0.000	(1, 0.266, 0.003)	0.000
X_{12}	(0, 0.050, 0.003)	0.018	(0.250, 0.058, 0.003)	0.954	(0.450, 0.121, 0.003)	0.028	(0.650, 0.156, 0.003)	0.000	(1, 0.255, 0.003)	0.000
X_{13}	(0, 0.083, 0.003)	0.000	(0.375, 0.083, 0.003)	0.000	(0.625, 0.167, 0.003)	0.000	(0.825, 0.192, 0.003)	0.107	(1, 0.238, 0.003)	0.893
X_{14}	(0, 0.010, 0.003)	0.000	(0.047, 0.010, 0.003)	0.000	(0.094, 0.026, 0.003)	0.003	(0.219, 0.060, 0.003)	0.475	(1, 0.303, 0.003)	0.522
X_{15}	(0, 0.067, 0.003)	0.000	(0.320, 0.073, 0.003)	0.000	(0.560, 0.150, 0.003)	0.000	(0.800, 0.192, 0.003)	0.027	(1, 0.238, 0.003)	0.973
X_{16}	(0,0.080,0.003)	0.782	(0.320, 0.073, 0.003)	0.218	(0.560, 0.080, 0.003)	0.000	(0.800, 0.080, 0.003)	0.000	(1,0.067,0.003)	0.000
X_{21}	(0, 0.067, 0.003)	1.000	(0.300, 0.067, 0.003)	0.000	(0.475, 0.125, 0.003)	0.000	(0.625, 0.146, 0.003)	0.000	(1, 0.260, 0.003)	0.000
X_{22}	(0, 0.083, 0.003)	0.915	(0.344, 0.073, 0.003)	0.085	(0.500, 0.130, 0.003)	0.000	(0.656, 0.154, 0.003)	0.000	(1, 0.257, 0.003)	0.000
X_{23}	(0, 0.067, 0.003)	0.000	(0.300, 0.067, 0.003)	0.000	(0.500, 0.133, 0.003)	0.035	(0.700, 0.167, 0.003)	0.930	(1, 0.250, 0.003)	0.035
X_{24}	(0, 0.067, 0.003)	0.000	(0.300, 0.067, 0.003)	0.000	(0.500, 0.133, 0.003)	0.003	(0.700, 0.167, 0.003)	0.932	(1, 0.250, 0.003)	0.065
X_{31}	(0, 0.083, 0.003)	0.000	(0.350, 0.075, 0.003)	0.000	(0.525, 0.138, 0.003)	0.019	(0.650, 0.148, 0.003)	0.745	(1, 0.259, 0.003)	0.236
X_{32}	(0, 0.111, 0.003)	0.000	(0.433, 0.089, 0.003)	0.026	(0.600, 0.156, 0.003)	0.396	(0.750, 0.172, 0.003)	0.550	(1,0.247,0.003)	0.028
X_{33}	(0, 0.083, 0.003)	0.000	(0.350, 0.075, 0.003)	0.000	(0.525, 0.138, 0.003)	0.001	(0.700, 0.165, 0.003)	0.025	(1, 0.251, 0.003)	0.974
X_{34}	(0, 0.167, 0.003)	0.000	(0.563, 0.104, 0.003)	0.077	(0.688, 0.177, 0.003)	0.431	(0.813, 0.182, 0.003)	0.458	(1, 0.242, 0.003)	0.034
X_{41}	(0, 0.067, 0.003)	0.000	(0.350, 0.083, 0.003)	0.000	(0.600, 0.158, 0.003)	0.035	(0.750, 0.171, 0.003)	0.391	(1,0.248,0.003)	0.574
X_{42}	(0, 0.111, 0.003)	0.000	(0.417, 0.083, 0.003)	0.000	(0.617, 0.164, 0.003)	0.029	(0.817, 190, 0.003)	0.426	(1, 0.238, 0.003)	0.545
X_{43}	(0, 0.056, 0.003)	0.000	(0.250, 0.056, 0.003)	0.000	(0.417, 0.111, 0.003)	0.002	(0.617, 0.150, 0.003)	0.823	(1, 0.258, 0.003)	0.175
X_{44}	(0, 0.067, 0.003)	0.000	(0.300, 0.067, 0.003)	0.000	(0.500, 0.133, 0.003)	0.016	(0.700, 0.167, 0.003)	0.616	(1, 0.250, 0.003)	0.368
CD could	descriptor, CM could	l membershil	d							



Notes: Figure(a) represents the quality level V of X_{31} with CG (0, 0.08333,0.003); Figure(b) represents the quality level IV of X_{31} with CG (0.525, 0.138, 0.003); Figure(c) represents the quality level II of X_{31} with CG (0.650, 0.148, 0.003); Figure (e) represents the level I of X_{31} with CG (1, 0.259, 0.003). Figure (f) is the comprehension of all the levels of X_{31} . All these figures are generated under the same condition $X_{(31)0} = 0.697$ presented in Table 3 and the cloud descriptor (CD) values in Table 4.

Fig. 6 Cloud models of each ranking grade of the factor X_{31} and cloud memberships with $X_{(31)0} = 0.697$

Table 5 Weight matrix for evaluation	Table 5	Weight	matrix	for	evaluation
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Ranking object	Assembly weight
Factor category	$A_X = (0.321, 0.232, 0.221, 0.226)$
Geological factors	$A_{X_1} = (0.155, 0.158, 0.171, 0.173, 0.180, 0.163)$
Engineering factors	$A_{X_2} = (0.207, 0.229, 0.288, 0.276)$
Environment factors	$A_{X_3} = (0.266, 0.240, 0.263, 0.231)$
Monitoring factors	$A_{X_4} = (0.236, 0.264, 0.239, 0.262)$

Taking account the weight matrix, we obtain the evaluations of the four categories:

$$R_{X} = \begin{bmatrix} R_{X_{1}} \\ R_{X_{2}} \\ R_{X_{3}} \\ R_{X_{4}} \end{bmatrix} = \begin{bmatrix} 0.281 & 0.191 & 0.005 & 0.105 & 0.418 \\ 0.407 & 0.029 & 0.011 & 0.525 & 0.028 \\ 0.000 & 0.024 & 0.200 & 0.443 & 0.333 \\ 0.000 & 0.000 & 0.021 & 0.561 & 0.418 \end{bmatrix}$$
(8)

Hence the comprehensive evaluation result can be calculated as:

 $B_X = A_X \oplus R_X = \begin{bmatrix} 0.185 & 0.073 & 0.053 & 0.380 & 0.309 \end{bmatrix}$ (9)

The elements in vector B_X represent the degree of the ranked slope in each ranking grade. The bigger the element value is, the more likely the slope is in the corresponding ranking grade.

The maximum element of B_X in Eq. (7) is 0.380 of ranking grade II with a small difference of 0.072 to grade I. The membership values of the other three grades are much smaller than those of grades II and I. If a maximum membership method is applied, the final evaluation is addressed to be grade II (stable) for this rock slope. In this case, however, some information would be omitted. In the final evaluation result B_X , the element of ranking grade II and I is 0.380 and 0.309, respectively; and that of the ranking grades V, IV and III add up to 0.311. The membership values indicate the possibilities of the slope stability in the corresponding ranking grade. Hence, we conclude that the stability status of the slope is between grade II and grade I, prone to II. This result is more practical than that of the maximum membership method for the purpose of slope reinforcement and treatments.

4.4 Validation and Discussion

One study of this slope carried out by the finite difference method (Hydro-China 2008) showed that the stability of the slope would be significantly improved by appropriate slope reinforcement and treatment measures. And due to grout replacement, the weak planes would not be the controlling factors to trigger potential slope failures. They concluded that the stability of this slope would be lying in class I in normal situations and class II in seismic condition with earthquake intensity <M8.0. Their conclusions are in good agreement with the results presented in this study. Moreover, field monitoring and manual inspection have all proved that this slope is under good stability situation.

Stability features of this slope have also been studied from the view of monitoring behaviors (Zhang et al. 2009). They investigated the deformation trend and its spatial distribution as well as the relationship between the deformation and the stability. They concluded that the excavating activities influenced the deformation of the slope with a very high depth (more than 80 m deep in the slope). The stress releasing process of the slope would continue and not come to be completely stable in a short time. In all, their study showed that this slope would be in a good global stability status with some local stability problems. These local stability problems would be probably caused by the influence of stress releasing of the rock mass as a result of the excavation activities. This is consistent with the result of this study.

Further, the extension method has also been applied for comprehensive stability evaluation of this rock slope (Tan et al. 2009). The extension method is developed from the extenics which is aimed at approaching contradictory problems in the science and engineering (Cai et al. 2003, 2013). In the application of extension method, slope stability evaluation is considered as a decision-making problem with many factors which are related to stability of slopes to be counted in a reasonable manner. The extension method provides the manner by which to incorporate the factors. The results showed that the safety status of the left abutment slope of Jinping 1 Hydropower Station was ranked in grade II prone to grade I. This result also accords with that of this study.

We can also observe some interesting findings from the cloud membership values of the four rows in R_X . The first row denotes the effects of geological factors, and the other three rows denote the effects of engineering factors, environmental factors and monitoring factors, respectively. The values of memberships in the first row indicate that the ranking of geological factors is most likely in grade I with moderate possibility in grade V and IV. This result is caused by the divergence of evaluating results of single factors.

The second row indicates that the ranked engineering factors have large divergent evaluating results. This result is caused by super slope height and sufficient slope treatments. The membership values show that the slope reinforcements and treatments have greatly improved the situation of the slope. The membership values in the third row show this slope is under favorable environmental situations. And the memberships of the monitoring factors show that this slope is under good situation of stability.

Therefore, according to the evaluation results in Eq. (8), the most unfavorable factors of the rock slope are the geological factors and engineering factors. Among the geological factors, the behaviors of the discontinuity material are most unfavorable and thus most important for the stability of the slope. Among the engineering factors, the slope height and slope angle appear as the most unfavorable factors to the stability of slope. Fortunately, thanks to the reinforcement measures, the stability of the slope is greatly improved and consequently situated in a favorable grade.

5 Concluding Remarks

This study investigates the potential capability of the cloud model-based approach for comprehensive stability evaluation of rock slopes. The evaluated results using this approach provide both fuzzy and probabilistic significance. The comprehensive evaluating results depend on the ranking standards and slope conditions. Also, experts' opinions can be quantitatively included in this strategy, which is particularly important in geotechnical engineering. The results show that the proposed strategy is feasible and practical for comprehensive evaluation of rock slope stability.

The most unfavorable factors to the stability of the rock slope are the behaviors of the discontinuity material, the slope height and slope angle. Fortunately, thanks to the reinforcement measures, the stability of the slope is greatly improved and consequently situated in a favorable condition.

Nevertheless, it must be convinced that the factors adopted for rock slope stability evaluation would be probably different due to data availability and slope conditions. The ranking grade standards would consequently change accordingly for different slopes in other areas with varying conditions. And the weight matrix of the factors would be varying if different experts could have been consulted.

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